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Equity Volatility Smile and Skew under a CEV-based Structural Leverage Model

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Abstract: The leverage effect is widely considered to be the major contributor to volatility skew in the stock option market. This study concentrates on explicit modeling of corporate leverage. The asset-equity relation is modeled by a perpetual American option, and the asymmetry of the asset return distribution is addressed by introducing a Constant Elasticity of Variance (CEV) asset dynamic. The model retains closed-form representation of firm equity with respect to asset and liability. This provides a convenient simplification to equity option pricing, so that the model can be calibrated to stock prices and the entire volatility structure. This model demonstrates how the volatility smile, as well as skew, can be accommodated by the structural model, and also successfully explains why low-leverage stocks could still have a non-trivial volatility structure. A cross-sectional study shows that the calibrated parameters effectively outline the financial characteristics of the leveraged firms. The credit quality measure generated from this model is also more informative in terms of explaining Credit Default Swap (CDS) spread movements.

Keywords: structural model, leverage, CEV, volatility skew and smile, calibration, free-boundary problem

1 Introduction

Since the introduction of the Black-Scholes option pricing model, the volatility smile and skew have quickly become a common pattern in the stock option market. One pervasive explanation to this phenomenon is the leverage effect, which concludes that a falling equity value is accompanied by an increasing equity volatility due to increased leverage. For this reason, the return distribution of equity exhibits asymmetry and fat-tail properties.

Various extensions to the Black-Scholes' Geometric Brownian Motion (GBM) equity dynamic have been proposed to match the volatility skew and smile. Some of these models are the Constant Elasticity of Variance (CEV) model (e.g. [1]), the Local Volatility model (e.g. [5]), the Stochastic Volatility model (e.g. [7]), the Jump Diffusion model (e.g. [14]) and Pure Jump Levy Process models (e.g. CGMY model in [3]). While many of these models are empirically successful, they mostly concentrate on building a delicate stochastic process for the equity value so that the volatility structure can be replicated and calibrated with a few parameters. The connection between the volatility structure and corporate leverage remains an area relatively less explored.

Since it is widely agreed that leverage information is coded into the implied volatility structure, an alternative model explicitly incorporating leverage has the potential to extract leverage information from the equity volatility structure, therefore further assisting fundamental analysis.

The quantitative modeling of corporate leverage is not an unexplored area at all. As early as 1974, Merton [13] modeled the corporate equity as a vanilla call option of the asset, with the strike equal to firm liability, assuming the liability is as simple as a single zero-coupon bond. Though the liability assumption and default dynamic are not perfectly realistic, Merton's revolutionary work reminds the world of an oft-ignored fact: the equity of a leveraged firm, which has long been considered a primitive security, is instead a derivative. Its value is driven by the asset dynamic and the leverage of the issuing corporation, and stock options are, as a result, compound options.

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There are various extensions to Merton's 1974 model. Leland's 1994 work [12], for example, models the firm's equity in a perpetual time frame, under the assumption of a constant and coupon-only liability structure. However, few of these models provided a practical strategy for obtaining crucial parameters to achieve agreement with market prices.

In the rest of this paper, the three most relevant studies linking a contingent claim leverage model to equity volatility skew are briefly reviewed. After that, a new CEV-based model allowing for an asymmetric asset dynamic is proposed. The model retains an analytically tractable format between the asset and equity relation, so that calibrating the leverage model to the market becomes practical. Empirical study shows the model's capability to produce not only the volatility skew but also the smile, and at the same time explains why the volatility structure could still be non-flat even in the absence of significant leverage. The calibrated parameters characterize crucial financial properties of the leveraged firms, and the credit quality measure shows better consistency to Credit Default Swap (CDS) spread movements.

2 Volatility Skew, Smile and Leverage

Equity holders leverage their business by borrowing and therefore putting themselves under liabilities. However, the limited liability nature of modern firms gives equity holders the right to declare default at any time, insulating them from any further claims but also taking away any existing value.

Equity holders tend to make optimal default decisions, and thus embed "optionality" into equity values. Equity value depends nonlinearly on the asset and the liability of the issuing firm. Stock options are therefore compound options, whose original source of randomness is the asset dynamic and the leverage.

Even if the asset return distribution is usually assumed to be symmetric as an expedience, the embedded optionality of equity introduces nonlinearity between asset return and equity return, which breaks the symmetry of the equity return distribution. This asymmetry property is the key to reproducing the implied volatility skew/smile observed in the stock option market.

Before formulating the previous logic into rigorous mathematics, three existing studies closely related to this work are reviewed.

The earliest study linking the contingent claim leverage model to volatility skew is by Toft and Prucyk in 1997 [15]. Based on the Leland 1994 model, Toft and Prucyk theoretically demonstrated that "the volatility skew is negatively related to leverage", and verified the conclusion by statistical test on empirical data. The symmetric asset return assumption remained throughout their study.

