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Time-Frequency Techniques using Band Limited Wavelets Applied to Detect Epileptic Events in EEG

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Abstract: In this work we present a novel methodology for detection and characterization of different types of epileptic events immersed in electroencephalogram signals. In a multiresolution analysis context, this technique uses band limited wavelets and modulated wavelet packets transform.

Keywords: EEG, epileptic events detection, band limited wavelet, modulated wavelet packets transform

1 Introduction

In the framework of a multiresolution analysis (MRA), the discrete wavelet transform allows to express a signal through a series of wavelets, implementing a time-scale technique that provides information of interest based on the coefficients of these series.

When a good temporal localization is achieved the frecuencial accuracy is usually lost. For instance, there are harmonic patterns temporarily immersed in a local phenomena of the signal that cannot be detected. However, it is possible to improve the frequency precision maintaining a good temporal localization when trigonometric wavelet packets are used.

In this article we propose time-frequency techniques based on band limited wavelets and show how they can be applied to automatic detection of epileptic events in deep electrode electroencephalograms (EEGs). These EEGs are part of the studies carried out in some epileptic patients resistant to drugs, who are candidates for resection surgery to remove the epileptogenic zone (the focus of seizures).

Since in studies of this type, the signal is acquired at several points in the brain (electrodes with contacts at different depths) for some days during which the patient remains hospitalized, it is of great importance to automate the analysis of these signals (EEGs channels) to collaborate with the subsequent visual study carried out by the specialists.

The development of algorithms for the detection and automatic classification of intercritical events and the prediction of epileptic seizures began some decades ago. Several methodologies have been proposed to address these problems, that continue to be of great interest. We can cite [2,4,5,8,17,18,19,20,22] among others.

This paper is organized as follows. In the next section we present the design of a bandwidth limited wavelet base, the associated energy profiles and some families of wavelet packets that allow us to refine the scheme in order to obtain a better frequency resolution. We apply the techniques developed in the processing of an EEG signal corresponding to an epileptic patient in Section 3. Finally, in Section 4, we state some conclusions.

2 Band limited wavelet

It is well known that several types of wavelet functions ψ generate MRA structures, combining in different proportions desirable localization, smoothness and symmetry properties. But is hard to obtain good accuracy of all these properties simultaneously.

If the associated conjugate filters are finite, as Daubechies case, the wavelets have compact support and consequently efficient computational calculus. But they are not smooth or have no symmetry properties. Moreover, they do not possess well and precise frequency

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localization or analytical expression. In consequence they are not suitable for some theoretical applications.

Cubic or fifth power spline wavelets are symmetrical, have a good balance between time-frequency localization and are associated with efficient numerical implementation methods. For certain type of applications they are very efficient and convenient, [1,3].

For time-frequency analysis or regularity studies, oscillating properties, i.e., infinite zero moments: $\int_{-\infty}^{+\infty} x^n \psi(x)\, dx = 0$, for all $n \in \mathbb{N}_{\geq 0}$, are indispensable.

We denote by $\widehat{\psi}$ the Fourier transform of $\psi \in L^2(\mathbb{R})$, defined by

$$\widehat{\psi}(\omega) = \int_{\mathbb{R}} \psi(x) e^{-i\omega x} dx.$$

The smoothness of $\widehat{\psi}$ guarantees the temporal localization. An efficient numerical implementation method is also desirable. In addition, the transform $\widehat{\psi}$ should be smooth and well localized in the band $\pi \leq |\omega| \leq 2\pi$. For these reasons our choice for time-frequency applications is a family of bandwidth limited wavelet proposed by Y. Meyer in [12]. Here we focus to our particular design and its respective properties, for more details see [7].

