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Used Better than Aged in mgf Ordering Class of Life Distribution with Application of Hypothesis Testing

S. E. Abu-Youssef[1,*], *Nahed S. A. Ali*[2] *and M. E. Bakr*[3]

[1] Department of Mathematics, Faculty of Science, Al-Azhar University, Nasr City (11884), Cairo, Egypt.
[2] Department of Mathematics, Faculty of Education, Ain Shams University, Cairo, Egypt
[3] Department of Basic Science, Giza Higher Institute for Engineering, Cairo, Egypt.

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Abstract: In this work a new class of life distributions, namely used better than aged in moment generating function (UBA_{mgf}) class of life distribution is introduced, Relations of this aging to other well-known aging classes and their applications to a shock model are discussed. Preservations of this aging concept under some reliability operations are also given. Testing exponentiality versus (UBA_{mgf}) class of life distribution is proposed. Pitman's asymptotic efficiencies of the test are calculated and compared with other tests. The percentiles of this test statistic are tabulated.

Keywords: UBA class of life distribution; Moment generating function ordering; Shock model application; Life testing.

1 Introduction

Needing to the reliability theory is very important because, it gives a common scientific language between the scientists working in various fields of aging studies. In relation to various aging characteristics statisticians divided the life distributions into classes such as increasing failure rate (IFR), increasing failure rate average (IFRA), new better than used (NBU), new better than used in convex ordering (NBUC), new better than used in expectation (NBUE), harmonic new better than used in expectation (HNBUE), new better than used in Laplace transform order (NBUL), used better than aged (UBA), used better than aged convex order upper tail (UBACT) and a lot of other classes. Cline [1] and others studied the connection between the class of age-smooth distributions and the class of distribution with sub-exponential tails which have many applications in queuing theory random walk and infinite divisibility.

Such aging classes are derived via several notions of comparison between random variables. So we introduce a new aging notion derived from the moment generating function order. Before we go into the details, let us quickly review some common notions of stochastic orderings and aging notions considered in this paper. If X and Y are two random variables with distributions F and G (survivals $\overline{F}$ and $\overline{G}$) respectively, then we say that X is smaller than Y in the:

(a)Usual stochastic order, denoted by $X \leq_{st} Y$ if

$$\overline{F}(x) \leq \overline{G}(x) \qquad \text{for all} \quad x.$$

(b)Increasing convex order, denoted by $X \leq_{icx} Y$ if

$$\int_x^\infty \overline{F}(u)du \leq \int_x^\infty \overline{G}(u)du.$$

(c)Increasing concave order, denoted by $X \leq_{icv} Y$ if

$$\int_0^x \overline{F}(u)du \leq \int_0^x \overline{G}(u)du.$$

* Corresponding author e-mail: abuyousf@hotmail.com

Another important ordering that has come to use in reliability and is the following:

A random variable X is smaller than a random variable Y with respect to moment generating function order (denoted by $X \leq_{mgf} Y$) if, and only, if

$$\int_0^\infty e^{sx} dF(x) \geq \int_0^\infty e^{sx} dG(x), \qquad s \geq 0 \tag{1}$$

It is easy to check that (1) is equivalent to

$$\int_0^\infty e^{sx} \overline{F}(x) dx \leq \int_0^\infty e^{sx} \overline{G}(x) dx. \tag{2}$$

On the other hand, in the context of lifetime distributions, some of the above orderings have been used to give characterizations and new definitions of aging classes. By aging, we mean the phenomenon whereby an older system has a shorter remaining lifetime, in some statistical sense, than a younger one [2].

We aim in this paper to introduce a new aging notion derived from the moment generating function ordering, namely used better than aged in moment generating function ordering (UBA_{mgf}) class of life distribution. Definition and relationships are given in section 2. In section 3, we discussed some closure properties to (UBA_{mgf}) such as convolution and formation of a coherent system. In section 4, applications of this aging to a shock model is given. Based on goodness of fit approach our test is constructed in section 5. Monte Carlo null distribution critical values are simulated and tabulated in Table 1 for sample sizes $n = 5(5)100$ using mathematica 8 program in section six. Finally, Pitman asymptotic efficiencies for linear failure rate (LFR), Weibull and Makeham distributions, which belong to the UBA_{mgf} class, are calculated in section 7.