Similar to Toft and Prucy, Hull et al. [9] made use of Merton's 1974 model, and theoretically derived the impact of leverage on volatility skew. From there they proposed a strategy of using two different implied volatilities to infer leverage and asset volatility. Instead of calibrating to the entire volatility skew, Hull et al. used a linear approximation to the skew based on the selection of a pair of strikes. The GBM asset dynamic assumption and Merton's single zero-coupon bond assumption remained in this research. The liability structure is mapped onto a single zero-coupon bond with a certain maturity, but how to reasonably specify this maturity remains an open question.

Chen and Kou [4] took the first step away from the GBM asset return assumption in the structural modeling of volatility skew. Chen and Kou extended Leland's model by introducing two-sided jumps into the GBM asset dynamic. By tuning parameters, Chen and Kou were able to produce various shapes of volatility skews. Similar to Toft and Prucyk, no strategy for selecting the model parameters to calibrate volatility skew as observed in the market is outlined.

In this research, the asset dynamic is modeled by a CEV process, which is asymmetric and at the same time retains a closed-form solution between the equity and asset dynamics. The entire volatility structure, together with current equity value, is used to imply the firm asset dynamic as well as the leverage. That is to say, the model, with proper parameters chosen, should reproduce the current market price of equity as well as volatility skew or smile. Liability is modeled as a perpetual constant, which is more realistic in that no maturity of liability needs to be specified.

This model takes into account the entire volatility structure, so that no manual selection of the strikes is necessary. The calibrated parameters, related to asset volatility, leverage and return asymmetry, reveal critical properties of the firm's fundamentals. The model also resolves a paradox many structural models are suffering from. When the leverage is insignificant, any GBM-based leverage model will produce a very flat volatility structure, which is inconsistent with the market. The CEV-based structural model could still produce volatility skews even in the absence of leverage simply because of the asymmetric asset return.

Another major contribution of this model is that it can accommodate not only the volatility skew but also the volatility smile. Volatility smile is observed in the stock option market with non-trivial probability. If the asset dynamic is modeled as a GBM, the leveraged equity

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model either produces a flat volatility structure when there is no substantial leverage, or a volatility skew when the leverage is significant. Introducing a non-It$\bar{o}$ asset dynamic will help to produce volatility smiles but make it extremely challenging to calibrate to the market. The CEV asset dynamic provides a balance between the diversity of volatility structure it could accommodate and mathematical tractability, which is highly desirable when calibrating to the market.

3 The CEV Leverage Model

The asset of a limited liability company (LLC) is the value of the tangible and intangible resources that could generate positive cash flows. The asset value under consideration here is the market value instead of the book value. The market value is modeled as a CEV process, which under equivalent martingale measure ($\mathbb{Q}$-measure) follows the SDE:

$$dV = rV dt + \phi V^{\alpha} d\tilde{W} \tag{1}$$

where V is the asset, r is the risk-free return assumed to be constant, ϕ is the CEV diffusion factor (not equivalent to GBM volatility), α is the elasticity and $\tilde{W}$ is the Brownian motion under $\mathbb{Q}$-measure.

Theoretically, α can take any positive value. When $\alpha = 1$, the CEV model degenerates to GBM, so the CEV model is a more generalized version of GBM. It is worth noting that the return distribution skews to the left when $0 < \alpha < 1$ and to the right when $\alpha > 1$. For more details about CEV model see [8].

The reason to choose the CEV model instead of GBM as asset dynamic is to introduce asymmetric asset return distribution. One major limitation of GBM asset dynamic in leverage firm modeling is that when leverage is low, the model naturally leads to a flat volatility structure. This is not desirable when considering stocks like Apple Inc. These firms have no essential borrowings compared to their cash holdings, but the stock option market may still display a volatility skew. An example of calibrating to Apple Inc.'s volatility skew under both GBM and CEV asset dynamic is given in the empirical study to justify the benefit of the CEV asset dynamic.

The other major drawback of GBM asset dynamic is that it cannot produce volatility smile. Even though the implied volatility skew dominates the stock option market, implied volatility smile can be observed with a non-trivial probability. An illustration of calibrating to volatility smile is provided in the empirical study, where one could see the calibration and implication of a smile and thus the value of having the flexibility to accommodate both the skew and smile within one single model.

Compared to stochastic volatility and jump diffusion models, the CEV model does not introduce an additional source of uncertainty and therefore keeps the market completeness. Compared to the pure jump Levy processes, the CEV model maintains a mathematically tractable form and connects smoothly to prevailing GBM models. This balance is the key to simplify the pricing of vanilla options so that they can be treated as simpler barrier options, rather than a compound option. The compound option is especially difficult to manage because the base layer, i.e. the equity-asset relation, is a perpetual option which cannot be easily approximated by numerical algorithms. To avoid constructing lattice on perpetual time horizon, a close-form solution of equity-asset relation is highly valuable.

