Journal of Statistics Applications & Probability
*An International Journal*

http://dx.doi.org/10.18576/jsap/090102

# Control Charts based on Exponentiated Mukherjee-Islam Distribution

*Amin Shaka Aunali* [1] [*], *Venkatesan D.* [1] *and Michele Gallo* [2]

[1] Department of Statistics, Annamalai University, India.
[2] Department of Social Science, University of Naples, L' Orientale, Italy

Received: 8 May 2019, Revised: 2 Nov. 2019, Accepted: 11 Nov. 2019.
Published online: 1 Mar. 2020.

**Abstract:** Manufacturing involves three stages: production, inspection and packaging. Production is performed, as follows: (i) Products of each component are inspected by sub-components working in parallel, and (ii) products inspected by each sub-component are packaged by a series of sub-sub-components. Control charts are beneficial for monitoring process statistics over time and detecting special causes which create variations that can be detected and verified. However, if the control chart indicates that the process has shifted, appropriate actions must be taken to regain control. Thus, a control chart indicates the right time or which observation corrective action should be taken. Utilizing of control charts designed for a normal distribution may be ineffective and increases the number of non-conforming products.
**Keywords:** Control Charts, Exponentiated Distributions, Mukherjee-Islam Distribution.

## 1 Introduction

It is impossible to inspect or test product quality which must be high from first stage. This requires that production process must be stable and that all persons involved in this process (i.e. operators, engineers, quality control personnel and administration) constantly strive to promote it and reduce the variability. Monitoring the online statistical process is an important tool to achieve this goal.

When a product meets or exceeds the customer's expectations, it must be stable and frequent. Specifically, the process must be able to operate with little variation in the target values or the nominal dimensions of the product's quality characteristics. SPC is a powerful set of tools that achieve the process stability and improve capability by reducing variability. Many real-life situations can be represented as a series-serial parallel system, i.e. a multi-component system, with each component having parallel components operating in parallel, each component comprising a series of sub-sub-components. Manufacturing comprises three phases: production, inspection and packaging. Production is performed, as follows: (i) Products of each component can be controlled by sub-components that work in parallel, and the products controlled by each sub-component are packed by several secondary sub-components.

Control charts are beneficial for monitoring process statistics over time and detecting special causes which create variations that can be detected and verified. However, if the control chart indicates that the process has shifted, appropriate actions must be taken to regain control. Thus, a control chart indicates the right time or which observation corrective action should be taken. The two control limits, i.e. the upper control limits (UCL) and the lower control limit (LCL), are used to monitor the process mean or variance. They reduce defective products and increase profits. Quality maintenance using control charts makes the industry well-reputed. Prof. Shewhart used control charts to monitor processes when quality adopted a normal distribution. However, several researchers developed different types of control charts using different types of distributions. Amin and Venkatesan (2017) compared of Bayesian method and classical charts and revealed small shifts in the control charts. In 2019, they discussed the recent developments in control charts techniques. Amin and Venkatesan (2019) proposed SPC using transmuted generalized uniform distribution.

Santiago and Smith (2013) proposed the control charts, i.e. the t-chart, when the time between events follows the exponential distribution. They used the variable transformation presented by Nelson (1994) to transform exponentially distributed data to an approximate normal data. Aslam *et al*. (2014) proposed a new control charts for the exponential

[*]Corresponding author e-mail: aunali473@gmail.com

distribution using the transformed variable and the repetitive sampling. Control charts are designed on the assumption that the quantitative trait of interest follows the normal distribution, which is not always the case in practice. The variable of interest may follow some non-normal distributions, such as an exponential distribution or gamma distribution or any other distribution. Utilizing control charts designed for normal distribution may not be workable in this situation and may increase the number of non-conforming products. In addition, the normal distribution is applied in situations where data are collected in subgroups, so that the central limit theorem can be applied when designing the control charts. In practice, it is impossible to collect data in groups. In many practical situations, classical distributions offer insufficient adjustment for real data. For example, if the data are asymmetric, the normal distribution is a bad option. Consequently, different generators have been proposed based on one or more parameters to generate new distributions. Adding the parameters, the distribution becomes more comprehensive and flexible for data modeling. The concept of two parameters to generate new distributions has been used. Thus, the exponentiated exponential distributions have been introduced by Gupta *et al*. (1998) where the family has two parameters, i.e. scale and shape. Some properties of the distribution were investigated by Gupta and Kundu (2001b) who observed that various properties of the new family are similar to those of Weibull or Gamma family. Gupta and Kundu (2001a, 2002) also examined the estimation and inference aspects of the distribution. Hassan (2005) proposed a class of goodness-of-fit tests for the distribution with estimated parameters. Pal *et al*. (2006) addressed the exponentiated Weibull family as an extension of the Weibull and exponentiated exponential families.

