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Coincidence and Common Fixed Point of Weakly Compatible Maps in Intuitionistic Fuzzy Metric Space

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Abstract: In this paper, we introduce new notion of common limit in the range property (CLR$_{fg}$) for a pair of self map which is a proper generalization of common limit in the range property (CLR_g) for a pair of self maps and discuss the existence of coincidence and common fixed point for weakly compatible self maps via property $(E.A.)$ and its variants in intuitionistic fuzzy metric space. Our results guarantee the existence of coincidence and common fixed point for noncompatible maps without closedness /completeness requirement of subspace even when all the maps are discontinuous. We also furnish an illustrative example in support of our results.

Keywords: Intuitionistic fuzzy metric space, weakly compatible, property $(E.A)$, CLR$_g$ property, CLR$_{fg}$ property, common property $(E.A)$, common fixed point and coincidence point.

1 Introduction

In 1984 Atanassov [3] introduced the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets [34] and later there has been much progress in the study of intuitionistic fuzzy sets (see for instance [2,3,8,13,16,17,19,27,33]). In 2004, Park [27] defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norms and continuous t-conorms as a generalization of fuzzy metric space due to George and Veeramani [6], which is useful in modeling some phenomena where it is necessary to study relationship between two probability functions. Alaca et al. [2] using the idea of intuitionistic fuzzy sets defined the notion of intuitionistic fuzzy metric space as Park [27] as a generalization of fuzzy metric space due to Kramosil and Michalek [12]. It has wide and diverse applications in the field of population dynamics, chaos control, computer programming, medicine, etc. and has direct physics motivation in the context of the two slit experiment as foundation of E-infinity of high energy physics, recently studied by El Naschie ([25,26]).

Aamri and Moutawakil [1] introduced the notion of property $(E.A.)$ which contains the class of compatible as well as noncompatible maps and this is the motivation to use the property $(E.A.)$ instead of compatibility or noncompatibility to find the existence of coincidence and common fixed point. Liu et al. [14] further improved it by common property $(E.A.)$. However property $(E.A.)$ and common property $(E.A.)$ always require closedness of subspace for the existence of coincidence and common fixed point. Recently, Sintunavarat et al. [32] introduced the notion of (CLR_g) property for a pair of self maps in fuzzy metric space and Chauhan et al. [4] introduced the notion of $JCLR_{ST}$ property for two pairs of self maps which even relax the closedness requirements of the underlying subspaces. In literature, many results have been proved in different settings such as metric space [1,7,10,11], probabilistic metric space [9], fuzzy metric space [4,5,6,15,18,23] and an intuitionistic fuzzy metric space [2,3,8,13,16,17,19,33] via property $(E.A.)$ and its variants.

It is well known that proofs of all the common fixed point theorems for a pair of weakly compatible self maps follow the same pattern.

1. To prove the existence of coincidence point for a pair of self maps.
2. To prove that this coincidence point is a common fixed point.
3. To prove uniqueness of common coincidence point.

Here first step is considered to be the most difficult part of the proof.

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In this paper, we introduce the new notion of common limit in the range property (CLR_{fg}) for a pair of self map which automatically gives the first step and does not require containment and closedness of underlying subspace.We further show that common limit in the range property (CLR_{fg}) is a proper generalization of common limit in the range property (CLR_{g}) for a pair of self map. Also we discuss the existence of coincidence and common fixed points for weakly compatible self maps via property $(E.A.)$ and its variants. Our results intuitionistically fuzzify and improve the results of Sedghi et al. [29] and guarantee the existence of coincidence and common fixed point for noncompatible maps even when all the maps are discontinuous. We also furnish an illustrative example in support of our results.

2 Preliminaries

The concepts of triangular norms (t-norms) and triangular conorms (t-conorms) are known as the axiomatic skelton that we use are characterization fuzzy intersections and union respectively. Menger [22] originally introduced these concepts in the study of statistical metric spaces.

Definition 2.1. [28] A binary operation $*:[0,1]\times[0,1]\to[0,1]$ is continuous t-norm if * satisfies the following conditions:for all $a,b,c,d\in[0,1]$,

(i) * is commutative and associative;
(ii) * is continuous;
(iii) $a*1=a$;
(iv) $a*b\leq c*d$ whenever $a\leq c$ and $b\leq d$.

Examples of t-norm are $a*b=min\{a,b\}$ and $a*b=ab$.

Definition 2.2. [28] A binary operation $\diamondsuit:[0,1]\times[0,1]\to[0,1]$ is continuous t-conorm if $\diamondsuit$ satisfies the following conditions: for all $a,b,c,d\in[0,1]$,

(i) $\diamondsuit$ is commutative and associative;
(ii) $\diamondsuit$ is continuous;
(iii) $a\diamondsuit 0=a$;
(iv) $a\diamondsuit b\geq c\diamondsuit d$ whenever $a\leq c$ and $b\leq d$.

Examples of t-conorm are $a\diamondsuit b=max\{a,b\}$ and $a\diamondsuit b=min\{1,a+b\}$.

Alaca et al. [2] using the idea of Intuitionistic fuzzy sets, defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norm and continuous t-conorms as:

Definition 2.3. [2] A 5-tuple $(X,M,N,*,\diamondsuit)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm, $\diamondsuit$ is a continuous t-conorm and M,N are fuzzy sets on $X^2\times[0,\infty)$ satisfying the following conditions: for all $x,y,z\in X$ and $t,s>0$;

(i) $M(x,y,t)+N(x,y,t)\leq 1$;
(ii) $M(x,y,0)=0$;
(iii) $M(x,y,t)=1$ if and only if $x=y$;
(iv) $M(x,y,t)=M(y,x,t)$;
(v) $M(x,y,t)*M(y,z,s)\leq M(x,z,t+s)$;
(vi) $M(x,y,.):[0,\infty)\to[0,1]$ is left continuous;
(vii) $lim_{t\to\infty}M(x,y,t)=1$;
(viii) $N(x,y,0)=1$;
(ix) $N(x,y,t)=0$ if and only if $x=y$;
(x) $N(x,y,t)=N(y,x,t)$;
(xi) $N(x,y,t)\diamondsuit N(y,z,s)\geq N(x,z,t+s)$;
(xii) $N(x,y,.):[0,\infty)\to[0,1]$ is right continuous;
(xiii) $lim_{t\to\infty}N(x,y,t)=0$.