The liability modeling is the most important and challenging part in structural modeling. Liability is the major factor to trigger default, thus it plays a key role in the equity valuation. Liability in practice could be very complicated, and therefore any mathematically tractable representation could not accurately replicate the real liability. Complicated modeling of liability pushes the model closer to reality but leaves a handful of parameters that cannot be determined with confidence.

Hull et al. 2004 work [9] provides a brand new perspective to liability modeling. The idea can be summarized as such: modeling the liability by a simple structure, and mapping the market assessment into this simple structure. More precisely, the equity and stock option traders compile detailed information of the liability and cast their opinions into the volatility structure. Hull et al. approximated the volatility skew linearly, and mapped the linearized volatility skew into a zero coupon bond maturing at some given date. This is accomplished by matching the equity dynamic to the linearized volatility skew.

Unlike Hull et al., a constant perpetual liability structure is adopted in this work. The major reason is to avoid the specification of maturity, which is not a natural input when evaluating corporate capital structure. The liability value (D) is considered to be constant in this model because it is a relatively stable component compared to asset and equity. Similar to constant interest rate assumption, this is not to exclude the possibility of changing liabilities, which can be accommodated by updating model parameter. The liability is implied from market data so that the structural model agrees with the stock price and equity volatility.

As mentioned in previous sections, the equity value of a LLC is not simply the net worth of asset value over the liability ($V - D$) but a more complicated derivative depending on asset and liability. This relation must be mathematically derived. Denote the equity value as E which is a function of asset value $E(V)$. To avoid

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unnecessary constraint imposed to the structure of $E(V)$, the equity value is first decomposed into:

$$E(V) = B(V) + K \tag{2}$$

a component $B(V)$ relevant to asset value, and a component K independent of asset value.

By the the no-arbitrage argument, the component $B(V)$ has to satisfy the following differential equation:

$$rV\frac{\partial B}{\partial V} + \frac{1}{2}\phi^2 V^{2\alpha}\frac{\partial^2 B}{\partial V^2} = rB \tag{3}$$

which specifies the martingale property of B. Note that by the constant liability assumption, B is stationary, i.e. $\partial B/\partial t = 0$.

To solve the above differential equation, proper boundary and limit conditions are required. The limit condition with ultimately large asset value is derived from the fact that the leveraged firm's equity value converges to net worth when leverages becomes less and less significant. This is represented by:

$$\lim_{V\to+\infty}(E(V) - (V - D)) = 0 \tag{4}$$

The default boundary, denoted by L , is a crucial component in this model. Different modeling of this boundary leads to significantly different solutions. In this study, we adopt the endogenous default assumption, which means the default boundary is assumed to be selected by the equity holders to maximize their own benefits.

Under endogenous default assumption, the exact location of default boundary is not known in advance but could be located by additional conditions that must be satisfied at that boundary. It is easy to see that at the level of default, the equity holder will have nothing according to law, so

$$E(L) = 0 \tag{5}$$

Given that $E(V)$ could be dynamically replicated by holding $\frac{\partial E}{\partial V}$ (i.e. Delta) shares of V, $E'(L)$ must also fall to zero at the default boundary because otherwise a further drop of asset value must lead to negative equity, which contradicts reality.

$$E'(L^+) = E'(L^-) = 0 \tag{6}$$

The logic above can be summarized into a free boundary problem:

$$\begin{cases} E(V) = B(V) + K \\ rV\frac{\partial B}{\partial V} + \frac{1}{2}\phi^2 V^{2\alpha}\frac{\partial^2 B}{\partial V^2} = rB \\ \lim_{V\to+\infty}(E(V) - (V - D)) = 0 \\ E(L) = 0 \\ E'(L^+) = E'(L^-) = 0 \end{cases} \tag{7}$$

Note that all specific solutions to equation (3) are combinations of two basic solutions denoted by $B_1(V)$ and $B_2(V)$. By nature of financial instruments, $B_1(V) = V$ must be one of the two solutions, which follows naturally after noticing that the simplest derivative is the underlying asset itself. Knowing one of the basic solutions, the second basic solution can be obtained by construction of $B_2(V) = V\centerdot g(V)$, which reduces (3) to

$$\frac{1}{2}\phi^2 V^{1+2\alpha} g'' + (rV^2 + \phi^2 V^{2\alpha})g' = 0$$

and leads to

$$B_2(V) = V\int_V^\infty \frac{1}{u^2} e^{-\frac{r}{\phi^2(1-\alpha)}(u^{2-2\alpha}+C)}du$$

The solution to the problem (7) is $K = -D$ and:

$$E(V) = V - D + DV\int_V^\infty \frac{1}{u^2} e^{-\frac{r}{\phi^2(1-\alpha)}(u^{2-2\alpha}-L^{2-2\alpha})}du \tag{8}$$

where L can be solved by root searching:

$$x - D + Dx\int_x^\infty \frac{1}{u^2} e^{-\frac{r}{\phi^2(1-\alpha)}(u^{2-2\alpha}-x^{2-2\alpha})}du = 0 \tag{9}$$

According to Ekström [6], the last term of equation (8) is the value of a perpetual American put option written on the asset V and with strike equal to liability D. This makes sense to the $E(V)$ equation, in that the equity holder is holding not only the net worth $V - D$ but also an additional protection security which allows them to exchange the asset V for an amount equal to liability D, and therefore cancels out the net worth position. This exchange makes sense only when the asset value is deeply below the liability, and the optimal exercise of such a perpetual American put option is exactly the endogenous default.