2.1 Wavelet design

In [7] we define, in frequency domain, the scaling function ϕ_α and the wavelet function ψ_α, as follows,

$$\widehat{\phi}_\alpha(\omega) = \begin{cases} 1 & |\omega| < \pi - \alpha \\ \frac{v_\alpha(\omega)}{\sqrt{v_\alpha^2(\omega) + v_\alpha^2(2\alpha - \omega)}} & \pi - \alpha < |\omega| < \pi + \alpha \\ 0 & |\omega| \geq \pi + \alpha \end{cases} \tag{1}$$

with

$$v_\alpha(\omega) = \begin{cases} \exp\left(- \frac{\left(\frac{\omega - \pi + \alpha}{2\alpha}\right)}{1 - \left(\frac{\omega - \pi + \alpha}{2\alpha}\right)^2} \right) & |\omega - \pi + \alpha| < 2\alpha \\ 0 & |\omega - \pi + \alpha| \geq 2\alpha \end{cases}$$

and using (1)

$$\widehat{\psi}_\alpha(\omega) = \sqrt{\widehat{\phi}_\alpha^2(\omega/2) - \widehat{\phi}_\alpha^2(\omega)}\; e^{-i\omega/2} \tag{2}$$

with parameter $\alpha \in (0, \pi/3]$.

We recall that $\psi_\alpha \in \mathscr{S}$, the Schwartz's class (infinitely derivable functions with exponential decay). In addition, the family

$$\{\psi_{jk}(x) = 2^{j/2}\, \psi_\alpha(2^j x - k), \;\; j,k \in \mathbb{Z}\}$$

is an orthonormal basis of $L^2(\mathbb{R})$ associated to the MRA generated by ϕ_α, well localized in both, time and frequency domains. Some graphics can be seen in the Figure 1.

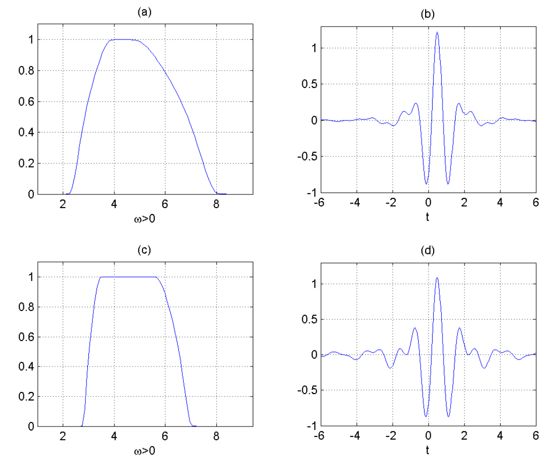

Fig. 1: (a) v_α with $\alpha = \pi/4$, (b) ψ_α with $\alpha = \pi/4$, (c) v_α with $\alpha = \pi/7$, (b) ψ_α with $\alpha = \pi/7$.

Then, for a signal s with finite energy we have the expansion formula

$$s(x) = \sum_{j\in\mathbb{Z}} \sum_{k\in\mathbb{Z}} c_{jk}\, \psi_{jk}(x) \tag{3}$$

for appropriate *wavelets coefficients*

$$c_{jk} = < s, \psi_{jk} > . \tag{4}$$

In time domain, ψ_α is well localized in the interval $[0,1]$ and punctually located in $1/2$. Thus, the wavelets ψ_{jk} are pointwise localized around each center $x_{jk} = \frac{1}{2^j}(k+1/2)$ and located in the intervals $\frac{1}{2^j}[k, k+1]$.

Based on the good localization properties, we can warranty that the synthesis information around each x_0 is practically determined by the wavelets ψ_{jk} of its around. More precisely, being $k_j(x_0) = [2^j x_0]$, for each j, where $[\cdot]$ is the floor function, we have the punctual synthesis of the signal s by the approximation formula (3)

$$s(x) \cong \sum_{j\in\mathbb{Z}} \sum_{k=k_j(x_0)-1}^{k_j(x_0)+1} \langle s, \psi_{jk} \rangle\; \psi_{jk}(x) \tag{5}$$

centered near x_0.

It is worth noting that the properties of s in x_0, such as regularity and some kind of singularities, are characterized by the wavelets localized in the *influence cone* of x_0 defined by

$$Q(x_0) = \{(j,k) \;/\; |k2^{-j} - x_0| < 1\} \tag{6}$$

that generates the sequences

$$s_j(x) = \sum_{|k2^{-j} - x_0| < 1} \langle s, \psi_{jk} \rangle\; \psi_{jk}(x) \tag{7}$$

convergent to x_0, for more details see [10].