Here, (M,N) is called an intuitionistic fuzzy metric space on X. The functions $M(x,y,t)$ and $N(x,y,t)$ denote the degree of nearness and the degree of non-nearness between x and y w.r.t. t respectively.

Remark 2.1. [2] Every fuzzy metric space $(X,M,*)$ is an intuitionistic fuzzy metric space of the form $(X,M,1-M,*,\diamondsuit)$ such that t-norm * and t-conorm $\diamondsuit$ are associated as $x\diamondsuit y=1-((1-x)*(1-y))$ for all $x,y\in X$ but the reverse implication is not true.

Remark 2.2. [2] In intuitionistic fuzzy metric space $(X,M,N,*,\diamondsuit)$, $M(x,y,.)$ is non-decreasing and $N(x,y,.)$ is non-increasing, for all $x,y\in X$.

Alaca et al. [2] introduced the following notions:

Definition 2.4. Let $(X,M,N,*,\diamondsuit)$ be an intuitionistic fuzzy metric space. Then

(a) a sequence $\{x_n\}$ in X is said to be Cauchy sequence if, for all $t>0$ and $p>0$,

$lim_{n\to\infty}M(x_{n+p},x_n,t)=1$ and $lim_{n\to\infty}N(x_{n+p},x_n,t)=0$.

(b) a sequence $\{x_n\}$ in X is said to be convergent to a point $x\in X$ if, for all $t>0$,

$lim_{n\to\infty}M(x_n,x,t)=1$ and $lim_{n\to\infty}N(x_n,x,t)=0$.

Definition 2.5. [11] A point $x\in X$ is a coincidence point of a pair of self maps (f,g) in an intuitionistic fuzzy metric space $(X,M,N,*,\diamondsuit)$ if $fx=gx$.

Definition 2.6. [11] A pair of self maps (f,g) in an intuitionistic fuzzy metric space $(X,M,N,*,\diamondsuit)$ are weakly compatible if they commute at their coincidence points; i.e., $fx=gx$ for some $x\in X$ implies that $fgx=gfx$.

Definition 2.7. [1] A pair of self maps (f,g) in an intuitionistic fuzzy metric space $(X,M,N,*,\diamondsuit)$ satisfy the property $(E.A)$ if there exists a sequence $\{x_n\}$ in X such that $lim_{n\to\infty}M(fx_n,z,t)=1$, $lim_{n\to\infty}M(gx_n,z,t)=1$ and $lim_{n\to\infty}N(fx_n,z,t)=1$, $lim_{n\to\infty}N(gx_n,z,t)=0$ for some $z\in X$.

Example 2.1. [1] Let $(X,M,N,*,\diamondsuit)$ be an intuitionistic fuzzy metric space where $X=[0,\infty)$ and let * be the continuous t-norm and $\diamondsuit$ be the continuous t-conorm defined by $a*b=ab$ and $a\diamondsuit b=min\{1,a+b\}$ respectively, for all $a,b\in[0,1]$. For each $t>0$ and $x,y\in X$, define (M,N) by $M(x,y,0)=0$, $M(x,y,t)=\frac{t}{t+|x-y|}$ and $N(x,y,0)=1$, $N(x,y,t)=\frac{|x-y|}{t+|x-y|}$. Define $f,g:X\to X$ by $fx=\frac{2x}{5}$ and $gx=\frac{x}{5}$ for all $x\in X$. Clearly, for sequence $\{x_n\}=\{\frac{1}{n}\}$, f and g satisfy property $(E.A)$.

It is well known that weak compatibility and property

$(E.A)$ are independent of each other [1].

Definition 2.8. [14] Two pairs of self maps (f,g) and (a,b) in an intuitionistic fuzzy metric space $(X,M,N,*,\diamondsuit)$ satisfy the common property $(E.A)$ if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that
$lim_{n\to\infty}M(fx_n,z,t) = lim_{n\to\infty}M(gx_n,z,t) = lim_{n\to\infty}M(ay_n,z,t) = lim_{n\to\infty}M(by_n,z,t) = 1$
and
$lim_{n\to\infty}N(fx_n,z,t) = lim_{n\to\infty}N(gx_n,z,t) = lim_{n\to\infty}N(ay_n,z,t) = lim_{n\to\infty}N(by_n,z,t) = 0$
for some $z \in X$.
Example 2.2.[14] Let $(X,M,N,*,\diamondsuit)$ be an intuitionistic fuzzy metric space as in Example 2.1 where $X = [-1,1]$. Define self maps f,g,a and b on X as $fx = \frac{x}{3}$, $ax = \frac{-x}{3}$, $gx = x$, $bx = -x$ for all $x \in X$. Then, with sequences $\{x_n\} = \{\frac{1}{n}\}$ and $\{y_n\} = \{\frac{-1}{n}\}$ in X, one can easily verify that $lim_{n\to\infty}M(fx_n,0,t) = lim_{n\to\infty}M(gx_n,0,t) = lim_{n\to\infty}M(ay_n,0,t) = lim_{n\to\infty}M(by_n,0,t) = 1$
and
$lim_{n\to\infty}N(fx_n,0,t) = lim_{n\to\infty}N(gx_n,0,t) = lim_{n\to\infty}N(ay_n,0,t) = lim_{n\to\infty}N(by_n,0,t) = 0.$
Therefore, pairs (f,g) and (a,b) satisfy the common property $(E.A.)$ property.
Definition 2.9. [32] A pair of self maps (f,g) in an intuitionistic fuzzy metric space $(X,M,N,*,\diamondsuit)$ satisfies the common limit in the range of g property (CLR_g) if there exists a sequence $\{x_n\}$ in X such that $lim_{n\to\infty}M(fx_n,gz,t) = 1$, $lim_{n\to\infty}M(gx_n,gz,t) = 1$ and $lim_{n\to\infty}N(fx_n,gz,t) = 1$, $lim_{n\to\infty}N(gx_n,gz,t) = 0$ for some $z \in X$.