Ekström [6] showed that

$$\lim_{\alpha\to 1} L = \frac{2r}{2r+\phi^2}D$$

and

$$\lim_{\alpha\to 1} DV\int_V^\infty \frac{1}{u^2} e^{-\frac{r}{\phi^2(1-\alpha)}(u^{2-2\alpha}-L^{2-2\alpha})}du$$
$$= (D-L)(\frac{V}{L})^{-2r/\phi^2}$$

which are the exercise barrier and price of a perpetual American put option under GBM asset dynamic. This is a highly desirable property because it ensures the $E(V)$ degenerates properly as $\alpha \to 1$. The details of the proof are not reproduced in this paper.

Given that the equity dynamic has been modeled by a stopped process with a closed-form solution, the vanilla option written on the equity can be treated as a down-and-out option (only with a more complicated pay-off), rather

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than being treated as a compound option. This simplifies the call option price to:

$$\begin{aligned} C_K &= \tilde{\mathbb{E}}[D(T)(E(V)-K)^+|\mathbb{F}_t] \\ &= \tilde{\mathbb{E}}[\mathbb{I}_{\tau>T}D(T)(V-(D+K)+ \\ DV &\int_V^\infty \frac{1}{u^2} e^{-\frac{r}{\phi^2(1-\alpha)}(u^{2-2\alpha}-L^{2-2\alpha})} du)^+|\mathbb{F}_t] \end{aligned} \tag{10}$$

where τ is the first hitting time of V through default barrier L. Unfortunately a closed form solution to (10) has not been found. However, thanks to the close-form solution of $E(V)$, (10) can still be solved by Monte-Carlo or by finite difference lattice so that calibration to the market becomes practical.

Given a set of strikes K and a corresponding implied volatility σ_K, the parameter set $\{V,\phi,D,\alpha\}$ can be calibrated so that C_K agrees to σ_K and $E(V)$ agrees to the stock price. Note that not all four parameters are independent, in that once ϕ, D and α are given, the choice of V is unique to ensure the stock price agreement. Therefore the implied volatility structure is calibrated by three parameters instead of four.

4 Empirical Study

In this section, the CEV-based structural model is first tuned to produce different shapes of volatility skews/smiles and then calibrated to a volatility smile observed on the market. Volatility skew calibration to a low leveraged firm (AAPL) is also demonstrated to illustrate the value of the CEV model's flexibility. A cross-sectional study is presented later to compare the fundamental characteristics of S&P-100 and NASDAQ-100 component companies. Finally, a rank-correlation study is provided to demonstrate the advantage of CEV-based model in terms of explaining the CDS movements.

Note that the CEV volatility ϕ is not comparable across candidates when V_0 or α are different. The adjusted volatility, $\phi V_0^{\alpha-1}$, is studied instead. Also, α is not comparable under different values of V_0. To overcome this difficulty, the cross-sectional test is based on a standardized asset value equal to \$ 1 million and adjusted call strikes proportional to the standardized stock price corresponding to the standardized asset value. Interest rate is assumed to be constant and flat at 2% for all the tests. All volatility smiles/skews are sampled at 1.5-year maturity for a balance between time horizon and liquidity. The vanilla call prices are simulated by 1,000-path and 1,000-step Monte Carlo. Paths are generated as a 10 dimension Sobol sequence Brownian-bridged to 1,000 steps. The fitting criteria is the sum of squared relative error of call prices. All of the data (equity, volatility cube, CDS curve and financial statements) is collected from Bloomberg.

4.1 Reproducing and calibrating to volatility smile

One appealing feature of the CEV-based structural model is its capability of producing different shapes of volatility skew or smiles. It is a valuable feature considering that volatility smile is not compatible to models whose underlying asset dynamic is log-normal.

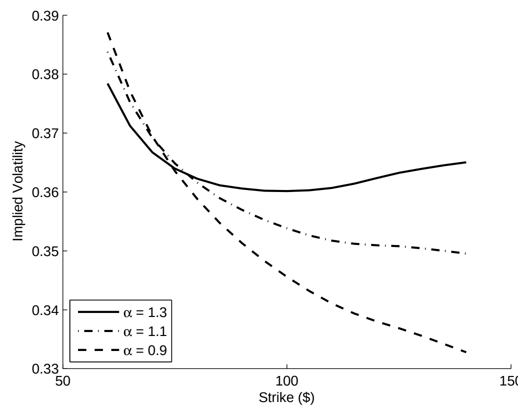

Fig. 1: Volatility skews and smiles under different elasticity when adjusted volatility is 0.3 and leverage is 0.2

Figure (1) illustrates the volatility skews/smiles under different elasticities (0.9, 1.1, and 1.3), whereas the adjusted volatility (0.3) and leverage (0.2) are held constant. When the elasticity is small, e.g. 0.9, the implied volatilities form a skew, which is commonly observed in the market. As the elasticity increases, the high-strike end of the implied volatility structure tilts up gradually. When elasticity reaches 1.3, the implied volatilities produce an obvious smile, where the implied volatilities are higher for both low and high strikes.