In frequency domain, the $|\widehat{\psi}_\alpha(\omega/2^j)|$ have compact support, determined by a smooth window function.

The wavelets transform are well localized on the two-sided frequency band

$$\Omega_j = \left\{2^j(\pi-\alpha) \le |\omega| \le 2^{j+1}(\pi+\alpha)\right\}. \qquad (8)$$

In contrast, they are not pointwise localized at any frequency $\omega \in \Omega_j$. Consequently, the wavelet coefficients (4) only give time-scale and not time-frequency information.

Suppose that at a level j we have M coefficients c_{jk} of the signal and

$$s_j(x) = \sum_{k=k_1}^{k_M} c_{jk}\, \psi_{jk}(x). \qquad (9)$$

Then its associated Fourier transform is

$$\begin{aligned}\widehat{s}_j(\omega) &= 2^{-j/2}\,\widehat{\psi}_\alpha(\omega/2^j) \sum_{k=k_1}^{k_M} c_{jk}\, e^{-i2^{-j}\omega k} \\ &= 2^{-j/2}\,|\widehat{\psi}_\alpha(\omega/2^j)| \sum_{k=k_1}^{k_M} c_{jk}\, e^{-i2^{-j}\omega(k+1/2)}. \end{aligned} \qquad (10)$$

Note that the last factor of (10) is a discrete Fourier transform that can give precision in M frequencies and, by design $|\widehat{\psi}_\alpha(\omega/2^j)|$ is almost one, then the formula (10) could be considered as a local time-frequency transform at level j.

This idea is not practical unless the frequencies are precisely defined and the transformation is properly formalized.

2.2 Local energy profiles

In numerical signal processing, only a finite number of scales j are available for the analysis. In this case, it will be only possible to estimate local and specific properties from the wavelet coefficients c_{jk}, (4).

In this subsection we define a tool that allows us to analyze, in a network of points, the energies of its neighborhoods, summarized in successive levels.

Let j_{max} the highest level compatible with the sampling frequency containing significant information. For each $x_k = x_{j_{max},k}$ and $j < j_{max}$ we define the *local energy profile* by the expression

$$E(j,x_k) = \sum_{l=j}^{j_{max}} \sum_{k=k_l(x_k)-1}^{k_l(x_k)+1} |c_{lk}|^2. \qquad (11)$$

Thus, decreasing successions are obtained

$$E(j-1,x_k) \ge E(j,x_k) \ge E(j_{max},x_k).$$

The analysis enables us to extract information about specific and local events.

2.3 Wavelet Packets

The wavelet transform is a time-scale technique that allows to extract and classify the information of interest from the coefficients of the expansion. It is particularly appropriate for detecting and characterizing singularities, or local and oscillating events. In this sense it is more efficient than the local Fourier analysis, see [6,11,12,21].

In addition, wavelets well localized in time, are not nearly-monochromatic functions, i.e., its Fourier transforms are not associated with a specific frequency. For this reason the atomic wavelet decomposition does not synthesize properly the information refered to the harmonic patterns that coexist with local events ([9,12, 13]). In the same analytical context, it is possible to refine the decomposition scheme enabling better frequency resolution. The elementary functions associated with these refined schemes are called *wavelet packets*.

There are several packets families in the literature, see [9,11,13]. In general, wavelet packets are elementary functions generated by appropriate linear combination of the wavelet basis functions. In particular, we are interested in functions

$$\theta_\lambda(x) = \sum_{(j,k)\in\Pi} b_{\lambda,j,k}\, \psi_{jk}(x) \qquad (12)$$

where Π is a subset of indexes, such that

$$\widehat{\theta}_\lambda(\omega) = 2^{-j/2}\,|\widehat{\psi}(\omega/2^j)| \sum_{(j,k)\in\Pi} b_{\lambda,j,k}\, e^{-i\omega(2k+1)/2^{j+1}}$$

are punctually localized in some frequency ω_λ.

Previously, for certain applications, we have developed and implemented packets of orthogonal wavelets from the application of Fourier matrices to the wavelets, [15,16].

This is the starting point for the design of the modulated wavelet packets that support current time-frequency techniques.