With a view to extend the (CLR_g) property to two pair of self maps, very recently Chauhan et. al. [4] define the $(JCLR_{gb})$ property (with respect to maps g and b) as follows:
Definition 2.10 [4] Two pairs of self maps (f,g) and (a,b) in an intuitionistic fuzzy metric space $(X,M,N,*,\diamondsuit)$ satisfy the $(JCLR_{gb})$ property (with respect to maps g and b) if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that
$lim_{n\to\infty}M(fx_n,gz,t) = lim_{n\to\infty}M(gx_n,gz,t) = lim_{n\to\infty}M(ay_n,gz,t) = lim_{n\to\infty}M(by_n,gz,t) = 1$
and
$lim_{n\to\infty}N(fx_n,Sz,t) = lim_{n\to\infty}N(gx_n,Sz,t) = lim_{n\to\infty}N(ay_n,Sz,t) = lim_{n\to\infty}N(by_n,Sz,t) = 0$
where $gz = bz$ for some $z \in X$.
It is interesting to note here that $(JCLR_{gb})$ property imply the common property $(E.A)$ but the reverse implication is not true in general.

Alaca [1] proved the following results:
Lemma 2.1. Let $(X,M,N,*,\diamondsuit)$ be intuitionistic fuzzy metric space. For all $x,y \in X$, $t > 0$, if for a number $k > 1$, $M(x,y,kt) \geq M(x,y,t)$ and $N(x,y,kt) \leq N(x,y,t)$ then $x = y$.
Lemma 2.2. Let $(X,M,N,*,\diamondsuit)$ be intuitionistic fuzzy metric space. For all $x,y \in X$, $t > 0$, if for a number $k > 1$, $M(y_{n+2},y_{n+1},t) \geq M(y_{n+1},y_n,kt)$, $N(y_{n+2},y_{n+1},t) \leq N(y_{n+1},y_n,kt)$, then $\{y_n\}$ is a Cauchy sequence in X.

3 Main Results

Let Φ be the set of all increasing and continuous functions $\phi : (0,1] \to (0,1]$ such that $\phi(t) > t$ for all $t \in (0,1]$ and Ψ be the set of all increasing and continuous functions $\psi : [0,1) \to [0,1)$ such that $\psi(t) < t$ for all $t \in [0,1)$.

Example 3.1. Let $\phi : (0,1] \to (0,1]$ and $\psi : [0,1) \to [0,1)$ defined by $\phi(t) = (t)^{\frac{1}{2}}$ and $\psi(z) = (z)^2$ for all $t \in (0,1]$ and $z \in [0,1)$. Clearly, $\phi \in \Phi$ and $\psi \in \Psi$.

Theorem 3.1. Let (f,g) be a pair of self maps in intuitionistic fuzzy metric space $(X,M,N,*,\diamondsuit)$ where * is a continuous t-norm and $\diamondsuit$ is a continuous t-conorm such that

$$(3.1)M(fx,fy,t) \geq \phi\{\min\{M(gx,gy,t), \sup_{t_1+t_2=\frac{2t}{k}} \min\{M(gx,fx,t_1),M(gy,fy,t_2)\}, \sup_{t_3+t_4=\frac{2t}{k}} max\{M(gx,fx,t_3),M(gy,fy,t_4)\}\}\}$$
$$N(fx,fy,t) \leq \psi\{\max\{N(gx,gy,t), \inf_{t_1+t_2=\frac{2t}{k}} \max\{N(gx,fx,t_1),N(gy,fy,t_2)\}, \inf_{t_3+t_4=\frac{2t}{k}} min\{N(gx,fx,t_3),N(gy,fy,t_4)\}\}\},$$

for all $x,y \in X$, $t > 0$, $\phi \in \Phi$ and $\psi \in \Psi$;
(3.2) $g(X)$ is a closed subspace of X;
(3.3) pair (f,g) satisfies the property $(E.A)$.
Then f and g have a unique common fixed point in X provided that the pair (f,g) is weakly compatible.
Proof. Since the pair (f,g) satisfies property $(E.A)$, then there exists a sequence $\{x_n\}$ in X such that $lim_{n\to\infty}M(fx_n,z,t) = lim_{n\to\infty}M(gx_n,z,t) = 1$ and $lim_{n\to\infty}N(fx_n,z,t) = lim_{n\to\infty}N(gx_n,z,t) = 0$, for some $z \in X$. As $g(X)$ is a closed subspace of X, there exists $u \in X$ such that $z = gu$. Therefore $lim_{n\to\infty}fx_n = lim_{n\to\infty}gx_n = z = gu$.
Firstly we claim that $fu = gu$. Suppose not, then there exists $t_0 > 0$ such that
$M(fu,gu,\frac{2t_0}{k}) > M(fu,gu,t_0)$
and
$N(fu,gu,\frac{2t_0}{k}) < N(fu,gu,t_0)$. (3.4)
The inequality (3.4) is always true when $fu \neq gu$. To support our claim, we suppose on contrary that (3.4) is not true all $t > 0$, i.e., $M(fu,gu,\frac{2t}{k}) = M(fu,gu,t)$
and
$N(fu,gu,\frac{2t}{k}) = N(fu,gu,t)$. (3.5)
Now, using equality (3.5) repeatedly, we get $M(fu,gu,t) = M(fu,gu,\frac{2}{k}t) = M(fu,gu,\frac{2^2}{k^2}t) = ... = M(fu,gu,\frac{2^n}{k^n}t) \to 1$ and $N(fu,gu,t) = N(fu,gu,\frac{2}{k}t) = N(fu,gu,\frac{2^2}{k^2}t) = ... = N(fu,gu,\frac{2^n}{k^n}t) \to 0$ as $n \to \infty$. This

gives, $M(fu,gu,t)=1$ and $N(fu,gu,t)=0$ for all $t>0$. Hence, $fu=gu$, which gives contradiction.
Therefore, inequality (3.4) is always true for some $t_0>0$.
Now, by using inequality (3.1), we have

$$M(fx_n,fu,t_0)\geq\phi\{\min\{M(gx_n,gu,t_0),\sup_{t_1+t_2=\frac{2t_0}{k}}\min\{M(gx_n,fx_n,t_1),M(gu,fu,t_2)\},\sup_{t_3+t_4=\frac{2t_0}{k}}\max\{M(gx_n,fu,t_3),M(gu,fx_n,t_4)\}\}\}$$

and

$$N(fx_n,fu,t_0)\leq\psi\{\max\{N(gx_n,gu,t_0),\inf_{t_1+t_2=\frac{2t_0}{k}}\max\{N(gx_n,fx_n,t_1),N(gu,fu,t_2)\},\inf_{t_3+t_4=\frac{2t_0}{k}}\min\{N(gx_n,fu,t_3),N(gu,fx_n,t_4)\}\}\}.$$