In the present paper, techniques of Exponentiated distributions are used for the Mukherjee-Islam distribution, which was introduced by Mukherjee and Islam (1983). Aafaq and Subramanian (2018) handled Exponentiated Mukherjee-Islam Distribution which is a finite range distribution and preferred to be used over more complex distributions such as Normal, Beta, Weibull, and other distributions, because of its simple form and easy use. Several distributions can be used to monitor the process control. The present paper demonstrates that control charts have been derived using the Exponentiated Mukherjee-Islam distribution to monitor the process. The organization of this paper is as follows:

Description of the distribution is provided in section Two. Section Three presents performance measures of the Exponentiated Mukherjee Islam distribution. Control Limits using the Exponentiated Mukherjee Islam distribution are addressed in section Four. A numerical example is covered in section Five. Section Six is dedicated to conclusion.

## 2 Description of the Distribution

### *2.1 Definition*

A random variable $X$ has an exponentiated distribution if its cumulative distribution function (cdf) is given by

$$F_\alpha = \left[H(x)\right]^\alpha; \qquad x \in R^1,\ \theta > 0 \tag{1}$$

with pdf of $X$ is given by

$$f_\alpha = \alpha\left[H(x)\right]^{\alpha-1} h(x)$$

### *2.2 Definition*

The density function (pdf) of Mukherjee Islam distribution is

$$h(x) = \frac{k}{\theta^k}; \qquad 0 < x < \theta;\ k > 0;\ \theta > 0 \tag{2}$$

and the respective cumulative distribution function (cdf) is

$$H(x) = \left(\frac{x}{\theta}\right)^k; \qquad 0 < x < \theta;\ k > 0;\ \theta > 0 \tag{3}$$

Where $k$ is the shape parameter and $\theta$ is the scale parameter.

Using equation (3) in (1), one can get the cdf of Exponentiated Mukherjee Islam distribution as follows:

$$F_\alpha(x) = \left[\left(\frac{x}{\theta}\right)^k\right]^\alpha; \qquad p,\ \theta > 0 \tag{4}$$

With its exponentiated density function

$$f_\alpha(x) = \frac{\alpha k x^{\alpha k-1}}{\theta^{\alpha k}}; \qquad \alpha > 0 \tag{5}$$

## 3 Performance measures of the Exponentiated Mukherjee Islam Distribution

From the above-mentioned exponentiated density function (p.d.f), one can derive the r[th] moment $E(X^r)$ of a Exponentiated Mukherjee-Islam distribution, so

$$\begin{aligned} E(X^r) &= \int_0^\theta x^r f(x)dx \\ &= \int_0^\theta x^r \frac{\alpha k x^{\alpha k-1}}{\theta^{\alpha k}} dx \end{aligned}$$

After simplifications, one can obtain

$$E(X^r) = \frac{\alpha k \theta^r}{r+\alpha k} \tag{6}$$

One can get the mean and second moment from equation (6) by substituting r=1 and r=2 respectively which are given by:

$$E(X) = \frac{\alpha k \theta}{1+\alpha k} \tag{7}$$

Then the second moment is given by

$$E(X^2) = \frac{\alpha k \theta^2}{2+\alpha k} \tag{8}$$

Hence, from equations (7) and (8), the variance $V(X)$ is given by

$$\begin{aligned} V(X) &= E(X^2) - \left[E(X)^2\right] \\ &= \frac{\alpha k \theta^2}{(2+\alpha k)} - \left[\frac{\alpha k \theta}{(1+\alpha k)}\right]^2 \\ &= \alpha k \theta \left[\frac{\theta}{(\alpha k+2)} - \frac{\alpha k \theta}{(\alpha k+1)^2}\right] \end{aligned}$$

Therefore,

$$V(X) = \alpha k \theta^2 \left[\frac{1}{(\alpha k+2)} - \frac{\alpha k}{(\alpha k+1)^2}\right] \tag{9}$$