2 Definitions and Preliminaries

In reliability theory, aging life is usually characterized by a nonnegative continuous random variable $X \geq 0$ representing equipment life with distribution function F and survival function $\overline{F}(t) = 1 - F(t)$ such that $F(0-) = 0$. One of the most important approaches to the study of aging is based on the concept of the residual life. For any random variable X, let $X_t = [X - t \mid X > t]$, $\quad t \in \{x : F(x) < 1)$, denote a random variable whose distribution is the same as the conditional distribution of $X - t$ given that $X > t$ and has survival function

$$\overline{F}_t(x) = \begin{cases} \dfrac{\overline{F}(x+t)}{\overline{F}(t)} & \overline{F}(t) > 0 \\ 0 & \overline{F}(t) = 0 \end{cases}$$

When X is the lifetime of a device which has a finite mean $\mu = E(X) = \int_0^\infty \overline{F}(u) du$, the mean of X_t is called mean residual life (MRL) and is given by

$$\mu(t) = E(X_t) = \frac{\int_t^\infty \overline{F}(u) du}{\overline{F}(t)}. \tag{3}$$

Further, the hazard rate of X is defined by

$$h(t) = -\frac{d}{dt} ln\overline{F}(t) = \frac{f(t)}{\overline{F}(t)}, \qquad t \geq 0, \quad \overline{F}(t) > 0,$$

where $f(t) = F'(t)$ is the probability density of X assuming it exist. Note that if $lim_{t\to\infty} h(t) = h(\infty)$ exists and is positive, then cf. Willmot and cai (2000)

$$\mu(\infty) = \lim_{t\to\infty} \mu(t) = \frac{1}{h(\infty)}.$$

Two classes of life distributions were introduced by [3] which are used better than aged (UBA) and used better than aged in expectation (UBAE) classes of life distribution. Precisely we have the following definitions:

Definition 1. *The distribution function F is said to be used better than aged (UBA) if $0 < \mu(\infty) < \infty$ and for all $x, t \geq 0$,*

$$\overline{F}(x+t) \geq \overline{F}(t) e^{-x/\mu(\infty)}, \qquad x, t \geq 0 \tag{4}$$

Definition 2. *The distribution function F is said to be used better than aged in expectation (UBAE) if $0<\mu(\infty)<\infty$ and for all $x,t\geq 0$,*

$$\mu(t)\geq\mu(\infty) \tag{5}$$

Remark that, F is UBA (UBAE) if and only if X_t converges in distribution to a random variable X_A (say) exponentially distributed with failure rate $1/\mu$ and

$$X_t\leq_{st}X_A,\ (E(X_t)\leq_{st}E(X_A)).$$

According to the above definitions we can deduce the following new definition for used better than aged in the moment generating function order as follows.

Definition 3. *The distribution function F is said to be used better than aged in the moment generating function order (UBA_{mgf}) if $0<\mu(\infty)<\infty$ and for all $x,t\geq 0$,*

$$\int_0^{\infty}e^{sx}\overline{F}(x+t)dx\geq\frac{\mu(\infty)}{1-s\mu(\infty)}\overline{F}(t),\ \ s\geq 0, \tag{6}$$

It is obvious that (6) is equivalent to $X_t\leq_{mgf}X_A$ for all $t\geq 0$.

To introduce the definition of the discrete UBA_{mgf} , let X be a discrete non-negative random variable such that $P(X=k)=p_k,\ k=0,1,2,\ldots$.

And let $\overline{P}_k=P(X>k),\ k\geq 1,\ \overline{P}_0=1$ denote the corresponding survival function.