Let $t_1=t_4=\varepsilon$ then $t_2=t_3=\frac{2t_0}{k}-\varepsilon$ where $\varepsilon\in(0,\frac{2t_0}{k})$ and $n\to\infty$, we get

$$\begin{aligned}M(gu,fu,t_0)&\geq\phi\{\min\{M(gu,gu,t_0),\\&\min\{M(gu,gu,\varepsilon),M(gu,fu,\frac{2t_0}{k}-\varepsilon)\},\\&\max\{M(gu,fu,\frac{2t_0}{k}-\varepsilon),M(gu,gu,\varepsilon)\}\}\}\end{aligned}$$

$$\begin{aligned}i.e.,\ M(gu,fu,t_0)&\geq\phi\{\min\{1,\\&\min\{1,M(gu,fu,\frac{2t_0}{k}-\varepsilon)\},\\&\max\{M(gu,fu,\frac{2t_0}{k}-\varepsilon),1\}\}\end{aligned}$$

$$\begin{aligned}i.e.,\ M(gu,fu,t_0)&\geq\phi(M(gu,fu,\frac{2t_0}{k}-\varepsilon))\\&>M(gu,fu,\frac{2t_0}{k}-\varepsilon)\end{aligned}$$

and

$$N(fx_n,fu,t_0)\leq\psi\{\max\{N(gx_n,gu,t_0),\inf_{t_1+t_2=\frac{2t_0}{k}}\max\{N(gx_n,fx_n,t_1),N(gu,fu,t_2)\},\inf_{t_3+t_4=\frac{2t_0}{k}}\min\{N(gx_n,fx_n,t_3),N(gu,fu,t_4)\}\}\}.$$

$$\begin{aligned}i.e.,\ N(gu,fu,t_0)&\leq\psi\{\max\{N(gu,gu,t_0),\\&\max\{N(gu,gu,\varepsilon),N(gu,fu,\frac{2t_0}{k}-\varepsilon)\},\\&\min\{N(gu,fu,\frac{2t_0}{k}-\varepsilon),N(gu,gu,\varepsilon)\}\}\}\end{aligned}$$

$$\begin{aligned}i.e.,\ N(gu,fu,t_0)&\leq\psi\{\min\{0,\\&\max\{0,N(gu,fu,\frac{2t_0}{k}-\varepsilon)\},\\&min\{N(gu,fu,\frac{2t_0}{k}-\varepsilon),0\}\}\}\end{aligned}$$

$$\begin{aligned}i.e.,\ N(gu,fu,t_0)&\leq\psi(N(gu,fu,\frac{2t_0}{k}-\varepsilon))\\&<M(gu,fu,\frac{2t_0}{k}-\varepsilon).\end{aligned}$$

As $\varepsilon\to0$, we get $M(gu,fu,t_0)>M(gu,fu,\frac{2t_0}{k}-\varepsilon)$ and $N(gu,fu,t_0)<N(gu,fu,\frac{2t_0}{k}-\varepsilon)$ which gives contradiction to (3.4). Therefore, $gu=fu=z$(say).
Since f and g are weakly compatible. Therefore, $fgu=gfu$ and then $ffu=fgu=gfu=ggu$. This gives, $fz=gz$.
Next, we claim that $fz=z$. Suppose not, then by (3.1), we get

$$M(fz,fu,t_0)\geq\phi\{\min\{M(gz,gu,t_0),\sup_{t_1+t_2=\frac{2t_0}{k}}\min\{M(gz,fz,t_1),M(gu,fu,t_2)\},\sup_{t_3+t_4=\frac{2t_0}{k}}\max\{M(gz,fu,t_3),M(gu,fz,t_4)\}\}\}$$

and

$$N(fz,fu,t_0)\leq\psi\{\max\{N(gz,gu,t_0),\inf_{t_1+t_2=\frac{2t_0}{k}}\max\{N(gz,fz,t_1),N(gu,fu,t_2)\},\inf_{t_3+t_4=\frac{2t_0}{k}}\min\{N(gz,fu,t_3),N(gu,fz,t_4)\}\}\}.$$

Let $t_1=t_3=\varepsilon$ then $t_2=t_4=\frac{2t_0}{k}-\varepsilon$ where $\varepsilon\in(0,\frac{2t_0}{k})$,

$$\begin{aligned}M(fz,z,t_0)&\geq\phi\{\min\{M(fz,z,t_0),\\&\min\{M(fz,fz,\varepsilon),M(z,z,\frac{2t_0}{k}-\varepsilon)\},\\&\max\{M(fz,z,\frac{2t_0}{k}-\varepsilon),M(z,fz,\varepsilon)\}\}\}\end{aligned}$$

and

$$\begin{aligned}N(fz,z,t_0)&\leq\psi\{\max\{N(fz,z,t_0),\\&\max\{N(fz,fz,\varepsilon),N(z,z,\frac{2t_0}{k}-\varepsilon)\},\\&\min\{N(fz,z,\frac{2t_0}{k}-\varepsilon),N(z,fz,\varepsilon)\}\}\}.\end{aligned}$$

