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Some Fixed Point Theorems for Symmetric Hausdorff Function on Hausdorff Spaces

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Abstract: The main objective of this paper is, to prove some fixed point theorems in Hausdorff spaces by using the symmetric Hausdorff function. Here, we are introducing a more generalized contractive condition for the existence and uniqueness of fixed point. Also, we are giving solution of some integral type mathematical models in Hausdorff spaces.

Keywords: Fixed point, Hausdorff space, symmetric Hausdorff function, integral type, contractive condition.

1 Introduction

Topology is an important area of mathematics with many applications in the domains of computer science and information sciences. The Hausdorff distance is commonly used as a similarity measure between two point sets. Authors in ([1], [11], [22]) utilized the concept of similarity measure between two point sets to discuss the shape matching in two and three dimensions. Nicolas Aspert et al [2] gave an idea for measuring errors between the surfaces by using Hausdorff distance. Kwan-Ho Lin et al [14] introduced a new spatially weighted Hausdorff distance measure for human face recognition.

Later, Facundo Mmoli [16] proposed and discussed certain modifications of the ideas concerning Gromov-Hausdorff distances in order to tackle the problems of shape matching and comparison. Riyaz Sikora et al [24] presented the design of more effective and efficient genetic algorithm based data mining techniques by using the concepts of Hausdorff distance measure.

After the introduction of Banach contraction principle in 1922, several authors extended, improved and generalized the result of Banach [4] in different ways. During the last few decades, many researchers from the field of applied sciences and engineering continued their studies on fixed point theory and many generalizations are emerging from it. Banach contraction principle [4] is also very useful for proving the existence and uniqueness of the solution to a variety of mathematical models.

Any metric space is Hausdorff metric space or easily Hausdorff Spaces in the induced topology. In early 80's, V. Popa [17] generalized the result of Banach from metric spaces to Hausdorff spaces and proved some unique fixed point theorems in Hausdorff spaces.

Theorem 1. *[17] Let $T : X \to X$ be a continuous mapping of a Hausdorff space X and let $f : X \times X \to [0, +\infty)$ be a continuous mapping such that, for each $x, y \in X$ and for each $x \neq y$,*

1. $f(x, y) \neq 0$
2. $f(Tx, Ty) \leq \alpha \left(\frac{f(x,Tx).f(y,Ty)}{f(x,y)} \right) + \beta (f(x, y))$
3. $f^2(x, y) \geq f(x, x).f(y, y)$

where, $\alpha, \beta > 0$ such that $\alpha + \beta < 1$. Also, if there exist some $x_0 \in X$ such that the sequence $x_n = T^n x_0$ has a convergent subsequence. Then T has a unique fixed point.

In 1984, Khan et al [13] established a new category of fixed point for single map by defining control function, which they called an altering distance function.

Definition 1. [13] *The function $\Phi : [0, +\infty) \to [0, +\infty)$ is called an altering distance function, if the following properties are satisfied:*

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1. *Φ is continuous and non- decreasing*
2. *$\Phi(t) = 0$ implies $t = 0$.*

In (2002), Branciari [5] gave an analogue of Banach contraction principle by defining Lebesgue- integrable function and proved a fixed point theorem satisfying contractive condition of integral type.

Definition 2.[5] *A function $\chi(t)$ is called a Lebesgue-integrable function if*

1. *$\chi : [0, +\infty) \to [0, +\infty)$ is Lebesgue summable for each compact of R_+*
2. *Its permitivity $A : [0, +\infty) \to [0, +\infty)$, as $A(t) = \int_0^t \chi(t)dt$, for all $t > 0$, is well defined, non decreasing and continuous*
3. *Moreover, if for each $\epsilon > 0$, $A(\epsilon) > 0$, this permitivity fulfill $A(t) = 0$ iff $t = 0$.*

After this result, many applications are being introduced by different researchers for an integral type inequalities. Some of them are listed in ([19],[20],[15],[23],[9]). Bessem Samet and Habib Yazidi generalized the results of Dass and Gupta [7] to an integral type inequality as stated below.