The discrete non-negative random variable X is said to be discrete used better than aged in moment generating function order (discrete UBA_{mgf}) if, and only, if

$$\sum_{k=0}^{\infty}\overline{P}_{k+i}z^k\geq\overline{P}_i\sum_{k=0}^{\infty}z^k,\ \ for\ all\ 0\leq z\leq 1\ and\ i=0,1,\ldots.$$

Now, since $X\leq_{st}X_A \Rightarrow X\leq_{mgf}X_A$.

Then, we have the following implication :

$$\begin{array}{ccccc}\text{IFR} & \subset & \text{UBA} & \subset & UBA_{mgf}\\ & & \cap & & \\ & & \text{UBAE} & & \end{array}$$

see [4] .

3 Preservation Results

As an important reliability operations, convolution, mixture and formation of coherent system of a certain class of life distribution is often paid much attention. It has been shown that UBA_{mgf} are closed under these operations.

a) **Convolution**

In the next theorem we establish the closure property for UBA_{mgf} under convolution.

Theorem 1. *UBA_{mgf} class of life distribution is closed under convolution operation.*

Proof. Suppose F_1 and F_2 are UBA_{mgf} , then we have

$$\begin{aligned}\int_0^{\infty}e^{sx}\overline{F}(x+t)dx &= \int_0^{\infty}\int_0^{\infty}e^{sx}\overline{F}_1(x+t-u)dF_2(u)dx\\ &= \int_0^{\infty}\int_0^{\infty}e^{sx}\overline{F}_1(x+t-u)dx\,dF_2(u)\\ &\geq \int_0^{\infty}\frac{\mu(\infty)}{1-s\mu(\infty)}\overline{F}_1(t-u)dF_2(u)dx\\ &= \frac{\mu(\infty)}{1-s\mu(\infty)}\overline{F}_1(t).\end{aligned}$$

Which proved that the UBA_{mgf} is closed under convolution operation, where

$$\overline{F}(x+t)=\int_0^{\infty}\overline{F}_1(x+t-u)dF_2(u).$$

b)**Formation of coherent systems using independent components**

A system is called coherent if:

i)Every component is relevant.

ii)The structure function, which represents the performance of the system in terms with performance of the component is increasing.

Design engineers give greater importance to coherency in building systems. For more details about coherent system see [5].

In the next theorem we establish the closure property of the UBA_{mgf} class under the formation of a coherent system operation.

Theorem 2. *A series system of nindependent UBA_{mgf} components is UBA_{mgf}.*

Proof. Let $X_1, X_2, \ldots, X_n$ be independent UBA_{mgf} then we have

$$\int_0^\infty e^{sx}\frac{p\left(min\left(X_1,\ldots,X_n\right)\geq y+t\right)}{p\left(min\left(X_1,\ldots,X_n\right)\geq t\right)}dx=\int_0^\infty\prod_{i=1}^n e^{sx}\frac{p\left(X_i\geq y+t\right)}{p\left(X_i\geq t\right)}dx=\int_0^\infty\prod_{i=1}^n e^{-sx}\frac{\overline{F}_i(y+t)}{\overline{F}_i(t)}dx.$$

Since F_i is UBAmgf class of life distribution, then we obtain

$$\int_0^\infty\prod_{i=1}^n e^{sx}\frac{\overline{F}_i(y+t)}{\overline{F}_i(t)}dx\geq\int_0^\infty\prod_{i=1}^n e^{sx}e^{-x/\mu_i(\infty)}dx=\int_0^\infty e^{sx}e^{-x\sum_{i=1}^n\frac{1}{\mu_i(\infty)}}dx$$

$$=\int_0^\infty e^{-\left(\left(\sum_{i=1}^n\frac{1}{\mu_i(\infty)}\right)-s\right)x}dx=\frac{1}{\sum_{i=1}^n\frac{1}{\mu_i(\infty)}-s}.$$

This completes the proof.

