Journal of Statistics Applications & Probability
An International Journal

http://dx.doi.org/10.18576/jsap/090309

A Probability Model for the Number of Female Child Births

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Received: 20 Jun. 2019, Revised: 20 Oct. 2019, Accepted: 24 Oct. 2019
Published online: 1 Nov. 2020

Abstract: We analysed the phenomenon of ‘son preference’ that is prevalent in the Indian society, through a probability model for the number of female childbirths among females of Indian society. In [1], a probability model for the number of female childbirths was applied to an observed set of data taken from NFHS-III (2005-06) for the seven North-East states of India. Some problem regarding the application of the model to the data set is found and the proper solution is suggested. The modified approach is illustrated to observed data set taken from NFHS-III(2005-06) for few states of India of different regions.

Keywords: Stopping Rule, Probability Model, Son Preference

1 Introduction

Birth of a female child is of major concern because of its apparent relationship with the level of fertility. The connection between female birth and fertility or vice versa is the root of many explanations of demographic transition and in the shaping of population distribution. Son preference is a worldwide phenomenon and perhaps it is more pronounced in Indian rural society than elsewhere. Son preference is widespread in several developing countries despite substantial improvements in education levels and economic development. Mothers derive large non-monetary benefits from giving birth to a son and therefore prefer boys to girls [2]. The distribution of the number of female childbirth in India is not following natural law. With increasing population and limiting resources, small family is preferred. High sex ratios at birth (108 boys to 100 girls or higher) are seen in China, Taiwan, South Korea, and parts of India and Vietnam. The imbalance is the result of son preference, accentuated by declining fertility [3]. In India, total fertility rate has declined noticeable from 3.4 child in 1992-93 to 2.2 child in 2015-16 [4]. This decline in fertility have intensified pressure to achieve their desired family sex composition [5,6,7]. Son preferences influence reproductive outcomes extensively and play an important role in settings, where notions of the ‘ideal family size’ are marked by a strong and persistent preference for sons [8]. The poor availability of health services is compounded by patriarchal gender and social norms that continue to restrict women’s reproductive options and in many cases dominate on women’s reproductive preferences [9]. These norms also limit women’s ability especially when young and newly married to access reproductive services. [10] provides empirical evidence from India that smaller families have a significantly higher proportion of sons; whereas, socially and economically disadvantaged couples not only want but also attain a higher proportion of sons if the effects of family size are controlled.

Vast literature is available on the analysis of son preference and its related issues in developing countries using large scale retrospective data provided by demographic health surveys. Few authors have analysed this phenomenon through a probability model. A probability model is an abstraction of the real world in which the relevant relations between the real elements are replaced by similar relations between mathematical entities. A model may be simple or extremely complicated depending upon the nature of the phenomenon under study. The social phenomena, where several social, cultural, psychological, and economic factors act and interact, are bound to be exceedingly complex. However, many times simple models based on reasonably good assumptions provide results that are interesting and have important policy implications [11,12]. Behaviour and trend of female childbirths may prove to be a powerful device of explaining changes

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and variation in populations. It is not so easy to find that the probability of birth is male (or female) with the application of probability modelling incorporating a parameter for sex preference. [13] proposed a model under the assumption that the probability of male birth remains constant across the population of women and also across the successive births for the same women and there is no sex-selective stopping of childbearing. If the probability is constant among women, then the distribution of male births follows the Binomial distribution. [14] has proposed a probability model for the pattern of male children, where family size and sex composition are dominated by strong son preference. [15] have developed a probability model for analysing the pattern of child death among all females. [1] used this model to analyse the number of female childbirth. The objective of this study is to illustrate limitations in the application of the model by [1] and hence relevant modifications are suggested. We analysed the phenomenon of 'son preference' that is prevalent in the Indian society, via the number of female childbirths. Keeping the primacy of model, here an attempt has been made to develop a probability model to explain the pattern of births of the female child for all females in the society. The applications of this model are illustrated through the real data taken from National Family Health Survey-III (NFHS-III) for five states (Bihar, Orissa, Rajasthan, West Bengal, and Tamil Nadu) of India. The estimates of the parameters are obtained and the suitability of the proposed discrete probability model is checked based on goodness of fit test using observed data.

