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Broadcasting Communication in High Degree Modified Chordal Rings Networks

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Abstract: The design of the interconnection network is one of the main research issues in distributed computing with regard to some specific parameters. This paper works on Modified Chordal Rings Degree Six (CHRm6) topology. Two broadcasting schemes for CHRm6 are proposed. The first scheme is for even source nodes and the second scheme is for odd source nodes. The proposed broadcasting schemes give upper and lower bound of broadcasting in CHRm6 depends on total number of nodes. We prove the lower bound on the broadcast time is $d+2$ where d is the diameter of the CHRm6.

Keywords: Chordal rings, interconnection, broadcasting, topology, delay, broadcast time.

1 Introduction

Interconnection networks can be modeled by graphs and graph theory plays an important field due to this representation graphs. There are a number of network topologies have been studied before such as ring, mesh, torus and hypercube. Every topology has specific goals and measured by some parameters. Chordal ring communication is one of the most efficient communication patterns for parallel interconnection. It has been discussed in various stages of development. In particular, chordal ring is an undirected circulant graph where nodes represent the processing elements and edges represent links between processing elements. In this paper we focus on processing elements as multiprocessors.

Chordal rings are attractive interconnection network topologies due to their simple structure and their short diameter. Other than diameter, there are some key features of interest such as degree, connectivity, structures, congestion, asymmetric and routing [1]. Routing algorithm is important to determine the set of paths that message may follow the approach of switching to identify the mode of operation and the mechanism to direct network resource distribution. Efficient routing algorithms must be based on simple and without routing tables or complex arbitration protocols [2], [3].

This paper is devoted to the broadcasting problems on Modified Chordal Rings Degree Six (CHRm6). Broadcasting is a process of disseminating a message that originating at originator node of an undirected graph to be delivered to all nodes in the network [4]. The interconnection network property is closely related to the problem of dissemination of message and is an interesting research area [3], [4]. The effective dissemination the message between multiprocessors is significant to ensure the best communication topology for parallel computing. Every node plays an important role in the network and was assuming that holds a piece of message.

In this paper, we look at the preliminaries of CHRm6 in Section 2. Broadcasting schemes were proposed in Section 3. Section 4 proved the boundary of broadcasting time for CHRm6. We make our conclusion and work recommendation in Section 5.

2 Preliminaries

Arden *et al*. [5] proposed the first Chordal Rings Degree 3 (CR3) while the first modified topology for higher degree

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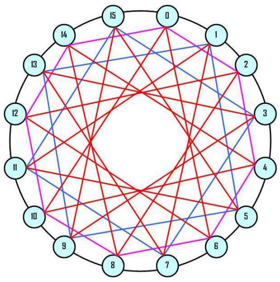

Fig. 1: CHRm6(16,1,2,4,6).

was analyzed by [6]. Definition 1 described about the connectivity of CHRm6 topology. An even nodes and odd nodes have different connection due to the Definition 1 as in Fig. 1.

Definition 1. *The modified degree six chordal rings called CHRm6 is an undirected circulant graphs. CHRm6 is denoted by* $CHRm6(N,s,h_1,h_2,h_3)$ *where* N *is the number of nodes,* s *is a ring edge with length 1, while* h_1,h_2 *and* h_3 *are chords by even lengths where* $h_1 < h_2 < h_3$*. CHRm6 consists of one ring with* N *nodes, where* N *is positive even number of nodes. Each even node,* i_{2k} *and odd node,* i_{2k+1} *are additionally connected to four nodes for* $0 \leq k < N/2$*.* i_{2k} *is connected to* $i_{(2k-h_1)(modN)}$ *,*$i_{(2k+h_1)(modN)}$ *,* $i_{(2k-h_3)(modN)}$ *and* $i_{(2k+h_3)(modN)}$*.* i_{2k+1} *is connected to* $i_{(2k+1-h_2)(modN)}$*,* $i_{(2k+1+h_2)(modN)}$*,* $i_{(2k+1-h_3)(modN)}$ *and* $i_{(2k+1+h_3)(modN)}$*. The values of* N *and* h_1*,* N *and* h_2*,* N *and* h_3 *must have* $\gcd(N,h_1,h_2,h_3) = 2$*.*

