Applied Mathematics & Information Sciences Letters
An International Journal

Gravitational Effect on Plane Waves in Generalized Thermo-microstretch Elastic Solid under Green Naghdi Theory

Mohamed I. A. Othman[1], *Sarhan. Y. Atwa*[1,2], *A. Jahangir*[3] *and A. Khan*[3]

[1]Faculty of Science, Department of Mathematics, Zagazig University, P.O. Box 44519, Zagazig, Egypt.
[2]Higher Institute of Engineering, Dep. of Eng. Math. and Physics, Shorouk Academy, Shorouk City, Egypt.
[3]Department of Mathematics, COMSATS, Institute of Information Technology, Islamabad, Pakistan.

Received: 4 Mar. 2013, Revised: 17 Mar. 2013, Accepted: 28 Mar. 2013
Published online: 1 May. 2013

Abstract: The present paper is aimed at studying the effect of gravity on the general model of the equations of the generalized thermo-microstretch for a homogeneous isotropic elastic half-space solid whose surface is subjected to a Mode-I crack problem. The problem is in the context of the Green and Naghdi theory (GN). The normal mode analysis is used to obtain the exact expressions for the displacement components, the force stresses, the temperature, the couple stresses and the microstress distribution. The variations of the considered variables through the horizontal distance are illustrated graphically. Comparisons are made with the results in the presence and absence of gravity with two cases: Case (1) for the generalized micropolar thermoelasticity elastic medium (without microstretch constants) between the both types (II, III). Case (2) for the generalized micropolar thermoelasticity elastic medium (without micropolar constants) between the both types II and III.

Keywords: Green and Naghdi theory, thermoelasticity, gravity, microstretch, Mode-I crack

1 Introduction

The linear theory of elasticity is of paramount importance in the stress analysis of steel, which is the commonest engineering structural material. To a lesser extent, the linear elasticity describes the mechanical behavior of the other common solid materials, e.g. concrete, wood and coal. However, the theory does not apply to study the behavior of many of the newly synthetic materials of the elastomer and polymer type, e.g. polymethyl-methacrylate (Perspex), polyethylene and polyvinyl chloride. The linear theory of micropolar elasticity is adequate to represent the behavior of such materials. For ultrasonic waves i.e. in the case of elastic vibrations characterized by high frequencies and small wavelengths, the influence of the body microstructure becomes significant. This influence of microstructure results in the development of new type of waves, which are not in the classical theory of elasticity. Metals, polymers, composites, solids, rocks, concrete is typical media with microstructures. More generally, most of the natural and man-made materials including engineering, geological and biological media possess a microstructure. Bhattacharyya and De [1], De and Sengupta [2] observed the effect of gravity in elastic media. Agarwal [3,4] studied respectively thermoelastic and magneto-thermoelastic plane wave propagation in an infinite non-rotating medium. Ailawalia [5,6] studied the gravitational effect along with the rotational effect on generalized thermo-elastic and generalized thermoplastic medium with two temperatures respectively. Mahmoud [7] discussed the effect of gravity on granular medium. Abed-Alla and Mahmoud [8] investigated the effect of gravity in magneto-thermo-viscoelastic media, and Sethi and Gupta [9] discussed the gravity effect in a thermo-viscoelastic media of higher order. These problems are based on the more realistic elastic model since earth; moon and all other planets have the strong gravitational effect.

Eringen and ?uhubi [10] and Eringen [11] developed the linear theory of micropolar elasticity. Eringen [12] introduced the theory of microstretch elastic solids. That

* Corresponding author e-mail: m_i_othman@yahoo.com

theory is a generalization of the theory of micropolar elasticity [11,12,13,14] and a special case of the micromorphic theory. The material points of microstretch elastic solids can stretch and contract independent of their transformations and gravities. The microstretch is used to characterize composite materials and various porous media [15]. The theory of thermo-microstretch elastic solids was introduced by Eringen [15]. The basic results in the theory of microstretch elastic solids were obtained in the literature [15,16,17,18]. An extensive review of the theory of thermo-microstretch elastic solids can be found in the book of Eringen's book [15]. In the framework of the theory of thermo-microstretch solids Eringen established a uniqueness theorem for the mixed initial-boundary value problem. This investigation was illustrated through the solution of one dimensional waves and comparing with lattice dynamical results. The asymptotic behavior of the solutions and an existence result were presented by Bofill and Quintanilla [19]. A reciprocal theorem and a representation of Galerkin type were presented by De Cicco and Nappa [20]. De Cicco and Nappa [21] extended the linear theory of thermo-microstretch elastic solids to permit the transmission of heat as thermal waves at finite speed. In Ref. [21], the uniqueness of the solution of the mixed initial-boundary-value problem is also investigated. The study is based on the entropy production inequality proposed by Green and Laws [22]. The coupled theory of thermoelasticity has been extended by including the thermal relaxation time in the constitutive equations by Lord and Shulman [23] and Green and Lindsay [24]. These theories eliminate the paradox of infinite velocity of heat propagation and are termed generalized theories of thermo-elasticity. Green and Naghdi [25,26] proposed another three models, which are subsequently referred to as GN-I, II and III models. The linearized version of model-I corresponds to the classical thermoelastic model-II the internal rate of production of entropy is taken to be identically zero implying no dissipation of thermal energy. This model admits un-damped thermoelastic waves in a thermoelastic material and is best known as the theory of thermo-elasticity without energy dissipation. Model-III includes the previous two models as special cases, and admits dissipation of energy in general. Othman and Song [27] studied the effect of rotation on the reflection of magneto-thermoelastic waves under thermoelasticity II. Othman and Song [28] investigated the reflection of plane waves from an elastic solid half-space under hydrostatic initial stress without energy dissipation. The normal mode analysis was used to obtain the exact expression for the temperature distribution, the thermal stresses and the displacement components. In the recent years, considerable efforts have been devoted to the study of failure and cracks in solids. This is due to the application of the latter generally in the industry and particularly in the fabrication of electronic components. Most of the studies of dynamical crack problem are done using the equations from coupled or even uncoupled theories of thermoelasticity [29,30]. This is suitable for most situations, in which longtime effects are sought. However, when short time behavior is important, as in many practical situations, the full system of generalized thermoelastic equations must be used [15]. The purpose of the present paper is to obtain the normal displacement, the temperature, the normal force stress and the tangential couple stress in a microstretch elastic solid under the effect of gravity. The problem of generalized thermo-microstretch in an infinite space weakened by a finite linear opening Mode-my crack is solved for the above variables. The distributions of the considered variables are represented graphically. A comparison of the temperature, the stresses and the displacements are carried out between the two types II, III for the propagation of waves in a semi-infinite microstretch elastic solid in the presence and absence of gravity.

2 Formulation of the Problem

We obtain the constitutive and the field equations for a linear isotropic generalized thermo-micro-stretch elastic solid in the absence of body forces. We use a rectangular coordinate system(x,y,z)having originated on the surface $y=0$ and$z-$ axis pointing vertically into the medium. The basic governing equations of linear generalized thermo-elasticity with gravity in the absence of body forces and heat sources are

$$(\lambda+\mu)(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 w}{\partial x\partial z})+(\mu+k)(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial z^2})-k\frac{\partial\phi_2}{\partial z}$$
$$\lambda_0\frac{\partial\phi^*}{\partial x}-\hat{\gamma}\frac{\partial T}{\partial x}+\rho g\frac{\partial w}{\partial x}=\rho\frac{\partial^2 u}{\partial t^2} \quad (1)$$

$$(\lambda+\mu)(\frac{\partial^2 u}{\partial x\partial z}+\frac{\partial^2 w}{\partial z^2})+(\mu+k)(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial z^2})+k\frac{\partial\phi_2}{\partial x}$$
$$+\lambda_0\frac{\partial\phi^*}{\partial z}-\hat{\gamma}\frac{\partial T}{\partial z}-\rho g\frac{\partial u}{\partial x}=\rho\frac{\partial^2 w}{\partial t^2}, \quad (2)$$

$$(\alpha+\beta+\gamma)\nabla(\nabla.\phi)-\gamma\nabla\times(\nabla\times\phi)+k(\nabla\times\mathbf{u})-2k\phi$$
$$=j\rho\frac{\partial^2\phi}{\partial t^2}, \quad (3)$$

$$\alpha_0\nabla^2\phi^*-\frac{1}{3}\lambda_1\phi^*-\frac{1}{3}\lambda_0(\nabla.\mathbf{u})+\frac{1}{3}\hat{\gamma}_1T=\frac{3}{2}\rho j\frac{\partial^2\phi^*}{\partial t^2}, \quad (4)$$

$$K\nabla^2T+K^*\nabla^2\dot{T}=\rho C_E\ddot{T}+\hat{\gamma}T_0\ddot{u}_{i,i}+\hat{\gamma}_1T_0\frac{\partial\phi^*}{\partial t}, \quad (5)$$

$$\sigma_{il}=(\lambda_0\phi^*+\lambda u_{r,r})\delta_{il}+(\mu+k)u_{l,i}+\mu u_{i,l}-k\varepsilon_{ilr}\phi_r$$
$$-\hat{\gamma}T\delta_{il}, \quad (6)$$

$$m_{il}=\alpha\phi_{r,r}\delta_{il}+\beta\phi_{i,l}+\gamma\phi_{l,i}, \quad (7)$$

$$\lambda_i = \alpha_0\, \phi^*_{,i} \tag{8}$$

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{9}$$

where T is the temperature above the reference temperature T_0 such that $\left|(T-T_0)/T_0\right| < 1, \lambda,\ \mu$ are the counterparts of Lame parameters, the components of displacement vector u are u_i, t is the time, σ_{ij} are the components of stress tensor, e, is the dilatation, e_{ij} are the components of strain tensor, j the micro inertia moment, k,α,β,γ are the micropolar constants, $\alpha_0, \lambda_0, \lambda_1$ are the microstretch elastic constants, ϕ^* is the scalar microstretch, ϕ is the rotation vector, m_{ij} is the couple stress tensor, δ_{ij} is the Kronecker delta, ε_{ijr} is the alternate tensor, the mass density is ρ, the specific heat at constant strain is C_E, the thermal conductivity is $K(\geq 0)$ and K^* material characteristic of the theory. The state of plane strain parallel to the xz-plane is defined by

$$u_1 = u\,(x,z,t), u_2 = 0, u_3 = w(x,z,t), \phi_1 = \phi_3 = 0,$$
$$\phi_2 = \phi_2(x,z,t), \phi^* = \phi^*(x,z,t), and \quad \Omega = (0,\Omega,0), \tag{10}$$

where,

$$\hat{\gamma} = (3\lambda\ +\ 2\mu\ +\ k)\,\alpha_{t_1}, \hat{\gamma}_1 = (3\lambda\ +\ 2\mu\ +\ k)\,\alpha_{t_2}$$
$$and\ \ \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \tag{11}$$