Theorem 2. *Let (X, d) be a complete metric space and f be a self-map of X such that for each $x, y \in X$,*

$$\int_0^{d(fx,fy)} \chi(t)dt \leq \alpha \int_0^{m(x,y)} \chi(t)dt + \beta \int_0^{d(x,y)} \chi(t)dt$$

and

$$m(x, y) = \frac{d(y, fy)[1 + d(x, fx)]}{[1 + d(x, y)]},$$

where $\alpha, \beta > 0$ are constants such that $\alpha + \beta < 1$ and $\chi(t)$ is a Lebesgue-integrable function, then f admits a unique fixed point .

Recently, Vishal Gupta et al [8] generalized the result of Rhoades [18] and proved a fixed point theorem satisfying a generalized weak contractive condition of integral type. Some results related to Hausdorff spaces are referred in ([3], [10], [6], [25]).

In 2011, Bessem Samet and Habib Yazidi [21] extend their own theorem (Theorem 2) and proved the following result satisfying a contractive condition of integral type in Hausdorff spaces.

Theorem 3. *[21] Let X be a Hausdorff space and $T : X \times X \to [0, +\infty)$ be a continuous mapping such that:*

$$T(x, y) \neq 0 \quad \forall \quad x, y \in X \quad and \quad x \neq y.$$

Let f be a continuous self-map of X satisfying the contractive condition such that for each $x, y \in X$,

$$\int_0^{T(fx,fy)} \chi(t)dt \leq \alpha \int_0^{m(x,y)} \chi(t)dt + \beta \int_0^{T(x,y)} \chi(t)dt$$

and

$$m(x, y) = \frac{T(y, fy)[1 + T(x, fx)]}{[1 + T(x, y)]},$$

where $\alpha, \beta > 0$ are constants such that $\alpha + \beta < 1$ and $\chi(t)$ is a Lebesgue-integrable function. If for some $x_0 \in X$, the sequence of iterates $f^n x_0$ has a subsequence $f^{n_k} x_0$ converging to $z \in X$. Then f admits a fixed point z.

2 Main Results

Here, first we will define symmetric Hausdorff function:

Definition 3. Let X be a Hausdorff space and $H : X \times X \to [0, +\infty)$ be a continuous mapping such that for all $x, y \in X$,

$$H(x, y) = 0 \quad iff \quad x = y,$$

then H is called symmetric Hausdorff function.

Now we are proving some unique fixed point theorems by using symmetric Hausdorff function:

Theorem 4. *Let $F : X \to X$ be a continuous mapping on Hausdorff spaces X and let H is a symmetric Hausdorff function satisfies:*

$$H(Fx, Fy) \leq \alpha (M(x, y)) + \beta (H(x, y)) \tag{1}$$

for all $x, y \in X, \alpha, \beta > 0$ and $\alpha + \beta < 1$, where

$$M(x, y) = max\Big\{H(x, y), H(x, Fx), H(y, Fy), \frac{H(x, Fx).H(y, Fy)}{H(x, y)}\Big\}. \tag{2}$$

Now, if there exist some $x_0 \in X$ such that the sequence $x_n = F^n x_0$ has a convergent subsequence. Then F has a unique fixed point.

Proof.- Let us choose $x_0 \in X$ such that $Fx_0 = x_1$. Now define a sequence $\{x_n\}$ in X such that $Fx_n = x_{n+1}$.
Step 1: Claim that $\{x_n\}$ has convergent subsequence and converges to some real numbers.
Now for each $n \geq 0$, Consider

$$\begin{aligned} H(x_n, x_{n+1}) &= H(Fx_{n-1}, Fx_n) \\ &\leq \alpha (M(x_{n-1}, x_n)) + \beta (H(x_{n-1}, x_n)), \end{aligned} \tag{3}$$

where, from equation (2)

$$\begin{aligned} M(x_{n-1}, x_n) &= max\Big\{H(x_{n-1}, x_n), H(x_{n-1}, Fx_{n-1}), \\ &H(x_n, Fx_n), \frac{H(x_{n-1}, Fx_{n-1}), H(x_n, Fx_n)}{H(x_{n-1}, x_n)}\Big\} \\ &= max\{H(x_n, x_{n+1}), H(x_{n-1}, x_n)\}. \end{aligned} \tag{4}$$