## 4 Control Limits using the Exponentiated Mukherjee Islam Distribution

A typical control chart has several values and control limit sets. Thus, when the process is under control, almost all points are within the upper control (UCL) and lower (LCL) limits [Duncan (1986) and Montgomery (2012)]. Therefore, from equations (7) and (9), the control limits are given by

$$Upper\ Control\ Limit\ (UCL) = \frac{\alpha k \theta}{1+\alpha k} + 3\theta \sqrt{\alpha k \left[ \frac{1}{(\alpha k + 2)} - \frac{\alpha k}{(\alpha k + 1)^2} \right]}$$

$$Centre\ Line\ (CL) = \frac{\alpha k \theta}{1+\alpha k}$$

$$Lower\ Control\ Limit\ (LCL) = \frac{\alpha k \theta}{1+\alpha k} - 3\theta \sqrt{\alpha k \left[ \frac{1}{(\alpha k + 2)} - \frac{\alpha k}{(\alpha k + 1)^2} \right]}$$

Where; $0 < x < \theta;\ \ k > 0;\ \ \theta > 0$

## 5 Numerical Illustrations

An example regarding the construction of control limits is considered for illustrating the applications of the proposed method. The control limits of the Exponentiated Mukherjee-Islam distribution are obtained using simulated data set for parameters *k*, *θ* and *α* (i.e. random variables). All the generated samples are reported in Table 1.

**Table 1.** Control limits using Exponentiated Mukherjee Islam Distribution.

| **θ** | **k** | **α=1** | | | **α=2** | | | **α=5** | | | **α=10** | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | **CL** | **LCL** | **UCL** | **CL** | **LCL** | **UCL** | **CL** | **LCL** | **UCL** | **CL** | **LCL** | **UCL** |
| 1 | 3 | 0.75 | 0.17 | 1.33 | 0.86 | 0.49 | 1.23 | 0.94 | 0.76 | 1.11 | 0.97 | 0.87 | 1.06 |
| | 5 | 0.83 | 0.41 | 1.26 | 0.91 | 0.66 | 1.16 | 0.96 | 0.85 | 1.07 | 0.98 | 0.92 | 1.04 |
| | 7 | 0.88 | 0.54 | 1.21 | 0.93 | 0.75 | 1.12 | 0.97 | 0.89 | 1.05 | 0.98 | 0.94 | 1.03 |
| | 9 | 0.90 | 0.63 | 1.17 | 0.95 | 0.8 | 1.1 | 0.98 | 0.91 | 1.04 | 0.99 | 0.96 | 1.02 |
| | 10 | 0.91 | 0.66 | 1.16 | 0.95 | 0.82 | 1.09 | 0.98 | 0.92 | 1.04 | 1 | 0.96 | 1.02 |
| 2 | 3 | 1.5 | 0.34 | 2.66 | 1.71 | 0.97 | 2.46 | 1.88 | 1.52 | 2.23 | 1.93 | 1.75 | 2.12 |
| | 5 | 1.67 | 0.82 | 2.51 | 1.82 | 1.32 | 2.32 | 1.92 | 1.70 | 2.15 | 1.96 | 1.84 | 2.08 |
| | 7 | 1.75 | 1.09 | 2.41 | 1.87 | 1.49 | 2.24 | 1.94 | 1.78 | 2.11 | 1.97 | 1.89 | 2.06 |
| | 9 | 1.80 | 1.26 | 2.34 | 1.89 | 1.59 | 2.19 | 1.96 | 1.83 | 2.08 | 1.98 | 1.91 | 2.04 |
| | 10 | 1.82 | 1.32 | 2.32 | 1.90 | 1.63 | 2.18 | 1.96 | 1.85 | 2.08 | 1.98 | 1.92 | 2.04 |
| 5 | 3 | 3.75 | 0.84 | 6.65 | 4.29 | 2.43 | 6.14 | 4.69 | 3.81 | 5.57 | 4.84 | 4.37 | 5.31 |
| | 5 | 4.17 | 2.05 | 6.28 | 4.55 | 3.30 | 5.79 | 4.81 | 4.25 | 5.36 | 4.9 | 4.61 | 5.2 |
| | 7 | 4.38 | 2.72 | 6.03 | 4.67 | 3.73 | 5.60 | 4.86 | 4.46 | 5.27 | 4.93 | 4.72 | 5.14 |
| | 9 | 4.5 | 3.14 | 5.86 | 4.74 | 3.99 | 5.49 | 4.89 | 4.57 | 5.21 | 4.95 | 4.78 | 5.11 |
| | 10 | 4.55 | 3.30 | 5.79 | 4.76 | 4.08 | 5.44 | 4.90 | 4.61 | 5.19 | 4.95 | 4.80 | 5.1 |
| 10 | 3 | 7.5 | 1.69 | 13.31 | 8.57 | 4.86 | 12.28 | 9.38 | 7.61 | 11.13 | 9.68 | 8.74 | 10.61 |
| | 5 | 8.33 | 4.11 | 12.56 | 9.09 | 6.60 | 11.58 | 9.61 | 8.50 | 10.72 | 9.80 | 9.23 | 10.38 |
| | 7 | 8.75 | 5.44 | 12.06 | 9.33 | 7.46 | 11.2 | 9.72 | 8.91 | 10.53 | 9.86 | 9.44 | 10.28 |
| | 9 | 9 | 6.29 | 11.71 | 9.47 | 7.98 | 10.97 | 9.78 | 9.14 | 10.42 | 9.89 | 9.56 | 10.27 |
| | 10 | 9.1 | 6.60 | 11.58 | 9.52 | 8.16 | 10.89 | 9.80 | 9.23 | 10.38 | 9.9 | 9.61 | 10.2 |