2 The Model

For completeness of the paper the probability model of [15,1] is explained here. Let a woman having n number of children. Thus, if X denotes the number of births of female child to a female and p be the probability of giving birth to the female child then the distribution of number of female child births to the females of given parity n follows the binomial distribution given as

$$P[X=x|n,p]=\binom{n}{x}p^x(1-p)^{n-x}, 0\le p\le 1, n>0,$$

where $x=0,1,2,\cdots,n$.

Let us assume that probability of having a female child may vary among females i.e., p follows beta distribution (due to its flexibility) with parameters a and b, which is given as

$$f(p)=\frac{1}{\beta(a,b)}p^{a-1}(1-p)^{b-1}, 0\le p\le 1, a,b>0.$$

Therefore, the joint distribution of x and p for given n is given by

$$\begin{aligned} P\left[X=x\bigcap P=p|n\right] &= P[X=x|n,p]\times f(p) \\ &= \binom{n}{x}p^x(1-p)^{n-x}\frac{1}{\beta(a,b)}p^{a-1}(1-p)^{b-1}. \end{aligned}$$

Therefore, the marginal distribution of X for fixed n, is written as

$$P[X=x|n]=\int_0^1\binom{n}{x}p^x(1-p)^{n-x}\frac{1}{\beta(a,b)}p^{a-1}(1-p)^{b-1}dp. \quad (1)$$

Further, in a cross sectional data, n the number of birth to a females is not fixed i.e. number of children ever born to a female is also a random variable and follows a Poisson distribution. Therefore, the distribution of number of births among woman is given as

$$P[n=k]=\frac{e^{-\lambda}\lambda^k}{k!},$$

where $k=0,1,2,\cdots,n$ and λ is the average parity. The joint distribution of X and n is now as follows

$$P\left[X=x\bigcap n=k\right]=P[X=x|n]\times P[n=k].$$

Hence, the marginal distribution of X is now given as

$$P[X=x]=\sum_{k=x}^{\infty}\int_0^1\binom{k}{x}p^x(1-p)^{k-x}\frac{1}{\beta(a,b)}p^{a-1}(1-p)^{b-1}dp\times\frac{e^{-\lambda}\lambda^k}{k!}. \tag{2}$$

After simplification, the equation (2) reduces to

$$P[X=x]=\frac{\lambda^x}{\beta(a,b)x!}\int_0^1 e^{-\lambda p}p^{a+x-1}(1-p)^{b-1}dp. \tag{3}$$

It is easy to verify that

$$\sum_{x=0}^{\infty}P[X=x]=1$$

Thus, $P[X=x]$ is a probability mass function for the birth of female child to females.

2.1 Estimation Procedure

The method of moments is used to estimate the unknown parameters (λ, a and b) of the model given in equation (3) for the distribution of the number of female child births to females of all parity. The method of moments provides estimates which are consistent and also this method is very simple in computation than the other methods. Therefore, the first three moments of the probability model given in equation (3) can be carried out as follows:

$$E(X)=\frac{\lambda a}{a+b},$$
$$E(X^2)=\frac{\lambda^2(a+1)a}{(a+b+1)(a+b)}+\frac{\lambda a}{a+b},$$
$$E(X^3)=\frac{\lambda^3(a+2)(a+1)a}{(a+b+2)(a+b+1)(a+b)}+\frac{3\lambda^2(a+1)a}{(a+b+1)(a+b)}+\frac{\lambda a}{a+b}.$$

