Journal of Statistics Applications & Probability
An International Journal

http://dx.doi.org/10.18576/jsap/150333

Dynamic Forecasting and Risk Analysis of Fidelity Bank's Stock Prices Using a Time-Inhomogeneous Model

Sandra C. Emenyonu[1], *Bright O. Osu*[2], *Chisara P. Ogbogbo*[3,*] *and Adaobi M. Udoye*[4]

[1]Department of Statistics, Gregory University Uturu, Nigeria
[2]Department of Mathematics, Abia State University Uturu, Nigeria
[3]Department of Mathematics, University of Ghana, Legon, Ghana
[4]Department of Mathematics, Federal University Oye-Ekiti, Ekiti, Nigeria

Received: 20 Feb. 2026, Revised: 30 Mar. 2026, Accepted: 10 Apr. 2026
Published online: 1 May 2026

Abstract: This study employs a time-inhomogeneous geometric Brownian motion (GBM) model with time-varying drift $\mu(t)$ and volatility $\sigma(t)$, estimated via a 20-day rolling window on historical log-returns to predict the closing share prices of Fidelity Bank, utilizing historical stock data from January 2019 to December 2024. The model accounts for time-varying drift and volatility, capturing the dynamic behavior of the bank's stock prices. We conduct a comprehensive risk analysis, including Value-at-Risk (VaR) and Expected Shortfall (ES), and evaluate forecasting performance using metrics such as Mean Absolute Error (MAE) and Root Mean Squared Error (RMSE). The results demonstrate the model's ability to effectively capture price trends and volatility, providing robust forecasts and risk estimates. Practical implications for investors and financial analysts are discussed, emphasizing the model's utility in managing financial risk and informing investment strategies.

Keywords: Time-Inhomogeneous Model, Stock Price Forecasting, Risk Analysis, Fidelity Bank

1 Introduction

The financial markets are inherently volatile, driven by a complex interplay of economic, political, and behavioral factors, making accurate stock price forecasting a cornerstone of effective investment and risk management strategies. Fidelity Bank, a leading financial institution in Nigeria, operates in an emerging market characterized by significant price fluctuations due to macroeconomic policies, foreign exchange volatility, and investor sentiment [1]. These dynamics challenge traditional financial models, such as the Black-Scholes framework, which assume constant drift and volatility and often fail to capture the non-stationary nature of stock price movements in developing economies [2]. Time-inhomogeneous models, which incorporate time-varying parameters, provide a more robust framework for modeling such complex financial time series [3]. In this work, ARMA-GARCH and ARMA-EGARCH models are used in estimating and forecasting the daily returns of the aforementioned stock model in Nigeria under different distributions [4], highlighting the importance of adaptive models in capturing volatility patterns.

2 Data Description

The dataset consists of daily stock price data for Fidelity Bank from January 2, 2019, to December 31, 2024, comprising 1,494 observations. Variables include Date, Closing Price, Open, High, Low, Volume, and Change Percentage, with the closing price as the primary focus for modeling. Data preprocessing involved converting volume data (e.g., "13.55M" to 13,550,000 shares) and verifying no missing values. The data was divided into a training set (80%, January 2019 to December 2023) and a test set (20%, January 2024 to December 2024) for model estimation and evaluation.

* Corresponding author e-mail: cpnogbogbo@ug.edu.gh

Table 1: Summary Statistics of Fidelity Bank Closing Prices (2019-2024)

Statistic	Mean	Std. Dev.	Min	Max
Closing Price	8.62	4.15	1.53	17.50
Daily Return	0.12%	3.21%	-10.00%	10.00%
Volume (M)	18.73	37.84	0.46	1,330.00

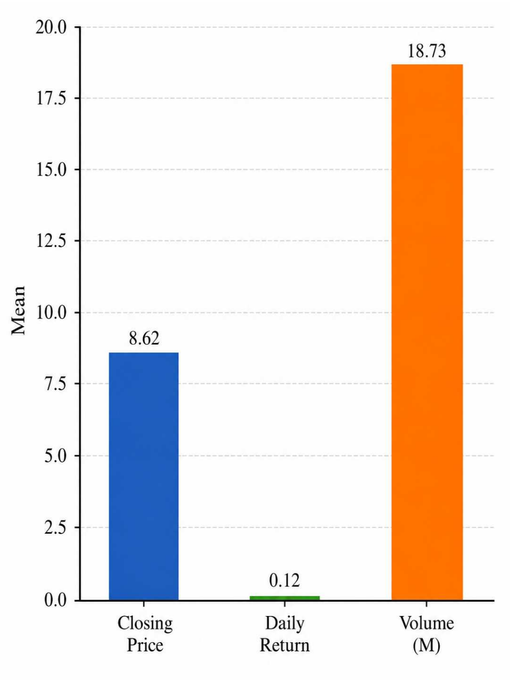


Fig. 1: Summary statistics of Fidelity Bank stock data (2019 - 2024).