Now, if $H(x_n, x_{n+1}) > H(x_{n-1}, x_n)$, then by using (3)

$$\begin{aligned} &H(x_n, x_{n+1}) \\ &\quad = H(Fx_{n-1}, Fx_n) \\ &\quad \leq \alpha(H(x_n, x_{n+1})) + \beta(H(x_{n-1}, x_n)) \\ &\quad \leq \frac{\beta}{1-\alpha}(H(x_{n-1}, x_n)) \\ &\quad < H(x_{n-1}, x_n). \end{aligned} \tag{5}$$

Also, if $H(x_n, x_{n+1}) \leq H(x_{n-1}, x_n)$, then from (3)

$$\begin{aligned} &H(x_n, x_{n+1}) \\ &\quad \leq (\alpha+\beta) H(x_{n-1}, x_n) < H(x_{n-1}, x_n) \end{aligned} \tag{6}$$

Repeating the above process n times , we get

$$H(x_n, x_{n+1}) < \ldots\ldots\ldots < H(x_0, x_1). \tag{7}$$

Thus we get a monotone sequence of non-negative real numbers which must converge with all its subsequence to some real number say z.

Step 2: Claim that $z \in X$ is a fixed point of F.
Suppose, $Fz \neq z$.
Consider

$$\begin{aligned} H(z, Fz) &= H\left(Fz, F^2 z\right) \\ &\leq \alpha(M(z, Fz)) + \beta(H(z, Fz)), \end{aligned} \tag{8}$$

Now

$$\begin{aligned} &M(z, Fz) \\ &\quad = max\Big\{H(z, Fz), H(z, Fz), H\left(Fz, F^2 z\right), \\ &\qquad \frac{H(z, Fz).H\left(Fz, F^2 z\right)}{H(z, Fz)}\Big\} \\ &\quad = max\left\{H(z, Fz), H\left(Fz, F^2 z\right)\right\} \\ &\quad < max\left\{H(z, Fz), H(z, Fz)\right\}, \end{aligned}$$

therefore form (8)

$$\begin{aligned} H(z, Fz) &\leq \alpha(H(z, Fz)) + \beta(H(z, Fz)) \\ &\leq (\alpha+\beta) H(z, Fz). \end{aligned}$$

This is a contradiction. Thus $Fz = z$. That is, $z \in X$ is a fixed point of F.

Step 3: Claim that z is a unique fixed point of F.
Suppose not, therefore there exists $w \in X$ such that $Fw = w$.
Now, again from (1)

$$\begin{aligned} H(z, w) &= H(Fz, Fw) \\ &\leq \alpha(M(z, w)) + \beta(H(z, w)), \end{aligned} \tag{9}$$

where

$$\begin{aligned} &M(z, w) \\ &\quad = max\Big\{H(z, w), H(z, Fz), H(w, Fw), \\ &\qquad \frac{H(z, Fz), H(w, Fw)}{H(z, w)}\Big\} \\ &\quad = max\left\{H(z, w), 0, 0, 0\right\} = H(z, w). \end{aligned} \tag{10}$$

From (9) and (10)

$$H(z, w) = \alpha(H(z, w)) + \beta(H(z, w)) < H(z, w),$$

which is a contradiction. Thus z is a unique fixed point of F. This completes the proof of Theorem 4.

Theorem 5. *Let F, G are continuous self mappings on Hausdorff spaces X and let H is a symmetric Hausdorff function satisfies,*

$$H(Fx, Gy) \leq \alpha(M(x, y)) + \beta(H(x, y)) \tag{11}$$

where, $\alpha, \beta > 0$ such that $\alpha + \beta < 1$.
Also

$$\begin{aligned} M(x, y) = max\Big\{&H(x, y), H(x, Fx), H(y, Gy), \\ &\frac{H(x, Fx).H(y, Gy)}{H(x, y)}\Big\}, \end{aligned} \tag{12}$$

then F and G have unique fixed point.