As $\varepsilon\to0$, we get

$$\begin{aligned}M(fz,z,t_0)&\geq\phi\{\min\{M(fz,z,t_0),1,\max\{M(z,fz,0),\\&M(fz,z,\frac{2t_0}{k}\}\}\}\end{aligned}$$

$$i.e., M(fz,z,t_0) \geq \phi\{\min\{M(fz,z,t_0),1, M(fz,z,\frac{2t_0}{k}\}\}$$

$$i.e., M(fz,z,t_0) \geq \phi(M(fz,z,t_0)) > M(fz,z,t_0)$$

and

$$N(fz,z,t_0) \leq \psi\{\max\{N(fz,z,t_0),0,\min\{N(z,fz,0), N(fz,z,\frac{2t_0}{k}\}\}\}$$

$$i.e., N(fz,z,t_0) \leq \psi\{\max\{N(fz,z,t_0),0, N(fz,z,\frac{2t_0}{k}\}\}$$

$$i.e., N(fz,z,t_0) \leq \psi N(fz,z,t_0) < N(fz,z,t_0),$$

a contradiction, hence, $fz = gz = z$. Therefore, z is a common fixed point of f and g.
For uniqueness; let w be another fixed point of f and g. Then by (3.1), we have

$$M(fz,fw,t_0) \geq \phi\{\min\{M(gz,gw,t_0), \sup_{t_1+t_2=\frac{2t_0}{k}} \min\{M(gz,fz,t_1),M(gw,fw,t_2)\}, \sup_{t_3+t_4=\frac{2t_0}{k}} \max\{M(gz,fw,t_3),M(gw,fz,t_4)\}\}\}$$

$$i.e., M(z,w,t_0) \geq \phi\{\min\{M(z,w,t_0), \sup_{t_1+t_2=\frac{2t_0}{k}} \min\{M(z,z,t_1),M(w,w,t_2)\}, \sup_{t_3+t_4=\frac{2t_0}{k}} \max\{M(z,w,t_3),M(w,z,t_4)\}\}\}$$

and

$$N(fz,fw,t_0) \leq \psi\{\max\{N(gz,gw,t_0), \inf_{t_1+t_2=\frac{2t_0}{k}} \max\{N(gz,fz,t_1),N(gw,fw,t_2)\}, \inf_{t_3+t_4=\frac{2t_0}{k}} \min\{N(gz,fw,t_3),N(gw,fz,t_4)\}\}\}$$

$$i.e., N(z,w,t_0) \leq \psi\{\max\{N(z,w,t_0), \inf_{t_1+t_2=\frac{2t_0}{k}} \max\{N(z,z,t_1),N(w,w,t_2)\}, \inf_{t_3+t_4=\frac{2t_0}{k}} \min\{N(z,w,t_3),N(w,z,t_4)\}\}\}.$$

Let $t_1 = t_3 = \varepsilon$ then $t_2 = t_4 = \frac{2t_0}{k} - \varepsilon$ where $\varepsilon \in (0, \frac{2t_0}{k})$, and as $\varepsilon \to 0$, we have
$M(z,w,t_0) \geq \phi\ (M(z,w,t_0)) > M(z,w,t_0)$ and

$$N(z,w,t_0) \leq \psi(N(z,w,t_0)) < N(z,w,t_0),$$

a contradiction, hence, $w = z$. It implies that f and g have a unique common fixed point in X.

Now we attempt to drop closedness of subspace from Theorem 3.1 using (CLR_g) property.

Theorem 3.2. Let (f,g) be a pair of self maps in intuitionistic fuzzy metric space $(X,M,N,*,\diamondsuit)$ where $*$ is a continuous t-norm and $\diamondsuit$ is a continuous t-conorm satisfying condition (3.1). If the pair (f,g) satisfies the (CLR_g) property then f and g have a unique common fixed point in X provided that the pair (f,g) is weakly compatible.

Proof. Since the pair (f,g) satisfies (CLR_g) property, then there exists a sequence $\{x_n\}$ in X such that $lim_{n\to\infty}M(fx_n,gu,t) = lim_{n\to\infty}M(gx_n,gu,t) = 1$ and $lim_{n\to\infty}N(fx_n,gu,t) = lim_{n\to\infty}N(gx_n,gu,t) = 0$ for some $u \in X$ as $n \to \infty$. Rest of the proof is same as Theorem 3.1.

Now, we introduce common limit in the range property (CLR_{fg}) for a pair of self maps as follow:

Definition 3.1. A pair of self maps (f,g) in an intuitionistic fuzzy metric space $(X,M,N,*,\diamondsuit)$ is said to satisfies common limit in the range property (CLR_{fg}) property, if there exists a sequence $\{x_n\}$ in X such that $lim_{n\to\infty}M(fx_n,gz,t) = lim_{n\to\infty}M(gx_n,gz,t) = 1$ and $lim_{n\to\infty}N(fx_n,gz,t) = lim_{n\to\infty}N(gx_n,gz,t) = 0$ where $fz = gz$ for some $z \in X$.

Example 3.2. Let $(X,M,N,*,\diamondsuit)$ be an intuitionistic fuzzy metric space as in Example 2.1 where $X = [0,\infty)$. Define $f,g : X \to X$ by $fx = \frac{2x}{5}$ and $gx = \frac{x}{5}$ for all $x \in X$. Clearly, for sequence $\{x_n\} = \{\frac{1}{n}\}$, f and g satisfy (CLR_{fg}) property as $lim_{n\to\infty}M(fx_n,g0,t) = lim_{n\to\infty}M(gx_n,g0,t) = 1$ and $lim_{n\to\infty}N(fx_n,g0,t) = lim_{n\to\infty}N(gx_n,g0,t) = 0$ where $f0 = g0$ and $0 \in X$.