4 Applications: Shock Model Application

Suppose that a device is subject to shocks. Let $N(t)$ be the number of shocks in time interval $(0,t\,]$. The kth shock arrives at time T_k . Let $X_k=T_{k+1}-T_k$ be the time between the kth and $(k+l)st$ shocks. We assume that $X_1, X_2, \ldots$ are mutually independent and identically distributed according to F. Let $a_k(t)=p\left(N(t)=k\right)$, $k=1,\,2,\ldots$ and let $\overline{P}_k$ be the probability of the device surviving k shocks. Then the survival probability of the system until time t is

$$\overline{H}\left(t\right)=\sum_{k=0}^\infty a_k\left(t\right)\overline{P}_k \quad .$$

Theorem 3. ***F*** *is UBA_{mgf} implies H is UBA_{mgf}.*

Proof. Observe that $\overline{H}\left(t\right)$ can be written in the form

$$\overline{H}\left(t\right)=\sum_{k=1}^\infty\overline{F}_k(t)\;p_k$$

Where $p_k=\overline{P}_{k-1}-\overline{P}_k,\;k=1,\,23,\,\ldots$ and F_k is the distribution function of T_k, and

$$\int_0^\infty e^{sx}\overline{H}\left(x+t\right)dx\;=\sum_{k=1}^\infty\int_0^\infty e^{sx}\overline{F}_k\left(x+t\right)\;p_k dx$$

$$\geq\frac{\mu(\infty)}{1-s\mu(\infty)}\sum_{k=1}^\infty\overline{F}_k\left(t\right)\;p_k,$$

Since F is UBA_{mgf}

$$=\frac{\mu(\infty)}{1-s\mu(\infty)}\overline{H}\left(t\right),$$

then H is UBA_{mgf}.

5 Testing Against UBA_{mgf}

This section is concerned with the construction of the proposed statistic as a U-statistic and discussing its asymptotic normality. Here, we hope to test the null hypothesis H_0 : F is exponential, against H_1 : F is UBA_{mgf}, and is not exponential. Non-parametric testing for classes of life distributions has been considered by many authors (see [6], [7], [8], [9]; [10] and [11].

According to Eq. (6) we may use the following as a measure of departure from H_0.

$$\delta(s) = E\left[\int_0^\infty e^{sx}\overline{F}(x+t)dx - \frac{\mu(\infty)}{1-s\mu(\infty)}\overline{F}(t)\right] = \int_0^\infty\left[\int_0^\infty e^{sx}\overline{F}(x+t)dx - \frac{\mu(\infty)}{1-s\mu(\infty)}\overline{F}(t)\right]dF_0(t).$$

The following theorem is essential for the development of our test statistic.

Theorem 4. *Let X be the UBA_{mgf} random variable with distribution function F; then based on the Goodness of fit approach technique,*

$$\delta(s) = \frac{1}{(1+s)}\left[\frac{1}{s}(\varphi(s)-1) - \frac{1}{(1-s\mu(\infty))}(1+\mu(\infty)-2\varphi(-1))\right] \tag{7}$$

where $\varphi(s) = \int_0^\infty e^{sx}dF(x)$.

Proof. Since

$$\delta(s) = \int_0^\infty\left[\int_0^\infty e^{sx}\overline{F}(x+t)dx - \frac{\mu(\infty)}{1-s\mu(\infty)}\overline{F}(t)\right]dF_0(t).$$

We can take $F_0(x) = 1-e^{-x}$, $x \geq 0$, then

$$\begin{aligned}\delta(s) &= \int_0^\infty\int_0^\infty e^{-t+su}\overline{F}(u+t)dudt - \frac{\mu(\infty)}{1-s\mu(\infty)}\int_0^\infty \overline{F}(t)e^{-t}\,dt \\ &= I_1 - I_2.\end{aligned}$$

Where,

$$\begin{aligned} I_1 &= \int_0^\infty\int_0^\infty e^{-t}e^{su}\overline{F}(u+t)dudt = \int_0^\infty\int_t^\infty e^{-t}e^{-s(t-x)}\overline{F}(x)dxdt \\ &= \frac{1}{s}\int_0^\infty e^{-t}(e^{st}-1)\overline{F}(t)\,dt = \frac{1}{s(1+s)}\left[(\varphi(s)-1) - s(1-\varphi(-1))\right]. \end{aligned} \tag{8}$$

And,

$$I_2 = \frac{\mu(\infty)}{1-s\mu(\infty)}\int_0^\infty \overline{F}(t)\;dF_0(t) = \frac{\mu(\infty)}{1-s\mu(\infty)}(1-\varphi(-1)). \tag{9}$$

From equations, (8) and (9), we obtain (7).