3 The Broadcasting Schemes for CHRm6

The minimum number units required to complete broadcasting from i is denoted as b(i) where i is a source node ($i_o$is for even source node and $i_1$is for odd source node) where $i_0,i_1 \in i$. The nodes are divided into two classes to yields symmetry.

Without loss of generality, it is assumed that node 0 is the originator for even source node and node 1 is the originator for odd source node. The maximum broadcast time of any node of CHRm6 is called the broadcast time, $b(CHRm6)$. The two following schemes are for node i_o (Scheme A) and for node i_1 (Scheme B) [4]. Broadcasting scheme for node i_1(Scheme A):

Scheme A: Even Source Node

1. The node 0 sends the message to the nodes in order of $+s,-s,+h_1,-h_1$ and $-h_3$.
2. A node that receives a message from $-s$ sends the message to $+s,+h_2,-h_2$ and $-h_3$.
3. A node that receives a message from $+s$ sends the message to $-s,+h_2$ and $-h_2$.
4. A node that receives a message from $-h_1$ sends the message to $-s$ and $-s$.
5. A node that receives a message from $+h_1$ sends the message to $+s$.

Scheme B: Odd Source Node

1. The node 1 sends the message to the nodes in order of $+s,-s,+h_2,-h_2$ and $-h_3$.
2. A node that receives a message from $-s$ sends the message to $+s,+h_1,-h_1$ and $-h_3$.
3. A node that receives a message from $+s$ sends the message to $-s,+h_2$ and $-h_2$.
4. A node that receives a message from $-h_2$ sends the message to $-s$ and $-s$.
5. A node that receives a message from $+h_2$ sends the message to $+s$.

These broadcast schemes A and B are implemented for the entire networks of CHRm6. The optimum free-table routing algorithms are implemented into these broadcasting schemes with condition that it will skip the path that is redundant and it will take the first path that it found [2]. It is based on layers shortest paths [3]. Fig. 2 and Fig. 3 show the illustration of broadcasting schemes in general for CHRm6.

We used the atomic structure as in Fig.4 and Fig.5 to show the above schemes completes the broadcast time at $d+2$. The broadcast model stated that a call involves only one informed and one uninformed node. However, there might be cases that two informed nodes might try to inform the same node because the above schemes do not check either the node that is being message forwarded already informed or not. There are three cases on how the message is forwarded [7].

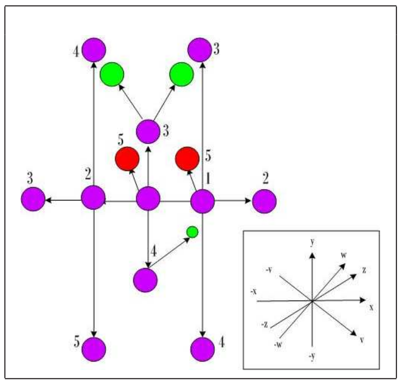

Fig. 2: Illustration of Broadcast Scheme A

Case 1. Let node i finished informing its neighbours and there is a new node i' to inform i again at time t. i' does not make the call to inform i then skip to the next node on its schedule.