Example 3.3. Let $(X,M,N,*,\diamondsuit)$ be an intuitionistic fuzzy metric space as in Example 2.1 where $X = [2,19)$. Define $f,g : X \to X$ by $f2 = 3, fx = 2$ if $x \in (3,19)$, $fx = 15$ if $x \in (2,3]$ and $g2 = 2, gx = \frac{(x+1)}{2}$ if $x \in (3,19), gx = 12$ if $x \in (2,3]$ for all $x \in X$.
Clearly, for sequence $\{x_n\} = \{3+\frac{1}{n}\}$, a pair of self map (f,g) satisfy (CLR_g) property as well as property $(E.A)$ since $lim_{n\to\infty}M(fx_n,g2,t) = lim_{n\to\infty}M(gx_n,g2,t) = 1$ and $lim_{n\to\infty}N(fx_n,g2,t) = lim_{n\to\infty}N(gx_n,g2,t) = 0$ where $g2 = 2 \in X$ but does not satisfy (CLR_{fg}) property as $g2 \neq f2$.
It is interesting to note that that (CLR_{fg})) property imply the (CLR_g)property and (CLR_g) property imply property $(E.A)$ but the reverse implication is not true. So, (CLR_{fg})property is more general than both (CLR_g) property as well as property $(E.A)$ for a pair of self maps.

Theorem 3.3. Let (f,g) be a pair of self maps in intuitionistic fuzzy metric space $(X,M,N,*,\diamondsuit)$ where $*$ is a continuous t-norm and $\diamondsuit$ is a continuous t-conorm satisfying condition (3.1). If the pair (f,g) satisfies the (CLR_{fg}) property then f and g have a unique common

fixed point in X provided that the pair (f,g) is weakly compatible.
Proof Since the pair (f,g) satisfies (CLR_{fg}) property property, then there exists a sequence $\{x_n\}$ in X such that $lim_{n\to\infty}M(fx_n,gu,t) = lim_{n\to\infty}M(gx_n,gu,t) = 1$ and $lim_{n\to\infty}N(fx_n,gu,t) = lim_{n\to\infty}N(gx_n,gu,t) = 0$ where $fu = gu = z(say)$ for some $u \in X$. Rest of the proof is same as Theorem 3.1.
The following example illustrates Theorem 3.3.

Example 3.4. Let $(X,M,N,*,\diamondsuit)$ be an intuitionistic fuzzy metric space where $a*b = a.b$ and $a\diamondsuit b = min\{1,a+b\}$ for all $a,b \in [0,1]$ and $X = [3,19)$. Let $\phi : (0,1] \to (0,1]$ and $\psi : [0,1) \to [0,1)$ defined by $\phi(t) = (t)^{\frac{1}{2}}$ and $\psi(z) = (z)^2$ for all $t \in (0,1]$ and $z \in [0,1)$. Clearly, $\phi \in \Phi$ and $\psi \in \Psi$. Define f and g on X as $fx = 3$ if $x = \{3\} \cup (5,19)$, $fx = 12$ if $x \in (3,5]$, and $g3 = 3$, $gx = 11$ if $x \in (3,5]$, $gx = \frac{x+1}{2}$ if $x \in (5,19)$. Then with sequences $\{x_n\} = \{5+\frac{1}{n}\}$ in X, we have $lim_{n\to\infty}M(fx_n,g3,t) = lim_{n\to\infty}M(gx_n,g3,t) = 1$ and $lim_{n\to\infty}N(fx_n,g3,t) = lim_{n\to\infty}N(gx_n,g3,t) = 0$, where $g3 = f3$. This shows that a pair (f,g) satisfies (CLR_{fg}) property. Also gX is not a closed subset of X. By a routine calculation, one may verify the condition (3.1). Thus, all the conditions of Theorem 3.3 are satisfied and $x = 3$ is the unique common fixed point of f and g. Also, f and g both are discontinuous at a common fixed point $x = 3$ and $f(X) \subsetneq g(X)$.

Remark 3.1. It is worth mentioning here that (CLR_{fg}) property neither require containment nor closedness of underlying subspace for the existence of common fixed for a pair of self map and is weaker than property $(E.A)$ and (CLR_g) property.
Theorem 3.4. Let a,b,f and g be four self maps in an intuitionistic fuzzy metric space $(X,M,N,*,\diamondsuit)$ where $*$ is a continuous t-norm and $\diamondsuit$ is a continuous t-conorm such that
(3.6)

$$\begin{aligned}M(ax,by,t) \geq \phi\{&\min\{M(fx,gy,t),\\ &\sup_{t_1+t_2=\frac{2t}{k}} \min\{M(ax,fx,t_1),M(gy,by,t_2)\},\\ &\sup_{t_3+t_4=\frac{2t}{k}} \max\{M(ax,gy,t_3),M(by,fx,t_4)\}\}\}\end{aligned}$$

and

$$\begin{aligned}N(ax,by,t) \leq \psi\{&\max\{N(fx,gy,t),\\ &\inf_{t_1+t_2=\frac{2t}{k}} \max\{N(ax,fx,t_1),N(gy,by,t_2)\},\\ &\inf_{t_3+t_4=\frac{2t}{k}} \min\{N(ax,gy,t_3),N(by,fx,t_4)\}\}\}\end{aligned}$$

for all $x,y \in X$, $t > 0$ and for some $1 \leq k < 2, \phi \in \Phi, \psi \in \Psi$.
If the pairs (a,f) and (b,g) satisfy the (CLR_{fg}) property, then a,b,f and g have a unique common fixed point in X provided that the pairs (a,f) and (b,g) are weakly compatible.
Proof. The pairs (a,f) and (b,g) satisfy the (CLR_{fg}) property, then there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that
$lim_{n\to\infty}M(ax_n,gu,t) = lim_{n\to\infty}M(fx_n,gu,t) = lim_{n\to\infty}M(by_n,gu,t) = lim_{n\to\infty}M(gy_n,gu,t) = 1$ and $lim_{n\to\infty}N(ax_n,gu,t) = lim_{n\to\infty}N(fx_n,gu,t) = lim_{n\to\infty}N(by_n,gu,t) = lim_{n\to\infty}N(gy_n,gu,t) = 0$ where $fu = gu$ for some $u \in X$.
Firstly we claim that $gu = bu$. Suppose not, then there exists $t_0 > 0$ such that
$M(gu,bu,\frac{2t_0}{k}) > M(gu,bu,t_0)$ and
$N(gu,bu,\frac{2t_0}{k}) < N(gu,bu,t_0)$. (3.7)

