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Testing EBU_{mgf} Class of Life Distributions based on Laplace Transform Technique

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Abstract: Atallah et al. [10] and Al-Gashgari et al. [9] achieved a new technique for testing exponentiality based on Laplace transform, in this paper we introduce a new test for testing exponentiality versus "exponential better than used in moment generating function ordering class"(EBU_{mgf}). By simulation, the critical values and the powers of the proposed test under various alternatives are calculated to assess the performance of the test. It is shown that the proposed test enjoys good power and performs better than some previous tests in terms of Pitman's asymptotic efficiencies for several alternative. Finally sets of real data are used as examples to illustrate the use of the proposed test in practical application.

Keywords: Moment generating function, EBU, Hypothesis test, Pitman's efficiency, Laplace transform. ***AMS Subject Classification:*** 62G10, 62N05,62G07,90B25.

1 Introduction

Equally important in reliability theory is the concept of aging. No aging means the age of the component has no effect on the distribution of its residual lifetime. Positive (negative) aging means that the age has, in some probabilistic sense, an adverse (beneficial) effect on the residual lifetime. Such aging could be positive, whereby a component wears out with time, or negative, whereby time has a beneficial effect on the residual lifetime. These notions of aging are captured through the well known monotonic aging classes like increasing failure rate (IFR), increasing failure rate average (IFRA), decreasing mean residual life (DMRL), new better than used (NBU), new better than used in expectation (NBUE) and harmonic new better than used in expectation (HNBUE). For definitions and interrelationships of these classes, see Barlow and Proschan [12] and Deshpande et al. [15].

The EBU class has been introduced by Elbatal [18]; he also discussed The closure properties under reliability operation, moment inequality, and heritage under shock model.

Definition 1.1 *X is exponential better (worse) than used (denoted by $X \in EBU$) If*

$$\bar{F}(x+t) \leq \bar{F}(t)e^{\frac{-x}{\mu}}, \quad \forall x,t \geq 0.$$

Statisticians and reliability analysts studied exponential better than used classes of life distributions from various points of view. Related paper dealing with EBU problems include Hendi et al. [23], Attia et al. [11], Abdul moniem [6], Hendi and AL-Ghufily [21] and AL-Ghufily 7,8.

Given two non-negative random variables X and Y, with survival functions $\bar{F}$ and $\bar{G}$, respectively, X is said to be smaller than Y in the moment generating function ordering (denoted by $X \leq_{mgf} Y$) if and only if,

$$\int_0^\infty e^{sx}\bar{F}(x)dx \leq \int_0^\infty e^{sy}\bar{G}(y)dy \qquad \text{for all} \quad s > 0.$$

Definition 1.2 *We say that X is exponential better than used in the moment generating function order (denoted by $X \in EBU_{mgf}$) if $X_t \leq_{mgf} Y \quad$ for all $\quad t > 0$, where Y is an exponential random variable with the same mean as X.*

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Equivalently, $X \in EBU_{mgf}$ *if and only if,*

$$\int_0^\infty e^{sx}\bar{F}(x+t)dx \leq \frac{\mu}{1-s\mu}\bar{F}(t), \quad \forall \quad 0 \leq s < \frac{1}{\mu}, \quad t \geq 0. \tag{1.1}$$

Note that, the definition 1.2 is motivated by comparing the moment generating function of the life time X_t of a component of age t with the moment generating function of another new life time Y of a component which is distributed exponentially with mean μ. EBU_{mgf} class developed first by Abbas [1] and subsequently by Gadallah 19.
In the current investigation, we present a procedure to test X is exponential versus it is EBU_{mgf} and not exponential in Section 2. In Section 3, the Pitman asymptotic efficiencies are calculated for some commonly used distributions in reliability. Monte Carlo null distribution critical points and the power estimates are simulated in Section 4. Finally our test is applied to two sets of real data in Section 5.

2 Testing Exponentiality

One of the oldest inference problems in reliability is testing exponentiality versus the most commonly known classes of aging distributions.For testing exponentiality versus NBU class see Hollander and Proschan [24], Koul [27], Alam and Basu [4], and Ahmad [2], among others. For testing NBUE we refer to Hollander and proschan [25], Koul and susarla [28] and Borges et al. [13], while testing versus HNBUE are discussed by Basu and Ebrahim [14], Ahmed [3] and Hendi et al. [22]. Testing versus NBUL are discussed by Diab et al. [17] and Diab [16]. Testing exponentiality versus NBU_{mgf} class was first taken up by Ahmad and Kayid [5]. This was followed by the works of Mahmoud and Gadallah [31].

