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Soft Photons and the Range of Electromagnetic Interactions in a Medium composed of Particles and Antiparticles

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Abstract: The minimum mass for soft photons is given in terms of the particle-anti-particle pairs in the medium. The physical relevance of the corresponding interaction range is verified.

Keywords: soft photons, wavelength, scaling, gravi-electromagnetic soliton.

1 Introduction

Interplanetary distances are known to satisfy a geometric rule initially known as Bode's law. The base of the geometric progression is a clear indication that wave motion equally causes the distances to form this progression as the laws of classical mechanics. It is evident that wavelengths of standing waves may be spaced by these distances and that a freezing of the configuration would result in the orbits of the solar system over a long interval. Secondary corrections to the geometric progression then place the planets at the nodes representing the aggregation of matter within a gravitational potential.

With the development of string theory to describe the fundamental nature of interactions, there exist solutions to the wave equation that define the motion essentially at microscopic scales. A semiclassical process of particle-antiparticle creation and annihilation yields times that are near to that predicted by the uncertainty principle, t_{He} [1]. A precise estimate of the duration of the splitting of the paths of the electron and positron represented by string paths yields two different times for virtual and real processes [2]. The duration of the real process is found to exceed t_{He}, and therefore, the electron and positron would form a real electromagnetic couple. The range of the interaction, however, is microscopic, and the large scaling of this effect is necessary.

A scaling factor which produces the inteplanetary distances is derived from a gravi-electromagnetic solitons that satisfies coupled gravitational and electromagnetic field equations [3,4]. The range of the gravitational and electromagnetic forces is infinite, and, therefore, it would be expected that these forces would describe much of the phenomena at astronomical distances. The infinite range of the force is equivalent to the masslessness of the photon. It is known, however, that the zero mass of the photon causes infrared divergences in quantum electrodynamics. This problem can be resolved by adding a Stückelberg term to the Lagrangian or introducting a lower limit to the soft photon mass. This mass then would correspond to a range of the interaction. If it is set equal to the combined mass of the electron and positron, a microscopic range results, and a scaling is found to be required for interplanetary distances. A much lower mass would yield the range at astronomical scales, although it is not related to the Planck mass.

2 Soft Photons

The origin of the Coulomb potential in electromagnetic interactions follows from the inverse Fourier transform of the photon propagator $\frac{g^{\mu\nu}}{q^2}$, since

$$\int dq \frac{g^{\mu\nu}(e\varepsilon_\mu^*)\,(e\varepsilon_\nu)}{q^2} e^{-iq\cdot x} = \varepsilon^* \cdot \varepsilon \frac{e^2}{r} = \frac{e^2}{r} \qquad (2.1)$$

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with $\varepsilon^* \cdot \varepsilon = 1$. The range of validity of the potential seems to exceed that of the interactions requiring the propagator. The low-momentum modes correspond to long-range modes of the wave propagator. The consistency of the $\frac{1}{r}$ potential depends on the solution to problems with infrared divergences in quantum electrodynamics.

The differential elements in the average energy and number densities are

$$\begin{aligned} d\overline{\mathscr{E}} &= k^0(|J_1(k)|^2 + |J_2(k)|^2)d\tilde{k}d\overline{n} \\ &= \frac{d\overline{\mathscr{E}}}{\hbar k^0} = \frac{1}{\hbar}(|J_1(k)|^2 + |J_2(k)|^2)d\tilde{k} \end{aligned} \qquad ((2.2))$$

and

$$\begin{aligned} \tilde{n} &= \frac{1}{\hbar}\int d\tilde{k}(|J_1(k)|^2 + |J_2(k)|^2) \\ &= \frac{1}{\hbar}\int \frac{d^3k}{(2\pi)^3 2k^0}(|J_1(k)|^2 + |J_2(k)|^2) \end{aligned} \qquad (2.3)$$

diverges because the integrand is singular in the limit $k^0 \to 0$.