2.3.1 *Trigonometric wavelet packets*

We consider again the orthogonal wavelet ψ_α defined in (2), localized on the two-side band, smooth and with fast decay. Its Fourier transform

$$\widehat{\psi}_\alpha(\omega) = |\widehat{\psi}_\alpha(\omega)|\, e^{-i\omega/2} \qquad (13)$$

is concentrated in the band $\pi \le |\omega| \le 2\pi$ and its module is almost constant in this interval.

Given a signal s, through a recursive algorithm, we compute the coefficients (4) and its projections in the wavelet subspace $W_j = \{\psi_{jk}, k \in \mathbb{Z}\}$:

$$s_j(x) = \sum_{k\in\mathbb{Z}} c_{jk}\psi_{jk}(x). \qquad (14)$$

On the other hand,

$$\widehat{s}_j(\omega) = 2^{-j/2}\,|\widehat{\psi}_\alpha(\omega/2^j)| \sum_{k\in\mathbb{Z}} c_{jk}\, e^{-i\omega(2k+1)/2^{j+1}}. \qquad (15)$$

In particular, the function $s(x) = e^{-i\omega_\lambda x}$ has nonzero projection in W_j, in the distributional sense, if and only if $\omega_\lambda \in \Omega_j$, defined in (8).

Then, sice the wavelet coefficients are

$$\langle e^{-i\omega_\lambda \cdot}, \psi_{jk}\rangle = 2^{-j/2}|\widehat{\psi}_\alpha(\omega_\lambda/2^j)|\, e^{i\,\omega_\lambda(2k+1)/2^{j+1}}$$

it results that

$$\widehat{s_j}(\omega) = 2^{-j/2}|\widehat{\psi}_\alpha(\omega_\lambda/2^j)| \sum_{k\in\mathbb{Z}} e^{-i(\omega-\omega_\lambda)(2k+1)/2^{j+1}}$$

is punctually localized in $\omega_0 = \omega_\lambda/2^j$, analogously to the FFT.

This suggests the design of elementary functions or trigonometric wavelet packets from sines and cosines of appropriate frequencies.

2.3.2 *Generating Functions*

Our strategy is to design elementary spanning functions in the subspace W_0. Its translations and scaling span frames for each subspace W_j.

For each $m \geq 1$, we define the *characteristic frequencies*:

$$\omega_{mh} = \pi + \frac{2h\pi}{2^m}; \quad 0 \leq h \leq 2^{m-1} \tag{16}$$

and consider the $2^m \times 2^m$ *orthogonal Fourier matrices*

$$F_m = \begin{pmatrix} \cdots & 2^{-m/2}\sin[\omega_{m0}(k+1/2)] & \cdots \\ \vdots & \vdots & \vdots \\ \cdots & 2^{(1-m)/2}\cos[\omega_{mh}(k+1/2)] & \cdots \\ \cdots & 2^{(1-m)/2}\sin[\omega_{mh}(k+1/2)] & \cdots \\ \vdots & \vdots & \vdots \\ \cdots & 2^{-m/2}\cos[\omega_{m2^{m-1}}(k+1/2)] & \cdots \end{pmatrix} \tag{17}$$

with $-2^{m-1} \leq k \leq 2^{m-1}-1,\ \ 0 \leq h \leq 2^{m-1}$.

Without lost of generality, we consider $j = 0$ and the corresponding subspace W_0. For $m \geq 0$, $0 \leq l \leq 2^m - 1$, we denote $F_m(l,k)$ the elements of the matrix (17), then the elementary functions defined by

$$\theta_{m,l}(x) = \sum_{k=-2^{m-1}}^{2^{m-1}-1} F_m(l,k)\psi_\alpha(x-k), \tag{18}$$

are localized in the time interval $[-2^{m-1}, 2^{m-1}]$.

Based on the matrices F_m and wavelets ψ_α properties, we can conclude that the families $\{\theta_{m,l}, 0 \leq l \leq 2^m - 1\}$ constitute an orthonormal basis of W_0 for each $m \geq 1$ and its Fourier transform

$$\widehat{\theta}_{m,l}(\omega) = |\widehat{\psi}_\alpha(\omega)| \sum_{k=-2^{m-1}}^{2^{m-1}-1} F_m(l,k)e^{-i\omega(k+1/2)} \tag{19}$$

are well localized in the two-side frequency band $\Omega_0 = \{\pi - \alpha \leq |\omega| \leq 2\pi + 2\alpha\}$. Moreover, the functions (19) are well localized in ω_{mh}, i. e., the characteristic frequencies defined in (16).