Another noticeable feature is that as the elasticity increases, its impact to the low-strike end of the implied volatility curve is very limited. This makes intuitive sense, since the low strike options cover scenarios in which the liability and default are the major concern. High strike options, on the other hand, reflects the growth perspective of the firm, which is mostly driven by the firm asset dynamic. It could be loosely interpreted as that the leverage governs the left end of volatility skew/smile, whereas the elasticity governs the right end. This also further validates why both leverage and asymmetry can be extracted from the implied volatilities at the same time.

Since the 1987 financial crisis, volatility skew has dominated the option market. However volatility smile can be observed with a non-trivial probability. Yahoo

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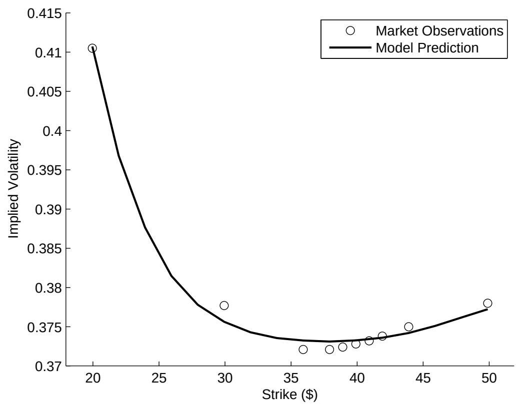

Fig. 2: Calibration to YAHOO Inc. volatility smile observed on Jan.17, 2014

Inc.'s implied volatility on Jan.17, 2014 is a typical volatility smile example. Figure (2) demonstrates how the CEV-based structural model could reproduce this volatility smile with high accuracy. The calibrated model comes with an adjusted volatility 31.36%, implied leverage 16.36% and elasticity 1.3044.

4.2 Volatility skew of low leverage firms

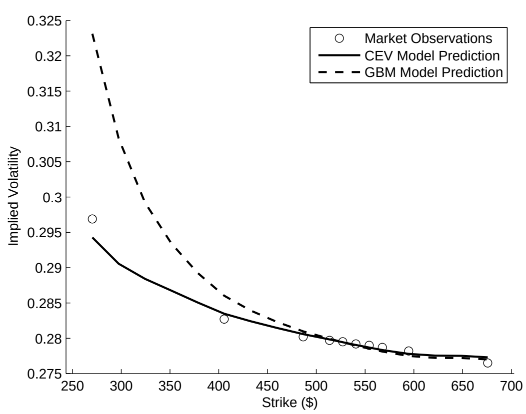

Fig. 3: AAPL volatility skew calibrated under both CEV and GBM asset dynamic

The benefit of picking CEV process as the asset dynamic is outstanding when modeling low leverage firms. Under GBM asset dynamic, lower leverage naturally means less volatility skew, which is incompatible with reality in some circumstances. Apple Inc. (NASDAQ: AAPL) for example, is known by its deep cash position, and therefore its effective leverage is very limited. However, a volatility skew is still observed from the option market. The skew is mostly due to the asymmetry of its asset return, rather than to the firm's leverage. If the skew is calibrated to a GBM asset dynamic, the implied leverage must be higher than reality to explain the skew. AAPL skew calibrated under the GBM and the CEV leverage model implies 6.4897% and 1.1166% respectively. From Figure (3), it can be easily observed that GBM produces an implied volatility much higher than the market quote on the low strike end. Both the implied leverage and the goodness of fit indicate that the GBM model is overstating the leverage, and the CEV model provides more flexibility to achieve better agreements to the market.

4.3 Parameter distribution analysis

The ideal test of effectiveness of implied parameters (i.e. V_0, D, ϕ and α) is comparing the parameters to fundamental research conclusions. However this analysis is difficult to approach because the information and workload required for fundamental research is far beyond the author's resources and specialties. On the other hand, the conclusions of fundamental research are difficult to quantify as well.

To overcome this difficulty, a cross-sectional approach is taken as an alternative. 99 components of S&P-100 index and 81 components of NASDAQ- 100 index (with 17 overlaps excluded) are selected as test candidates to ensure strong capitalization, market liquidity and data accuracy. From the fundamental point of view, components of two indices should be notably different because S&P-100 components are mostly mature and stable companies whereas the NASDAQ-100 components have a much higher emphasis on high risk and high growth potential stocks. These two types of companies should differ in terms of business uncertainty, effective leverage and asset return asymmetry.

