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Improved $(\frac{G'}{G})$ -expansion Method for Elliptic-like Equation and General Traveling Wave Solutions of Some Class of NLPDEs

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Abstract: In this paper, a simple transformation technique is used to reduce some class of nonlinear partial differential equation to the elliptic-like equation F”(ξ)-rF(ξ)-sF(ξ)=0, and then the periodic and solitary wave solutions of some class of NLPDEs are constructed by using improved (G’/G) -expansion method. Some new travelling wave solutions involving parameters, expressed by three types of functions which are the hyperbolic functions, the trigonometric functions and the rational functions. The solitary wave solutions are derived from the hyperbolic function solutions.

Keywords: Improved $(\frac{G'}{G})$ -expansion method, Nonlinear partial differential equation, Solitary wave solution.

1 Introduction

It is well known that nonlinear evolution equations (NLEEs) are often presented to describe the motion of the isolated waves, localized in a small part of space, in many fields like hydrodynamic, plasma physics and nonlinear optic. Many powerful methods have been proposed to obtain exact solutions of nonlinear evolution equations, such as inverse scattering method [1], Backlund transformation method [2,3]. Hirota’s bilinear scheme [4, 5]. homogeneous balance method [6]. Riccati equation rational expansion method [7,8]. the tanh-method [9,10]. and so on.

In recent years, with the development of symbolic computation packages like Maple and Mathematica, which enable us to perform the tedious and complex computation on computer The $(\frac{G'}{G})$-expansion method proposed by Wang et al. [11]. is one of the most effective direct methods to obtain travelling wave solutions of a large number of NLEEs,such as the KdV equation,the mKdV equation, the variant Boussinesq equations,the Hirota–Satsuma equations,and so on .Later, the further developed methods named the generalized $(\frac{G'}{G})$-expansion method,the modified $(\frac{G'}{G})$-expansion method and the extended $(\frac{G'}{G})$-expansion method have been proposed in Refs. [12,13,14]. respectively. As we know, when using the direct method, the choice of an appropriate ansatz is great importance . In this paper, by introducing a new general ansatze, we propose the improved $(\frac{G'}{G})$-expansion method, which can be used to obtain travelling wave solutions of NLEEs.

2 Description of the improved $(\frac{G'}{G})$-expansion method

Suppose that a nonlinear evolution equation, say in two independent variables x and t, is given by

$$N(u,u_t,u_x,u_{tt},u_{xx},u_{xt},\ldots)=0, \tag{2.1}$$

where $u = u(x,t)$ is an unknown function, N is a polynomial in $u = u(x,t)$ and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved. To determine u explicitly, we take the following five steps:

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Step 1:
Use the travelling wave transformation:

$$u(x,t) = u(\xi), \qquad \xi = x - wt, \tag{2.2}$$

where w is a constant to be determined latter. Then, the *NLEE* (2.1) is reduced to a nonlinear ordinary differential equation (*NLODE*) for $u = u(\xi)$

$$N(u, -wu', u', w^2u'', u'', -wu'', \ldots) = 0. \tag{2.3}$$

Step 2:
We suppose that the *NLODE* (2.3) has the following solution:

$$u(\xi) = \sum_{i=-m}^{-1} \frac{a_i(\frac{G'}{G})^i}{(1+\beta(\frac{G'}{G}))^i} + a_0 + \sum_{i=1}^{m} \frac{a_i(\frac{G'}{G})^i}{(1+\beta(\frac{G'}{G}))^i}, \tag{2.4}$$

where β, a_i $(i = -m, -m+1, \ldots, m-1, m)$ are constants to be determined later, m is a positive integer, and $G = G(\xi)$ satisfies the following second order linear ordinary differential equation($LODE$) :

$$G'' + \mu G = 0, \tag{2.5}$$

where μ is a real constant. The general solutions of Eq. (2.5) can be listed as follows.When $\mu < 0$, we obtain the hyperbolic function solution of Eq. (2.5)

$$G(\xi) = A_1 \cosh\sqrt{-\mu}\xi + A_2 \sinh\sqrt{-\mu}\xi, \tag{2.6}$$

where A_1 and A_2 are arbitrary constants.When $\mu > 0$, we obtain the trigonometric function solution of Eq. (2.5)

$$G(\xi) = A_1 \sin\sqrt{\mu}\xi + A_2 \cos\sqrt{\mu}\xi, \tag{2.7}$$

where A_1 and A_2 are arbitrary constants.When $\mu = 0$, we obtain the rational function solution of Eq. (2.5)