Let μ_1', μ_2' and μ_3' denote the first, second and third raw moments about origin for the data. Therefore, we can replace $E(X), E(X^2)$ and $E(X^3)$ by μ_1', μ_2' and μ_3' respectively in the above equations. Hence, we get the three equations with three unknown parameters λ, a and b as given below:

$$\mu_1'=\frac{\lambda a}{a+b}, \tag{4}$$
$$\mu_2'=\frac{\lambda^2(a+1)a}{(a+b+1)(a+b)}+\frac{\lambda a}{a+b}, \tag{5}$$
$$\mu_3'=\frac{\lambda^3(a+2)(a+1)a}{(a+b+2)(a+b+1)(a+b)}+\frac{3\lambda^2(a+1)a}{(a+b+1)(a+b)}+\frac{\lambda a}{a+b}. \tag{6}$$

[1] applied the above proposed model for the number of female child births to the data obtained from the National Family Health Survey (2005-06) for the states known as Seven Sisters of India but by considering *only those females in the study who have given birth to at least one child*. Accordingly, λ was taken as the mean number of children ever born to females having at least one child birth and it was estimated as:

$$\lambda=\frac{E}{N-n_0}. \tag{7}$$

where, E is the total number of births and N is the total number of females considered for study and n_0 is the total number of childless females in a particular category so that $N-n_0$ is the number of females having at least one child ever born.

Hence, by solving the equations (4) to (7), the values of the unknown parameters $a, b,$and λ was obtained.

2.2 The Problem

The probability model was derived for the number of female births to females of all parities. But, while applying the model, [1] considered only those females who have given birth to at least one child. This should not have been the case. So, the first modification that we have done here is that we have considered for the study those women also who have no births. In doing so, we are adhering to the assumptions of the model.

In cross-sectional data, there are females of varying marital duration. Concerning the problem of analysing the number of female childbirths among females, it is pertinent that the above model should be applied specifically to a marriage cohort of women who have been given at least that much exposure to marriage so that the probability of giving birth to a child is high, i.e., it is not a good idea to include in the study the women having a marital duration of one or two months. So, the second modification that we have done over here is that we have considered the females having a marital duration greater than seven years to ensure homogeneity.

So, now with the above mentioned modifications, we have taken λ as the mean number of children ever born to females having exposed to at least seven years of marriage and estimating it as follows:

$$\lambda = \frac{E}{n}. \tag{8}$$

where,

– E is the total number of births
– n is the number of females (of all parity) having exposed to at least seven years of marital duration.

Hence, by solving the equations (4), (5), (6) and (8), we can obtain the estimates of the unknown parameters $a, b,$and λ.

3 Application and Results

In order to formally understand the effect of the modifications stated above, we have carried out a comparative study of [1] and the above proposed modification to the data obtained from the National Family Health Survey (2005-06) for the five states (Bihar, Orissa, Rajasthan, West Bengal, and Tamil Nadu) of India [16]. To apply the above proposed model for the number of female childbirths to the data obtained from NFHS-III for the states by considering the females of all parity who have been exposed to at least seven years of marital duration. The observed and expected frequencies of females according to the number of female childbirths for the five different states for the method of [1] without any modification. The modifications mentioned in the above section are presented in Tables 1, 3, 5, 7, and 9. With the approach used in the [1], the values of λ, i.e. the mean number of children ever born to females having at least one child ever born vary from 2.418 to 3.809; 'a'' vary as 3.020 to 11.691 and; 'b'' vary as 3.169 to 13.045. While, for the model with modifications, the values of λ, i.e. the mean number of children ever born to females of all parity having exposed to at least seven years of marital duration vary from 2.561 to 4.864; 'a'' vary as 5.448 to 8.760 and; 'b'' vary as 5.370 to 11.240. The tables also show the values of the calculated chi-square test statistic obtained from the data of the states. As expected, calculated χ^2 values for [1] are very high and one can easily say that it is not able to capture the reality. When doing it the other way (i.e. accounting modification), the calculated χ^2 values are substantially decreased, but are still insignificant.

The observed and expected frequency curves of females according to the number of female childbirths of all parities obtained from the model incorporating the modifications for the five different states of India are given in Figures 2. Probability distributions of beta distribution each having the parameter values equal to the values of a and b that we have obtained by applying the model incorporating the modifications to the data of the particular state. The plots are given in Figure 1.