Table 1 shows that the fixed value of the parameter *θ* and *α*, the control limits increase whenever the parameter *k* increases. In addition, when parameter *θ* with fixed parameter *α* increases, the control limits increase. The Exponentiated Mukherjee Islam (EMI) Control Chart is shown in Fig. 1 for $\theta = 5$, $\alpha = 2$ and $k$=3.

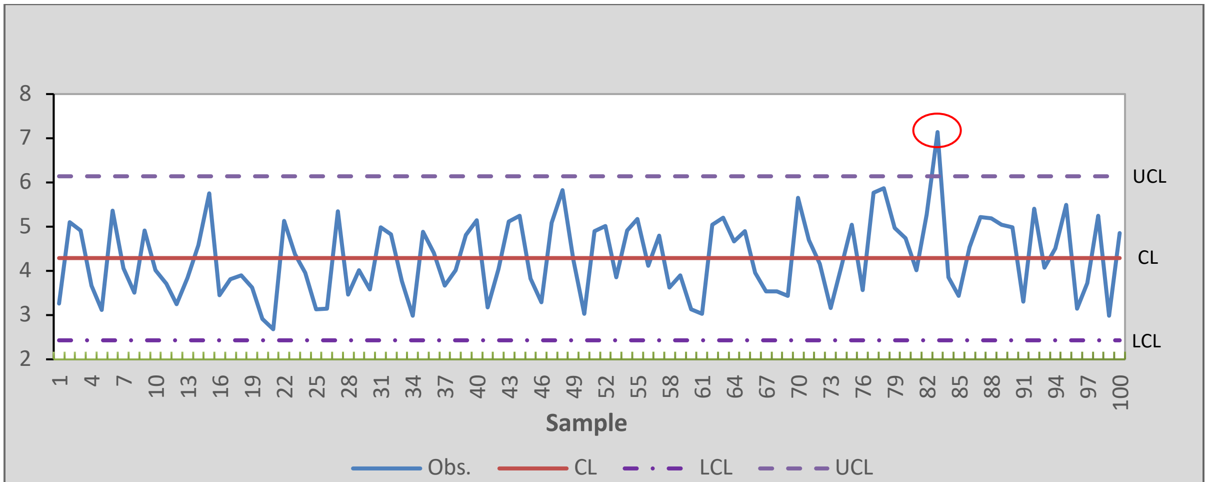


**Fig. 1: EMI Control Chart with k=3, θ=5 and α=2**

Moreover, the observations of the process control must be $0 < x < \theta$; $k > 0$; $\theta > 0$ otherwise, the process will be out of control. That is based on manufacturing products. Manufacturing engineers should fix the parameter's value based on what type of data they are working with. Table 1 reveals that the more the parameters $k$ and $\theta$ increase, the higher control limits.