$$G(\xi) = A_1 + A_2\xi \tag{2.8}$$

where A_1 and A_2 are arbitrary constants.
Step 3:
Determine the positive integer m by balancing the highest order derivative and nonlinear terms in Eq. (2.3).
Step 4:
Substituting (2.4) along with Eq. (2.5) into Eq. (2.3) and then setting all the coefficients of $(\frac{G'}{G})$ of the resulting system's numerator to zero, yields a set of over-determined nonlinear algebraic equations for w, β and $a_i(i = -m, -m+1, \ldots, m-1, m)$.
Step 5:
Assuming that the constants w, β, $a_i(i = -m, -m+1, \ldots, m-1, m)$ can be obtained by solving the algebraic equations in Step 4, then substituting these constants and the known general solutions of Eq. (2.5) into (2.4), we can obtain the explicit solutions of Eq. (2.1) immediately.

3 Exact solutions of the elliptic-like equation

Let us consider the following elliptic-like equation

$$F''(\xi) - rF(\xi) - sF^3(\xi) = 0, \tag{3.1}$$

where r and s are arbitrary constants.

By balancing the highest order derivative terms and nonlinear terms in Eq. (3.1), we find that Eq. (3.1) own the solutions in the form

$$F(\xi) = a_0 + \frac{a_1(\frac{G'}{G})}{1+\beta(\frac{G'}{G})} + \frac{b_1(1+\beta(\frac{G'}{G}))}{(\frac{G'}{G})}, \tag{3.2}$$

where $G = G(\xi)$ satisfies Eq. (2.5),β, a_0, a_1, b_1, r and s are constants to be determined latter.

Substituting (3.2) along with Eq. (2.5) into Eq. (3.1) and then setting all the coefficients of $(\frac{G'}{G})$ of the resulting system's numerator to zero, yields a set of over-determined nonlinear algebraic equations for β, a_0, a_1, b_1, r and s . Solving the over-determined algebraic equations by Maple or Mathematica, we can obtain the following results:
Case 1:

$$a_0 = -\sqrt{\tfrac{2}{s}}\mu\beta, \quad a_1 = 0, \quad b_1 = \sqrt{\tfrac{2}{s}}\mu, \quad \mu = \tfrac{r}{2}$$

Case 2:

$$a_0 = \sqrt{\tfrac{2}{s}}\mu\beta, \quad a_1 = 0, \quad b_1 = -\sqrt{\tfrac{2}{s}}\mu, \quad \mu = \tfrac{r}{2}$$

Case 3:

$$a_0 = \sqrt{\tfrac{2}{s}}\mu\beta, \quad a_1 = -\sqrt{\tfrac{2}{s}} - \sqrt{\tfrac{2}{s}}\mu\beta^2, \quad b_1 = 0, \quad \mu = \tfrac{r}{2}$$

Case 4:

$$a_0 = -\sqrt{\tfrac{2}{s}}\mu\beta, \quad a_1 = \sqrt{\tfrac{2}{s}} + \sqrt{\tfrac{2}{s}}\mu\beta^2, \quad b_1 = 0, \quad \mu = \tfrac{r}{2}$$

Using Case 1,2, (3.2) and the general solutions of Eq. (2.5), we can find the following travelling wave solutions of elliptic-like equation(3.1)When $\mu < 0$ and $r < 0$, we obtain the hyperbolic function solutions of Eq. (3.1)

$$\begin{aligned} F(\xi) = \pm\sqrt{\frac{2}{s}}\mu\beta \pm \frac{\sqrt{\frac{2}{s}}\mu(\sqrt{-\mu}}{(A_1 \sinh\sqrt{-\mu}\xi + A_2 \cosh\sqrt{-\mu}\xi)} \\ \times[(A_1 + \beta A_2\sqrt{-\mu})\cosh\sqrt{-\mu}\xi \\ +(A_2 + \beta A_1\sqrt{-\mu})\sinh\sqrt{-\mu}\xi], \end{aligned} \tag{3.3}$$

where β, A_1, A_2 are arbitrary constants and $r < 0$

In particular, when setting $\beta = A_1 = 0$, $A_2 \neq 0$, the solutions (3.3) can be written as

$$F(\xi) = \pm\sqrt{\frac{-r}{s}} \tanh\sqrt{\frac{-r}{2}}\xi. \tag{3.4}$$

Setting again $\beta = 0, A_1 > 0, A_1^2 > A_2^2$ the following kink-shaped solution of Eq. (3.1)

$$F(\xi) = \pm\sqrt{\frac{-r}{s}} \tanh(\sqrt{\frac{-r}{2}}\xi + \xi_0), \tag{3.5}$$

where $\xi_0 = \tanh^{-1}\frac{A_1}{A_2}$.