4 Discussion

From the results obtained, we may reach on the conclusion that though in [1], the considered probability model is suitable to describe the distribution of the number of female childbirths to females in some of the sub-domains in the Seven Sisters. We are not able to give any obvious explanation for it, however, one important reason for getting such large values of χ^2 maybe that for very large values of total frequency, even slight departure from reality may yield high value of χ^2 [17,18, 19]. But here, from figure 2 very clearly observe that, in general, the fits are all unsatisfactory. From all the tables (see table 1, 3, 5, 7 and 9) it is evident that the cell frequency of 1 and 2 is the main cause behind the insignificant result. In obtaining the distributions of female childbirths, the most important factor for which an allowance must be made is the '**stopping rule behaviour**' whereby the couples who have accomplished their ideal composition of children simply stop reproducing. Some couples want just one or two children and who have no further children once they have one or more sons, even without having produced a daughter. On the other hand, there are couples with one or more daughters and they may opt to have another child in the hope of having a boy and in the process contributing more girls. A natural question may arise: Does this phenomenon of stopping rule of ‘son preference’ alter the distribution of childbirths? To answer the question, a cross-tabulation of the frequency distribution of the total number of children ever born and the total number of daughters ever born to observe the two variables simultaneously are presented in Tables 2, 4, 6, 8, 10 for all five states under consideration. We know that the probability of giving birth to a female child is almost equal to the probability of giving birth to a male child. This means that if, for example, we consider the case of a single birth then the number of male births should be equal to that of female births. But this is not the case in reality!. We can very well observe this fact from Table 2. For example, if the total children ever born is, say 1, then there are 106 male births and 73 female births. If $p \sim \frac{1}{2}$ then the number of male births should have been equal to that of female births. But, here the situation is different! This can also be observed in the case when total children born is 2. The phenomenon of ‘**son preference**’ affecting the ‘**stopping rule**’ behaviour of the Indian society seems to be the strongest reason behind it. Tables 4,6, 8, and 10 show that all the other states also exhibit similar behaviour. Figure 1 depicts the distribution of p i.e. probability of having a female birth in different states under study. This clearly illustrates that the probability of p varies from female to female. Although this variation is mixture of biological and sociological behaviour.

One of the limitations of the model lies in the assumptions of Poisson distribution about the form of distribution of birth among women. If the populations have different patterns of childbirth, this model could not explain the characteristics of that population. This assumption is strong and that the departure from the assumption is sufficient to produce the poor fit. But, the Poisson distribution applies because of its simplicity and range of variability. Attempts can be made to consider more logically justified distribution so that the model could be considered to be more realistic. One more important point to note here is that, as nowadays, fertility is going down more couples plan to have only one or two children. The couples who opt to have one child are likely to want a son as they are living in a society with a strong preference for sons. But, if a female child is born, it is highly likely that they may opt to have another child in the hope of having a boy. And if a male child is born then they are likely to stop as they have achieved the desired number of son and daughter. This seems to be a plausible explanation behind the observed frequency for one female child being too high. These findings infer that there is gender preference for children as also have been evidenced in several studies. This is not the case for this state only.

5 Conclusion

Two modifications applied in the probability model for female child birth is proposed and it was found that it fits better in terms of χ^2 statistics. The development of a more realistic model will undoubtedly enlarge the scope of the contributions that have been made so far. It is always possible to elaborate theories so that the observations are fitted more closely and a more satisfactory model is obtained. Within this framework, we would like to focus on how we can overcome the above limitations in the model so that the modified model takes account of the finer details of the pattern of female childbirth. The most obvious amendment concerns the inclusion of ‘**stopping rule**’ behaviour, being governed by ‘**son preference**’, in the model. As already discussed, parents’ preference for sons is common in our country. Sons are preferred because they have a higher wage-earning capacity, they continue the family line and they usually take responsibility for the care of parents in illness and old age. In India, there is also a specific local reason for son preference, and i.e. the expense of the dowry. It is hoped that if we can include this factor of gender bias, which is also responsible for the decline in female ratio, in our model, the model may then describe the data well and hence may become more closer to reality [20] .