However, these transforms do not have an appropriate decay and present undesirable sidelobes.

For this reason, we consider the Dirichlet's kernel of order $M = 2^{m-p}$ with $p < m$ and its 2^p powers, $p \in \mathbb{N}_0$:

$$D_M(\omega) = 1 + 2\sum_{n=1}^{M}\cos(n\omega) = \frac{\sin((M+1/2)\omega)}{\sin(\omega/2)} \tag{20}$$

designing appropriate normalized weights $\mu_{mk}^{(p)}$, we define the elementary functions:

$$\theta_{m,l}^{(p)}(x) = \sum_{k=-2^{m-1}}^{2^{m-1}-1} \mu_{mk}^{(p)} F_m(l,k)\psi_\alpha(x-k) \tag{21}$$

or in the frequency domain

$$\widehat{\theta_{m,l}^{(p)}}(\omega) = |\widehat{\psi}_\alpha(\omega)| \sum_{k=-2^{m-1}}^{2^{m-1}-1} \mu_{mk}^{(p)} F_m(l,k)e^{-i\omega(k+1/2)}. \tag{22}$$

2.3.3 *Modulated Packet Frames*

From (21), and fixing m, p such that $0 \leq p < m$, in each subspace W_j, we generate the family of *modulated wavelet packets*

$$\{\theta_{m,l,j,n}^{(p)}(x) = 2^{j/2}\theta_{m,l}^{(p)}\left(2^j x - n\right), 0 \leq l \leq 2^m - 1, n \in \mathbb{Z}\}. \tag{23}$$

The parameter n indicates the translations of 2^m wavelets ψ_{jk} involved in each packet. More precisely, the range $n - 2^{m-1} \leq k \leq n + 2^{m-1} - 1$ corresponds to the 2^m successive wavelets immersed in the function $\theta_{m,l,j,n}$.

In each subspace W_j the family (23) is over complete. Moreover, constitutes a Parseval frame of the subspace W_j, [11,21]. Therefore, an appropriate structure is necessary to ensure an efficient implementation and stable reconstruction formulae.

Then, if $s_j \in W_j$,

$$s_j(x) = \sum_{l,n} < s(x), \theta_{m,l,j,n}^{(p)} > \theta_{m,l,j,n}^{(p)}(x) \tag{24}$$

the packets coefficients are calculated from the wavelet coefficients (4) by

$$< s(x), \theta_{m,l,j,n}^{(p)} > = \sum_{k=-2^{m-1}}^{2^{m-1}-1} \mu_{mk}^{(p)} F_m(l,k)\, c_{jk+n}. \tag{25}$$

Note that the new wavelet coefficients (25) are discrete convolutions of the original wavelet coefficients and the result of refinement schemes in frequency, in each space W_j.

The parameter m determines $2^{m-1}+1$ characteristic angular frequencies in the interval $[\pi, 2\pi]2^j$ and can be chosen for each level j, according to the needs of the analysis. Usually in the applications we take $m = 4,5,6$ specifying in this way $9,17,33$ characteristic angular frequencies, respectively. The other parameter could be chosen as $p = 2$ or $p = 4$, obtaining very good frequency resolution.

Furthermore, the functions $\theta^{(p)}_{m,l,j,n}$ are well localized in the temporal interval

$$X_{m,j,n} = \left[\frac{1}{2^j}(n-2^{m-1}),\ \frac{1}{2^j}(n+2^{m-1})\right] \qquad (26)$$

and pointwise localized around the center $[\frac{n}{2^j}]$. Its Fourier transforms $\widehat{\theta}^{(p)}_{m,l,j,n}$ are located in the bilateral frequency band (8). More precisely, around the frequencies:

$$\omega_{m,j,h} = 2^j \omega_{mh} \qquad (27)$$

and decay like the 2^p power of the kernel (20), i.e., $(D_M(\omega))^{2^p}$.