Let $X_1, X_2, \ldots, X_n$ be a random sample from the distribution function F. For generality, we assume $\mu(\infty)$ is known and equal one. The empirical estimator $\widehat{\delta}(s)$ of our test statistic can be obtained as follows:

$$\widehat{\delta}_n(s) = \frac{1}{(s+1)}\sum_i\left\{\frac{1}{s}(e^{sX_i}-1) - \frac{2}{(1-s)}(1-e^{-X_i})\right\}.$$

To make the test is invariant, let

$$\widehat{\Delta}_n(s) = \frac{\widehat{\delta}_n(s)}{\overline{X}}.$$

Let us rewrite $\widehat{\delta}$ as follows,

$$\widehat{\Delta}_n(s) = \frac{1}{\overline{X}n}\sum_i \emptyset(X_i)$$

where

$$\emptyset(X_i) = \frac{1}{(1+s)}\{\frac{1}{s}\left(e^{sX_i}-1\right) - \frac{2}{(1-s)}\left(1-e^{-X_i}\right)\}.$$

To find the limiting distribution of $\widehat{\delta}(s)$ we resort to the U-statistic theory and practice (Lee 1990). Set

$$\emptyset(X_1) = \frac{1}{(1+s)}\{\frac{1}{s}\left(e^{sX_1}-1\right) - \frac{2}{(1-s)}\left(1-e^{-X_1}\right)\}.$$

Then, $\widehat{\Delta}_n(s)$ is equivalent to U-statistic given by:

$$U_n = \frac{1}{\binom{n}{1}}\sum_i \emptyset(X_i).$$

The following theorem summarizes the asymptotic normality of $\widehat{\delta}_n(s)$.

Theorem 5. *(i)As $n\to\infty$, $\sqrt{n}\,(\widehat{\delta}_{mg_n}(s) - \delta_{mg}(s))$ is asymptotically normal with mean 0 and variance $\sigma^2(s)$ where,*

$$\sigma^2(s) = E\left(\frac{1}{(1+s)}\left\{\frac{1}{s}(e^{sx}-1) - \frac{2}{(1-s)}(1-e^{-x})\right\}\right)^2$$

(ii)Under H_0, the variance is

$$\sigma_0^2(s) = \frac{2}{3(s-2)(2s-1)(s-1)^2}.$$

Proof. (i)Using standard U-statistic theory, Lee (1990), and direct calculations, we get

$$\mu_0 = \int_0^\infty \left(\frac{1}{(1+s)}\left\{\frac{1}{s}(e^{sx}-1) - \frac{2}{(1-s)}(1-e^{-x})\right\}\right)e^{-x}dx = 0,$$

and

$$\sigma^2(s) = Var\left[\widehat{\delta}_n(s_{mg})\right] = E(\frac{1}{(1+s)}\left\{\frac{1}{s}(e^{sx}-1) - \frac{2}{(1-s)}(1-e^{-x})\right\})^2.$$

(ii)Under H_0, the variance is given by

$$\sigma_0^2(s) = \int_0^\infty \left(\frac{1}{(1+s)}\left\{\frac{1}{s}(e^{sx}-1) - \frac{2}{(1-s)}(1-e^{-x})\right\}\right)^2 e^{-x}dx = \frac{2}{3(s-2)(2s-1)(s-1)^2}.$$

6 Monte Carlo Null Distribution Critical Points

Based on 10000 generated samples from the standard exponential distribution the Monte Carlo null distribution critical values of our test $\widehat{\delta}(0.01)$ are simulated and tabulated, where n = 5(5)100 in Table 1. Mathematica 8 program is used.