Using Case 3,4, (3.2) and the general solutions of Eq. (2.5), we can find the following travelling wave solutions of elliptic-like equation(3.1)

$$F(\xi) = \pm\sqrt{\frac{2}{s}}\mu\beta \pm \frac{(\sqrt{\frac{2}{s}}+\sqrt{\frac{2}{s}}\mu\beta^2)(\sqrt{-\mu}(A_1\sinh\sqrt{-\mu}\xi + A_2\cosh\sqrt{-\mu}\xi))}{((A_1+\beta A_2\sqrt{-\mu})\cosh\sqrt{-\mu}\xi + (A_2+\beta A_1\sqrt{-\mu})\sinh\sqrt{-\mu}\xi}, \tag{3.6}$$

where β, A_1, A_2 are arbitrary constants

In particular, when setting $\beta = A_1 = 0$, $A_2 \neq 0$, the solutions (3.6) can be written as

$$F(\xi) = \pm\sqrt{\frac{-r}{s}}\coth\sqrt{\frac{-r}{2}}\xi. \tag{3.7}$$

Setting again $\beta = 0, A_1 > 0, A_1^2 > A_2^2$ the following singular soliton solution of Eq. (3.1)

$$F(\xi) = \pm\sqrt{\frac{-r}{s}}\coth(\sqrt{\frac{-r}{2}}\xi + \xi_0), \tag{3.8}$$

where $\xi_0 = \tanh^{-1}\frac{A_1}{A_2}$.

When $\mu > 0$ and $r > 0$ we get the trigonometric function solutions of Eq. (3.1)

$$\begin{aligned}F(\xi) = \pm\sqrt{\frac{2}{s}}\mu\beta \pm \frac{\sqrt{\frac{2}{s}}\mu}{\sqrt{\mu}(A_1\cos\sqrt{\mu}\xi - A_2\sin\sqrt{\mu}\xi)}\\ \times[((A_1-\beta A_2\sqrt{\mu})\sin\sqrt{\mu}\xi\\ +(A_2+\beta A_1\sqrt{\mu})\cos\sqrt{\mu}\xi)].\end{aligned} \tag{3.9}$$

In particular, when setting $\beta = A_1 = 0$, $A_2 \neq 0$, the solutions (3.9) can be written as

$$F(\xi) = \pm\sqrt{\frac{r}{s}}\cot\sqrt{\frac{r}{2}}\xi. \tag{3.10}$$

Setting again $\beta = 0, A_1 > 0, A_1^2 > A_2^2$ the following periodic solutions of Eq. (3.1)

$$F(\xi) = \pm\sqrt{\frac{r}{s}}\cot(\sqrt{\frac{r}{2}}\xi + \xi_1), \tag{3.11}$$

where $\xi_1 = \tan^{-1}\frac{A_2}{A_1}$.

Using Case 3,4, (3.2) and the general solutions of Eq. (2.5), we can find the following periodic solutions of elliptic-like equation(3.1)

$$F(\xi) = \pm\sqrt{\frac{2}{s}}\mu\beta \pm \frac{(\sqrt{\frac{2}{s}}+\sqrt{\frac{2}{s}}\mu\beta^2)(\sqrt{\mu}(A_1\cos\sqrt{\mu}\xi - A_2\sin\sqrt{\mu}\xi))}{((A_1-\beta A_2\sqrt{\mu})\sin\sqrt{\mu}\xi + (A_2+\beta A_1\sqrt{\mu})\cos\sqrt{\mu}\xi)}, \tag{3.12}$$

where β, A_1, A_2 are arbitrary constants.

Setting $\beta = 0, A_1 > 0, A_1^2 > A_2^2$ the following singular soliton solution of Eq. (3.1)

$$F(\xi) = \pm\sqrt{\frac{r}{s}}\tan(\sqrt{\frac{r}{2}}\xi + \xi_1), \tag{3.13}$$

where $\xi_1 = \tan^{-1}\frac{A_2}{A_1}$.

When $\mu = 0$ and $r = 0$ we get the rational function solutions of Eq. (3.1)

$$F(\xi) = \pm\sqrt{\frac{2}{s}}(\frac{A_2}{A_1 + A_2\xi + A_2\beta}), \tag{3.14}$$

where β, A_1, A_2 are arbitrary constants.