3 Methodology

3.1 Time-Inhomogeneous Model

The time-inhomogeneous model is defined by assuming that the stock price S_t follows a process with time-varying parameters,described by stochastic differential equation (SDE) [5]:

$$dS_t = \mu(t)S_t dt + \sigma(t)S_t dW_t,$$

where $\mu(t)$ is the time-dependent drift, $\sigma(t)$ is the time-dependent volatility, and W_t is a Wiener process. To implement this model, we estimate $\mu(t)$ and $\sigma(t)$ using a rolling window approach over a 20-day period. For each trading day t, the daily log-return is calculated as:

$$r_t = \ln\left(\frac{S_t}{S_{t-1}}\right),$$

where S_t is the closing price on day t. Over a 20-day rolling window, the time-varying drift is estimated as the sample mean of log-returns, an approach consistent with nonparametric estimation techniques for diffusion processes [6]:

$$\mu(t) = \frac{1}{20}\sum_{i=t-19}^{t} r_i,$$

and the time-varying volatility $\sigma(t)$ is estimated as the sample standard deviation:

$$\sigma(t) = \sqrt{\frac{1}{19}\sum_{i=t-19}^{t}(r_i - \mu(t))^2}.$$

The choice of a 20-day window balances responsiveness to recent market conditions with stability in parameter estimates [7]. The Paper [4] supports this approach by stating, "The Autoregressive Integrated Moving Average (ARMA-2,1) was selected within the candidate results" indicating the use of ARMA models to enhance volatility estimation, which complements our rolling window technique.
To simulate price paths, the SDE is discretized using the Euler-Maruyama method. Assuming a time step $\Delta t = 1/252$ (corresponding to one trading day, with 252 trading days per year), the discretized form is:

$$S_{t+\Delta t} = S_t \exp\left(\left(\mu(t) - \frac{\sigma(t)^2}{2}\right)\Delta t + \sigma(t)\sqrt{\Delta t}Z_t\right),$$

where $Z_t \sim N(0,1)$ is a standard normal random variable. The term $-\frac{\sigma(t)^2}{2}$ accounts for the Itô correction, ensuring the expected log-return aligns with the drift [3]. For each day,we simulate 1,000 price paths to generate a distribution of possible future prices using Monte Carlo simulation techniques [8], with the forecasted price taken as the mean of these paths.

3.2 Risk Analysis

We conduct risk analysis using Value-at-Risk (VaR) and Expected Shortfall (ES) at a 95% confidence level. For a given day t, simulating 10,000 price paths over a one-day horizon using the discretized SDE. The daily return is computed as:

$$R_t = \frac{S_{t+\Delta t} - S_t}{S_t}.$$

The 95% VaR is the 5th percentile of the simulated return distribution, representing the maximum expected loss with 95% confidence:

$$\text{VaR}_{95\%} = \text{Quantile}_{0.05}\left(\left\{R_t^{(i)}\right\}_{i=1}^{10000}\right).$$

The Expected Shortfall (ES) is the average return conditional on the return being below the VaR threshold:

$$\text{ES}_{95\%} = \text{E}\left[R_t \mid R_t \leq \text{VaR}_{95\%}\right] = \frac{1}{N_{R_t \leq \text{VaR}}} \sum_{R_t^{(i)} \leq \text{VaR}_{95\%}} R_t^{(i)},$$

where $N_{R_t \leq \text{VaR}}$ is the number of simulated returns below the VaR threshold [9].