The constants $\hat{\gamma}$ and $\hat{\gamma}_1$ depend on the mechanical as well as the thermal properties of the body and the dot denote the partial derivative with respect to time, $\alpha_{t_1}, \alpha_{t_2}$ are the coefficients of linear thermal expansions. The constitutive relation can be written as

$$\sigma_{xx} = \lambda_0\phi^* + (\lambda + 2\mu + k)\,\frac{\partial u}{\partial x} + \lambda\frac{\partial w}{\partial z} - \hat{\gamma}T, \tag{12}$$

$$\sigma_{zz} = \lambda_0\phi^* + (\lambda + 2\mu + k)\,\frac{\partial w}{\partial z} + \lambda\frac{\partial u}{\partial x} - \hat{\gamma}T, \tag{13}$$

$$\sigma_{xz} = \mu\frac{\partial u}{\partial z} + (\mu + k)\,\frac{\partial w}{\partial x} + k\phi_2, \tag{14}$$

$$\sigma_{zx} = \mu\frac{\partial w}{\partial x} + (\mu + k)\,\frac{\partial u}{\partial z} + k\phi_2, \tag{15}$$

$$m_{xy} = \gamma\frac{\partial \phi_2}{\partial x}, \tag{16}$$

$$m_{zy} = \gamma\frac{\partial \phi_2}{\partial z}, \tag{17}$$

For convenience, the following non-dimensional variables are used:

$$\bar{x}_i = \frac{\omega^*}{c_2}\, x_i,\ \bar{u}_i = \frac{\rho c_2 \omega^*}{\hat{\gamma}T_0}\, u_i,\ \bar{t} = \omega^* t,\ \ \bar{T} = \frac{T}{T_0},$$
$$\bar{\sigma}_{ij} = \frac{\sigma_{ij}}{\hat{\gamma}T_0},\ \bar{m}_{ij} = \frac{\omega^*}{c_2\hat{\gamma}T_0} m_{ij},\ \bar{\phi}_2 = \frac{\rho c_2^2}{\hat{\gamma}T_0}\,\phi_2,$$
$$\bar{\lambda}_3 = \frac{\omega^*}{c_2\hat{\gamma}T_0}\lambda_3, \bar{\phi}^* = \frac{\rho\, c_2^2}{\hat{\gamma}T_0}\,\phi^*, \omega^* = \frac{\rho\, C_E c_2^2}{K^*},$$
$$\bar{g} = \frac{g}{c_2\omega^*}, c_2^2 = \frac{\mu}{\rho}. \tag{18}$$

Using equation (18) then, equations (1)-(5) become (dropping the dashed for convenience)

$$\frac{\partial^2 u}{\partial t^2} = \frac{(\mu+k)}{\rho\, c_2^2}\nabla^2 u + \frac{(\mu+\lambda)}{\rho\, c_2^2}\frac{\partial e}{\partial x} - \frac{k}{\rho\, c_2^2}\frac{\partial\, \phi_2}{\partial z}$$
$$+ \frac{\lambda_0}{\rho\, c_2^2}\frac{\partial \phi^*}{\partial x} - \frac{\partial T}{\partial x} + g\frac{\partial w}{\partial x} \tag{19}$$

$$\frac{\partial^2 w}{\partial t^2} = \frac{(\mu+k)}{\rho\, c_2^2}\nabla^2 w + \frac{(\mu+\lambda)}{\rho\, c_2^2}\frac{\partial\, e}{\partial z} + \frac{k}{\rho\, c_2^2}\frac{\partial\, \phi_2}{\partial x}$$
$$+ \frac{\lambda_0}{\rho\, c_2^2}\frac{\partial\, \phi^*}{\partial z} - \frac{\partial T}{\partial z} - g\frac{\partial u}{\partial x} \tag{20}$$

$$\frac{j\rho\, c_2^2}{\gamma}\frac{\partial^2\phi_2}{\partial t^2} = \nabla^2\phi_2 - \frac{2\,k\, c_2^2}{\gamma\,\omega^*}\phi_2 + \frac{k c_2^2}{\gamma\,\omega^{*2}}(\frac{\partial\, u}{\partial\, z} - \frac{\partial\, w}{\partial\, x}), \tag{21}$$

$$(\frac{c_3^2}{c_2^2}\nabla^2 - \frac{c_4^2}{\omega^{*2}} - \frac{\partial^2}{\partial t^2})\,\phi^* - \frac{c_5^2}{\omega^{*2}}\, e + a_9 T = 0, \tag{22}$$

$$\varepsilon_2\,(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}) + \varepsilon_3(\frac{\partial^2 \dot{T}}{\partial x^2} + \frac{\partial^2 \dot{T}}{\partial z^2}) = \ \ \ddot{T}\ + \varepsilon_1\,\ddot{e} + \varepsilon_4\frac{\partial\, \phi^*}{\partial t} \tag{23}$$

where

$$c_3^2 = \frac{2\alpha_0}{3\rho\, j}, c_4^2 = \frac{2\lambda_1}{9\rho\, j}, c_5^2 = \frac{2\lambda_0}{9\rho\, j},\ \varepsilon_1 = \frac{\hat{\gamma}^2\, T_0}{\rho^2 C_E C_2^2},$$
$$\varepsilon_2 = \frac{K}{\rho C_E C_2^2}, \varepsilon_3 = \frac{K^*\omega^*}{\rho C_E C_2^2}, \varepsilon_4 = \frac{\hat{\gamma}\hat{\gamma}_1 T_0}{\rho^2 C_E\,\omega^* C_2^2}. \tag{24}$$

Assuming the scalar potential functions and defined by the relations in the non-dimensional form:

$$u = \frac{\partial\, R}{\partial\, x}\ +\ \frac{\partial\, \psi}{\partial\, z}, w = \frac{\partial\, R}{\partial\, z}\ -\ \frac{\partial\, \psi}{\partial\, x} \tag{25}$$

,

$$e = \nabla^2 R. \tag{26}$$

Using equation (25) in equations (19-23), we obtain.

$$[\nabla^2 -\ a_0\frac{\partial^2}{\partial\, t^2}]\, R - a_0 T +\ a_1\phi^* - g\, a_0\,\frac{\partial\, \psi}{\partial\, x}\ =\ 0, \tag{27}$$

$$[\nabla^2 - a_2\frac{\partial^2}{\partial\, t^2}]\,\psi -\ a_3\phi_2 + g\, a_2\frac{\partial\, R}{\partial x} = 0, \tag{28}$$

$$[\nabla^2 - 2a_4 - a_5\frac{\partial^2}{\partial t^2}]\phi_2 + a_4\nabla^2\psi = 0, \qquad (29)$$

$$[a_6\nabla^2 - a_7 - \frac{\partial^2}{\partial t^2}]\phi^* - a_8\nabla^2 R + a_9 T = 0, \qquad (30)$$

$$\varepsilon_2\nabla^2 T + \varepsilon_3\nabla^2\dot{T} = \ddot{T} + \varepsilon_1\nabla^2\ddot{R} + \varepsilon_4\frac{\partial\phi^*}{\partial t}, \qquad (31)$$

where

$$c_1^2 = \frac{\lambda+2\mu+k}{\rho}, a_0 = \frac{c_2^2}{c_1^2}, a_1 = \frac{\lambda_0}{\lambda+2\mu+k}, a_2 = \frac{\rho c_2^2}{\mu+k},$$
$$a_3 = \frac{k}{\mu+k}, a_4 = \frac{kc_2^2}{\gamma\omega^{*2}}, a_5 = \frac{\rho jc_2^2}{\gamma}, a_6 = \frac{c_3^2}{c_2^2}, a_7 = \frac{c_4^2}{\omega^{*2}},$$
$$a_8 = \frac{c_5^2}{\omega^{*2}} \ and \ a_9 = \frac{2\hat{\gamma}_1 c_2^2}{9\hat{\gamma} j\omega^{*2}} \qquad (32)$$