4 Exact solutions of some class of NLPDEs

In this section, we use three examples to illustrate the applicability of our method to solve NLPDEs.

Example 1:

Let us first consider a class of nonlinear partial differential equations[15, 16]:

$$iu_t + n(u_{xx} + \alpha_1 u_{yy}) + \beta_1 |u|^2 u + \gamma_1 uv = 0, \tag{4.1a}$$
$$\alpha_2 v_{tt} + (v_{xx} - \beta_2 v_{yy}) + \gamma_2(|u|^2)_{xx} = 0, \tag{4.1b}$$

where $n, \alpha_i, \beta_i, \gamma_i$ $(i = 1, 2)$ are real constants and $n \neq 0, \beta_1 \neq 0, \gamma_1 \neq 0, \gamma_2 \neq 0$.

The important cases of Eqs. (4.1) are as follows. In fact, if one takes

$$v = 0, u_x = 0, n = 1,$$

then Eqs. (4.1) represent the nonlinear Schrodinger equation

$$iu_t + \alpha_1 u_{yy} + \beta_1 |u|^2 u = 0. \tag{4.2}$$

Also, if one takes

$$v = v(x,t), n = 1, \alpha_1 = 0, \beta_1 = -2k, \tag{4.3a}$$
$$\beta_2 = 0, \gamma_1 = 2, \alpha_2 = -1, \gamma_2 = -1, \tag{4.3b}$$

then Eqs. (4.1) become the generalized Zakharov (GZ) equations [17]

$$iu_t + u_{xx} - 2k|u|^2 u + 2uv = 0, \tag{4.4a}$$
$$v_{tt} - v_{xx} + (|u|^2)_{xx} = 0. \tag{4.4b}$$

If one takes

$$n = \frac{1}{2}\sigma^2, \alpha_1 = 2n, \beta_1 = k, \gamma_1 = -1, \tag{4.5a}$$
$$\alpha_2 = 0, \beta_2 = \alpha_1, \gamma_2 = -2k, \sigma^2 = \pm 1, \tag{4.5b}$$

then Eqs. (4.1) is the Davey–Stewartson (DS) equations [18]

$$iu_t + \frac{1}{2}\sigma^2(u_{xx} + \sigma^2 u_{yy}) + k|u|^2 u - uv = 0, \quad (4.6a)$$

$$v_{xx} - \sigma^2 v_{yy} - 2k(|u|^2)_{xx} = 0. \quad (4.6b)$$

For our purpose, we introduce the following transformations:

$$u = e^{i\theta}F(\xi), v = V(\xi),$$
$$\theta = px + qy + kt, \xi = x + cy + dt, \quad (4.7)$$

where p,q,k,c and d are real constants.

Substituting (4.7) into (4.1), we can know that $d = -2n(p + \alpha_1 qc)$, and F,V satisfy the following system:

$$-(k + p^2 n + n\alpha_1 q^2)F + (n + n\alpha_1 c^2)F''$$

$$+\beta_1 F^3 + \gamma_1 FV = 0, \quad (4.8a)$$

$$(\alpha_2 d^2 - \beta_2 c^2 + 1)V'' + \gamma_2 (F^2)'' = 0. \quad (4.8b)$$

Integrating Eq. (4.8b) twice with respect to ξ and taking the integration constant as zero yields

$$V = -\frac{\gamma_2}{\alpha_2 d^2 - \beta_2 c^2 + 1}F^2. \quad (4.9)$$

Substituting Eq. (4.9) into Eq. (4.8a) yields

$$F''(\xi) - rF(\xi) - sF^3(\xi) = 0, \quad (4.10)$$

where

$$r = \frac{k + p^2 n + n\alpha_1 q^2}{n + n\alpha_1 c^2}, s = \frac{-\beta_1(\alpha_2 d^2 - \beta_2 c^2 + 1) + \gamma_1\gamma_2}{(n + n\alpha_1 c^2)(\alpha_2 d^2 - \beta_2 c^2 + 1)}.$$

Then the solutions of Eqs. (4.1) are

$$\begin{cases} u(x,y,t) = e^{i(px+qy+kt)}F(\xi), \\ v(x,y,t) = -\frac{\gamma_2}{\alpha_2 d^2 - \beta_2 c^2 + 1}F^2(\xi). \end{cases} \quad (4.11)$$

The expression $F(\xi)$ appearing in these solutions is given by relations (3.3)–(3.14) , where $\xi = x + cy + dt, d = -2n(p + \alpha_1 qc)$ p,q,c are real constants.