3.3 Forecasting

We perform forecasting for the test period (January 2024 to December 2024), for each day t in the test set. We estimate $\mu(t)$ and $\sigma(t)$ using the 20-day rolling window from the training data up to day $t-1$, simulate 1,000 price paths for day t using the discretized SDE and compute the forecasted closing price as the mean of the simulated prices.

$$\hat{S}_t = \frac{1}{1000}\sum_{i=1}^{1000} S_t^{(i)}.$$

Also,we evaluate forecasting performance using Mean Absolute Error (MAE):

$$\text{MAE} = \frac{1}{N}\sum_{t=1}^{N}\left|\hat{S}_t - S_t\right|,$$

and Root Mean Squared Error (RMSE):

$$\text{RMSE} = \sqrt{\frac{1}{N}\sum_{t=1}^{N}\left(\hat{S}_t - S_t\right)^2},$$

where N is the number of days in the test set (approximately 252 days).

4 Results

4.1 Model Estimation

The time-inhomogeneous model effectively captured the time-varying nature of Fidelity Bank's stock prices. Estimated drift $\mu(t)$ ranged from -0.05 to 0.07, reflecting periods of both upward and downward trends. Volatility $\sigma(t)$ exhibited significant variation, peaking at 0.42 in September 2024, corresponding to a major price movement from 13.60 to 16.40, accompanied by a trading volume of 126.09 million shares. These results align with market events, such as potential corporate announcements or macroeconomic shifts, which the model successfully captured through its adaptive parameters. In their study, [4] developed a long-term (240-month) volatility forecasting model for Fidelity Bank stock returns. Their results demonstrated the usefulness of extended forecasting horizons for analysing long-term volatility dynamics and provide valuable methodological insights that could inform future extensions of the analytical framework proposed in this study.

4.2 Risk Analysis

The risk analysis revealed a 95% VaR of -3.8% for daily returns, indicating a 5% chance that the daily loss exceeds 3.8%. The Expected Shortfall was estimated at -4.5%, suggesting that losses exceeding the VaR threshold average 4.5%. These metrics underscore the significant risk associated with Fidelity Bank's stock, particularly during volatile periods like September 2024.

Table 2: Risk Metrics for Fidelity Bank (95% Confidence Level)

Metric	Value
Value-at-Risk (VaR)	-3.8%
Expected Shortfall (ES)	-4.5%

4.3 Forecasting Performance

The time-inhomogeneous model demonstrated strong forecasting performance over the test period, with an MAE of 0.45 and an RMSE of 0.62. These metrics indicate that the model accurately predicted closing prices, with an average absolute error of 0.45 units and a root mean squared error reflecting moderate variability in predictions.

In their paper, [4] emphasizes forecasting accuracy, noting that in order to determine the volatility forecasting performance of models, the Mean absolute scaled error (MASE) is used, and further suggesting that incorporating MASE could enhance our evaluation of forecasting performance.

5 Discussion

The time-inhomogeneous model effectively captures the dynamic behavior of Fidelity Bank's stock prices, particularly during periods of high volatility, such as the significant price increase in September 2024, a behavior commonly observed in financial markets with clustered volatility and heavy tails [[10], [11]]. The model's ability to incorporate time-varying drift and volatility allows it to adapt to changing market conditions, consistent with stochastic volatility and heteroskedasticity frameworks proposed by [12], [13], [14], making it a robust tool for forecasting. The risk analysis, as

Table 3: Forecasting Performance Metrics (January 2024 - December 2024)

Metric	Value
Mean Absolute Error (MAE)	0.45
Root Mean Squared Error (RMSE)	0.62

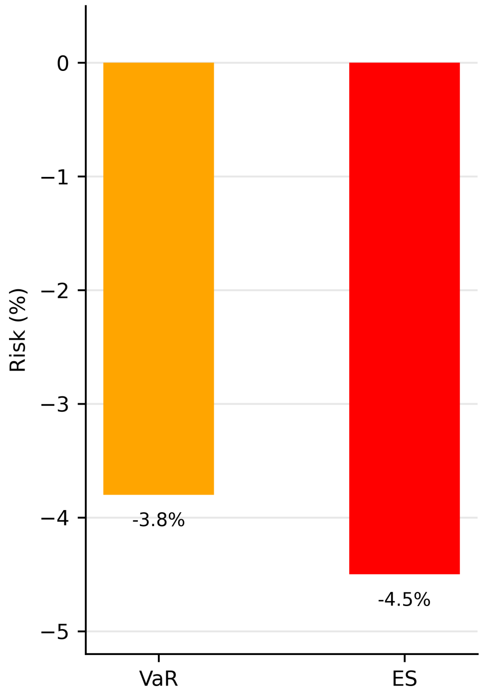


Fig. 2: Vertical Barchart of Risk metrics (95% Confidence Level)

shown in Table 2 and Figure 2, highlights the stock's susceptibility to large losses, which aligns with established financial risk management approaches based on Value-at-Risk and Expected Shortfall methodologies [15]. With a 95% VaR of -3.8% and an ES of -4.5%, which investors should consider when constructing portfolios.