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this article.

Table 1: Bihar: Expected & Observed distributions of female child birth

Number of female child births	Rai et. al Observed	Rai et. al Expected	Proposed Observed	Proposed Expected
0	480	546	290	309
1	892	756	591	543
2	601	639	509	530
3	364	413	353	376
4	224	222	223	215
5	104	103	104	105
6	59	42	59	45
7	14	16	14	17
8 or more	5	6	5	8
Total	2743	2743	2148	2148
Estimates	χ^2= 50.274 λ=3.809	a=3.020 b=3.169	χ^2= 16.116 λ=4.864	a=6.027 b=7.734

Table 2: Bihar: Distribution of female child according to total children ever born

		Total children ever born						
		0	1	2	3	4	5 or more	Total(%)
Total daughters ever born	0	51	44	88	72	21	14	290(13.50)
	1	0	50	149	196	123	73	591(27.51)
	2	0	0	38	113	171	187	509(23.70)
	3	0	0	0	32	87	234	353(16.43)
	4	0	0	0	0	13	210	223(10.38)
	5 or more	0	0	0	0	0	182	182(08.42)
Total		51	94	275	413	415	900	2148

Table 3: Orissa: Expected & Observed distributions of female child birth

Number of female child births	Rai et. al Observed	Rai et. al Expected	Proposed Observed	Proposed Expected
0	741	808	507	536
1	1202	1053	850	785
2	648	720	562	596
3	318	342	301	312
4	137	126	136	126
5	36	39	36	41
6+	19	13	19	15
Total	3101	3101	2411	2411
Estimates	χ^2= 40.676 λ=2.935	a=11.691 b=13.045	χ^2=12.53 λ=3.285	a=5.448 b=6.827

Table 4: Orissa: Distribution of female child according to total children ever born

		Total children ever born						
		0	1	2	3	4	5 or more	Total(%)
Total daughters ever born	0	68	106	206	85	29	13	507(21.03)
	1	0	73	318	282	120	57	850(35.26)
	2	0	0	72	209	174	107	562(23.31)
	3	0	0	0	54	97	150	301(12.48)
	4	0	0	0	0	21	115	136(05.64)
	5 or more	0	0	0	0	0	55	55(02.28)
Total		68	179	596	630	441	497	2411

Table 5: Rajasthan: Expected & Observed distributions of female child birth

Number of female child births	Rai et. al Observed	Rai et. al Expected	Proposed Observed	Proposed Expected
0	539	614	362	393
1	985	844	689	638
2	658	668	587	566
3	319	392	310	361
4	183	187	179	183
5	84	76	83	79
6	42	28	42	29
7 or more	11	12	11	14
Total	2821	2821	2263	2263
Estimates	χ^2= 55.798 λ=3.608	a=4.260 b=4.883	χ^2= 21.58 λ=4.057	a=6.648 b=7.693

Table 6: Rajasthan: Distribution of female child according to total children ever born

		Total children ever born						
		0	1	2	3	4	5 or more	Total(%)
Total daughters ever born	0	49	47	152	70	24	20	362(16.00)
	1	0	29	200	256	118	86	689(30.45)
	2	0	0	42	154	192	199	587(25.94)
	3	0	0	0	28	81	201	310(13 .70)
	4	0	0	0	0	14	165	179(07.91)
	5 or more	0	0	0	0	0	136	136(06.01)
Total		49	76	394	508	429	807	2263

Table 7: West Bengal: Expected & Observed distributions of female child birth

Number of female child births	Rai et. al Observed	Rai et. al Expected	Proposed Observed	Proposed Expected
0	1268	1403	890	969
1	1903	1644	1387	1246
2	977	1042	847	867
3	407	470	395	429
4	152	168	152	168
5	52	49	52	55
6 or more	33	16	33	22
Total	4792	4792	3756	3756
Estimates	χ^2= 99.028 λ=2.665	a=6.916 b=7.420	χ^2= 42.818 λ=2.947	a=7.009 b=7.492