Example 2. We consider the Maccari system [19,20]:

$$\begin{cases} iu_t + u_{xx} + uv = 0, \\ v_t + v_y + (|u|^2)_x = 0. \end{cases} \quad (4.12)$$

Maccari system was derived from the Kadomtsev–Petviashvili equation by Attilio Maccari,and discussed its Lax pairs explicitly demonstrated.

Using the wave transformations

$$u = e^{i(px+qy+kt)}F(\xi), v = V(\xi), \quad (4.13)$$

where $\xi = x + cy + dt$, amplitude $F(\xi)$ is a real function, where p,q,k and c are real constants. Substituting (4.13) into (4.12),we
can know that

$$F'' - (k + p^2)F + FV = 0, \quad (4.14a)$$

$$(c + d)V' + 2FF'' = 0. \quad (4.14b)$$

Integrating Eq. (4.14b) with respect to ξ and taking the integration constant as zero yields

$$-(c + d)V = F^2, \quad (4.15)$$

Substituting Eq. (4.15) into Eq. (4.14a) yields

$$F''(\xi) - rF(\xi) - sF^3(\xi) = 0, \quad (4.16)$$

where $r = k + p^2, s = \frac{1}{c+d}$.
Then the solutions of Eqs. (4.12) are

$$\begin{cases} u(x,y,t) = e^{i(px+qy+kt)}F(\xi), \\ v(x,y,t) = -\frac{1}{c+d}F^2(\xi). \end{cases} \quad (4.17)$$

The expression $F(\xi)$ appearing in these solutions is given by relations (3.3)–(3.14) , where $\xi = x + cy + dt$, p,q,c,k,d are real constants and $c \neq -d$.

Example 3. Let us first consider a new integrable coupled nonlinear schrodinger equations [21]

$$i\phi_t + \chi\phi_{xx} \mp 2\mu(\frac{|\phi|^2 + |\omega|^2}{|\phi|^2|\omega|^2})\phi = R_1, \quad (4.18a)$$

$$i\omega_t + \chi\omega_{xx} \mp 2\mu(\frac{|\phi|^2 + |\omega|^2}{|\phi|^2|\omega|^2})\omega = R_2, \quad (4.18b)$$

where the perturbative terms $R1$ and $R2$, and the real parameters χ and μ are, respectively, defined as follows

$$R_1 = \frac{2\chi\phi_x^2}{\phi}, R_2 = \frac{2\chi\omega_x^2}{\omega}, \chi = \frac{\alpha + \delta}{\alpha(\alpha - \delta)}, \mu = \frac{\alpha^2 - \delta^2}{\alpha^2\delta},$$

with α and δ as two real constants.

For our purpose, we introduce the following transformations:

$$\phi(x,t) = e^{-i(kx+lt+\delta_1)}F^{-1}(\xi),$$
$$\omega(x,t) = e^{-i(kx+lt+\delta_2)}F^{-1}(\xi), \quad (4.19)$$

where $\xi = ax + 2k\chi at, F(\xi)$ is a real function, k,l and a are the real parameters. The phase constants δ_1 and δ_2 represent the complex envelopes ϕ and ω. Substituting (4.19) into (4.18), yields

$$F''(\xi) - rF(\xi) - sF^3(\xi) = 0,$$

where

$$r = \frac{l+\chi k^2}{a^2\chi}, s = \frac{\mp 4\mu}{a^2\chi}.$$

Then the solutions of Eqs. (4.18) are

$$\begin{cases} u(x,y,t) = e^{-i(kx+lt+\delta_1)}F^{-1}(\xi), \\ v(x,y,t) = e^{-i(kx+lt+\delta_2)}F^{-1}(\xi). \end{cases} \tag{4.20}$$

The expression $F(\xi)$ appearing in these solutions is given by relations (3.3)–(3.14) , where $\xi = ax + 2k\chi at$, k, l and a are the real constants.

5 Conclusions

The improved $(\frac{G'}{G})$ -expansion method is applied successfully for solving the system of a class of nonlinear partial differential equations, a new integrable coupled nonlinear Schrodinger equations, the Maccari system.These exact solutions include the hyperbolic function solutions, trigonometric function solutions and rational function solutions. When the parameters are taken as special values, the solitary wave solutions are derived from the hyperbolic function solutions.This method has more advantages: it is direct and concise.

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