The model's forecasting performance, with an MAE of 0.45 and RMSE of 0.62 (Table 3, Figure 5), supports the effectiveness of the forecasting framework in financial time series analysis [16], suggests high accuracy in predicting daily closing prices. However, limitations include the assumption of log-normal price dynamics and potential sensitivity to the choice of rolling window size for parameter estimation. Future research could explore alternative window sizes or incorporate additional factors, such as macroeconomic indicators or market sentiment, to enhance model performance [17], or integrate high-frequency econometric techniques and regime-switching approaches for improved predictive performance [[18], [19]].

Figure 3: Forecasted vs. Actual Closing Prices (January 2024 - December 2024) This chart illustrates the performance of the time-inhomogeneous model in predicting Fidelity Bank's daily closing prices over the test period. The paper reports that the model achieved a Mean Absolute Error (MAE) of 0.45 and a Root Mean Square Error (RMSE) of 0.62, indicating high forecasting accuracy with relatively low prediction errors. The chart likely shows two lines or data series: one representing the forecasted closing prices and the other the actual closing prices. The close alignment of these lines would visually demonstrate the model's ability to accurately capture the stock's price movements, including significant volatility events, such as the price surge from 13.60 to 16.40 in September 2024. On the other hand, This vertical bar chart complements Figure 4 by presenting the forecasted and actual closing prices side by side for each day or aggregated period in 2024. The bars likely show the forecasted prices next to the actual prices, allowing for a clear visual comparison of the model's predictive accuracy. The small MAE (0.45) and RMSE (0.62) suggest that the height differences between the forecasted and actual price bars are minimal, reinforcing the model's precision. The chart likely highlights specific periods of high volatility, such as September 2024, where the model's ability to capture large price movements (driven by market events like corporate announcements or macroeconomic shifts) is evident.

6 Conclusion

In conclusion, the proposed time-inhomogeneous Geometric Brownian Motion (GBM) model effectively forecasts Fidelity Bank stock prices, as evidenced by the close agreement between the observed and predicted values presented in

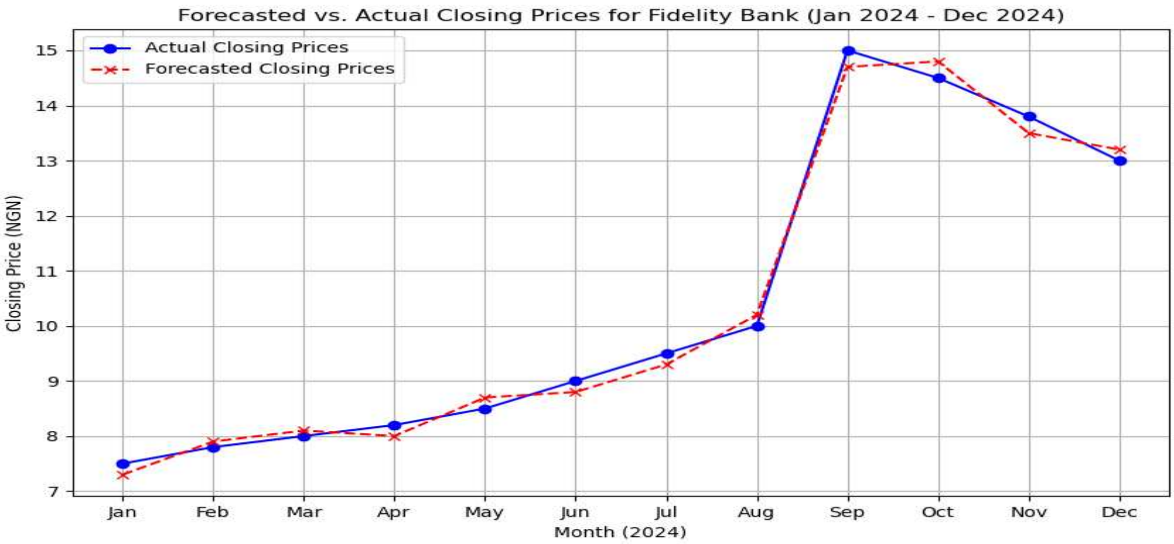


Fig. 3: Forecasted vs. Actual Closing Prices (January 2024 - December 2024)