To benchmark the effectiveness of the CEV-based structural model, the original Merton's model is implemented under suggestions of Jones et al. in 1984 [11] (Merton-JMR), where the asset value (V_0), asset volatility (σ_V) are obtained from equity (E_0), equity volatility (σ_E) and liability (D) input by jointly solving $E_0 = C(V_0, D, \sigma_V)$ and $E_0\sigma_E = \frac{\partial E}{\partial V}V_0\sigma_V$, where function $C()$ is the Black-Schole's vanilla call valuation. Here σ_E is approximated by the 30-day realized volatility of the stock return and D is approximated by KMV's suggestion of short-term liability plus one half of long-term liability [2].

The distribution comparison is based on Kernel smoothing function estimation. The difference between

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distributions is measured by the Hellinger distance to overcome possible outlier problems commonly seen in financial data. The Hellinger distance between density function $f(x)$ and $g(x)$ is defined as:

$$H < f,g >= 1 - \int \sqrt{f(x)g(x)}dx$$

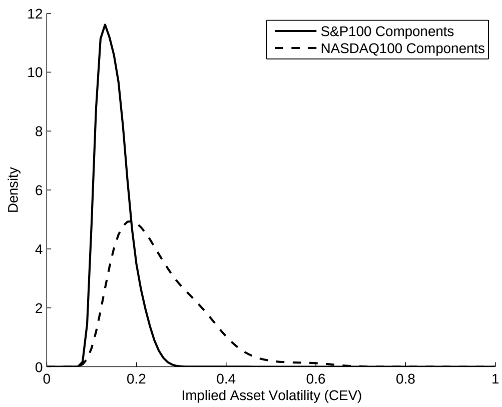

Fig. 4: Kernel smoothing function estimation of firm volatility under CEV implementation

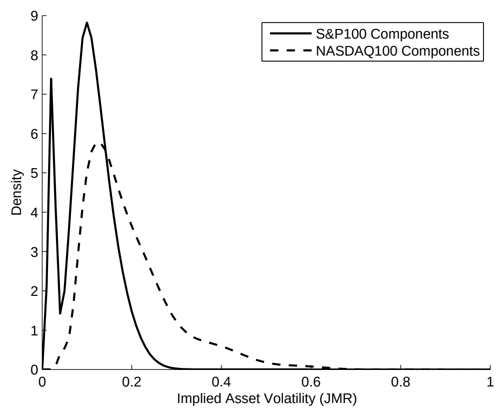

Fig. 5: Kernel smoothing function estimation of firm volatility under JMR implementation

Unlike in GBM, the CEV volatility factor ϕ is not directly comparable between candidates because of different α and V_0. The adjusted volatility $\sigma = \phi V_0^{\alpha-1}$, which is the instantaneous volatility of asset return, is compared instead. Figure (4) and (5) shows the Kernel smoothing function estimations of asset volatility under both CEV and JMR implementations. It is remarkable that the adjusted asset volatility of S&P-100 companies clusters around 10% whereas for NASDAQ-100 companies it is more dispersed to the higher volatility zone and with a fat tail on the right. This is consistent with the fact that stable businesses usually have lower and similar uncertainty whereas the growing businesses tend to have higher and more diversified volatility due to the business model diversity. The Merton-JMR implementation captures a similar pattern, but with much lower confidence. The Hellinger distribution distance between S&P-100 and NASDAQ-100 for the CEV implementation is 0.2899 compared to 0.2022 in the JMR implementation. Therefore, both implementations correctly capture the differences in business uncertainty, while CEV implementation demonstrates an advantage in producing more differentiated volatility distributions.

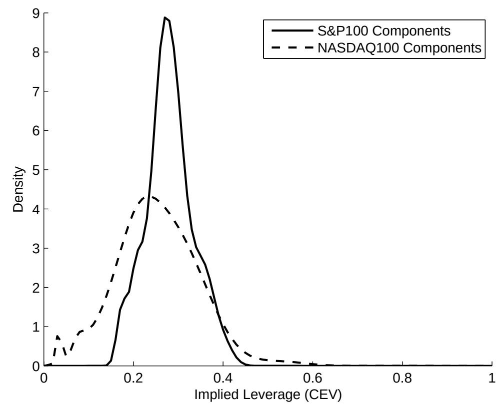

Fig. 6: Kernel smoothing function estimation of implied firm leverage under CEV implementation

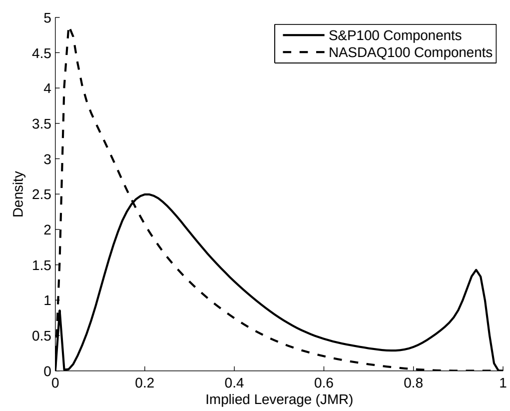

Fig. 7: Kernel smoothing function estimation of implied firm leverage under JMR implementation