3 The solution of the problem

The solution of the considering physical variables can be decomposed in terms of normal modes and are given in the following form:

$$[R, \psi, \phi^*, \phi_2, \sigma_{il}, m_{il}, T, \lambda_z](x,z,t) = [\bar{R}, \bar{\psi}, \bar{\phi}^*, \bar{\phi}_2, \bar{\sigma}_{il}, \bar{m}_{il}, \bar{T}, \bar{\lambda}_z](x)exp(\omega t + i bz). \qquad (33)$$

where$[\bar{R}, \bar{\psi}, \bar{\phi}^*, \bar{\phi}_2, \bar{\sigma}_{il}, \bar{m}_{il}, \bar{T}, \bar{\lambda}_z](x)$ are the amplitudes of the functions,ω is a complex constant and b is the wave number in the z direction. Using equation (33), then equations (27-31) become

$$(D^2 - A_1)\bar{R} - a_0\bar{T} + a_1\bar{\phi}^* - A_2 D\bar{\psi} = 0, \qquad (34)$$

$$(D^2 - A_3)\bar{\psi} - a_3\bar{\phi}_2 + A_4 D\bar{R} = 0, \qquad (35)$$

$$(D^2 - A_5)\bar{\phi}_2 + a_4(D^2 - b^2)\bar{\psi} = 0, \qquad (36)$$

$$(a_6 D^2 - A_6)\bar{\phi}^* - a_8(D^2 - b^2)\bar{R} + a_9\bar{T} = 0, \qquad (37)$$

$$[\varepsilon(D^2 - b^2) - \omega^2]\bar{T} - \varepsilon_1\omega^2(D^2 - b^2)\bar{R} - \varepsilon_4\omega\bar{\phi}^* = 0. \qquad (38)$$

Where

$$D = \frac{d}{dx}, A_1 = b^2 + a_0\omega^2, A_2 = ga_0, A_3 = b^2 + a_2\omega^2,$$
$$A_4 = ga_2, A_5 = b^2 + 2a_4 + a_5\omega^2, A_6 = b^2 a_6 + a_7 + \omega^2$$

Eliminating, $\bar{\phi}_2$, $\bar{\psi}, \bar{R}, \bar{T}and\bar{\phi}^*$ in equations (34-38), we get the following tenth order ordinary differential equation for variables

$$[D^{10} - AD^8 + BD^6 - CD^4 + ED^2 - F]\{\bar{\phi}_2(x), \bar{\psi}(x), \bar{R}(x), \bar{T}(x), \bar{\phi}^*(x)\} = 0. \qquad (39)$$

Equation (39) can be factored as

$$(D^2 - k_1^2)(D^2 - k_2^2)(D^2 - k_3^2)(D^2 - k_4^2)(D^2 - k_5^2)\{\bar{\phi}_2(x), \bar{\psi}(x), \bar{R}(x), \bar{T}(x), \bar{\phi}^*(x)\} = 0, \qquad (40)$$

where

$$A = g_{18}/\varepsilon g_{17}, B = g_{19}/\varepsilon g_{17}, C = g_{20}/\varepsilon g_{17},$$
$$E = g_{21}/\varepsilon g_{17}, F = g_{22}/\varepsilon g_{17}, g_1 = \varepsilon_4\omega - a_1\varepsilon_1\omega^2,$$
$$g_2 = -\varepsilon_4\omega A_1 + a_1\varepsilon_1\omega^2 b^2, g_3 = a_1\varepsilon b^2 + a_1\omega^2 + a_0\varepsilon_4\omega,$$
$$g_4 = A_2\varepsilon_4\omega, g_5 = A_3 + A_5 - a_3 a_4, g_6 = A_3 A_5 - a_3 a_4 b^2,$$
$$g_7 = g_2 - g_1 g_5 + A_4 g_4, g_8 = -g_2 g_5 + g_1 g_6 - A_4 g_4 A_5,$$
$$g_9 = g_2 g_6, g_{10} = -g_3 - a_1\varepsilon g_5, g_{11} = g_3 g_5 + a_1\varepsilon g_6,$$
$$g_{12} = g_3 g_6, g_{13} = a_6(\varepsilon b^2 + \omega^2)$$
$$+A_6\varepsilon, g_{14} = \varepsilon_4 a_9\omega$$
$$+A_6(\varepsilon b^2 + \omega^2), g_{15} = \varepsilon_1 a_6\omega^2 b^2 + A_6\varepsilon_1\omega^2 - a_8\varepsilon_4\omega,$$
$$g_{16} = \varepsilon_4 a_8\omega b^2 - \varepsilon_1\omega^2 A_6 b^2, g_{17} = (a_6 g_1 + a_1\varepsilon_1\omega^2 a_6),$$
$$g_{18} = -a_6(\varepsilon g_7 + \varepsilon_1\omega^2 g_{10}) + g_{13}g_1 + a_1\varepsilon g_{15},$$
$$g_{19} = a_6(\varepsilon g_8 + \varepsilon_1\omega^2 g_{11}) - g_{13}g_7 + g_{14}g_1$$
$$-a_1\varepsilon g_{16} - g_{10}g_{15}, g_{20} = -a_6\varepsilon g_9 + g_{13}g_8 - g_{14}g_7$$
$$+g_{10}g_{16} + g_{11}g_{15} + g_{12}a_6\varepsilon_1\omega^2, g_{21} = -g_{13}g_9 + g_{14}g_8$$
$$-g_{11}g_{16} + g_{12}g_{15}, g_{22} = -g_{14}g_9 - g_{12}g_{16}.$$

The solution of equation (39) has the form

$$\bar{R} = \sum_{n=1}^{5} M_n e^{-k_n x} \qquad (41)$$

$$\bar{T} = \sum_{n=1}^{5} M'_n e^{-k_n x} \qquad (42)$$

$$\bar{\psi} = \sum_{n=1}^{5} M''_n e^{-k_n x} \qquad (43)$$

$$\bar{\phi}_2 = \sum_{n=1}^{5} M'''_n e^{-k_n x} \qquad (44)$$

$$\bar{\phi}^* = \sum_{n=1}^{5} M''''_n e^{-k_n x} \qquad (45)$$

whereM_n, M'_n, M''_n, M'''_n and M''''_n,are some parameters, k_n^2, $(n = 1,2,3,4,5)$ are the roots of the characteristic equation of equation (39). Using equations (41-45) in equations (34-38), we get the following relations

$$\bar{T} = \sum_{n=1}^{5} H_{1n} M_n e^{-k_n x} \qquad (46)$$

$$\bar{\psi} = \sum_{n=1}^{5} H_{2n} M_n e^{-k_n x} \qquad (47)$$

$$\bar{\phi}_2 = \sum_{n=1}^{5} H_{3n} M_n e^{-k_n x} \qquad (48)$$

$$\bar{\phi}^{*} = \sum_{n=1}^{5} H_{4n} M_n e^{-k_n x} \tag{49}$$

Where

$$H_{1n} = [k_n^4 a_6 \varepsilon_1 \omega^2 - k_n^2 g_{15} - g_{16}] / [k_n^4 a_6 \varepsilon - k_n^2 g_{13} + g_{14}], \tag{50}$$

$$H_{2n} = [A_4 k_n (k_n^2 - A_5)] / [k_n^4 - k_n^2 g_5 + g_6], \tag{51}$$

$$H_{3n} = -[a_4 A_4 k_n (k_n^2 - b^2)] / [k_n^4 - k_n^2 g_5 + g_6], \tag{52}$$

$$H_{4n} = [a_8 (k_n^2 - b^2) - a_9 H_{1n}] / [a_6 k_n^2 - A_6] \tag{53}$$

4 Boundary conditions

The plane boundary subjects to an instantaneous normal point force and the boundary surface is isothermal. The boundary conditions on the vertical plane $y = 0$and in the beginning of the crack, at $x = 0$ are shown in Fig. 1:

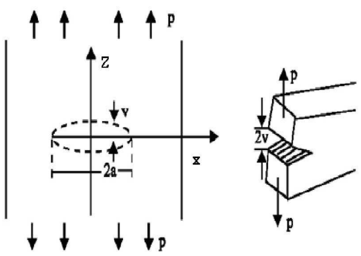

Fig. 1: Displacement of an external Mode-I crack.

(1) The mechanical boundary condition is that the surface of the half-space obeys

$$\sigma_{zz} = -\, p(z,t), \quad |x| < a \tag{54}$$

$$\sigma_{xz} = 0, \qquad -\infty < x < \infty \tag{55}$$

$$\sigma_{xx} = 0, \qquad -\infty < x < \infty \tag{56}$$

$$\lambda_z = 0 \qquad -\infty < x < \infty \tag{57}$$

(2) The thermal boundary condition is that the surface of the half-space is subjected to a thermal shock,

$$T = f(z,t), \qquad |x| < a \tag{58}$$

Using equations (25), (12-17) with the non-dimensional boundary conditions and using equations (46-49), we obtain the expressions for the displacement components, the force stress, the coupled stress and the temperature distribution of the microstretch generalized thermoelastic medium as follows:

$$\bar{u} = \sum_{n=1}^{5} (-k_n + ibH_{2n}) M_n e^{-k_n x} \tag{59}$$

$$\bar{w} = \sum_{n=1}^{5} (ib + k_n H_{2n}) M_n e^{-k_n x} \tag{60}$$

$$\bar{\sigma}_{xx} = \sum_{n=1}^{5} H_{5n} M_n e^{-k_n x} \tag{61}$$

$$\bar{\sigma}_{zz} = \sum_{n=1}^{5} H_{6n} M_n e^{-k_n x} \tag{62}$$

$$\bar{\sigma}_{xz} = \sum_{n=1}^{5} H_{7n} M_n e^{-k_n x} \tag{63}$$

$$\bar{\sigma}_{zx} = \sum_{n=1}^{5} H_{8n} M_n e^{-k_n x} \tag{64}$$

$$\bar{m}_{xy} = \sum_{n=1}^{5} -a_{15} k_n H_{3n} M_n e^{-k_n x} \tag{65}$$

$$\bar{\lambda}_z = \sum_{n=1}^{5} a_{16} H_{4n} M_n e^{-k_n x}, \tag{66}$$

here,

$$a_{10} = \frac{\lambda_0}{\rho C_2^2}, \quad a_{11} = \frac{C_1^2}{C_2^2}, \quad a_{12} = \frac{\lambda}{\rho C_2^2}, \quad a_{13} = \frac{\mu + k}{\rho C_2^2},$$

$$a_{14} = \frac{k}{\rho C_2^2}, a_{15} = \frac{\gamma \omega^{*2}}{\rho C_2^4}, \quad a_{16} = \frac{\alpha_0 \omega^{*}}{\rho C_2^3}.$$

$$H_{5n} = a_{10} H_{4n} - k_n a_{11} (-k_n + ibH_{2n}) + iba_{12} (ib + k_n H_{2n}) - H_{1n} \tag{67}$$

$$H_{6n} = a_{10} H_{4n} + iba_{11} (ib + k_n H_{2n}) - k_n a_{12} (-k_n + ibH_{2n}) - H_{1n} \tag{68}$$

$$H_{7n} = ib(ibH_{2n} - k_n) - a_{13} k_n (ib + k_n H_{2n}) + a_{14} H_{3n} \tag{69}$$

$$H_{8n} = -\, k_n (ib + k_n H_{2n}) + iba_{13} (ibH_{2n} - k_n) + a_{14} H_{3n} \tag{70}$$

Applying the boundary conditions (54-58) at the surface of the plate, we obtain a system of five equations. After applying the inverse of matrix method,

$$\begin{pmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \end{pmatrix} = \begin{pmatrix} H_{61} & H_{62} & H_{63} & H_{64} & H_{65} \\ H_{71} & H_{72} & H_{73} & H_{74} & H_{75} \\ H_{51} & H_{52} & H_{53} & H_{54} & H_{55} \\ k_1 H_{41} & k_1 H_{42} & k_1 H_{43} & k_1 H_{44} & k_1 H_{45} \\ H_{11} & H_{12} & H_{13} & H_{14} & H_{15} \end{pmatrix}^{-1} \begin{pmatrix} -\bar{p} \\ 0 \\ 0 \\ 0 \\ \bar{f} \end{pmatrix} \tag{71}$$

we obtain the values of the five constants $M_n, n = 1,2,3,4,5$ Hence, we obtain the expressions for the displacements, the force stress, the coupled stress and the temperature distribution of the microstretch generalized thermoelastic medium.

5 Particular Cases

Case 1: The corresponding equations for the generalized micropolar thermoelasticity elastic medium (without microstretch constants) can be obtained from the above mentioned cases by taking:

$$\alpha_0 = \lambda_0 = \lambda_1 = \phi^* = 0 \tag{72}$$

After substituting equation (72) in the equations (1-7) and use equations (18-25) and (33) we get

$$(D^2 - A_1)R - A_2 D\psi - a_0 T = 0, \tag{73}$$

$$(D^2 - A_3)\bar{\psi} + A_4 D\bar{R} - a_3\bar{\phi}_2 = 0, \tag{74}$$

$$[D^2 - A_5]\,\phi_2 + a_4\,(D^2 - b^2)\psi = 0, \tag{75}$$

$$[\varepsilon\,(D^2 - b^2) - \omega^2]T = \varepsilon_1\omega^2(D^2 - b^2)R. \tag{76}$$

Eliminating $\bar{\phi}_2, \bar{\psi}$ in equations (73-76), we get the following eighth order ordinary differential equation for $\bar{\phi}_2, \bar{\psi}, \bar{R}$ and $\bar{T}$

$$\left[D^8 - AD^6 + BD^4 - CD^2 + E\right]\{\bar{\phi}_2, \bar{\psi}, \bar{R}, \bar{T}\}(x) = 0 \tag{77}$$

Equation (77) can be factorized as

$$(D^2 - k_1^2)(D^2 - k_2^2)(D^2 - k_3^2)(D^2 - k_4^2)\{\bar{\phi}_2, \bar{\psi}, \bar{R}, \bar{T}\}(x) = 0. \tag{78}$$

Here,
$A = g'_9/\varepsilon\,, B = g'_{10}/\varepsilon\,, C = g'_{11}/\varepsilon\,, E = g'_{12}/\varepsilon\,,$

$g'_1 = a_0\varepsilon_1\omega^2 + \varepsilon b^2 + \omega^2 + \varepsilon A_1,$
$g'_2 = a_0\varepsilon_1\omega^2 b^2 + \varepsilon b^2 A_1 + \omega^2 A_1,$
$g'_3 = \varepsilon b^2 + \omega^2,$
$g'_4 = A_3 + A_5 - a_3 a_4,$
$g'_5 = A_3 A_5 - a_3 a_4 b^2,$
$g'_6 = g'_2 + A_3 g'_1 - A_2 A_4 g'_3,$
$g'_7 = A_3 g'_2,$
$g'_8 = g'_1 + A_3\varepsilon - A_2 A_4\varepsilon$
$g'_9 = \varepsilon g'_4 + g'_1 - A_2 A_4\varepsilon,$
$g'_{10} = \varepsilon g'_{55} + g'_1 g'_4 + g'_2 - A_2 A_4(\varepsilon A_5 + g'_3),$
$g'_{11} = g'_1 g'_{55} + g'_2 g'_4 - A_2 A_4 A_5 g'_3,$

The solution of equation (77), has the form

$$\bar{R} = \sum_{n=1}^{4} Z_n e^{-k_n x}, \tag{79}$$

$$\bar{T} = \sum_{n=1}^{4} H'_{1n} Z_n e^{-k_n x}, \tag{80}$$

$$\bar{\psi} = \sum_{n=1}^{4} H'_{2n} Z_n e^{-k_n x}, \tag{81}$$

$$\bar{\phi}_2 = \sum_{n=1}^{4} H'_{3n} Z_n e^{-k_n x}, \tag{82}$$

where Z_n are some parameters, k_n^2, $(n = 1,2,3,4)$ are the roots of the characteristic equation of equation (77). Using equations (79-82) in equations (73-76), we get the following relations

$$H'_{1n} = [\varepsilon_1\omega^2(k_n^2 - b^2)]/[\varepsilon\,(k_n^2 - b^2) - \omega^2], \tag{83}$$

$$H'_{2n} = [-k_n^4\varepsilon + k_n^2 g'_1 - g'_2]/[A_2 k_n(\varepsilon k_n^2 - g'_3)], \tag{84}$$

$$H'_{3n} = [-k_n^6\varepsilon + k_n^4 g'_8 - k_n^2 g'_6 + g'_7]/[a_3 A_2 k_n(\varepsilon k_n^2 - g'_3)]. \tag{85}$$

Using equations (72), (12-17), (18) with the non-dimensional boundary conditions and using equations (46-49), we obtain the expressions of the displacement components, the force stress and the coupled stress distribution for generalized micropolar thermoelastic medium (without microstretch) as follows:

$$\bar{u} = \sum_{n=1}^{4} (-k_n + ibH'_{2n})\,Z_n\,e^{-k_n x}, \tag{86}$$

$$\bar{w} = \sum_{n=1}^{4} (ib + k_n H'_{2n})\,Z_n\,e^{-k_n x}, \tag{87}$$

$$\bar{\sigma}_{xx} = \sum_{n=1}^{4} H'_{4n} Z_n e^{-k_n x}, \tag{88}$$

$$\bar{\sigma}_{zz} = \sum_{n=1}^{4} H'_{5n} Z_n e^{-k_n x} \tag{89}$$

$$\bar{\sigma}_{xz} = \sum_{n=1}^{4} H'_{6n} Z_n\,e^{-k_n x}, \tag{90}$$

$$\bar{\sigma}_{zx} = \sum_{n=1}^{4} H'_{7n} Z_n\,e^{-k_n x}, \tag{91}$$

$$\bar{m}_{xy} = \sum_{n=1}^{4} -a_{15} k_n H'_{3n} Z_n e^{-k_n x}, \tag{92}$$

where

$$H'_{4n} = -k_n a_{11}(-k_n + ibH'_{2n}) + iba_{12}(ib + k_n H'_{2n}) - H'_{1n}, \tag{93}$$

$$H'_{5n} = iba_{11}(ib + k_n H'_{2n}) - k_n a_{12}(-k_n + ibH'_{2n}) - H'_{1n}, \tag{94}$$

$$H'_{6n} = ib\,(ibH'_{2n} - k_n) - a_{13}k_n(ib + k_nH'_{2n}) + a_{14}H'_{3n}, \quad (95)$$

$$H'_{7n} = -\,k_n(ib + k_nH'_{2n}) + iba_{13}(ibH'_{2n} - k_n) + a_{14}H'_{3n}. \quad (96)$$

Applying the boundary conditions (54) -(56) and (58) at the surface$x = 0$ of the plate, we obtain a system of four equations.