Table 8: West Bengal:Distribution of female child according to total children ever born

		Total children ever born						
		0	1	2	3	4	5 or more	Total(%)
Total daughters ever born	0	105	328	309	100	32	16	890(23.70)
	1	0	248	621	327	123	68	1387(36.93)
	2	0	0	196	326	191	134	847(22.55)
	3	0	0	0	85	147	163	395(10.52)
	4	0	0	0	0	31	121	152(04.05)
	5 or more	0	0	0	0	0	85	85(02.26)
Total		105	576	1126	838	524	587	3756

Table 9: Tamil Nadu: Expected & Observed distributions of female child birth

Number of female child births	Rai et. al Observed	Rai et. al Expected	Proposed Observed	Proposed Expected
0	1092	1860	853	1251
1	1785	1379	1337	1168
2	901	620	785	603
3	276	214	262	225
4	75	61	74	68
5 or more	25	20	25	22
Total	4154	4154	3336	3336
	χ^2= 593.222 λ=2.418	a=3.183 b=5.676	χ^2= 219.235 λ=2.561	a=5.954 b=8.852

Table 10: Tamil Nadu: Distribution of female child according to total children ever born

		Total children ever born						
		0	1	2	3	4	5 or more	Total(%)
Total daughters ever born	0	132	170	386	128	27	10	853(25.57)
	1	0	145	719	359	97	17	1337(40.08)
	2	0	0	251	350	132	52	785(23.53)
	3	0	0	0	95	113	54	262(07.85)
	4	0	0	0	0	27	47	74(02.22)
	5 or more	0	0	0	0	0	25	25(00.75)
Total		132	315	1356	932	396	205	3336

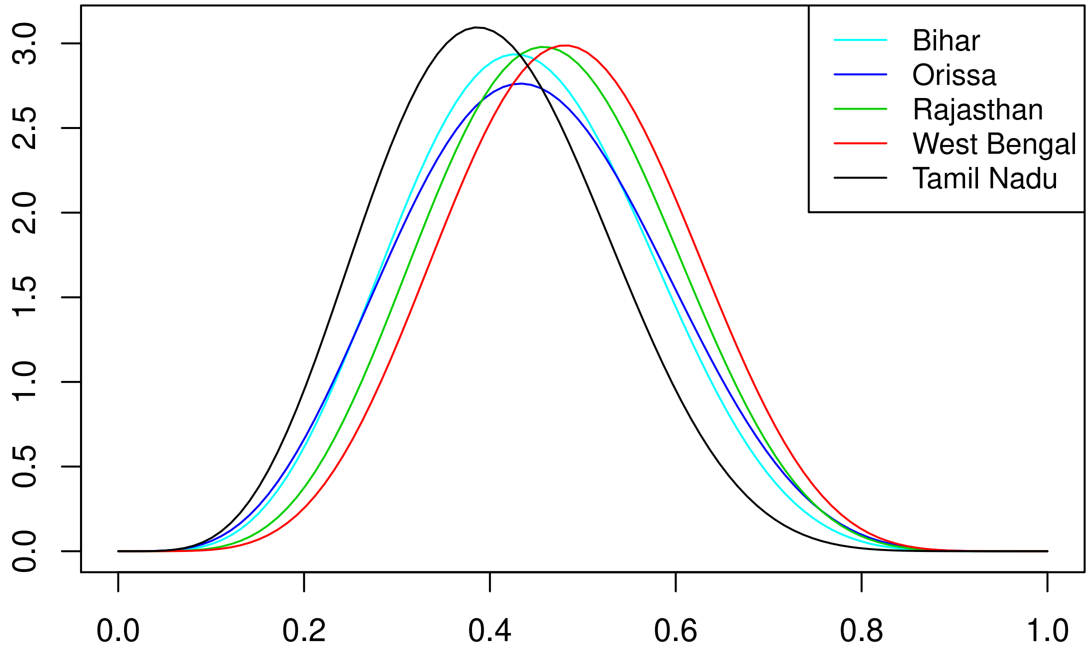

Fig. 1: Variation in probability of having a female child among females (p) for the five different states