Our goal in this section is to present a test statistic based on Laplace transform for testing $H_0 : \delta(s,\beta) = 0$ versus $H_1 : \delta(s,\beta) > 0$. Using (1.1) the measure of departure can be defined as

$$\delta(s,\beta) = \mu\int_0^\infty e^{-\beta t}\bar{F}(t)dt - (1-s\mu)\int_0^\infty\int_0^\infty e^{sx-\beta t}\bar{F}(x+t)dxdt.$$

The following lemma is essential for the development of our test statistic.

Lemma 2.1*If* $\phi(\beta) = \int_0^\infty e^{-\beta x}dF(x)$ *then*

$$\delta(s,\beta) = s(\beta\mu+1)(1-\phi(\beta)) - \beta(s\mu-1)(1-\phi(-s)).$$

*Proof.*Note that

$$\begin{aligned}\delta(s,\beta) &= \mu\int_0^\infty e^{-\beta t}\bar{F}(t)dt - (1-s\mu)\int_0^\infty\int_0^\infty e^{sx-\beta t}\bar{F}(x+t)dxdt \\ &= \mu I_1 - (1-s\mu)I_2.\end{aligned}$$

One can show that

$$I_1 = \int_0^\infty e^{-\beta t}\bar{F}(t)dt = \frac{1}{\beta}(1-\phi(\beta)),$$

and

$$\begin{aligned}I_2 &= \int_0^\infty\int_0^\infty e^{sx-\beta t}\bar{F}(x+t)dxdt = \int_0^\infty\int_t^\infty e^{-\beta t}e^{s(u-t)}\bar{F}(u)dudt \\ &= \frac{-1}{\beta s(\beta+s)}[\beta(1-\phi(-s))+s(1-\phi(\beta))].\end{aligned}$$

Thus the result follows.

To make the test scale invariant, we let $\delta_1(s,\beta) = \frac{\delta(s,\beta)}{\mu^2}$.
Note that under $H_0 : \delta_1(s,\beta) = 0$, while under $H_1 : \delta_1(s,\beta) > 0$.
To estimate $\delta_1(s,\beta)$, let $X_1, X_2, X_3, \cdots, X_n$ be a random sample from F, so the empirical form of $\delta_1(s,\beta)$ is

$$\hat{\delta}_{1n}(s,\beta) = \frac{1}{n^2 X^2}\sum_{i=1}^n\sum_{j=1}^n [s(\beta X_i+1)(1-e^{-\beta X_j}) - \beta(sX_i-1)(1-e^{sX_j})]. \tag{2.1}$$

By defining

$$\phi(X_1,X_2) = s(1+\beta X_1)(1-e^{-\beta X_2}) - \beta(sX_1-1)(1-e^{sX_2}),$$

and define the symmetric kernel

$$\psi(X_1\,,X_2) = \frac{1}{2}[\,\phi(X_1\,,X_2)+\phi(X_2\,,X_1)]$$

This leads to $\hat{\delta}_{1n}(s,\beta)$ is equivalent to U- statistic

$$U_n = \frac{1}{\binom{n}{2}}\sum_R \phi(X_i\,,X_j).$$

The next result summarizes the asymptotic normality of $\hat{\delta}_{1n}(s,\beta)$.

Theorem 2.1*As $n\to\infty$, $\sqrt{n}(\hat{\delta}_{1n}(s,\beta)-\delta_1(s,\beta))$ is asymptotically normal with mean 0 and variance is σ^2 given in* (2.5). *Under H_0 , the variance is reduced to* (2.6).