Proof.- Choose $x_0 \in X$ such that $Fx_0 = x_1$ and $Gx_1 = x_2$. Now construct a sequence $\{x_n\}$ in X such that

$$Fx_{2n} = x_{2n+1} \text{ and } Gx_{2n+1} = x_{2n+2}.$$

Now for each $n \geq 0$, Consider

$$\begin{aligned} &H(x_{2n+1}, x_{2n+2}) \\ &\quad = H(Fx_{2n}, Gx_{2n+1}) \\ &\quad \leq \alpha(M(x_{2n}, x_{2n+1})) + \beta(H(x_{2n}, x_{2n+1})), \end{aligned} \tag{13}$$

where from equation (5)

$$\begin{aligned} &M(x_{2n}, x_{2n+1}) \\ &\quad = max\Big\{H(x_{2n}, x_{2n+1}), H(x_{2n}, Fx_{2n}), \\ &\qquad H(x_{2n+1}, Gx_{2n+1}), \\ &\qquad \frac{H(x_{2n}, Fx_{2n}).H(x_{2n+1}, Gx_{2n+1})}{H(x_{2n}, x_{2n+1})}\Big\} \\ &\quad = max\left\{H(x_{2n}, x_{2n+1}), H(x_{2n+1}, x_{2n+2})\right\}. \end{aligned} \tag{14}$$

Now, if $H(x_{2n+1}, x_{2n+2}) > H(x_{2n}, x_{2n+1})$ then by using (13),

$$\begin{aligned} &H(x_{2n+1}, x_{2n+2}) \\ &\quad \leq \alpha(H(x_{2n+1}, x_{2n+2})) + \beta(H(x_{2n+1}, x_{2n+2})) \\ &\quad \leq \frac{\beta}{1-\alpha}(H(x_{2n}, x_{2n+1})) \\ &\quad < H(x_{2n}, x_{2n+1}). \end{aligned} \tag{15}$$

Also, if $H(x_{2n+1}, x_{2n+2}) \leq H(x_{2n}, x_{2n+1})$ then again from (13),

$$\begin{aligned} H(x_{2n+1}, x_{2n+2}) &\leq (\alpha+\beta) H(x_{2n}, x_{2n+1}) \\ &< H(x_{2n}, x_{2n+1}). \end{aligned} \tag{16}$$

Repeating the above process n times , we get

$$H(x_{2n+1}, x_{2n+2}) < \ldots\ldots\ldots\ldots < H(x_0, x_1).$$

Thus we get a monotone sequence $\{x_n\}$ of non-negative real numbers which must converge with all its subsequence to some real no z.

Now, we show that z is a fixed point of F and G. First we show that z is fixed point of F. Suppose, $Fz \neq z$.
Consider $\{x_n\}$ has a subsequence $\{x_{2n_k}\}$ converge to some real number z.
Therefore,

$$\begin{aligned} H(z, Fz) &= H\left[\lim_{k\to\infty}\{x_{2n_k}\}, F\left(\lim_{k\to\infty}\{x_{2n_k}\}\right)\right] \\ &= H\left[\lim_{k\to\infty}\{x_{2n_k}\}, \lim_{k\to\infty}\{x_{2n_k+1}\}\right] \\ &= \lim_{k\to\infty} H[\{x_{2n_k+1}\}, \{x_{2n_k+2}\}] \\ &= H\left[\lim_{k\to\infty}\{x_{2n_k+1}\}, \lim_{k\to\infty}\{x_{2n_k+2}\}\right] \\ &= H\left[F\lim_{k\to\infty}\{x_{2n_k}\}, GF\left(\lim_{k\to\infty}\{x_{2n_k}\}\right)\right] \\ &= H(Fz, GFz). \end{aligned} \tag{17}$$

Now, if $z \neq Fz$ then from (11),

$$H(Fz, GFz) \leq \alpha(M(z, Fz)) + \beta(H(z, Fz)). \tag{18}$$

Now, from (5)

$$\begin{aligned} M(z, Fz) &= max\{H(z, Fz), H(Fz, GFz)\} \\ &< max\{H(z, Fz), H(z, Fz)\} \\ &< H(z, Fz). \end{aligned} \tag{19}$$

Hence from (17), (18) and (19)

$$\begin{aligned} H(z, Fz) &< \alpha(H(z, Fz)) + \beta(H(z, Fz)) \\ &= (\alpha+\beta)H(z, Fz) < H(z, Fz) \\ &< H(z, Fz), \end{aligned} \tag{20}$$

which is a contradiction. Thus z is a fixed point of F. Analogously, we can show that z is fixed point of G.