Table 1: The Upper Percentile Points of $\widehat{\delta}(0.01)$ with 10000 Replications

n	90%	95%	99%
5	0.227032	0.319497	0.442451
10	0.195124	0.239401	0.321086
15	0.156921	0.207935	0.28445
20	0.14302	0.193264	0.287329
25	0.122356	0.158973	0.234162
30	0.119859	0.153288	0.211601
35	0.11309	0.147247	0.200039
40	0.100245	0.136108	0.192219
45	0.102163	0.1314	0.183553
50	0.0934178	0.12904	0.180371
55	0.100963	0.125296	0.168596
60	0.0925389	0.118794	0.174871
65	0.0891563	0.108883	0.156322
70	0.084098	0.112696	0.157575
75	0.0770758	0.0973109	0.143918
80	0.0824498	0.103958	0.141885
85	0.0803769	0.098325	0.13703
90	0.0743475	0.100896	0.131009
95	0.0710787	0.0895947	0.123556
100	0.0760844	0.0930191	0.127766

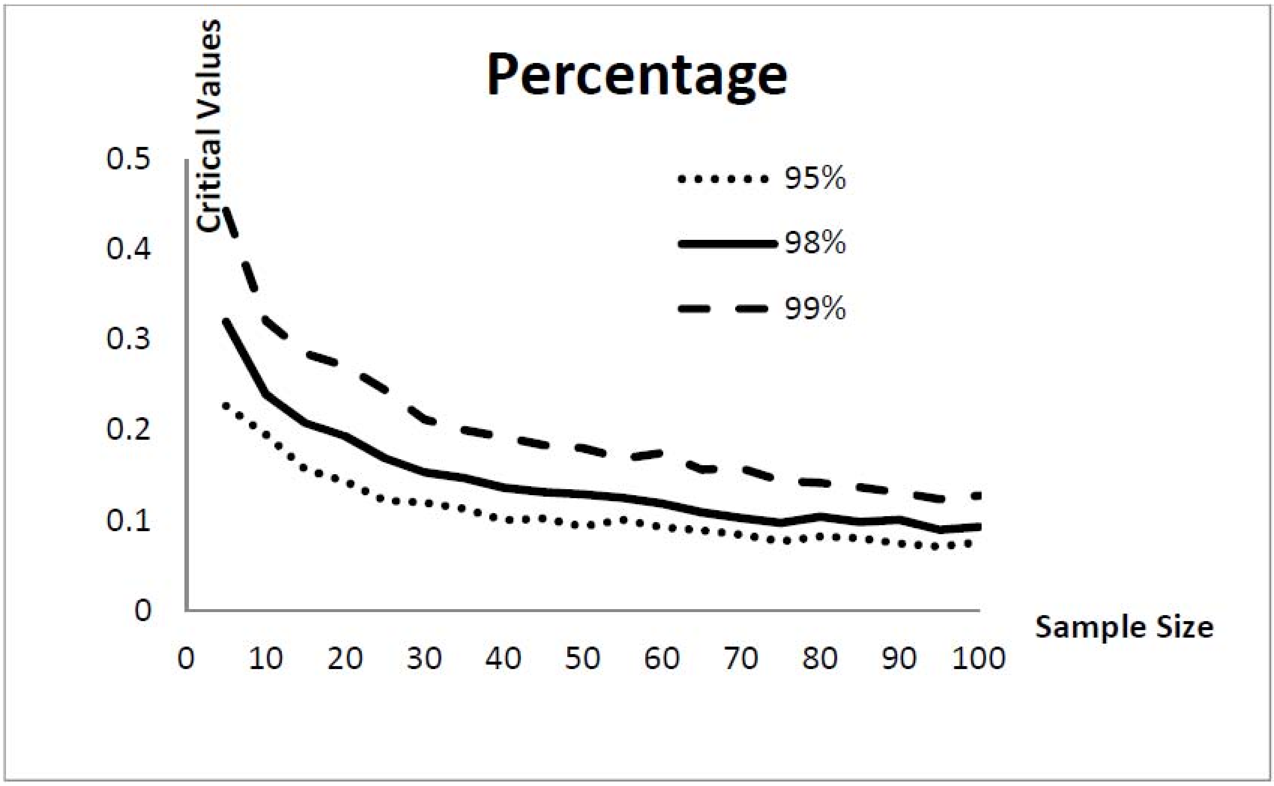

Fig. 1: The Relation Between Sample Size and Critical Values.

It is clear from Table 1 and Fig. 1 , the critical values decrease as the sample size increases and they increase as the confidence level increases.

7 Pittman Asymptotic Relative Efficiency (ARE):

Since the above test statistic $\widehat{\delta}(s)=\dfrac{\delta}{\overline{X}}$ is new and no other tests are known for this class (UBAL). We may compare our test to the other classes. Here we choose the test $\Delta_{\theta,(1)}$ presented by [2005] and $\delta_{F_n}^{(2)}$ presented [12] for (NBAFR) class of life distribution. Then comparisons are achieved by using Pitman asymptotic relative efficiency PARE, which is defined as follows:

Let T_{1n} and T_{2n} be two statistics, then PARE of T_{1n} relative to T_{2n} is defined by