One major difference between CEV and Merton-JMR implementation is the treatment of liabilities. Liabilities are readable from financial reports. However these values are highly unreliable in many respects. For example, stable companies tend to make long-term rolling borrowings, which pump up their liability book values

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without putting them under significant financial stress. These liabilities are usually adjusted down in fundamental analysis. Growing companies, on the other hand, tend to make short-term borrowings, and their capability to roll over these debts depends highly on their short-term performance. These liabilities are usually the default triggers for growing companies and should not be adjusted down. Off-balance-sheet items further complicate the liability analysis by its hidden and diversified nature.

CEV implementation is devoted to imply liability from market data, and therefore taking into account the professional adjustments made by fundamental traders. JMR implementation can only make very crude adjustments. Figure (6) and (7) shows the Kernel smoothing function estimation comparison of implied leverage under CEV and JMR implementation respectively. Once again, CEV implementation suggests concentrated leverage distribution for S&P-100 companies and more dispersed distribution for NASDAQ-100 companies. Additionally, NASDAQ-100 companies show slightly lower leverage, which is also consistent with reality. The JMR implementation produces misleading results in terms of leverage. A significant number of S&P-100 companies have leverage close to one, and NASDAQ-100 companies show high concentration at very low leverage. This might be true when reading the financial statements, but the implication is less meaningful or even misleading when trying to understand the firm's fundamentals. Even though JMR implementation shows a higher Hellinger distribution distance than CEV (0.1904 vs 0.1193), it should not be considered as an advantage due to its misleading implication.

The distribution of elasticity factor α is also illustrative in revealing the firm characteristic. Figure (8) shows the Kernel smoothing function estimation under CEV implementation (note that this measure is unavailable under JMR implementation). For NASDAQ-100 stocks, a strong clustering around $\alpha = 0.7$ and a weaker clustering around $\alpha = 1.2$ are very noticeable. The strong clustering to lower elasticity illustrates that a fat left tail in asset return distribution is expected by the market for most of the growth company, whose valuation is rich and downside is large; whereas the weaker clustering to higher elasticity corresponds to the fewer companies with moderate current valuation but with strong growth potential that has not yet been priced in. The distribution of S&P-100 asset return elasticity is more dispersed, which is consistent to the fact that these stocks are mostly fairly valued, and the elasticity purely reflects the nature of the business' profitability. The Hellinger distance between elasticities is 0.1407.

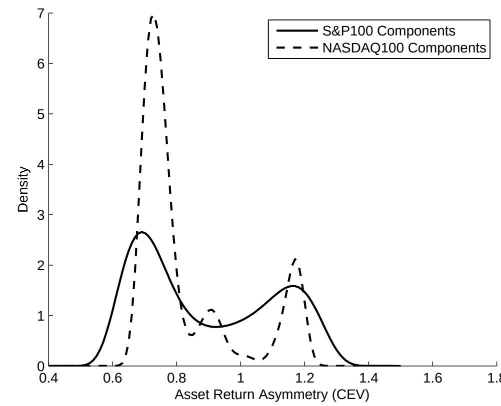

Fig. 8: Kernel smoothing function estimation of asset return elasticity (α) for CEV implementation

4.4 Rank consistency analysis between expected default loss and CDS spread

Given the calibrated parameters, the probability of default (PD) under CEV implementation can be calculated as the first hitting probability by solving the Kolmogorov backward equation:

$$\begin{cases} \frac{\partial P}{\partial t} + rV\frac{\partial P}{\partial V} + \frac{1}{2}\phi^2 V^{2\alpha}\frac{\partial^2 P}{\partial V^2} = 0 \\ P(V=L,t) = 1 \\ P(V=\infty,t) = 0 \\ P(V,t=T) = \mathbb{I}_{\{V=L\}} \end{cases} \tag{11}$$

The expected default loss (EDL) is then the product between loss given default $1-L$ and the probability of default.

It is very challenging to match the implied EDL to CDS spread for several reasons. For one reason, the pricing measure used to generate EDL is calibrated only to vanilla options, and there could be a significant misalignment between the option market and CDS market. A careful joint calibration is required to bridge this gap. Other reasons are also reported e.g. in [10] and [9]. Therefore an alternative approach of day-by-day analysis is taken instead. A good credit risk measure should be able to produce ranking consistency to CDS spreads, and the movements of CDS spread should also be captured by the credit risk measure. Due to the limited availability of CDS and volatility skew data, it is extremely tedious to apply this test on a large sample. Only two individual stocks with observable CDS movements in the study period are reported here.