Uniqueness -: Claim that z is a unique fixed point of F and G.
For this, suppose that there exists another fixed point say w such that $Fw = w$ and $Gw = w$.
Consider

$$\begin{aligned} H(z, w) &= H(Fz, Gw) \\ &\leq \alpha(M(z, w)) + \beta(H(z, w)), \end{aligned} \tag{21}$$

where

$$\begin{aligned} M(z, w) &= max\Big\{H(z, w), H(z, Fz), H(w, Gw), \\ &\qquad \frac{H(z, Fz).H(w, Gw)}{H(z, w)}\Big\} \\ &= H(z, w). \end{aligned} \tag{22}$$

Thus, from (21) and (22)

$$\begin{aligned} H(z, w) &\leq \alpha(H(z, w)) + \beta(H(z, w)) \\ &\leq (\alpha+\beta)H(z, w) \\ &< H(z, w), \end{aligned} \tag{23}$$

which is a contradiction. Thus z is a unique fixed point of F and G. This completes the proof.

Theorem 6. *Let $F : X \to X$ be a continuous mapping on Hausdorff spaces X and let H is a symmetric Hausdorff function satisfies,*

$$H(Fx, Fy) \leq \Phi(M(x, y)), \tag{24}$$

where Φ is an altering distance function and

$$\begin{aligned} M(x, y) = max\Big\{&H(x, y), H(x, Fx), H(y, Fy), \\ &\frac{H(x, Fx).H(y, Fy)}{H(x, y)}\Big\}. \end{aligned} \tag{25}$$

If there exist some $x_0 \in X$ such that the sequence $x_n = F^n x_0$ has a convergent subsequence. Then F has a unique fixed point.

Proof.- For each $n \geq 0$, let us take

$$\begin{aligned} H(x_n, x_{n+1}) &= H(Fx_{n-1}, Fx_n) \\ &\leq \Phi(M(x_{n-1}, x_n)) \end{aligned} \tag{26}$$

using equation (25), We have

$$\begin{aligned} &M(x_{n-1}, x_n) \\ &\quad = max\{H(x_n, x_{n+1}), H(x_{n-1}, x_n)\}. \end{aligned} \tag{27}$$

Now, if $H(x_n, x_{n+1}) > H(x_{n-1}, x_n)$ then by using (26),

$$H(x_n, x_{n+1}) \leq \Phi(H(x_n, x_{n+1})) < H(x_n, x_{n+1}) \tag{28}$$

which is a contradiction. Thus $H(x_n, x_{n+1}) \leq H(x_{n-1}, x_n)$. Therefore from (26), (27) and (28)

$$H(x_n, x_{n+1}) \leq \Phi(H(x_{n-1}, x_n)) < H(x_{n-1}, x_n). \tag{29}$$

Repeating the above process n times , we get

$$H(x_n, x_{n+1}) < \ldots\ldots\ldots\ldots < H(x_0, x_1).$$

Thus, we get a monotone sequence of non-negative real numbers which must be convergent with all its subsequence to some real number z
Next we Claim that $z \in X$ is a fixed point of F.

Suppose $Fz \neq z$.
Consider

$$H(z,Fz) = H\left(Fz,F^2z\right) \leq \Phi((M(z,Fz))). \tag{30}$$

Now

$$\begin{aligned} &M(z,Fz) \\ &= max\left\{H(z,Fz), H(z,Fz), H\left(Fz,F^2z\right), \frac{H(z,Fz).H\left(Fz,F^2z\right)}{H(z,Fz)}\right\} \\ &= max\left\{H(z,Fz), H\left(Fz,F^2z\right)\right\} \\ &< max\left\{H(z,Fz), H(z,Fz)\right\}. \end{aligned}$$

Hence from (30)

$$H(z,Fz) \leq \Phi((H(z,Fz))) < H(z,Fz),$$

this is a contradiction. Thus $Fz = z$. That is, $z \in X$ is a fixed point of F.

Uniqueness -: Claim that z is a unique fixed point of F. Suppose not, therefore there exists $w \in X$ such that $Fw = w$.
Again from (24),

$$H(z,w) = H(Fz,Fw) \leq \Phi((M(z,w))), \tag{31}$$

where

$$\begin{aligned} &M(z,w) \\ &= max\left\{H(z,w), H(z,Fz), H(w,Fw), \frac{H(z,Fz), H(w,Fw)}{H(z,w)}\right\} \\ &= max\left\{H(z,w), 0, 0, 0\right\} \\ &= H(z,w). \end{aligned} \tag{32}$$

From (31) and (32)

$$H(z,w) = \Phi(H(z,w)) < H(z,w).$$

Thus z is a unique fixed point of F. This completes the proof of theorem 6.