*Proof.*Let

$$\begin{aligned}\eta_1(X_1) &= E[\phi(X_1\,,X_2)\,|X_1\,] \\ &= \frac{s\beta}{1+\beta}(1+\beta X_1)+\frac{s\beta}{1-s}(sX_1-1),\end{aligned} \tag{2.2}$$

and

$$\begin{aligned}\eta_2(X_2) &= E[\phi(X_1\,,X_2)\,|X_2\,] \\ &= s(\beta+1)(1-e^{-\beta X_2})-\beta(s-1)(1-e^{sX_2}).\end{aligned} \tag{2.3}$$

Considering $\eta(X) = \eta_1(X_1)+\eta_2(X_2)$, gives

$$\eta(X) = \left\{\frac{s\beta(\beta+s)}{(1+\beta)(1-s)}X - s(1+\beta)e^{-\beta X} - \beta(1-s)e^{sX} - \frac{s\beta(\beta+s)}{(1+\beta)(1-s)} + \beta + s\right\}. \tag{2.4}$$

In view of (2.4), the variance is

$$\sigma^2 = Var\left\{\frac{s\beta(\beta+s)}{(1+\beta)(1-s)}X - s(1+\beta)e^{-\beta X} - \beta(1-s)e^{sX}\right\}. \tag{2.5}$$

Under H_0 it is easy to prove that $\mu_0 = E[\eta(X)] = 0$ and the variance σ_0^2 reduces to

$$\sigma_0^2 = \frac{s^2\beta^2(\beta+s)^2(2s^2\beta^2+\beta-s+1)}{(1+\beta)^2(1-s)^2(1+2\beta)(1-2s)(1+\beta-s)}. \tag{2.6}$$

3 The Pitman Asymptotic Efficiencies (PAEs)

To judge on the quality of this procedures, we evaluate its Pitman asymptotic efficiencies (PAEs) for some commonly used distributions in reliability, these are:

1.Linear failure rate family (LFR): $\bar{F}_\theta(x) = \exp(-x-\frac{\theta}{2}x^2)\,, x>0\,, \theta\geq 0$.
2.Makeham family:$\bar{F}_\theta(x) = \exp(-x+\theta(x+e^{-x}-1))\,, x>0\,, \theta\geq 0$.
3.Weibull family: $\bar{F}_\theta(x) = \exp(-x^\theta)\,, x>0\,, \theta>0$.

The PAE is defined by

$$PAE(\delta(s,\beta)) = \frac{1}{\sigma_0}\left|\frac{d\delta_\theta(s,\beta)}{d\theta}\right|_{\theta\to\theta_0},$$

Table 1: Pitman asymptotic efficiencies for various values of s and β

	$\delta(s,\beta)$					U_n	δ_3	$\delta^{(2)}_{F_n}$
	s	$\beta=0.3$	$\beta=0.4$	$\beta=0.6$	$\beta=0.8$			
LFR	0.02	0.97862	0.96574	0.93787	0.90982	0.433	0.408	0.217
	0.12	0.99753	0.99315	0.98106	0.96734			
	0.22	0.99113	0.99381	0.99431	0.99145			
	0.32	0.93223	0.93966	0.94809	0.95152			
	0.42	0.74232	0.75062	0.76055	0.76515			
Makeham	0.02	0.27296	0.27772	0.28397	0.28720	0.144	0.039	0.144
	0.12	0.25933	0.264968	0.273106	0.278336			
	0.22	0.23788	0.24376	0.25250	0.25839			
	0.32	0.20393	0.20928	0.21714	0.22236			
	0.42	0.14540	0.14918	0.15452	0.15786			
Weibull	0.02	1.0975	1.12052	1.15531	1.11797	0.132	0.170	0.05
	0.12	1.04136	1.06662	1.10625	1.13569			
	0.22	0.956637	0.981852	1.02153	1.05103			
	0.32	0.82440	0.84672	0.88104	0.90566			
	0.42	0.59395	0.60951	0.63236	0.64763			

where

$$\delta_\theta(s,\beta) = s(\beta\mu_\theta+1)(1-\phi_\theta(\beta)) - \beta(s\mu_\theta-1)(1-\phi_\theta(-s)).$$