An efficient signal representation tool is thus achieved as a superposition of waves associated to a defined frequency.

3 Experimental results. EEG time-frequency analysis

Interictal events are epileptiform discharges that can be observed between seizures, during record of the brain electrical activity of epileptic patients.

The International Federation of Societies for Electroencephalography and Clinical Neurophysiology (IFSECN) define: “Epileptiform patterns (epileptiform discharge or activity): transients distinguishable from background activity, with a characteristic spiky morphology, typically, but neither exclusively nor invariably, found in interictal EEGs of people with epilepsy”, [14]. Each type of event is usually depicted in terms of its morphological characteristics, such as amplitude, duration, sharpness, and emergence from its background. The detection of these patterns, as well as the beginning and the propagation of the seizure between EEG channels, collaborate in the process of localization of the seizure focus in the brain.

In this section we apply the signal representation tools developed previously in the processing of EEG signals corresponding to electroencephalograms taken with deep implantation electrodes. This type of studies is only carried out in patients who are candidates for surgery to remove the epileptogenic focus. We show the performance of this methodology in detection and classification of different types of events: *spike, sharp waves* and *“spike and wave”*.

Figure 2(a) shows a typical electroencephalographic recording, corresponding to one of the 17 channels, with $N = 2^{19}$ data and sampling frequency $\nu = 200$ Hz. According to the specialist, there are several events mentioned above immersed in this signal. Figure 2(b) shows its corresponding power spectrum. We can see that the relative information is in the spectral band of less than 40 Hz. The frequency peak observed at 50 Hz corresponds to an artifact generated by the alternating current present in the electric power supply in the data acquisition process.

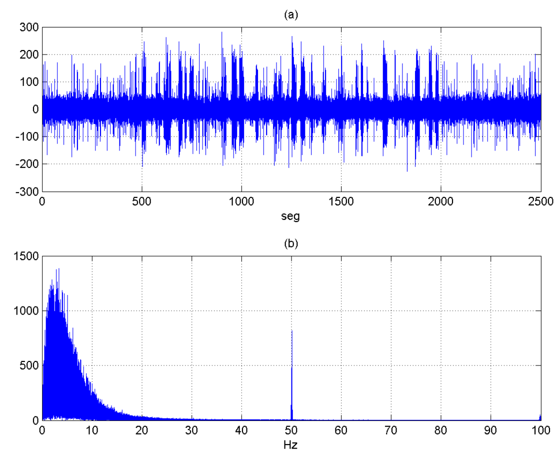

Fig. 2: (a) Typical EEG signal. (b) Its module Fourier transform.

The wavelet analysis is applied between levels $j = -11$ and $j = -2$. The energy distribution is shown in Table 1. We can observe that its energy is practically localized on levels $j = -6,-5,-4$, corresponding to the frequency band [1.5625,12.5000] Hz.

Table 1: Energy distribution corresponding to one of the EEG signal channel

level j	energy %	frecuency band Hz
-11	0.06	[0.0488, 0.0977]
-10	0.12	[0.0977, 0.1953]
-9	0.20	[0.1953, 0.3906]
-8	0.94	[0.3906, 0.7813]
-7	6.22	[0.7813, 1.5625]
-6	23.40	[1.5625, 3.1250]
-5	54.26	[3.1250 6.2500]
-4	13.72	[6.2500, 12.5000]
-3	0.64	[12.5000, 25.0000]
-2	0.21	[25.0000, 50.0000]
-1	0.00	[50.0000, 100.0000]

3.1 Spike detection

In Figure 3(a) we can see the temporal interval [1570, 1610] sec of the EEG signal, where the specialist detected, for instance, six spike events (marked with arrows), i. e., transients clearly different from background activity, with pointed peak at a conventional paper speed or time scale, and a duration from 20 to under 70 msec, [14]. The local energy profiles defined in (11) allow us to clearly localize this type of event, see Figure 3(b). Finally, to complete the analysis, modulated wavelet packets defined in subsection (2.3.3) are used on level $j = -4$, with $(m, p) = (5, 3)$, obtaining very good frequency resolution. The resulting dominant frequencies are displayed on the Figure 3(c).