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \end{pmatrix} = \begin{pmatrix} H'_{51} & H'_{52} & H'_{53} & H'_{54} \\ H'_{61} & H'_{62} & H'_{63} & H'_{64} \\ H'_{41} & H'_{42} & H'_{43} & H'_{44} \\ H_{11} & H_{12} & H_{13} & H_{14} \end{pmatrix}^{-1} \begin{pmatrix} -\overline{p} \\ 0 \\ 0 \\ \overline{f} \end{pmatrix} \quad (97)$$

After applying the inverse of matrix method, we obtain the values of the four constantsZ_n $;n = 1,2,3,4$

Case 2: The corresponding equations for the generalized micropolar thermoelasticity elastic medium (without micropolar constants) can be obtained from the above mentioned cases by taking:

$$k = \alpha = \beta = \gamma = 0 \quad (98)$$

Substituting equations (98) in equations (1-7) and use equations (18), (25) and (33) we get

$$(D^2 - A_1)\overline{R} - a_0\,\overline{T} + a_1\,\overline{\phi}^* - A_2D\,\overline{\psi} = 0, \quad (99)$$

$$(D^2 - A_3)\,\overline{\psi} + A_4D\overline{R} = 0, \quad (100)$$

$$(a_6D^2 - A_6)\overline{\phi}^* - a_8(D^2 - b^2)\overline{R} + a_9\overline{T} = 0 \quad (101)$$

$$[\varepsilon\,(D^2 - b^2) - \omega^2]\,\overline{T} - \varepsilon_1\omega^2(D^2 - b^2)\overline{R} - \varepsilon_4\omega\,\overline{\phi}^* = 0. \quad (102)$$

Eliminating$\overline{\psi}, \overline{R},, \overline{T}\,and\,\overline{\phi}^*$in equations (99-102), we get the following eight order ordinary differential equations

$$[D^8 - AD^6 + BD^4 - ED^2 + F]\ \{\overline{\psi}, \overline{R}, \overline{T}, \overline{\phi}^*\}\ (x) = 0. \quad (103)$$

Equation (103) can be factored as

$$(D^2 - k_1^2)\,(D^2 - k_2^2)(\,D^2 - k_3^2)\,(\,D^2 - k_4^2)$$
$$\{\overline{\psi}(x), \overline{R}(x), \overline{T}(x), \overline{\phi}^*(x)\} = 0, \quad (104)$$

where

$A = g''_{18}/g''_{17}, B = g''_{19}/g''_{17}, E = g''_{20}/g''_{17},$
$F = g''_{21}/g''_{17}, g''_1 = \varepsilon_4\omega - a_1\varepsilon_1\omega^2, g''_2 = -\,\varepsilon_4\omega A_1$
$+ a_1\varepsilon_1\omega^2b^2, g''_3 = a_1\varepsilon\,b^2 + a_1\omega^2 + a_0\varepsilon_4\omega\,, g''_4 = A_2\varepsilon_4\omega,$
$g''_5 = A_3 - a_3a_4, g''_6 = -a_3a_4b^2, g''_7 = g_2 - g_1g_5 + g_4A_4,$
$g''_8 = -g_2g_5, g''_9 = g''_2g''_6, g''_{10} = -g''_3 - a_1\varepsilon\,g''_5, g''_{11} = g''_3g''_5$
$+ a_1\varepsilon\,g''_6, g''_{12} = g''_3g''_6, g''_{13} = a_6(\varepsilon\,b^2 + \omega^2) + A_6\varepsilon,$
$g''_{14} = \varepsilon_4a_9\omega + A_6(\varepsilon\,b^2 + \omega^2),$
$g''_{15} = \varepsilon_1a_6\omega^2b^2 + A_6\varepsilon_1\omega^2 - a_8\varepsilon_4\omega,$
$g''_{16} = \varepsilon_4a_8\omega b^2 - \varepsilon_1\omega^2A_6b^2, g''_{17} = \varepsilon(a_6g_1 + a_1\varepsilon_1\omega^2a_6),$
$g''_{18} = -\,a_6(\varepsilon\,g_7 + \varepsilon_1\omega^2g''_{10}) + g''_{13}g''_1 + a_1\varepsilon\,g''_{15},$
$g''_{19} = a_6(\varepsilon\,g''_8 + \varepsilon_1\omega^2g''_{11}) - g''_{13}g''_7 + g''_{14}g''_1 - a_1\varepsilon g''_{16}$
$-g''_{10}g''_{15}, g''_{20} = g''_{13}g''_8 - g''_{14}g''_7 + g''_{10}g''_{16} + g''_{11}g''_{15},$
$g''_{21} = g''_{14}g''_8 - g''_{11}g''_{16}.$

The solution of equation (103), has the form

$$\overline{R} = \sum_{n=1}^{4} G_n\,e^{-\,k_nx}, \quad (105)$$

$$\overline{T} = \sum_{n=1}^{4} H''_{1n}G_ne^{-\,k_nx}, \quad (106)$$

$$\overline{\psi} = \sum_{n=1}^{4} H''_{2n}G_ne^{-\,k_nx}, \quad (107)$$

$$\overline{\phi}^* = \sum_{n=1}^{4} H''_{3n}G_n\,e^{-\,k_nx}, \quad (108)$$

whereG_n are some parameters, k_n^2, $(n = 1,2,3,4)$are the roots of the characteristic equation of equation (103). Where

$$H''_{1n} = [k_n^4\,a_6\varepsilon_1\omega^2 - k_n^2\,g''_{15} - g''_{16}]/[k_n^4\,a_6\varepsilon - k_n^2\,g''_{13} + g''_{14}] \quad (109)$$

$$H''_{2n} = A_4k_n/[k_n^2 - A_3], \quad (110)$$

$$H''_{3n} = [a_8(k_n^2 - b^2) - a_9\,H''_{1n}]/[a_6k_n^2 - A_5] \quad (111)$$

Using equations (98), (12-17) , (18) with the non-dimensional boundary conditions and using equations (46-49), we obtain the expressions of the displacement components, the force stress and the micro-stress distribution for generalized thermoelastic medium (without micropolar) as follows:

$$\overline{u} = \sum_{n=1}^{4} (-k_n + ibH''_{2n})\,G_n\,e^{-\,k_nx}, \quad (112)$$

$$\overline{w} = \sum_{n=1}^{4} (ib + k_nH''_{2n})\,G_ne^{-\,k_nx}, \quad (113)$$

$$\bar{\sigma}_{xx} = \sum_{n=1}^{4} H''_{5n} G_n e^{-k_n x}, \tag{114}$$

$$\bar{\sigma}_{zz} = \sum_{n=1}^{4} H''_{6n} G_n e^{-k_n x}, \tag{115}$$

$$\bar{\sigma}_{xz} = \sum_{n=1}^{4} H''_{7n} G_n e^{-k_n x}, \tag{116}$$

$$\bar{\sigma}_{zx} = \sum_{n=1}^{4} H''_{8n} G_n e^{-k_n x}, \tag{117}$$

$$\bar{\lambda}_z = -\sum_{n=1}^{4} k_n a_{16} H''_{3n} G_n e^{-k_n x}. \tag{118}$$

Here

$$a_{10} = \frac{\lambda_0}{\rho C_2^2}, a_{11} = \frac{C_1^2}{C_2^2}, a_{12} = \frac{\lambda}{\rho C_2^2}, a_{13} = \frac{\mu}{\rho C_2^2} = 1,$$
$$a_{16} = \frac{\alpha_0 \omega^*}{\rho C_2^3},$$

$$H''_{5n} = a_{10} H''_{3n} - k_n a_{11}\left(-k_n + ibH''_{2n}\right) + iba_{12}\left(ib + k_n H''_{2n}\right) - H''_{1n} \tag{119}$$

$$H''_{6n} = a_{10} H''_{3n} + iba_{11}\left(ib + k_n H''_{2n}\right) - k_n a_{12}\left(-k_n + ibH''_{2n}\right) - H''_{1n} \tag{120}$$

$$H''_{7n} = ib(ibH''_{2n} - k_n) - a_{13} k_n \left(ib + k_n H''_{2n}\right) \tag{121}$$

$$H''_{8n} = -k_n (ib + k_n H''_{2n}) + iba_{13}\left(ibH''_{2n} - k_n\right) \tag{122}$$

Applying the boundary conditions (54-56) and (58) at the surface $x = 0$ of the plate, we obtain a system of four equations. After applying the inverse of matrix method,

$$\begin{pmatrix} G_1 \\ G_2 \\ G_3 \\ G_4 \end{pmatrix} = \begin{pmatrix} H''_{61} & H''_{62} & H''_{63} & H''_{64} \\ H''_{71} & H''_{72} & H''_{73} & H''_{74} \\ H''_{51} & H''_{52} & H''_{53} & H''_{54} \\ H''_{11} & H''_{12} & H''_{13} & H''_{14} \end{pmatrix}^{-1} \begin{pmatrix} -\bar{p} \\ 0 \\ 0 \\ \bar{f} \end{pmatrix} \tag{123}$$

We obtain the values of the four constants. Hence, we obtain the expressions for the displacements, the force stress, the microstress and the temperature distribution of the generalized thermo-microstretch elastic medium.