The $PAE(\delta(s,\beta))$ can be written as,

$$\text{PAE}(\delta(s,\beta),F) = \frac{1}{\sigma_0}\left|s\beta\mu'_\theta(\phi_\theta(-s)-\phi_\theta(\beta)) - s(\beta\mu_\theta+1)\phi'_\theta(\beta) + \beta(s\mu_\theta-1)\phi'_\theta(-s)\right|,$$

where $\phi'_\theta(\beta) = \int_0^\infty e^{-\beta x} dF'_\theta(x)$ and $\mu'_\theta = \int_0^\infty \overline{F}'_\theta(x)dx$.
After some mathematical calculations we get

$$\text{PAE}(\delta(s,\beta),LFR) = \frac{1}{\sigma_0}\left|\frac{\beta s(\beta s+1)(\beta+s)}{(1+\beta)^2(1-s)^2}\right|_{\theta\to 0},$$

$$\text{PAE}(\delta(s,\beta),Makeham) = \frac{1}{\sigma_0}\left|\frac{\beta s(\beta s+2)(\beta+s)}{2(1+\beta)(2+\beta)(1-s)(2-s)}\right|_{\theta\to 0}$$

and

$$\text{PAE}(\delta(s,\beta),Weibull) = \frac{1}{\sigma_0}\left|\begin{array}{l} s(\beta+1)\int_0^\infty (x-1)e^{-(1+\beta)x} lnx dx + \beta(1-s)\int_0^\infty (x-1)e^{-(1-s)x} lnx dx \\ -\dfrac{s\beta(s+\beta)}{(1-s)(1+\beta)}\int_0^\infty xe^{-x} lnx dx - (s+\beta) \end{array}\right|_{\theta\to 1}.$$

Table 1 gives the efficiencies of our proposed test $\delta(s,\beta)$ for various values of s,β comparing with the tests given by Kango [26](U_n), Mugdadi and Ahmad [32] (δ_3) and Mahmoud and Abdul Alim [30] ($\delta^{(2)}_{F_n}$).

One can note that our test is more efficient for all used alternatives.

4 Monte Carlo Null Distribution Critical Points

In practice, simulated percentiles are commonly used by applied statisticians and reliability analyst. Next, we simulate the Monte Carlo null distribution critical points for $\hat{\delta}_{1n}(s,\beta)$ in (2.1) based on 10000 simulated sample 3(1)50 from the standard exponential distributions. Table 2 gives these percentile points of the statistics $\hat{\delta}_{1n}(s,\beta)$ at $s=0.12$ and $\beta=0.8$. In view of Table 2, it is noticed that the critical values are increasing as the confidence level increasing and is almost decreasing as the sample size increasing.

Table 2: Critical values of statistic $\hat{\hat{\delta}}_{1n}(s,\beta)$ at $s = 0.12$ and $\beta = 0.8$

n	0.01	0.05	0.10	0.90	0.95	0.99	n	0.01	0.05	0.10	0.90	0.95	0.99
3	-0.029	-0.017	-0.008	0.028	0.031	0.036	27	-0.020	-0.012	-0.008	0.010	0.012	0.015
4	-0.034	-0.016	-0.009	0.024	0.027	0.032	28	-0.017	-0.012	-0.008	0.009	0.011	0.015
5	-0.032	-0.016	-0.009	0.022	0.024	0.028	29	-0.021	-0.012	-0.008	0.010	0.012	0.015
6	-0.038	-0.017	-0.010	0.019	0.022	0.025	30	-0.019	-0.011	-0.008	0.009	0.011	0.014
7	-0.038	-0.017	-0.010	0.019	0.021	0.025	31	-0.016	-0.011	-0.008	0.009	0.011	0.014
8	-0.032	-0.017	-0.010	0.018	0.021	0.025	32	-0.017	-0.011	-0.008	0.009	0.011	0.015
9	-0.032	-0.016	-0.009	0.017	0.020	0.025	33	-0.016	-0.012	-0.008	0.009	0.011	0.014
10	-0.028	-0.015	-0.009	0.016	0.019	0.023	34	-0.016	-0.011	-0.007	0.009	0.011	0.014
11	-0.026	-0.016	-0.010	0.015	0.018	0.023	35	-0.017	-0.011	-0.007	0.009	0.010	0.013
12	-0.025	-0.013	-0.008	0.015	0.016	0.021	36	-0.015	-0.010	-0.007	0.009	0.010	0.013
13	-0.028	-0.015	-0.009	0.014	0.016	0.021	37	-0.015	-0.010	-0.007	0.008	0.010	0.013
14	-0.024	-0.013	-0.009	0.014	0.016	0.020	38	-0.015	-0.010	-0.007	0.009	0.010	0.013
15	-0.024	-0.015	-0.009	0.013	0.015	0.019	39	-0.016	-0.010	-0.007	0.008	0.010	0.013
16	-0.024	-0.014	-0.010	0.013	0.015	0.019	40	-0.016	-0.009	-0.007	0.008	0.010	0.013
17	-0.026	-0.013	-0.009	0.012	0.015	0.018	41	-0.015	-0.010	-0.007	0.008	0.010	0.013
18	-0.022	-0.013	-0.008	0.012	0.014	0.017	42	-0.015	-0.010	-0.006	0.008	0.010	0.012
19	-0.021	-0.013	-0.009	0.012	0.014	0.017	43	-0.015	-0.010	-0.007	0.008	0.010	0.012
20	-0.021	-0.013	-0.008	0.012	0.013	0.017	44	-0.016	-0.009	-0.007	0.008	0.009	0.012
21	-0.020	-0.012	-0.008	0.011	0.013	0.017	45	-0.014	-0.009	-0.007	0.008	0.009	0.012
22	-0.019	-0.012	-0.009	0.011	0.013	0.017	46	-0.016	-0.009	-0.006	0.008	0.009	0.012
23	-0.021	-0.012	-0.008	0.011	0.013	0.016	47	-0.014	-0.008	-0.006	0.008	0.009	0.012
24	-0.019	-0.012	-0.009	0.010	0.013	0.016	48	-0.016	-0.009	-0.006	0.008	0.009	0.013
25	-0.018	-0.012	-0.008	0.010	0.012	0.015	49	-0.014	-0.009	-0.006	0.007	0.009	0.011
26	-0.018	-0.012	-0.008	0.010	0.012	0.015	50	-0.014	-0.009	-0.007	0.007	0.009	0.012