## 6 Conclusion

In this article, the process control is developed using an Exponentiated Mukherjee Islam Distribution. A novel algorithm is given for sentencing the process while manufacturing. The Control limits are given for the Exponentiated Mukherjee Islam Distribution with different values of parameters $k$, $\theta$ and $\alpha$. Table is constructed to help to select the parameters based on the type of data the manufacturing engineer handles. The control chart is drawn by considering the parameters $k$=3; $\theta$=5 and $\alpha$= 2, where the 83rd observation is shown out of control because it is greater than the value of parameter $\theta$. Hence it is advisable to use process data less than the value of parameter $\theta$.

**Acknowledgement:** Sincere thanks go to the referees and Editor for their positive and insightful comments which improve the paper.

## References

[1] A. R. Aafaq and C. Subramanian. Exponentiated Mukherjee-Islam Distribution. *Journal of Statistics Applications & Probability, An International Journal*, **7 (2)**, 357-361 (2018).

[2] A. S. Hassan. Goodness-of-fit for the generalized exponential distribution. *InterStat, Electronic Journal*, 1-15 (2005).

[3] D. C. Montgomery, Introduction to Statistical Quality Control, *Wiley*, New York, 7th ed., (2012).

[4] S. P. Mukherjee and A. Islam, A finite-range distribution of finite times. *Naval Research Logistics Quarterly*, **30 (3)**, 487-491 (1983).

[5] L. S. Nelson. The Shewhart control chart-tests for special causes. *Journal of Quality Technology*, **16**, 238-239 (1984).

[6] L. S. Nelson. A control chart for parts-per-million nonconforming items. *Journal of Quality Technology*, **26**, 239-240 (1994).

[7] M. Pal; M. M. Ali and J. Woo. Exponentiated Weibull Distribution. *Statistica*, **66 (2)**, 139-147 (2006).

[8] E. Santiago and J. Smith. Control Charts Based on the Exponential Distribution: Adapting Runs Rules for the t Chart. *Quality Engineering*, **25 (2)**, 85-96 (2013).

[9] S. A. Amin and D. Venkatesan. Comparison of Bayesian Method and Classical Charts in Detection of Small Shifts in the Control Charts. *International Journal of Operations Research and Optimization,* **8 (1)**, 23-35 (2017).

[10] S. A. Amin and D. Venkatesan. Recent Developments in Control Charts Techniques. *Universal Review Journal*, **8 (4)**, 746-756, (2019).

[11] S. A. Amin and D. Venkatesan. SPC Using Transmuted Generalized Uniform Distribution. *Journal of Information and Computational Science,* 9 (8), 332-339 (2019).

[12] M. Aslam; M. Azam and C. H. Jun. A new exponentially weighted moving average sign chart using repetitive sampling. *Journal of*

*Process Control*, **24 (7)**, 1149-1153 (2014).

[13] A. J. Duncan. Quality Control and Industrial Statistics. Irwin, Homewood, IL, 5th ed., (1986).

[14] R. C. Gupta; P. L. Gupta and R. D. Gupta. Modeling failure time data by Lehman alternatives. *Communications in Statistics, Theory and Methods*, **27 (4)**, 887-904 (1998).

[15] R. D. Gupta and D. Kundu. Exponentiated Exponential Family: An Alternative to Gamma and Weibull. *Biometrical Journal*, **43 (1)**, 117-130 (2001a).

[16] R. D. Gupta and D. Kundu. Generalized exponential distribution: different method of estimations. *Journal of Statistical Computation and Simulation*, **69**, 315-338 (2001b).

[17] R. D. Gupta and D. Kundu, Generalized Exponential Distribution: Statistical Inferences. *Journal of Statistical Theory and Applications*, **1**, 101-118 (2002).

**Amin Shaka Aunali,** is young PhD research scientist in Statistics at Annamalai University, India. His research interests are in the areas of SQC, Queueing Theory, Bayesian Inference, Probability Theory and Design of Experiment. He has published research articles in reputed international and national journals.

**Venkatesan D.,** is currently working as a Professor in Statistics, Annamalai University, India. His research interests are in the areas of Time Series Analysis, Bayesian Inference, SQC. He has published 45 research articles in reputed international and national journals.

**Michele Gallo**, is currently working as a Professor of Statistics, Department of Social Science, University of Naples – L' Orientale, Italy. His research interests are in the areas of Stochastic Process and its Application, Time Series, SQC, Biostatistics, Multidimensional data analysis. He has published research articles in reputed international and national journals.