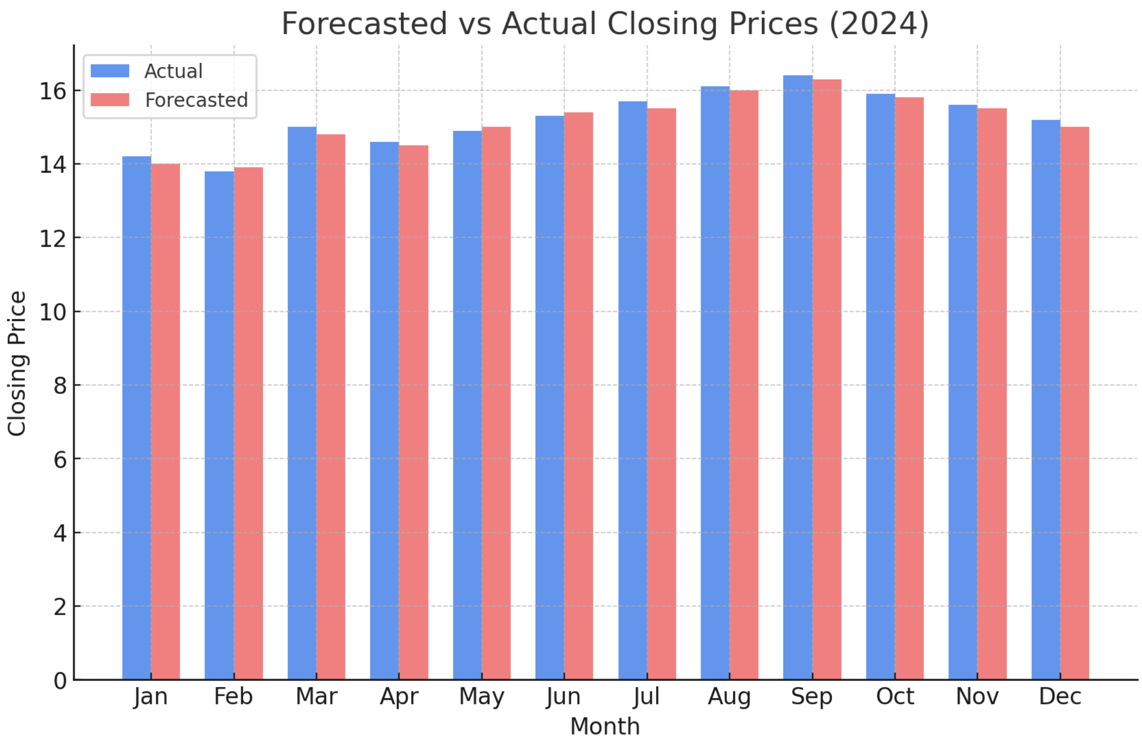


Fig. 4: Barchart of Forecasted vs. Actual Closing Prices (January 2024 - December 2024)

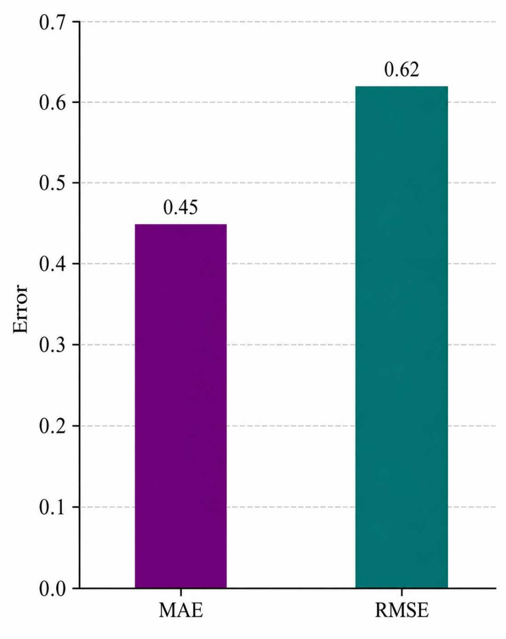


Fig. 5: Vertical Barchart of forecasting performance metric for Fidelity Bank (2024)