4.1 The Power Estimates

The powers estimate of the test statistic $\hat{\hat{\delta}}_{1n}(s,\beta)$ are shown in Tables 3 and 4 at the significant levels $\alpha = 0.05$ and $\alpha = 0.01$ respectively. These powers estimated for LFR, Makeham and Weibull distributions based on 10000 simulated samples for sizes $n = 10, 20$ and 30.

Table 3: Powers estimates at $\alpha = 0.05$

	n	$\theta = 2$	$\theta = 3$	$\theta = 4$
LFR	10	0.255	0.510	0.591
	20	0.412	0.754	0.824
	30	0.586	0.902	0.933
Makeham	10	0.910	0.900	0.915
	20	0.992	0.992	0.989
	30	1.000	1.000	1.000
Weibull	10	0.755	0.993	1.000
	20	0.980	1.000	1.000
	30	0.999	1.000	1.000

Table 4: Powers estimates at $\alpha = 0.01$

	n	$\theta = 2$	$\theta = 3$	$\theta = 4$
LFR	10	0.180	0.273	0.339
	20	0.410	0.567	0.663
	30	0.613	0.766	0.845
Makeham	10	0.808	0.820	0.826
	20	0.981	0.977	0.970
	30	0.997	1.000	0.997
Weibull	10	0.352	0.915	0.993
	20	0.917	1.000	1.000
	30	0.991	1.000	1.000

It is clear from Tables 3 and 4 that our test has good powers for Makeham and weibull distributions and acceptance powers for LFR distributions. The powers estimate increase as the the sample size increases. The powers are getting as greater as the class departs the exponential distribution.

5 Numerical Examples

Example 5.1*Consider the data given in Attia et al. [11], these data represent 39 liver cancers patients taken from Elminia cancer center Ministry of Health - Egypt.*
It is easily to show that $\hat{\hat{\delta}}_{1n}(s,\beta) = 0.0167$ *and this value exceeds the tabulated critical value in Table 2. It is evident that at the significant level 0.05 this data set has* EBU_{mgf} *property.*

Example 5.2*Consider the data, which represent the remission times for the placebo of 21 patients (Lawless [29], p.5):*
It was found that $\hat{\hat{\delta}}_{1n}(s,\beta) = 0.0131$ *which is greater than the tabulated critical value in Table 2. Then we reject* H_0 *which states that the data set has exponential property.*

6 Discussion

Testing exponentiality is becoming increasingly popular in lifetime analysis and reliability studies, in this paper we introduced a new test for testing exponentiality versus ”exponential Better than Used in moment generating function ordering class” based on Laplace transform. The Pitman asymptotic efficiency of this test is calculated for some alternative distributions and compared with other tests for exponentiality. The critical values and the powers of the proposed test are calculated. Finally, the proposed test is applied to some real data.

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