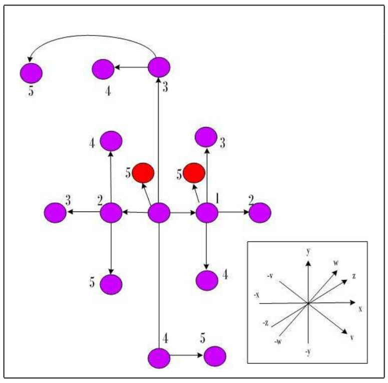

Fig. 3: Illustration of Broadcast Scheme B

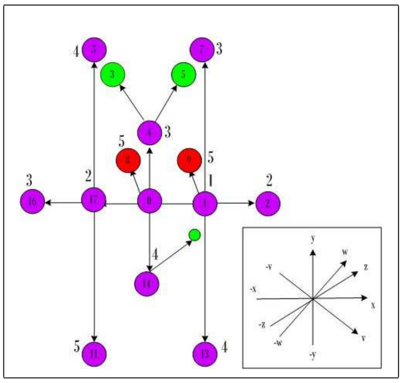

Fig. 4: Illustration of Broadcast Scheme A for CHRm6 (18,1,4,6,8)

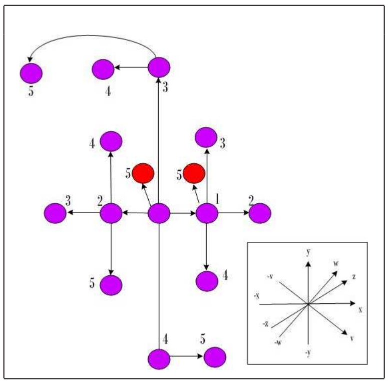

Fig. 5: Illustration of Broadcast Scheme B for CHRm6 (18,1,4,6,8)

Case 2. Let node i is informs but it is still in the process of informing its neighbours. Another node i' is considering to inform i at time t. Let a be the link through which i would inform its neighbor at time $t+1$. Let b be the link where i' would send message to i. There are three cases has to take into account. First if $|a| < |b|$ or $a = -b$, then i does not change its policy of forwarding message and i does not inform i. If $|a| > |b|$ then i continue its policy of forwarding message at time $t+1$ based on the Scheme A and Scheme B and assume that it is informed by i' at time t, but i' does not send the message to $a = b$ then i and i' skip forwarding the message to each other (referred to redundant step) and proceed with the next step on their message forwarding schedule.

Case 3. Let two or more nodes try to inform another node i. Let L_a be the set of links that would inform i simultaneously. Only one link $l_j \in L_a$ is chosen to inform i, if $|l| < |l_i|$for all $l_i \in L_a$ where $i = j$. If i is already informed, the node that informing i is considered as $i' = i + l_j$. If there are two links l_j and $-l_j$ such that $\left|l_j\right| < |l_i|$ for all values of $i = j$, thus either one of them is chosen. Both of the schemes imply that all the nodes $i \pm s$ or $i \pm h$ where $h_1, h_2, h_3 \in h$ are informed when the node i forwards the message to its neighbours. There is nothing to be done if a node is informed and finished forwarding the message to all of its uninformed neighbours.

Theorem 1 explains the scenario of the informed node but is still in active is attempted to be informed again. If a node was considering to inform i via the link a at time t then all the neighbors $i+e$ of i where $|e| > |a|$ are all informed by time $t+1$ the latest.

Theorem 3.1. When a node i receives a message via a at time t then the nodes b where $|b| > |a|$, will be informed at the latest by the time $t+1$.

Proof. Let the node i is informed at time t and $i+c$ where $|c| > |a|$ is not informed at $t+1$. Since i receives the message through the link a then the node $j = i+a$ is the sender of the message. Node j on its turn receives the message from j' through link a' such that $|a'| \geq |a|$. There should be a node that got informed via link c' such that $c' = |c|$ along the path of the nodes which inform each other ending in j and i. Assume this node as x.

Let $\sum$ is the series of calls that starts at x, informs i in T time units by following a path P of length L, path P takes a node x to i.