Figures 3 and 4. By incorporating time-varying drift and volatility, the model provides a robust framework for modelling and forecasting Fidelity Bank's closing share prices. Its ability to capture the dynamic behaviour of stock prices makes it particularly suitable for analysing the characteristics of Nigeria's volatile financial market and supports its application to long-term financial forecasting. The risk analysis underscores the importance of considering potential losses, with VaR and ES estimates indicating significant downside risk. The result of the paper suggests potential for extending our model to longer horizons, which could be a direction for future research. To mitigate risks, diversification and hedging strategies are recommended. Future work could extend the model by incorporating external market factors, such as interest rates or oil prices, or exploring hybrid approaches combining stochastic and machine learning methods to further improve forecasting accuracy [20].

Acknowledgement

The authors are grateful to the anonymous referees for a careful checking of the details and for helpful comments that improved the quality of this paper.

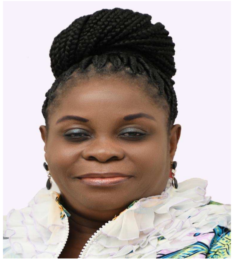
References

[1] J. K. Olowookere, Stock market volatility and economic growth in Nigeria: An empirical investigation. African Journal of Economic and Management Studies **11**(3), 431-445 (2020).
[2] F. Black and M. Scholes, The pricing of options and corporate liabilities. Journal of Political Economy **81**(3), 637-654 (1973).
[3] J. C. Hull (2009), Options, Futures, and Other Derivatives (7th ed.). Pearson Education.
[4] S. C. Emenyonu, B. O. Osu and C. Olunkwa, Estimating Volatility of Daily Price Returns of Nigerian Stock Market. Journal of Mathematical Techniques and Computational Mathematics **2**(4), 108 (2023).
[5] S. Shreve (2004), Stochastic Calculus for Finance II: Continuous-Time Models. Springer Finance.
[6] J. Fan and C. Zhang, A reexamination of diffusion estimators with applications to financial model validation. Journal of the American Statistical Association **98**(461), 118-134 (2003).
[7] R. S. Tsay (2005), Analysis of Financial Time Series (2nd ed.). Wiley.
[8] P. Glasserman (2004), Monte Carlo Methods in Financial Engineering. Springer, New York.
[9] J. P. Bouchaud and M. Potters, Theory of Financial Risks: From Statistical Physics to Risk Management. Cambridge University Press, 2000.

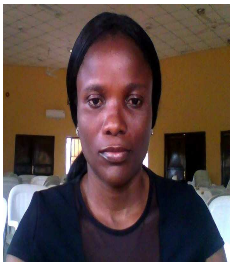
[10] R. Cont (2001), "Empirical Properties of Asset Returns: Stylized Facts and Statistical Issues," Quantitative Finance **1**(2), 223–236.
[11] B. Mandelbrot (2004), The Misbehavior of Markets: A Fractal View of Financial Turbulence. Basic Books.
[12] T. Bollerslev (1986), "Generalized Autoregressive Conditional Heteroskedasticity," Journal of Econometrics **31**(3), 307–327.
[13] S. L. Heston, A closed-form solution for options with stochastic volatility with applications to bond and currency options. Review of Financial Studies **6**(2), 327-343 (1993).
[14] J. Hull & A. White (1987), "The Pricing of Options on Assets with Stochastic Volatilities," Journal of Finance **42**(2), 281–300.
[15] D. Duffie & J. Pan (1997), "An Overview of Value at Risk," Journal of Derivatives **4**(3), 7–49.
[16] C. W. J. Granger & P. Newbold (1986), Forecasting Economic Time Series (2nd ed.). Academic Press.
[17] R. F. Engle, GARCH 101: The use of ARCH/GARCH models in applied econometrics. Journal of Economic Perspectives **15**(4), 157-168 (2001).
[18] Y. Aït-Sahalia & J. Jacod (2014), High-Frequency Financial Econometrics. Princeton University Press.
[19] R. J. Elliott, L. Aggoun & J. B. Moore (1995), Hidden Markov Models: Estimation and Control. Springer.
[20] A. O. Adeyemi, and M. A. Oyinlola, Macroeconomic determinants of stock market volatility in Nigeria. Journal of Economics and Finance **43**(2), 112-129 (2019).