The larger number of soft photons, however, supports the model of an amorphous amount of matter accumulating at large distances. The infrared divergence can be regulated by introducing an infinitesimal mass in the Stückelberg gauge. Since

$$\begin{aligned} \overline{n} &= -\int d^4x d^4y [A_{in}^{(-)}(x)j(x), A_{in}^{(+)}(y)j(y)] \\ &= -\int \frac{d^3k}{(2\pi)^3 2k^0} J(k)J^*(k)|_{k^0=\sqrt{k^2+m^2}} \end{aligned} \qquad (2.4)$$

and $J \cdot J^* \sim \frac{1}{k_0^2} = \frac{1}{(|\mathbf{k}|^2+m^2)^{\frac{1}{2}}}$ for an accelerated charge,

$$\begin{aligned} \overline{n} &\sim \int_0^{k_{max}} \frac{dkk^2}{(k^2+\mu^2)^{\frac{3}{2}}} \sim ln\,\frac{k_{max}}{\mu}\overline{\mathscr{E}} \\ &\sim \int_0^{k_{max}} \frac{dkk^2}{k^2+\mu^2} \sim k_{max} - \frac{\mu\pi}{2} \end{aligned} \qquad (2.5)$$

such that $lim_{\mu\to 0}\overline{\mathscr{E}} = k_{max}$ and $lim_{\mu\to 0}\overline{n} = \infty$.

Although the photon is massless, the energy of a distribution of photons should have a lower bound. This lower bound would be determined by momentum conservation and the masses of the charged particles emitting and absorbing these photons. Conservation of momentum for internal loops does not prevent an infinite range of the momentum variable in the integral. However, an electron-positron pair has the combined rest mass 1.022 MeV/c^2, and, in a vacuum polarization graph with photon lines between fermion loops, the minimum photon energy can be set equal to 1.022 MeV. Again, the probability would be maximized for soft photons while the divergence is removed. The length scale corresponding to the energy is

$$\frac{\hbar c}{1.022\ MeV} = \frac{0.197\ GeV - fm}{1.022\ \times 10^{-3} GeV} = 1.927 \times 10^{-13} m. \qquad (2.6)$$

The scaling factor necessary for planetary distances may be introduced through an exponential coefficient in a gravi-electromagnetic soliton metric.

A lower limit for the photon energy such that the range of the electromagnetic interaction reached astronomical distances would be $1.443258 \times 10^{-18} eV$, which is less than the rest energy of any known particle. Furthermore, there is no connection between this value and Planck scale physics, and a theoretical basis for the minimum energy would have to be developed.

3 Magnification of Microscopic Scales to Astronomical Distances.

The semiclassical time for the process [1] of the splitting and joining of the paths of the electron and positron is found to be

$$t_{ve}^{(1)} = \frac{R_{max}}{\left[1+\frac{\rho_*^2}{8\left(\frac{g}{2}\right)^2}\right]^{\frac{1}{3}}}\int_0^1 \frac{d\tilde{\zeta}}{\sqrt{1-\tilde{\zeta}^{\frac{3}{2}}}} + \frac{R_{max}}{c}\int_{\left[1+\frac{\rho_*^2}{8\left(\frac{g}{2}\right)^2}\right]^{-\frac{1}{3}}}^1 \frac{d\zeta}{\sqrt{1-\zeta^{\frac{3}{2}}}} \qquad (3.1)$$

where R_{max} is the maximum distance between the particle and antiparticle during the interval, $\rho_* = \frac{R_{max}}{ct_{He}}$ and g is the Landé g factor. The value of ρ_* had been taken to be nearly $\frac{1}{2}$ in a previous derivation [2]. The value of ρ_* is based on the length of the expected paths of the electron and positron. A distribution of paths would yield a range of values from nearly zero to $ct_{path\ max}$ such that the time required for the entire process is less than t_{He}. The distance covered by light during the first half of the process would be $c\frac{t_{He}}{2}$, which would bound the distance travelled by the electron or positron. The use of this distance is supported the mediation by a photon in quantum electrodynamics. Then $R_{max} \approx c\frac{t_{He}}{2}$, $\rho_* \approx \frac{1}{2}$ and $1 + \frac{\rho_*^2}{8\left(\frac{g}{2}\right)^2} \approx 1.0625$. It is shown that