$$e(T_{1n},T_{2n})=\frac{\mu_1'(\theta_0)}{\sigma_1(\theta_0)}\Big/\frac{\mu_2'(\theta_0)}{\sigma_2(\theta_0)}.$$

Where:

$$\mu_i'(\theta_0)=\lim_{n\to\infty}\frac{\partial}{\partial\theta}E(T_{n_i})\Big|_{\theta\to\theta_0},\qquad \text{and}\qquad \sigma_i^2(\theta_0)=\lim_{n\to\infty}var(T_{n_i}).$$

Three of the most commonly used alternatives they are:

(i)Linear failure rate family

$$\overline{F}_1(x)=e^{-x-\frac{x^2}{2}\theta},\qquad \theta,x\geq 0. \tag{10}$$

(ii)Weibull family:

$$\overline{F}_2(x)=e^{-x^\theta},\qquad \theta\geq 1,,x\geq 0. \tag{11}$$

(iii)Makeham family:

$$\overline{F}_2(x)=e^{-x-\theta(x+e^{-x}-1)},\qquad \theta,x\geq 0. \tag{12}$$

Note that H_0 (the exponential distribution) is attained at $\theta=0$ in (i) and (iii) and $\theta=1$in (ii). The Pitman's asymptotic efficiency (PAE) of $\widehat{\Delta}(s)$ is equal to

$$PAE\left(\widehat{\delta}(s)\right)=\frac{\left|\frac{\partial}{\partial\theta}\delta(s)\right|_{\theta\to\theta_0}}{\sigma_0(s)}=\frac{1}{\sigma_0(s)}\left|\frac{1}{s(1+s)}\int_0^\infty e^{-sx}d\overline{F}_{\theta_0}'(x)-\frac{2}{(1+s)(1-s)}\int_0^\infty e^{-x}d\overline{F}_{\theta_0}'(x)\right|$$

Where $\overline{F}_{\theta_0}'(x)=\dfrac{d}{d\theta}\overline{F}_\theta(u)\Big|_{\theta\to\theta_0}$

This leads to:

(i)PAE in case of the linear failure rate distribution:

$$PAE\left(\widehat{\delta}(s)\right)=\frac{1}{\sigma_0(0.01)}\left|\frac{-1}{0.0101}\int_0^\infty e^{-sx}d\left(\frac{-x^2}{2}e^{-x}\right)-\frac{1}{0.9999}\int_0^\infty e^{-x}d\left(\frac{-x^2}{2}e^{-x}\right)\right|=1.43$$