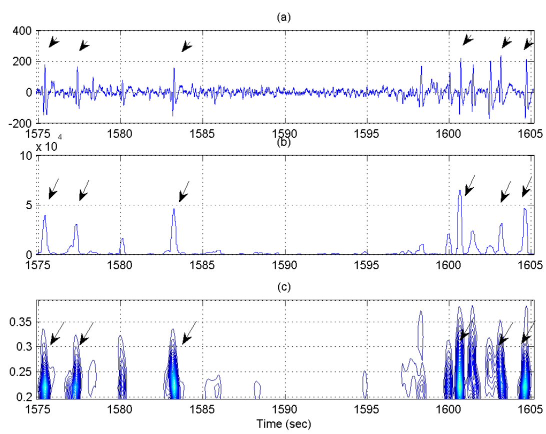

Fig. 3: (a) EEG signal in the temporal range [1570, 1610] sec and spikes detection. (b) Local energy profile on levels $j = -4, -3$, (c) Representation of the dominant frequencies on level $j = -4$.

We consider another portion of the same signal lasting 6 seconds, where the specialist detected multiple spike complex or a train of spikes (marked with arrows), see Figure 4(a). Similar results were obtained. The local profile on levels $j = -4, -3$ and the result of the application of modulated wavelet packets, with $(m, p) = (5, 3)$, can be seen in the Figure 4(b-c) respectively.

3.2 "Spike and wave" detection

An "spike and wave" event is a pattern consisting of a spike followed by a slow wave, [14]. In this case we consider a section of the same signal in which the specialist detected a train of spike and wave, see Figure 5(a). Figure 5(b) shows the local profile at $j = -5, -4, -3$. The application of wavelet packets allow us to characterize the dominant frequencies on levels $j = -4, -3$, $(m, p) = (5, 3)$, as seen in Figure 5(c-d). We observe how these packets can distinguish frequency details in each scale j and its temporal variations with high precision.

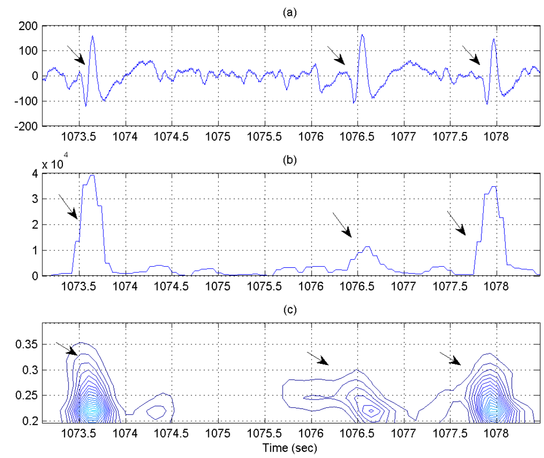

Fig. 4: (a) EEG signal in the temporal interval [1073, 1079] sec and spikes detection. (b) Local energy profile on levels $j = -4, -3$. (c) Representation of the dominant frequencies on level $j = -4$.

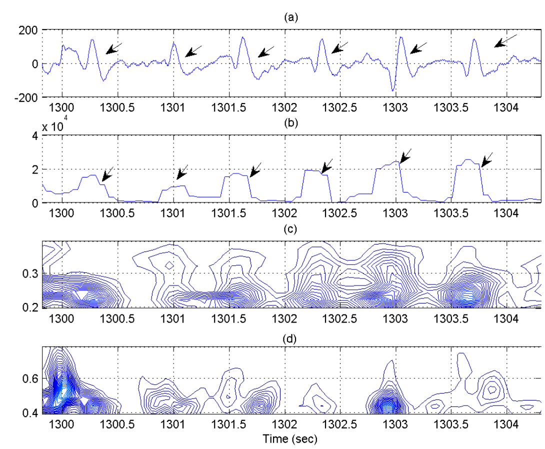

Fig. 5: (a) EEG signal in the temporal range [1299.5, 1304.5] sec and a train of "spike and wave" detection. (b) Local energy profile on leves $j = -4, -3$. (c) Representation of the dominant frequencies on level $j = -4$. (d) Representation of the dominant frequencies on level $j = -3$.