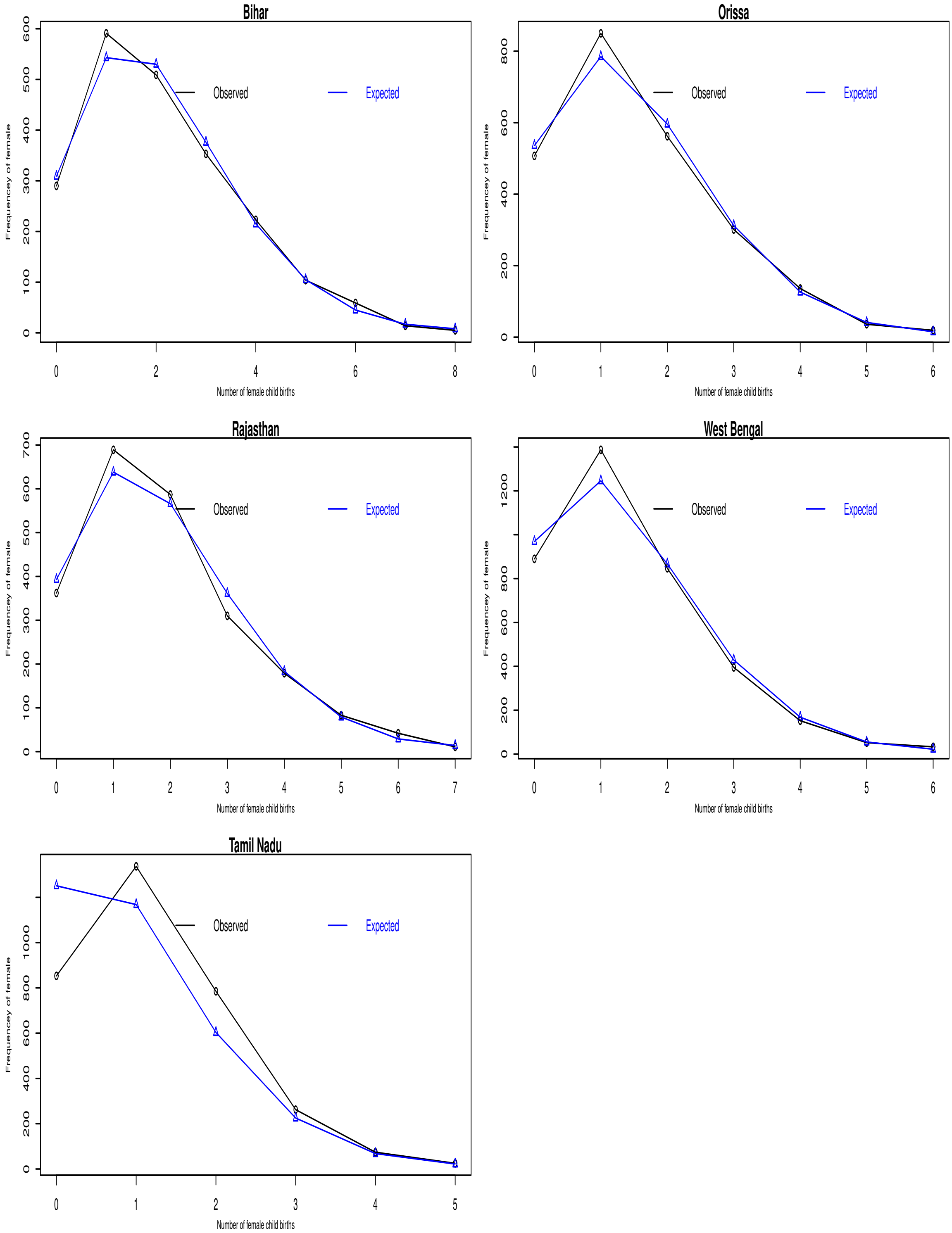

Fig. 2: Observed and expected frequency curves for all states

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