(ii)PAE in case of the Weibull distribution:

$$PAE\left(\widehat{\delta}(s)\right)=\frac{1}{\sigma_0(0.01)}\left|\frac{-1}{0.0101}\int_0^\infty e^{-sx}d\left(-xln|x|e^{-x}\right)-\frac{2}{0.9999}\int_0^\infty e^{-x}d\left(-xln|x|e^{-x}\right)\right|=0.5972$$

(iii)PAE in case of the Makeham distribution.

$$PAE\left(\widehat{\delta}(s)\right)=\frac{1}{\sigma_0(0.01)}\left|\frac{-1}{0.0101}\int_0^\infty e^{-sx}d\left((1-x-e^{-x})e^{-x}\right)-\frac{1}{0.9999}\int_0^\infty e^{-x}d\left((1-x-e^{-x})e^{-x}\right)\right|=0.1019$$

Direct calculations of PAE of $\Delta_{\theta,(1)}$, $\delta_{F_n}^{(2)}$ and $\widehat{\delta}(s)$ are summarized in table 2, the efficiencies in table shows clearly our U-statistic $\widehat{\delta}(s)$ perform well for F_1, F_2 *and* F_3.

In Table 3, we give PARE's of $\widehat{\delta}(s)$ with respect to $\Delta_{\theta,(1)}$and $\delta_{F_n}^{(2)}$ whose PAE are mentioned in Table 2.

It is clear from Table 3 that the statistic $\widehat{\delta}(s)$ perform well for $\overline{F}_1$, $\overline{F}_2$ and $\overline{F}_3$ and it is more efficient than both $\Delta_{\theta,(1)}$ and $\delta_{F_n}^{(2)}$ for all cases mentioned above. Hence our class of life distribution, which deals the much larger, is better and also simpler.

Table 2: PAE of $\Delta_{\theta,(1)}$, $\delta_{F_n}^{(2)}$ and $\widehat{\delta}(s)$

Distribution	$\Delta_{\theta,(1)}$	$\delta_{F_n}^{(2)}$	$\widehat{\delta}(s)$
LFR	0.408	0.217	1.3
Weibull	0.170	0.050	0.969
Makeham	0.0395	0.144	0.86

Table 3: PARE of $\widehat{\delta}(s)$ with respect to $\Delta_{\theta,(1)}$and $\delta_{F_n}^{(2)}$.

Distribution	$e(\widehat{\delta}(s),\Delta_{\theta,(1)})$	$e\left(\widehat{\delta}(s),\delta_{F_n}^{(2)}\right)$
LFR	3.18	5.99
Weibull	5.7	19.38
Makeham	21.77	5.97

8 Conclusion

A new class of life distributions namely used better than aged in moment generating function (UBA_{mgf}) is derived. Our proposed aging class of life distribution is the largest from some other well-known aging classes. The applications of UBA_{mgf} to a shock model are presented. The preservations of this aging concept under some reliability operations are also given. Testing exponentiality versus UBA_{mgf} class of life distribution is proposed. Pitman's asymptotic efficiencies of the test are calculated and showed our test statistic of our class of life distribution is much larger , perform better and also simpler.

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S. E. Abu-Youssef is presently employed professor of Statistics and head of the Depart of Mathematics, Al Azhar University. He is referee of several journals in the branch of mathematical statistics. His research interests include Theory of reliability, Statistical inference, ordered statistics and Bayesian analysis. He is published more than 70 papers in reputed international journals of Statistics.

Nahed S. A. Ali is a teacher of statistics in the faculty of education, Ain Shams university. Her research interests are in the areas of non-parametric statistics, linear models, saddle point, and reliability

M. E. Bakr is a Lecture of Mathematics at Basic Science Department, Giza Higher Institute for Engineering and Modern Technology, Cairo, Egypt. He received his PhD. in Mathematical Statistics in 2018 from Al Azhar University, Cairo, Egypt. His research interests include: Classes of life distributions and their Hypothesis tests.