$$\begin{aligned} t_{ve}^{(1)} &\doteq 2\frac{R_{max}}{c}\int_0^1 \frac{d\tilde{\zeta}}{\sqrt{1-\tilde{\zeta}^{\frac{3}{2}}}}\left[\frac{1}{(1.0625)^{\frac{1}{3}}}\right. \\ &\qquad \left. + \frac{4}{3}\int_0^{arc\ sin\left(\left(1-\frac{1}{(1.0625)^{\frac{1}{2}}}\right)^{\frac{1}{2}}\right)} (cos\ \theta)^{\frac{1}{3}}\right] \\ &= (1.0625)^{\frac{1}{3}}(2.71788 \times 10^{-22} sec)\left[\frac{1}{(1.0625)^{\frac{1}{3}}} + \frac{4}{3}\frac{0.173374}{1.7247}\right] \\ &= (1.0625)^{\frac{1}{3}}(2.71788 \times 10^{-22} sec)(1.1140399) \\ &\simeq 3.0894897 \times 10^{-22} sec \end{aligned} \qquad (3.2)$$

where the factor of $(1.0625)^{\frac{1}{3}}$ arises in the new expression for R_{max}, the maximum separation between the electron and the positron.

The string amplitude represents the creation and annihilation of an electron-positron pair if the time for the completion of one loop is is less than the Heisenberg time of $\frac{\frac{\hbar}{2}}{2m_ec^2}$. A larger average value for the splitting and joining of the paths of the electron and positron follows a weighted sum of times for higher-genus trajectories with the coefficients given by the absolute squares of the amplitudes, which would equal

$$\tilde{\alpha}_1 t_{ve}^{(1)} + \left(\frac{1}{24.3497748}\right)^2 (2t_{ve}^{(1)}) + \left(\frac{1}{24.3497748}\right)^3 (3t_{ve}^{(1)}) + \dots \tag{3.3}$$

where the sum of the coefficients equals one. Since

$$\tilde{\alpha}_1 = \left[\frac{1-\frac{2}{(24.3497748)^2}}{1-\frac{1}{(24.3497748)^2}}\right]^{\frac{1}{2}} = 0.9991548 \tag{3.4}$$

Then

$$\begin{aligned} t_{ve} &= (\tilde{\alpha}_1^2 + 0.00338337144)t_{ve}^{(1)} \quad = 1.0016938\, t_{ve}^{(1)} \\ &= 3.0947226 \times 10^{-22}\, sec. \end{aligned} \tag{3.5}$$

The sum of the weighted times for a real process of splitting and joining of the electron and positron paths would be

$$\alpha_1 t_{ve}^{(1)} + \frac{1}{24.3497748}(2t_{ve}^{(1)})\frac{1}{(24.3497748)^2}(3t_{ve}^{(1)}) + \dots \tag{3.6}$$

with the sum of the coefficients being one, such that

$$\alpha_1 = \frac{1-\frac{2}{24.3497748}}{1-\frac{1}{24.3497748}} = 0.9571730344911 \tag{3.7}$$

It follows that

$$\begin{aligned} t_{ve} &= (0.9571730344911 + 0.08748807999248)t_{ve}^{(1)} \\ &\doteq (1.04466111)t_{ve}^{(1)} = 3.2274697 \times 10^{-22}\, sec. \end{aligned} \tag{3.8}$$

The duration of the real process is greater than the Heisenberg time, t_{He}, and the splitting and joining of the paths would produce a string network rather than a virtual amplitude.