6 Numerical Results and Discussions

In order to illustrate our theoretical results obtained in the preceding section and to compare various theories of thermoelasticity, we now present some numerical results. In the calculation, we take a magnesium crystal as in Eringen [10] as the material subjected to mechanical and thermal disturbances. Since, ω is a complex constant, we take $\omega = \omega_0 + i\zeta$ with $\omega_0 = -2.5$ and $\zeta = 1$. The physical constants used are:

$$\begin{aligned}
&\rho = 1.74 \times 10^3 kgm^{-3}, j = 0.2 \times 10^{-21} m^3, \quad z = 0.6,\\
&T_0 = 293K, \mu = 4.0 \times 10^{11} kgm^{-1}s^{-2}, k = 1 \times 10^{11} kgm^{-1}s^{-2},\\
&\gamma = 0.779 \times 10^{-8} kgms^{-2}, K^* = 0.1 \times 10^{-3} W m^{-1} K^{-1},\\
&K = 1.3 \times 10^{-4} W m^{-1} K^{-1}, \lambda_0 = 0.5 \times 10^{11} kgm^{-1}s^{-2},\\
&\lambda_1 = 0.1 \times 10^9 kgm^{-1}s^{-2}, \alpha_0 = 0.779 \times 10^{-4} kgm^{-1}s^{-2},\\
&\lambda = 9.4 \times 10^{11} kgm^{-1}s^{-2}, \bar{p} = 2, t = 0.1, \bar{f} = 1,\\
&b = 2.1, \quad \varepsilon_1 = 0.68, \quad \varepsilon_2 = 0.9,
\end{aligned}$$

The variation in the field quantities i.e., the vertical component of displacement w, the temperature distribution T the normal stress σ_{zz} and the micro-stress λ_z against the distance x are studied numerically in the context of GN theories (types II and III) for different values of gravity $g = 9.8$ and $g = 0$ i.e., in the presence and absence of gravity effect. These are represented in 2-D and 3-D. Figs. 2-12 are representing the above mentioned variation in 2D for general microstretch material and two particular cases: (1) without microstretch and with micropolar effect, (2) with microstretch and without micropolar effect. These 2-D figures are obtained for the plane $z = 0.1$, in these figures the solid line and dashed line are for GN-II for $g = 0, 9.8$ respectively, the dashed with dot line and dotted line are for GN-III for $g = 0, 9.8$ respectively. Figs. 13-20 illustrate the 3-D curves of physical quantities for GN-II and III in the presence of gravity effect. Fig. 2 gives variation in the displacement component versus the distance x for GN-II and III. The curves which obtained are having continuous behavior. An interesting feature of gravity is, the curve obtained by GN-II is higher than that obtained by GN-III in the presence of gravity for $0.42 \le x < 5$, and the curve obtained by GN-III for $g = 0$ is lower than that obtained by GN-II for $0.3 \le x < 3.5$, after and before these values both curves moves with the same value. The curve obtained by GN-II for $g = 9.8$ is higher than that obtained by GN-II for $g = 0$ in the range $0 \le x < 1.2$ and remains below for the values of x above this range. The same type of behavior is obtained by GN-III for almost the value $0 \le x < 0.96$, finally all curves converges to zero. Fig. 3 shows the temperature distribution plotted against under the both types of GN theory II and III for $g = 0, 9.8$. Graph represented curves obtained by GN-III for $g = 9.8$ being above the curve for $g = 0$ under $0 \le x < 4.18$. For GN-II, under this theory gravity has a decreasing effect on temperature distribution in the range $0 \le x < 3.3$ after these ranges of horizontal distance all curves moves with same result and finally converges to zero. Fig. 4 gives the normal stress distribution σ_{zz} against x for GN-II and III under different values of gravity $g = 9.8, 0$. Curves obtained by both GN theories under both values of gravity are having coincident starting point. Gravity is having an increasing effect on the normal stress distribution in both types of GN theories. After $2.8 \le x$ all four curves started moving very close to each other and finally converge to zero. Fig. 5 gives variation in the microstressλ_z versus$x$ by GN-II and III for$g = 0, 9.8$. Starting point for all curves are the same, it can also be seen from the figure that the gravity has a decreasing effect on microstress for both GN-II and III i.e., for the case $g = 0$curve of being higher than that of$g = 9.8$. For$g = 0$ the curve obtained by GN-III is below the presence by GN-II but in presence of gravitational effect curves of GN-III is higher than GN-II. Finally for sufficiently large values of horizontal distance all curves converge to zero. From the above discussion it can be concluded that the gravity has a decreasing effect for the microstress and the normal stress for both GN-theories. Figs. 6-8 represent the change in the vertical component of displacement, the temperature and the normal stress distribution with respect to distance$x$, for GN-II and III with conditions of gravity$g = 0, 9.8$, for without microstretch and with micropolar effect (NMST). Fig. 6 gives the graphical representation of the relation between displacement w and distance$x$. It can be seen from the figure that, the gravity has a decreasing effect on the vertical component of displacement for GN-II and GN-III under$0 < x \leq 1.33$ and respectively and having an increasing effect after this range up to$x < 2.9$. For $g = 0$ the curves of GN-II and III are moving very close to each other but the curve obtained by GN-II is higher than the curve obtained by GN-III. For $g = 9.8$curve in placement distribution in the case of GN-III is higher than GN-II for$x < 2.9$ and opposite result for higher values of horizontal displacement. In Fig. 7 the temperature distribution is studied for GN-II and III for$g = 0, 9.8$ versus the horizontal distance$x$. All four curves have the same starting point; gravity has an increasing effect in GN-III it can be seen from the figure that distance between both curves increasing for $x < 1$ and after this range of horizontal distance both curves of started closing each other finally both curves move with the same value for$3.5 < x$. In GN-II gravity is having a decreasing effect maximum effect of gravity is obtained for $x \approx 0.8$ after this the values of both curves with and without gravity started closing and finally join and respond with the same value to change in horizontal distance. The normal stresses against horizontal distance represented in Fig. 8 for type II and III under two different values of gravity. The temperature distribution curves of normal stresses have the same starting point. Gravity is having an increasing effect for both theories of GN (II and III). For$g = 0$ curve obtained by GN-II is higher than the curve obtained by GN-II for the range$0 < x \leq 2.8$. For the condition$g = 9.8$ the curve obtained by GN-III is higher than the curve obtained by GN-II in the range of $0 < x \leq 1.73$ and after this normal stress distribution in GN-II is higher. Finally for $3.75 < x$ all curves converges to zero. Figs. 9-12 represent a variation of the field quantities i.e., the displacement component, the temperature, the normal stress and the microstress versus the distance x . For GN-II and III and $g = 0, 9.8$ the material selected is generalized thermo-micro-stress elastic medium without micropolar effect (WMT). Fig. 9 represents the variation in versus for the above mentioned values of gravity. During $g = 0$ the curves obtained are very close to each other with the curve of GN-II is higher than the curve of GN-III. For the second value of gravity i.e., $g = 9.8$ the curve obtained by GN-III is below than that the curve obtained by GN-II in the ranges $0.7 < x \leq 3.4$and higher in the range$0.3 < x \leq 0.7$ before and after these values of horizontal distance both theories gave same distribution curve i.e., the curves obtained for with dissipation and without dissipation are same. All the curves obtained by GN-II converge to zero in $2.5 < x$. The temperature distribution versus of the non micropolar material is shown in Fig. 10 for$g = 0$ and$g = 9.8$. During the case of GN-II gravity is having decreasing effect on temperature distribution in the medium while for the case of GN-III the gravity is having an increasing effect on temperature through the medium. During both conditions of gravity temperature distribution for in the case of without dissipation is more than with dissipation i.e., temperature distribution for GN-II is higher than without distribution. Fig. 11 gives the curves of the normal stress versus distance under the same values of gravity under GN-theories and different conditions of gravity. During the cause of this material both theories are having an increasing effect of gravity. Maximum effect of gravity is found approximately at $x = 1.6$. For$g = 0$ the curve obtained by GN-III is higher than the curve obtained by GN-II i.e., normal stress distribution during the case of energy dissipation is greater that temperature distribution for without energy dissipation. The same type of result found for9.8 , finally all converges to zero. Fig. 12 represents the results of the micro-stress versus$x$ for GN-II and III under the same values of gravity. For the case of GN-II gravity has an increasing effect on the microstress in the range. Gravity has a dual type behavior for GN-III, for sufficiently large of horizontal distance curves converges to zero. Figs. 13-20 present 3-D graphs for the displacement w the temperature T the normal stress σ_{zz} and the microstress λ_z versus the distance$x$ for GN-II and III under the effect of gravity (i.e., $g = 9.8$) for the material of the generalized thermo-microstretch elastic (GTMSE) medium. These 3-D graphs are very important to understand the variety of field quantities with respect to vertical distance In $xy-$plane the curves are represented by lines with different color where the red represents the peak that curve obtained while relating with both horizontal and vertical displacement components, the blue line represents the lowest position which curve obtained and green line gives the value of horizontal and vertical distance at which physical variable obtained value equal to zero.

7 Conclusion

1. The curves in GN theory of types II and III decrease exponentially with the increasing x, which indicates that the thermoelastic waves are unattended and

non-dispersive, while purely thermoelastic waves undergo both attenuation and dispersion.

2. The presence of the microstretch plays a significant role in all the physical quantities.

3. In most of the figures, the physical quantities based on the GN theory of type II are lower compared with those based on the GN theory of type III.

4. Analytical solutions based upon normal mode analysis of the thermoelasticity problem in solids have been developed and utilized.

5. The radial and axial distributions of the temperature were estimated at different distances from the crack edge.

6. The stress distributions, the tangential coupled stress and the values of microstress were evaluated as functions of the distance from the crack edge.

7. Crack dimensions are significant to elucidate the mechanical structure of the solid.

8. It can be concluded that a change of volume is attended with a change of the temperature, while the effect of the deformation upon the temperature distribution is the subject of the theory of thermoelasticity.