The market data of American International Group Inc. (NYSE:AIG) and Simon Property Group Inc. (NYSE:SPG) between Jan.17, 2014 and Feb.28, 2014 are analyzed. Figure (9) and (11) shows the scatter plot of

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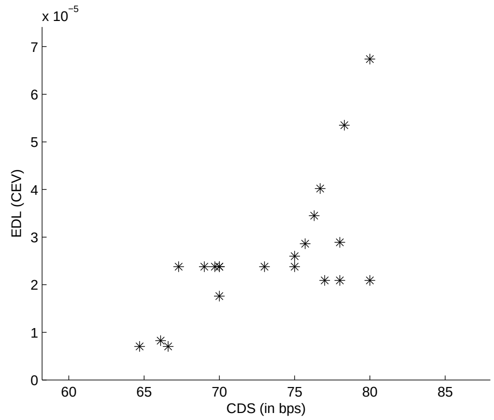

Fig. 9: Relationship between CEV EDL and CDS, NYSE:AIG

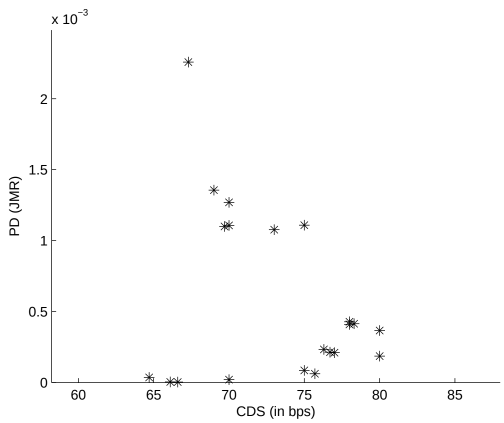

Fig. 10: Relationship between JMR PD and CDS, NYSE:AIG

5-year EDL v.s. 5-year CDS spread under CEV implementation for AIG and SPG respectively, whereas Figure (10) and (12) shows the scatter plot of 5-year probability of default v.s. 5-year CDS spread under Merton's JMR implementation for AIG and SPG respectively. It is quite noticeable that under CEV implementation, the EDL shows a much stronger ranking consistency with CDS, compared to the PD under JMR implementation. The Kendall rank correlation measure also confirms this observation: CEV implementation leads with 0.5327 Kendall τ v.s. JMR's -0.2284 in the SPG case, and 0.5045 v.s. -0.0290 in the AIG case. This suggests that for a particular firm, the EDL produced by CEV implementation works more desirably in capturing the daily movements of the CDS spreads.

5 Conclusions

In this paper, an alternative structural model is proposed with asymmetric asset return distribution. The model retains closed-form representation of equity value with respect to asset and liability. The model can accommodate more diversified volatility structures including skews as well as smiles. The closed form solution enables the calibration of the model to the entire implied volatility structure. The calibrated parameters, including implied leverage, business uncertainties and asset return asymmetry, provide an effective outline of companies' fundamental characteristics. Cross-sectional study shows that S&P-100 companies tend to have similar implied leverages, lower business uncertainties and more diversified asset return asymmetry, whereas the NASDAQ-100 companies tend to have lower but more dispersed distribution of implied leverages, higher business uncertainties and strong clustering to left skewed asset return asymmetry. The expected default loss, as a credit quality measure, provides much stronger ranking consistency to CDS spread compared to Merton's probability of default.

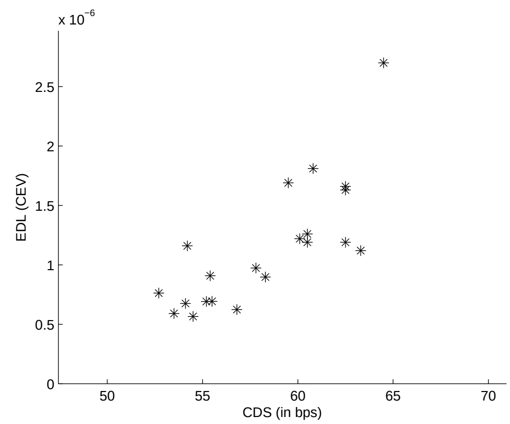

Fig. 11: Relationship between CEV EDL and CDS, NYSE:SPG

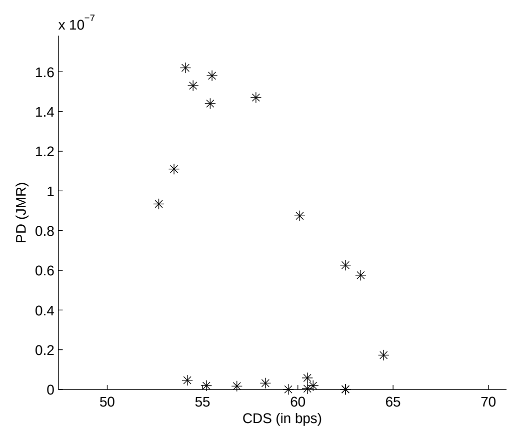

Fig. 12: Relationship between JMR PD and CDS, NYSE:SPG

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