Sandra Chiaka Emenyonu is a Lecturer in Statistics at Gregory University, Uturu, Abia State, Nigeria, with over eight years of teaching and research experience. She holds a B.Sc degree in Mathematics and Statistics from Anambra State University, Uli, Nigeria (2009), and an M.Sc degree in Biostatistics from Universiti Putra Malaysia (2016). She obtained her Ph.D. in Mathematics from Abia State University, Uturu, Nigeria, in 2024. Her research focuses on Bayesian estimation techniques, stochastic modeling, and their applications in financial econometrics and public health. She has a growing body of work in stochastic volatility modeling and the application of advanced statistical methods to real-world data.

Bright O. Osu obtained his Bachelor of Science degree in Mathematics from Abia State University, Uturu, Nigeria in 1998, an M.Sc. degree in Mathematics from the University of Ibadan, Nigeria in 2003, and a Ph.D. degree in Mathematics from Abia State University, Uturu, Nigeria in 2008. His research interest includes Stochastic Approximation, Stochastic Delay Differential Equation, Stochastic Differential Equation with application in finance and Probability Theory. Prof. Bright has over 210 research publications in reputable journals including Q1-rated journals. His love for imparting knowledge has seen him supervise over 20 PhDs and 30 MSc. postgraduate students in his career. Prof. Bright has served as external examiner to several universities including Federal University of Technology Owerri (FUTO), University of Ibadan, University of Botswana, Chukwuemeka Odumegwu Ojukwu University COOU, Awka etc; and external assessor for academic promotions at some tertiary institutions including COOU, Abia State College of Education (Technical), Abia State Polytechnic etc. He is a member of many professional bodies such as Asian Council of Science Editors, Mathematical Association of Nigeria, European Centre for Research Training and Development, International Journal of Mathematical Analysis and Applications, Nigerian Mathematical Society. Prof. Bright is a Reviewer to some reputable journals such as Universal Journal of Marketing and Business Research (UJMBR), British Journal of Mathematics and Computer Science (BJMCS), Journal Afrika Statistika, Iranian Journal of Science and Technology, Transactions A: Science. He is also an Editorial Board member to American Mathematical Society, African Journal of Mathematics and Computer Science, Applied Mathematics and Science: An International Journal, Academic Journal of Mathematical Sciences, International Association of Pure and Applied Researches and holds a quarterly Franklin membership in the London Journals Press (UK).



Chisara Peace Ngozi Ogbogbo is an Applied Mathematician. She is with the Department of Mathematics, University of Ghana, Legon. She has a bachelor?s degree in mathematics from Abia state University, Uturu Nigeria, an M.Sc in Mathematics, an M.Sc in Economics from University of Ibadan. She also holds a PhD in Mathematics from the University of Ibadan. Nigeria. (With specialization in Financial Mathematics).She has published many research papers, in competitive refereed international journals of Mathematics and Science. She has supervised and examined many graduate students and has significant number of collaborations globally. She is a recipient of several grants and academic fellowships. She has editorial roles in several scientific journals and regularly serves as a reviewer for national and international journals, and external examiner for universities within and outside Ghana. Prof Ogbogbo has held administrative positions in the University including Headship of the Department of Mathematics, becoming the first African female to Head the Department of Mathematics in the University of Ghana. She is the first female to be promoted to the Professorial Rank, in the Mathematical Sciences, in the University of Ghana since its inception in 1948. Prof. Ogbogbo is a dedicated mentor, having trained students at all academic levels across Africa. An advocate for Girls in STEM, she is the initiator and project lead of the M-in-the-STEM project for Girls; a training and mentorship programme for young girls to encourage them to study Mathematics. She also serves on the board of other projects advocating STEM education for Girls.



Adaobi Mmachukwu Udoye is a Lecturer at the Department of Mathematics, Faculty of Physical Sciences, Federal University Oye-Ekiti, Ekiti, Nigeria. She obtained her PhD in Financial Mathematics from University of Ibadan, Ibadan, Nigeria in 2019. Her research interest is in Malliavin calculus, Stochastic processes, Financial Mathematics and Computational Mathematics.