A distinction arises between the Compton wavelength of an electron-positron pair, the length scale derived from the inverse of the minimum photon energy resulting from regularization of infrared divergence and the extent of bounded surfaces in a string network. The inverse of the infimum of the soft photon energy may be related to the extent of a genus-four surface because there is a resolution to the problem of infrared divergences through string theory.

The magnification of the microscopic distances that may be achieved in a gravi-electromagnetic soliton is precise. Given a pair annihilation length of $4.796 \times 10^{-14} cm$, planetary distances can be reached after a scaling by a factor of 2.9087×10^{26}. The gravi-electomagnetic soliton metric,

$$\begin{aligned} ds^2 &= f(t,z)(dz^2 - dt^2) + g_{ab}dx^a dx^b f(t,z) \\ &= \frac{D_1}{\sqrt{q_0 sinh\, t\, cosh\, z}} e^{k^2 q_0^2 sinh^2 t cosh^2 z} \\ D_1 &= (sinh\, t\, cosh\, \sigma + v_0^2 cosh\, t)^2 + (cosh\, z\, sin\, \tau - v_0 sinh\, z)^2 \\ \sigma &= 2kq_0 sinh\, z - 2s_0 \tau = 2kq_0 cosh\, t - 2t_0 + \frac{\pi}{2} \\ g_{11} &\sim cosh\, z(cosh(sinh\, z))^2 e^{2kq_0 cosh\, t\, sinh\, z} \qquad z \gg 1 \\ g_{12} &\sim cosh\, z\, (cosh(sinh\, z))^2 e^{-2kq_0 cosh\, t sinh\, z} \qquad z \gg 1 \\ g_{22} &\sim cosh\, z\, (cosh(sinh\, z)) \qquad z \gg 1. \end{aligned} \tag{3.9}$$

and the radial distance $\mathscr{R}$, such that the electromagnetic potential decreases as $\frac{1}{\mathscr{R}}$, is given by $e^{kq_0 sinh\, z}$.[3,4]. The value of z yielding the scaling factor is $z \approx 4.1098 - ln(kq_0)$ [2]. Therefore, it represents a surface of genus four for strings on the hadronic scale. A greater weighting for surfaces of this genus has been found in a study of an analytic representation of superdiffeomorphisms of super-Riemann surfaces, for which there exists a measure for universal Teichmüller space [5]. These results support a model of the space-time manifold consisting of string networks [6].

4 Conclusion

Three distances are presented for setting the range of the gravitational and electromagnetic interactions that would govern the motion of the planets in the solar system. The removal of the infrared divergence in quantum electrodynamics requires a lower limit to the mass which causes the range of the interaction to be finite. This mass might be selected to be the combined mass of the electron and the positron, yielding a Compton wavelength of $1.97 \times 10^{-13} m$. A magnification by 7.0812×10^{25} gives a standard interplanetary distance. The pair annihilation length in a central magnetic field would be $4.796 \times 10^{-14} cm$ and a scaling by 2.9087×10^{26} is necessary.

Infrared divergences are removed in superstring theory and reappear in the field theory limit. The size of the superstring serves as a regulator of the infinity. Although surfaces of fixed genus g formed by strings of the Planck scale would yield multiple of $\ell_P = 1.6 \times 10^{-33} cm$, hadronic strings with a size of $\mathscr{O}(10^{-15} m)$ would give distances of $\mathscr{O}(g \cdot 10^{-15} m)$.

The scaling factor in the metric of the gravi-electromagnetic soliton expands the range to astronomical distances. Each of the three distances can be increase by a factor of approximately the same order. Beginning with superstring theory, the third mechanism requires a scaling from Planck lengths to hadronic distances. A theoretical explanation follows from a study of the independence of the cut-off on the genus of Riemann surfaces contributing to the physical processes with respect to conformal transformations.

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