9. The values of all the physical quantities converge to zero with an increase in the distance x and all functions are continuous.

10. The presence of gravity plays a significant role in all the physical quantities.

References

[1] S. Bhattacharyya and S.N. De, Surface waves in viscoelastic media under the influence of gravity, Australian Journal of Physics. **30**, 347-353 (1977).

[2] S.N. De and P.R. Sengupta, Plane influence of gravity on wave propagation in elastic layer, Journal of the Acoustical Society of America **55**, 919-921 (1974).

[3] V.K. Agarwal, On plane waves in generalized thermoelasticity, Acta Mechanica **31**, 185-198 (1979)

[4] V.K. Agarwal, On electromagneto-thermoelastic plane waves, Acta Mechanica **34**, 181-191 (1979)

[5] P. Ailawalia, and N.S. Narah, Effect of rotation under the influence of gravity due to various sources in a generalized thermoelastic medium, Advances in Applied Mathematics and Mechanics **2**, 810-827 (2010).

[6] P. Ailawalia, G. Khurana, and S. Kumar, Effect of rotation in a generalized thermoelastic medium with two temperatures under the influence of gravity, International Journal of Applied Mathematics and Mechanics **5**, 99-116 (2009).

[7] S.R. Mahmoud, Effect of rotation, gravity field and initial stress on generalized magneto-thermoelastic Rayleigh waves in a granular medium, Applied Mathematical Sciences **5**, 2013-2032 (2011).

[8] A.M. Abd-Alla and S.R. Mahmoud, Magneto-thermo-viscoelastic interaction in an unbounded non-homogeneous body with a spherical cavity subjected to a periodic loading, Applied Mathematical Sciences **5**, 1431-1447 (2011).

[9] M. Sethi and K.C. Gupta, Surface waves in non-homogeneous, general thermo, viscoelastic media of higher order under influence of gravity and couple-stress, International Journal of Applied Mathematics and Mechanics **7**, 1-20 (2011).

[10] A.C. Eringen and E.S. Suhubi, Non linear theory of simple micropolar solids, International Journal of Engineering Science 2, 1-18 (1964).

[11] A.C. Eringen, Linear theory of micropolar elasticity, Journal of Mathematics and Mechanics **15**, 909-923 (1966).

[12] A.C. Eringen, Micropolar elastic solids with stretch, Ari Kitabevi Matbassi, Istanbul **24**, 1-18 (1971).

[13] A.C. Eringen, Theory of micropolar elasticity. In: H. Liebowitz (ED.) Fracture, Vol. II, Academic Press, New York, (1968).

[14] A.C. Eringen, Theory of thermo-microstretch elastic solids, International Journal of Engineering Science **28**, 1291-1301 (1990).

[15] A.C. Eringen, Microcontinuum field theories I: Foundation and solids, Springer-Verlag, New York, Berlin, Heidelberg, 1999.

[16] D. Iesan and L. Nappa, On the plane strain of microstretch elastic solids, International Journal of Engineering Science **39**, 1815-1835 (2001).

[17] D. Iesan and A. Pompei, On the equilibrium theory of microstretch elastic solids, International Journal of Engineering Science, **33**, pp. 399-410, 1995.

[18] S. De Cicco, Stress concentration effects in microstretch elastic bodies, International Journal of Engineering Science **41**, 187-199 (2003).

[19] F. Bofill and R. Quintanilla, Some qualitative results for the linear theory of thermo-microstretch elastic solids, International Journal of Engineering Science **33**, 2115-2125 (1995).

[20] S. De Cicco and L. Nappa, Some results in the linear theory of thermo-microstretch elastic solids, Journal of Mathematics and Mechanics **5**, 467-482 (2000).

[21] S. De Cicco and L. Nappa, On the theory of thermo-microstretch elastic solids, Journal of Thermal Stresses **22**, 565-580 (1999).

[22] A.E. Green and N. Laws, On the entropy production inequality, Archive for Rational Mechanics and Analysis **45**, 47-59 (1972).

[23] H.W. Lord and Y. ?hulman, A generalized dynamical theory of thermoelasticity, Journal of the Mechanics and Physics of Solids **15**, 299-306 (1967).

[24] A.E. Green and K. A. Lindsay, Thermoelasticity, Journal of Elasticity **2**, 1-7 (1972).

[25] A.E. Green and P. M. Naghdi, On undamped heat waves in an elastic solid, Journal of Thermal Stresses **15**, 253-264, (1992).

[26] A.E. Green and P.M. Naghdi, Thermoelasticity without energy dissipation, Journal of Elasticity **31**, 189-209 (1993).

[27] M.I.A. Othman and Y.Q. Song, The effect of rotation on the reflection of magneto-thermo-elastic waves under thermoelasticity without energy dissipation, Acta Mechanica **184**, 189-204 (2006).

[28] M. I. A. Othman, and Y.Q. Song, Reflection of plane waves from an elastic solid half-space under hydrostatic initial stress without energy dissipation, International Journal of Solids and Structures **44**, 5651-5664 (2007).

[29] D. Hasanyan, L. Librescu, Z. Qin and R. Young, Thermoelastic cracked plates carrying non-stationary

electrical current, Journal of Thermal Stresses **28**, 729-745 (2005).
[30] A. Elfalaky, and A. A. Abdel-Halim, A mode-I crack problem for an infinite space in thermo- elasticity, Journal Applied Sciences **6**, 598-606 (2006).

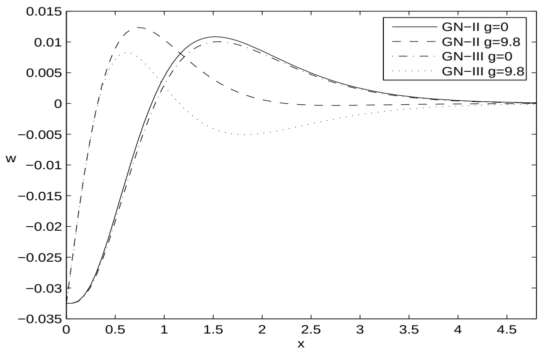

Fig. 2: Vertical displacement distribution of different gravity for medium (GTMSE)

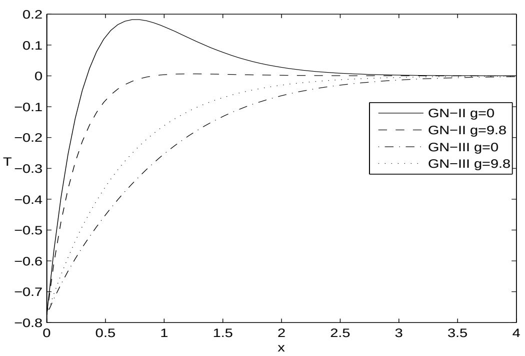

Fig. 3: Temperature distribution of different gravity for medium (GTMSE))

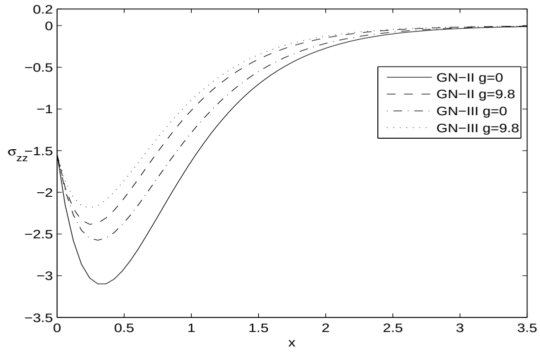

Fig. 4: Normal stress distribution of different gravity for medium (GTMSE)

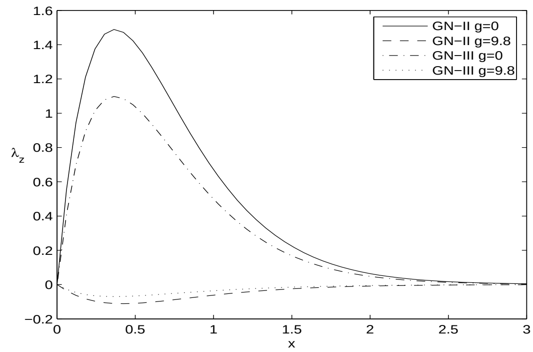

Fig. 5: Microstress distribution of different gravity for medium (GTMSE)

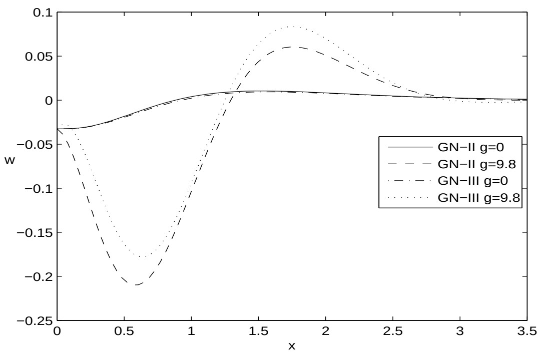

Fig. 6: vertical displacement distribution of different gravity for medium (NMST)

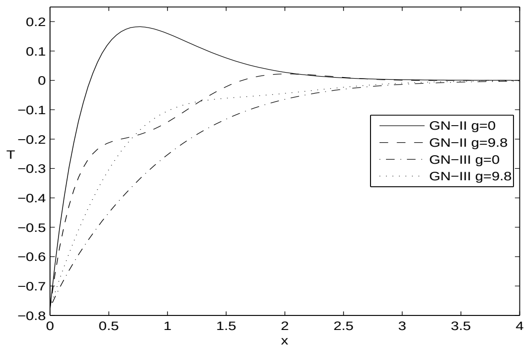

Fig. 7: Temperature distribution of different gravity for medium (NMST)