Corollary 1. *Let $F,G : X \rightarrow X$ are continuous mappings on Hausdorff spaces X and let H is a symmetric Hausdorff function satisfies,*

$$H(Fx,Gy) \leq \Phi(M(x,y)),$$

where Φ is an altering distance function and

$$M(x,y) = max\left\{H(x,y), H(x,Fx), H(y,Gy), \frac{H(x,Fx).H(y,Gy)}{H(x,y)}\right\}.$$

If there exist some $x_0 \in X$ such that the sequence $x_n = F^n x_0$ has a convergent subsequence. Then F and G have unique fixed point.

Proof.- Proof of result evidentially follows from theorem 5.

Theorem 7. *Let $F : X \rightarrow X$ be a continuous mapping on Hausdorff spaces X and let H is a symmetric Hausdorff function satisfies:*

$$\int_0^{H(Fx,Fy)} \chi(t)dt \leq \alpha \int_0^{M(x,y)} \chi(t)dt + \beta \int_0^{H(x,y)} \chi(t)dt,$$

where $\chi(t)$ is a Lebesgue- integrable function, $\alpha, \beta > 0$ and $\alpha + \beta < 1$. Also

$$M(x,y) = max\left\{H(x,y), H(x,Fx), H(y,Fy), \frac{H(x,Fx).H(y,Fy)}{H(x,y)}\right\}.$$

If there exist some $x_0 \in X$ such that the sequence $x_n = F^n x_0$ has a convergent subsequence. Then F has a unique fixed point.

Proof.- By assuming $\chi(t) = 1$ and using theorem 4, we obtained the desired result.

Theorem 8. *Let $F,G : X \rightarrow X$ are continuous mappings on Hausdorff spaces X and let H is a symmetric Hausdorff function satisfies:*

$$\int_0^{H(Fx,Gy)} \chi(t)dt \leq \alpha \int_0^{M(x,y)} \chi(t)dt + \beta \int_0^{H(x,y)} \chi(t)dt,$$

where $\chi(t)$ is a Lebesgue- integrable function, $\alpha, \beta > 0$ and $\alpha + \beta < 1$.
Also

$$M(x,y) = max\left\{H(x,y), H(x,Fx), H(y,Gy), \frac{H(x,Fx).H(y,Gy)}{H(x,y)}\right\},$$

then F and G have unique fixed point.

Proof.- Take $\chi(t) = 1$ and using theorem 5, result follows directly.

Theorem 9. *Let $F : X \rightarrow X$ be a continuous mapping on Hausdorff spaces X and let H is a symmetric Hausdorff function satisfies:*

$$\int_0^{H(Fx,Fy)} \chi(t)dt \leq \Phi\left(\int_0^{M(x,y)} \chi(t)dt\right),$$

where Φ is an altering distance function, $\chi(t)$ is a Lebesgue- integrable function and

$$M(x,y) = max\left\{H(x,y), H(x,Fx), H(y,Fy), \frac{H(x,Fx).H(y,Fy)}{H(x,y)}\right\}.$$

If there exist some $x_0 \in X$ such that the sequence $x_n = F^n x_0$ has a convergent subsequence. Then F has a unique fixed point.

Proof.- By assuming $\chi(t) = 1$ and using theorem 6, we obtained the result immediately.

3 Conclusion

Usually, the distance between two point sets A and B is defined as $D(A,B) = min_{a\in A}\{min_{b\in B}\{d(a,b)\}\}$. But this definition of distance can become quite unsatisfactory for many problems of sciences and engineering such as shape matching, Image comparison etc. To overcome these difficulties Hausdorff distance is introduced. It is basically the maximum distance of a set to the nearest point in the other set. It provides a means of determining the resemblance of one point set to other by examining the friction of points in one set that lie near to the points in the other set. It has been widely used by various authors in different fields such as shape matching [1], Image comparison [16], Human face recognition [14], data mining [24] and many more.

In this study, a restricted Hausdorff distance function is introduced and some fixed point results, by using iterative and convergence criteria, have been established. The implementation and comparison of this restricted Hausdorff distance function with other existing distances is an open area under discussion for the future research.

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