The inequality (3.7) is always true when $gu \neq bu$. To support our claim, we suppose on contrary that (3.7) is not true all $t > 0$, i.e., $M(gu,bu,\frac{2t}{k}) = M(gu,bu,t)$ and $N(gu,bu,\frac{2t}{k}) = N(gu,bu,t)$. (3.8)
Now, using equality (3.8) repeatedly, we get $M(gu,bu,t) = M(gu,bu,\frac{2t}{k}) = M(gu,bu,(\frac{2}{k})^2t) = ... = M(gu,bu,(\frac{2}{k})^nt) \to 1$ and $N(gu,bu,t) = N(gu,bu,\frac{2t}{k}) = N(gu,bu,(\frac{2}{k})^2t) = ... = N(gu,bu,(\frac{2}{k})^nt) \to 0$, as $n \to \infty$. This gives, $M(gu,bu,t) = 1$ and $N(gu,bu,t) = 0$ for all $t > 0$. Hence, $gu = bu$, which gives contradiction. Therefore, inequality (3.7) is always true for some $t_0 > 0$.
Using (3.6), take $x = x_n, y = u$, we get

$$\begin{aligned}M(ax_n,bu,t_0) \geq \phi\{&\min\{M(fx_n,gu,t_0),\\ &\sup_{t_1+t_2=\frac{2t_0}{k}} \min\{M(ax_n,fx_n,t_1),M(gu,bu,t_2)\},\\ &\sup_{t_3+t_4=\frac{2t_0}{k}} \max\{M(ax_n,gu,t_3),M(bu,fx_n,t_4)\}\}\}\end{aligned}$$

and

$$\begin{aligned}N(ax_n,bu,t_0) \leq \psi\{&\max\{N(fx_n,gu,t_0),\\ &\inf_{t_1+t_2=\frac{2t_0}{k}} \max\{N(ax_n,fx_n,t_1),N(gu,bu,t_2)\},\\ &\inf_{t_3+t_4=\frac{2t_0}{k}} \min\{N(ax_n,gy_n,t_3),N(bu,fx_n,t_4)\}\}\}.\end{aligned}$$

Let $t_1 = t_3 = \varepsilon$ then $t_2 = t_4 = \frac{2t_0}{k} - \varepsilon$ where $\varepsilon \in (0,\frac{2t_0}{k})$, and $n \to \infty$, we get

$$\begin{aligned}M(gu,bu,t_0) &\geq \phi\{M(gu,bu,\frac{2t_0}{k}-\varepsilon)\}\\ &> M(gu,bu,\frac{2t_0}{k}-\varepsilon)\end{aligned}$$

and

$$\begin{aligned}N(gu,bu,t_0) &\leq \psi\{N(gu,bu,\frac{2t_0}{k}-\varepsilon)\}\\ &< N(gu,bu,\frac{2t_0}{k}-\varepsilon).\end{aligned}$$

As $\varepsilon \to 0$, we get $M(gu,bu,t_0) \geq M(gu,bu,\frac{2t_0}{k})$ and $N(gu,bu,t_0) \leq N(gu,bu,\frac{2t_0}{k})$
which gives contradiction, hence $gu = bu$. Next, we show that $au = gu$. Suppose not, then again as done above,

there exists $t_0 > 0$ such that $M(au,gu,\frac{2t_0}{k}) > M(au,gu,t_0)$ and $N(au,gu,\frac{2t_0}{k}) < N(au,gu,t_0)$. (3.9)

Using (3.6), take $x = u, y = y_n$, we get

$$M(au,by_n,t_0) \geq \phi\{\min\{M(fu,gy_n,t_0), \sup_{t_1+t_2=\frac{2t_0}{k}} \min\{M(au,fu,t_1),M(gy_n,by_n,t_2)\}, \sup_{t_3+t_4=\frac{2t_0}{k}} \max\{M(au,gy_n,t_3),M(by_n,fu,t_4)\}\}\}$$

and

$$N(au,by_n,t_0) \leq \psi\{\max\{N(fu,gy_n,t_0), \inf_{t_1+t_2=\frac{2t}{k}} \max\{N(au,fu,t_1),N(gy_n,by_n,t_2)\}, inf_{t_3+t_4=\frac{2t_0}{k}} \min\{N(au,gy_n,t_3),N(by_n,fu,t_4)\}\}\}.$$

Let $t_2 = t_4 = \varepsilon$ then $t_1 = t_3 = \frac{2t_0}{k} - \varepsilon$ where $\varepsilon \in (0,\frac{2t_0}{k})$, and $n \to \infty$, we get

$$M(au,gu,t_0) \geq \phi\{M(au,gu,\frac{2t_0}{k} - \varepsilon)\} > M(au,gu,\frac{2t_0}{k} - \varepsilon)$$

and

$$N(au,gu,t_0) \leq \psi\{N(au,gu,\frac{2t_0}{k} - \varepsilon)\} < N(au,gu,\frac{2t_0}{k} - \varepsilon).$$

As $\varepsilon \to 0$, we get
$M(au,gu,t_0) \geq M(au,gu,\frac{2t_0}{k})$
and
$N(au,gu,t_0) \leq N(gu,bu,\frac{2t_0}{k})$
which gives contradiction, hence $au = gu$. So, $au = bu = fu = gu = z$ (say). Since the pair (a,f) is weakly compatible, $afu = fau$ and then $az = fz$. Similarly, as the pair (b,g) is weakly compatible, $bgu = gbu$ and then $gz = bz$.