Path P takes a node y to $y+i-x$. If $c = -c$ then it is assumed that x_1 and x_2 be two nodes such that $x_1 - c = x$ and $x - c' = c_2$. Clearly since x is informed by its c' then x_1 is the node that informs x. After being informed, x first informs x_2 by its $-c'$. Since x and x_2 are informed by their c' link then the same series of calls are done by both of them. It took T time units for x to inform i after being informed. Since x_2 is informed one time unit after x is informed then it will take $T+1$ time unit before $x_2 + i - x = (x - c') + i - x$ is informed. Note that $x - c' + i - x = i - c'$ but since $c = -c'$, the node $i+c$ will be informed by the time $T+1$. This is contradicts the initial assumption.

If $c' = c$ then let x_1 be the node such that $x_1 - c' = x$. Thus x_1 is the one who informs x. But it is not known through which node x_1 is informed, this imply that x_1 might not inform the node $x_1 + i - x = c$ in T time units after being informed. The order in which x uses its links depends on the link through which it is informed. If x is informed by link e such that $ce > 0$(i.e both links had the same sign) then after x_1 is informed and after it forwards the message through $-c'$ the order of the links that c_1 will use will be the same as that of x. Hence, T time units after x_1 is informed, the node $x_1 + i - x$ will be informed. If $ce < 0$ (i.e the links have different signs) then after $x_1$sends the message via $-c'$ the order of the remaining links used by x_1 will not be the same e as that of x. There can be a maximum of 2 time units difference between the time at which x_1 forwards the message along the same link a. Given that x_1 is informed one time unit before x and adding the possible 2 time units difference. We conclude that the node $x_1 + i - x = x + c' + i - x = i + c' = i + c$ will be informed at most one time unit after i is informed. This is again contradicts the initial assumption.

4 Boundary for CHRm6 Broadcasting Time

4.1 Lower Bound for CHRm6 Broadcasting Time

The lower bound of the broadcast time of CHRm6 which has at least 14 nodes that are all at a distance d from a source node, i (even source node or odd source node). Our results are based on theorem 2 that will be proved.

Theorem 4.1. The broadcast time of CHRm6, $b(CHRm6 \geq 2)$ at distance d from the originator,i

Proof. Let we assume that $d+1$ time unit will inform $d+2$ nodes. There can be only one node, i_d at distance d from the originator, i_o. Let $P = \{i_{0,} i_1, \ldots, i_d\}$ be the path from i_o to i_d. Note that there are $d+1$ nodes on this path. Furthermore, node i_a got informed at time a and informed node $i_{(a+1)}$ at time $a+1$. The same node at time $a+2$ can inform a new node which can start a chain of calls which can inform a node at distance d from i_0 at the time $d+1$. Since there are $d+1$ nodes on the path P, then there can be exactly $d+1$ nodes which can be informed at time $d+1$. Therefore it can be $d+2$ informed nodes at time $d+1$.

The shortest distance between source node (i_o for even source node and i_1 for odd source node) and destination node j in the optimal graph of CHRm6 can be found only by examining the shortest distance between i_o to j and i_1 to j. This is because CHRm6 is symmetric when we divide into two classes of nodes (even and odd nodes).

Theorem 4.2. A lower bound on the broadcast time of CHRm6 is $d+2$ where d is distance.

Proof. According to Fig. 4 and Fig. 5, the number of nodes at distance d from i_o or i_1 is much more than $d+3$. It is concluded that $d+2$ is a lower bound on the broadcast time of CHRm6.

4.2 An Upper Bound on CHRm6 Broadcasting Time

The proposed broadcasting schemes give the upper bound of broadcast in CHRm6 which is depends on N. The optimal CHRm6 can be constructed based on N and thus the upper bound of the broadcast time will be determined depends on maximum distance it can go towards achieve optimality.

5 Conclusion

This paper presents the broadcast problems from theoretical aspect. The atomic structure of CHRm6 is used to obtain the results in broadcasting. The broadcast schemes assure that every node in CHRm6 is informed by the time at least $d+2$ and above. The broadcast schemes are based on N. This theory concept is based on no nodes or links failure. Future research may deal with nodes or links failure.

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