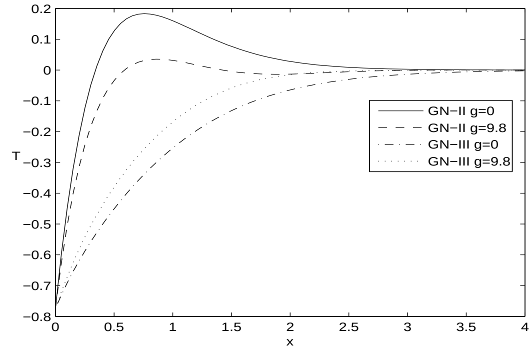

Fig. 10: Temperature distribution of different gravity for medium (WMT)

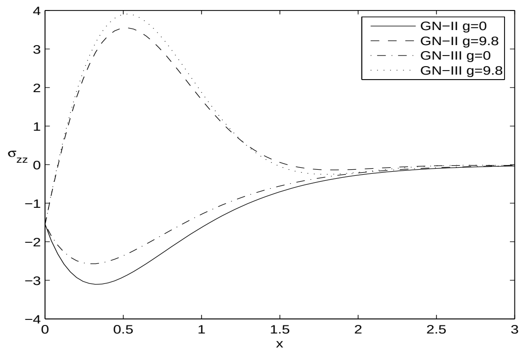

Fig. 8: Normal stress distribution of different gravity for medium (NMST)

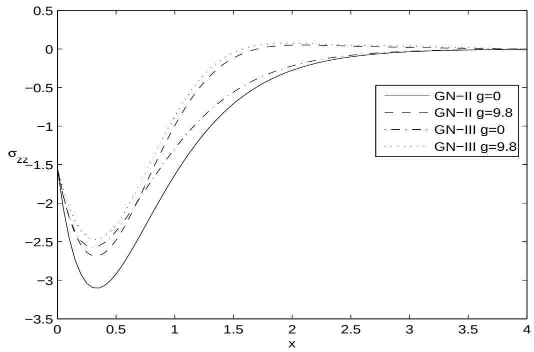

Fig. 11: Normal stress distribution of different gravity for medium (WMT)

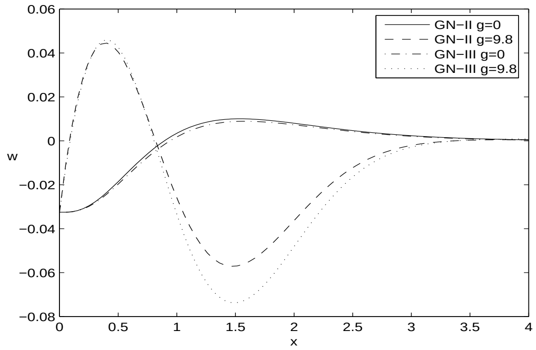

Fig. 9: Normal displacement distribution of different gravity for medium (WMT)

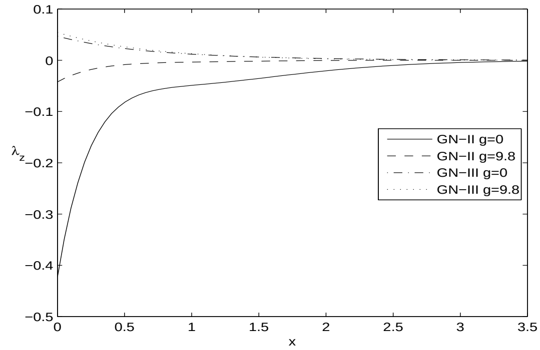

Fig. 12: Microstress distribution of different gravity for medium (WMT)

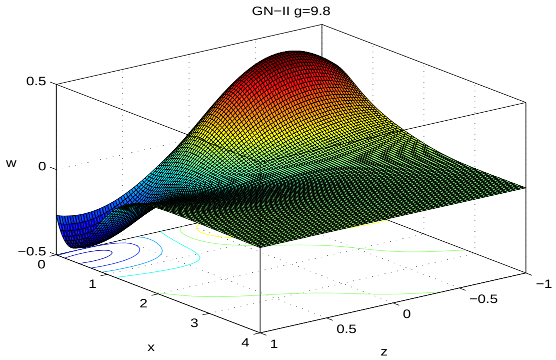

Fig. 13: Microstress distribution of different gravity for medium (WMT)

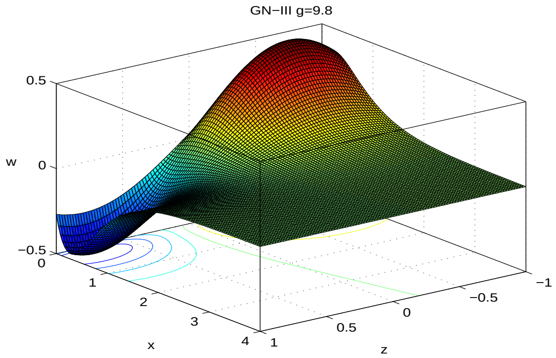

Fig. 14: Microstress distribution of different gravity for medium (WMT)

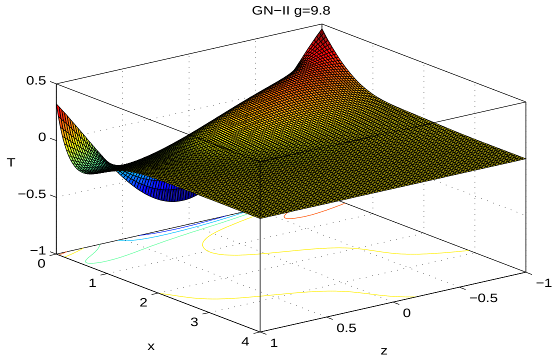

Fig. 15: Microstress distribution of different gravity for medium (WMT)

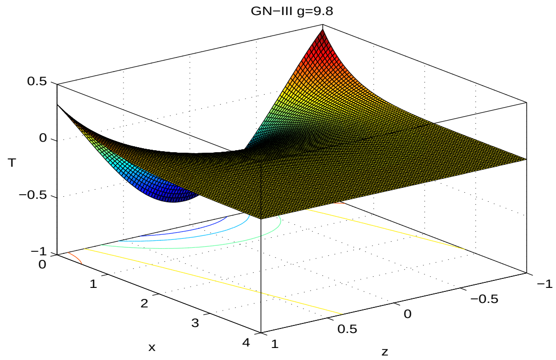

Fig. 16: Microstress distribution of different gravity for medium (WMT)

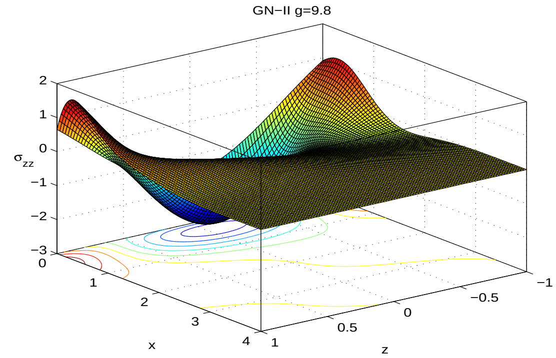

Fig. 17: Microstress distribution of different gravity for medium (WMT)

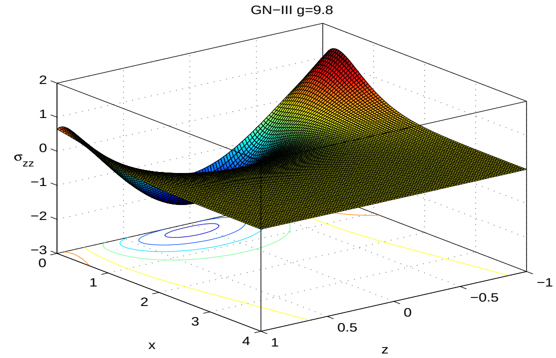

Fig. 18: Microstress distribution of different gravity for medium (WMT)

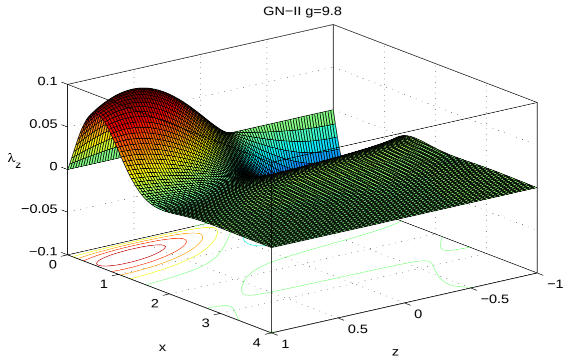

Fig. 19: Microstress distribution of different gravity for medium (WMT)

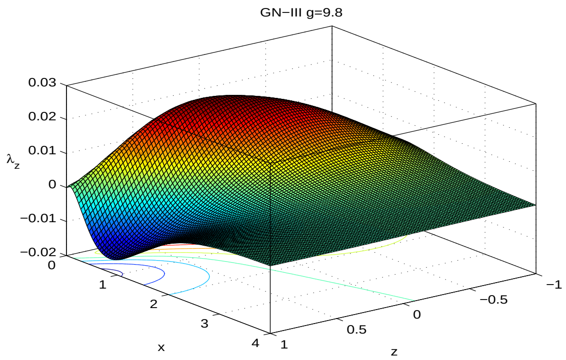

Fig. 20: Microstress distribution of different gravity for medium (WMT)