Next, we claim that $az = z$, suppose not. Then by (3.6), take $x = z, y = u$, we get

$$M(az,bu,t_0) \geq \phi\{\min\{M(fz,gu,t_0), \sup_{t_1+t_2=\frac{2t_0}{k}} \min\{M(az,fz,t_1),M(gu,bu,t_2)\}, \sup_{t_3+t_4=\frac{2t_0}{k}} \max\{M(az,gu,t_3),M(bu,fz,t_4)\}\}\}$$

and

$$N(az,bu,t_0) \leq \psi\{\max\{N(fz,gu,t_0), \inf_{t_1+t_2=\frac{2t_0}{k}} \max\{N(az,fz,t_1),N(gu,bu,t_2)\}, \inf_{t_3+t_4=\frac{2t_0}{k}} \min\{N(az,gu,t_3),N(bu,fz,t_4)\}\}\}.$$

Let $t_1 = t_3 = \varepsilon$ then $t_2 = t_4 = \frac{2t_0}{k} - \varepsilon$ where $\varepsilon \in (0,\frac{2t_0}{k})$, $n \to \infty$ and $\varepsilon \to 0$, we get

$M(az,z,t_0) \geq M(z,az,t_0)$
and
$N(az,z,t_0) \leq N(z,az,t_0)$

a contradiction, hence, $az = bz = z$. Therefore, z is a common fixed point of a and b. Similarly, we prove that $fz = gz = z$ by taking $x = u, y = z$ in (3.6). Therefore, we conclude that $z = az = bz = fz = gz$, this implies that a,b,f and g have common fixed point in X. For uniqueness: we can easily prove uniqueness of fixed point of a,b,f and g by using (3.6).

Finally, we conclude this paper by furnishing example to demonstrate the validity of Theorem 3.4 besides exhibiting its superiority over earlier relevant results.

Example 3.5. Let $(X,M,N,*,\diamondsuit)$ be an intuitionistic fuzzy metric space where $a*b = a.b$ and $a\diamondsuit b = min\{1,a+b\}$ for all $a,b \in [0,1]$ and $X = [3,19)$. Let $\phi : (0,1] \to (0,1]$ and $\psi : [0,1) \to [0,1)$ defined by $\phi(t) = (t)^{\frac{1}{2}}$ and $\psi(z) = (z)^2$ for all $t \in (0,1]$ and $z \in [0,1)$. Clearly, $\phi \in \Phi$ and $\psi \in \Psi$. Define a,b,f and g by
$ax = 1$ if $x \in \{1\} \cup (3,15)$, $ax = x+11$ if $x \in (1,3]$, $bx = 1$ if $x \in \{1\} \cup (3,15)$, $bx = x+5$ if $x \in (1,3]$, $f1 = 1$, $fx = 6$ if $x \in (1,3]$, $fx = \frac{x+1}{4}$ if $x \in (3,15)$ and $g1 = 1$, $gx = 11$ if $x \in (1,3]$, $gx = x-2$ if $x \in (3,15)$.

Take $\{x_n\} = \{y_n\} = \{3+\frac{1}{n}\}$, clearly
$lim_{n\to\infty}M(ax_n,g1,t) = lim_{n\to\infty}M(fx_n,g1,t) = lim_{n\to\infty}M(by_n,g1,t) = lim_{n\to\infty}M(gy_n,g1,t) = 1$ and $lim_{n\to\infty}N(ax_n,g1,t) = lim_{n\to\infty}N(fx_n,g1,t) = lim_{n\to\infty}N(by_n,g1,t) = lim_{n\to\infty}N(gy_n,g1,t) = 0$

where $f1 = g1$ for some $1 \in X$. Thus, (a,f) and (b,g) satisfies CLR_fg property. Also, $aX = \{1\} \cup (12,14]$, $bX = \{1\} \cup (6,8]$, $fX = [1,4) \cup \{6\}$, $gX = (1,13)$ and condition (3.6) is satisfied by maps a,b,f and g. Thus, the maps a,b,f and g satisfy all conditions of Theorem 3.4. Hence, a,b,f and g have a unique common fixed point $x = 1$. Moreover it should be noted that aX,bX,fX and gX are not closed subspaces of X. Also, $aX \subsetneq gX$ and $bX \subsetneq fX$. Also, a,b,f and g are all discontinuous maps at a common fixed point $x = 1$.

Remark 3.2. Sedghi et al. [29] proved a common fixed point theorem for a pair of weakly compatible self maps using containment and closedness of subspace. Our Theorem 3.1 intuitionistically fuzzify their result without containment of subspace. Moreover, in Theorem 3.2 and Theorem 3.3 we have even removed the closedness of subspace and in Theorem 3.4 we extended their result to two pairs of self maps without using containment and closedness of subspace. Also it is interesting to note that self maps are discontinuous. Hence, our results intuitionistically fuzzify, improve and extend the similar results existing in literature without containment, continuity and closedness requirement of subspaces for a pair as well as two pairs of self maps.

Remark 3.3. It is interesting to point out here that Theorems 3.1, 3.2, 3.3 and 3.4 still remain valid if weak compatibility requirement of a pair self maps for the existence of common fixed point is replaced by any one of the following: (i) R-weakly commuting property,
(ii) pointwise R-weakly commuting maps,
(ii) R-weakly commuting property of type (A_g),
(iii) R-weakly commuting property of type (A_f),
(iv) R-weakly commuting property of type (P),
(v) compatiblity of type (A),
(vi) compatiblity of type (B),
(vii) compatibility of type (C),
(viii) compatibility of type (P),
(ix) subcompatibility,
(x) occasionally weak compatibility,
and several others weaker forms of commutativity existing in literature. Actually all these notions coincide with weak compatibility in the presence of unique point of coincidence of underlying self maps yet all these are distinct from each other. For development of weaker forms of commuting maps and relationship between them one may refer to Singh et al. [31] and Murthy [24].

4 Conclusion

Motivated by the applications of intuitionistic fuzzy metric space in population dynamics, chaos control, computer programming, medicine etc. , we framed suitable conditions to ensure the existence of common fixed point in intuitionistic fuzzy metric space for a discontinuous pairs of self mappings by introducing the new notion - common limit in the range property (CLR_{fg}) for a pair of self mappings and thereby improving and intuitionistically fuzzifing the results of Sedghi et al. [29]. It is interesting to note that our Examples cannot be covered by all those coincidence and common fixed point theorems which require containment of range space and continuity of involved pair of self mappings along with completeness (or closedness) of underlying space/subspace. It is worth mentioning here that several authors claimed to have introduced some weaker notions of commuting mappings, weak compatibility is still the minimal and the most widely used notion among all weaker notions of commutativity.

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