3.3 Sharp wave detection

A sharp wave is a transient, clearly recognizable from background activity, with pointed peak at a conventional paper speed or time scale, and duration of 70±200 msec, [14].

A similar analysis in the temporal range [220, 229] sec is performed and the results are shown in the Figure 6.

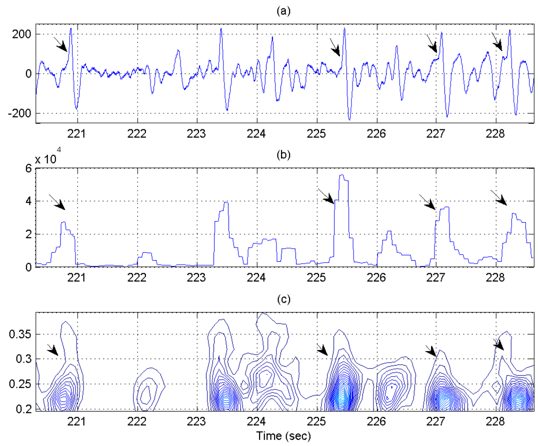

Fig. 6: (a) EEG signal in the temporal range [220, 229] sec with sharp wave detected. (b) Local energy profile on levels $j = -4, -2$. (c) Representation of the dominant frequencies on level $j = -4$.

3.4 Spike vs "spike and wave" detection

Finally, we consider a new segment of the same EEG signal, in the temporal interval [1305, 1314] sec, in which the specialist detected two spikes (marked with arrows) and three "spikes and wave" (marked with ellipses), see Figure 7(a).

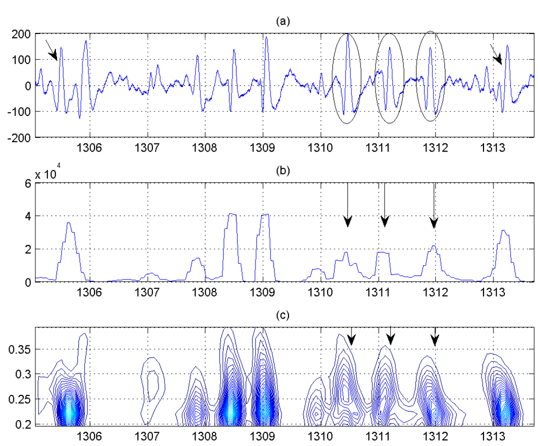

Fig. 7: (a) EEG signal in the temporal range [1305, 1314] sec with spikes and "spike and wave" detected. (b) Local energy profile on levels $j = -4, -3$. (c) Representation of the dominant frequencies on level $j = -4$.

In Figure 7(b) the local profile on levels $j = -4, -3$ can be observed. In it, four events of similar structures are distinguished. Two of them have been marked by the specialist as spikes. The rest other transient wave recognized as "spike and wave", are also characterized in the profile as lower than previous peaks. Once again, we observe that these packets can distinguish frequency details in scale $j = -4$ and its temporal variations with high precision, see Figure 7(c). In this case,we choose $(m, p) = (4, 2)$.

4 Conclusions

The electroencephalogram is a traditional procedure to investigate the abnormal functioning of the brain activity. Particularly long-term EEGs with depth electrodes are part of the studies carried out in some epileptic patients resistant to drugs that are candidates for surgical removal of the epileptogenic focus. The analysis of the signal of the different EEG channels: detection of epileptic events, paroxysms, beginning of the seizures and their propagation among the EEG's channels, provide important information needed to identify the epileptic focus in the brain.

In this work we have proposed a method to detect different interictal epileptic events in EEG recordings based mainly on a band limited wavelet and modulated wavelet packets representation. We emphasized the approach on modulated wavelet packets due to its notorious efficiency extracting features from non-stationary signals, allowing simultaneous analysis in time and frequency domains. At the same time implementing local profiles allow us to accurately detect the temporal location of events. The method was applied to real clinical EEG data of epileptic patients. The promising results suggest that this processing approach in both, temporal and frequency domains can be a real help to identify fast EEG transients.

Finally, we hope that these new developments can be adapted to help solve the epileptic seizure prediction problem.

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