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Characterizations of *b*-Soft Separation Axioms in Soft Topological Spaces

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Abstract: Many scientists have studied and improved the soft set theory, which is initiated by Molodtsov [33] and easily applied to many problems having uncertainties from social life. The main purpose of our paper, is to introduce new soft separation axioms based on the b-open soft sets which are more general than of the open soft sets. We show that, the properties of soft b-T_i-spaces $(i = 1,2)$ are soft topological properties under the bijection and irresolute open soft mapping. Also, the property of being soft b-regular and soft b-normal are soft topological properties under bijection, irresolute soft and irresolute open soft functions. Further, we show that the properties of being soft b-T_i-spaces $(i = 1,2,3,4)$ are hereditary properties.

Keywords: Soft set, Soft topological space, Soft interior, Soft closure, Open soft, Closed soft, Soft b- separation axioms, Soft b-T_i-spaces $(i = 1,2,3,4)$, Soft b-regular, Soft b-normal, b-irresolute soft functions, Irresolute b-open soft functions.

1 Introduction

In real life situation, the problems in economics, engineering, social sciences, medical science etc. do not always involve crisp data. So, we cannot successfully use the traditional classical methods because of various types of uncertainties presented in these problems. To exceed these uncertainties, some kinds of theories were given like theory of fuzzy set, intuitionistic fuzzy set, rough set, bipolar fuzzy set, i.e. which we can use as mathematical tools for dealings with uncertainties. But, all these theories have their inherent difficulties. The reason for these difficulties Molodtsov [33] initiated the concept of soft set theory as a new mathematical tool for dealing with uncertainties which is free from the above difficulties. In [33,34], Molodtsov successfully applied the soft theory in several directions, such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability, theory of measurement, and so on. After presentation of the operations of soft sets [31], the properties and applications of soft set theory have been studied increasingly [6,27,34]. Xiao et al.[44] and Pei and Miao [37] discussed the relationship between soft sets and information systems. They showed that soft sets are a class of special information systems. In recent years, many interesting applications of soft set theory have been expanded by embedding the ideas of fuzzy sets [4,5,9,16, 25,29,30,31,32,34,35,47]. To develop soft set theory, the operations of the soft sets are redefined and a uni-int decision making method was constructed by using these new operations [10].

Recently, in 2011, Shabir and Naz [40] initiated the study of soft topological spaces. They defined soft topology as a collection τ of soft sets over X. Consequently, they defined basic notions of soft topological spaces such as open soft and closed soft sets, soft subspace, soft closure, soft nbd of a point, soft separation axioms, soft regular spaces and soft normal spaces and established their several properties. Min in [43] investigate some properties of these soft separation axioms. In [17], Kandil et al. introduced some soft operations such as semi open soft, pre open soft, α-open soft and β-open soft and investigated their properties in detail. Kandil et al. [24] introduced the notion of soft semi separation axioms. In particular they study the properties of the soft semi regular spaces and soft semi normal spaces. The notion of soft ideal was initiated for the first time by Kandil et al.[20]. They also introduced the concept of soft local function. These concepts are discussed with a view to find

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new soft topologies from the original one, called soft topological spaces with soft ideal $(X,\tau,E,\tilde{I})$. Applications to various fields were further investigated by Kandil et al. [18,19,21,22,23,26]. The notion of supra soft topological spaces was initiated for the first time by El-sheikh and Abd El-latif [13]. They also introduced new different types of subsets of supra soft topological spaces and study the relations between them in detail. The notion of b-open soft sets was initiated for the first time by El-sheikh and Abd El-latif [12], which is extended by Abd El-latif et al. in [1]. Maji et al. [29] initiated the study involving both fuzzy sets and soft sets. In [8] the notion of fuzzy soft set was introduced as a fuzzy generalization of soft sets and some basic properties of fuzzy soft sets are discussed in detail. Then, many scientists such as X. Yang et al. [45], improved the concept of fuzziness of soft sets. In [2], Karal and Ahmed defined the notion of a mapping on classes of fuzzy soft sets, which is a fundamental important in fuzzy soft set theory, to improve this work and they studied properties of fuzzy soft images and fuzzy soft inverse images of fuzzy soft sets. Chang [11] introduced the concept of fuzzy topology on a set X by axiomatizing a collection $\mathfrak{T}$ of fuzzy subsets of X. Tanay et al. [41] introduced the definition of fuzzy soft topology over a subset of the initial universe set while Roy and Samanta [39] gave the definition f fuzzy soft topology over the initial universe set. Some fuzzy soft topological properties based on fuzzy semi open soft sets namely, fuzzy semi open soft sets, fuzzy semi closed soft sets, fuzzy semi soft interior, fuzzy semi soft closure fuzzy semi separation axioms and fuzzy soft semi connectedness, were introduced by Kandil et al. in [16,25].

The purpose of this paper, is to introduce the notion of soft b-separation axioms. In particular we study the properties of the soft b-regular spaces and soft b-normal spaces. We show that if x_E is b-closed soft set for all $x\in X$ in a soft topological space (X,τ,E), then (X,τ,E) is soft b-T_1-space. Also, we show that if a soft topological space (X,τ,E) is soft b-T_3-space, then $\forall\ x\in X$, x_E is b-closed soft set. This paper, not only can form the theoretical basis for further applications of topology on soft sets, but also lead to the development of information systems.

2 Preliminaries

Definition 2.1.[33] Let X be an initial universe and E be a set of parameters. Let $P(X)$ denote the power set of X and A be a non-empty subset of E. A pair (F,A) denoted by F_A is called a soft set over X , where F is a mapping given by $F:A\rightarrow P(X)$. In other words, a soft set over X is a parametrized family of subsets of the universe X. For a particular $e\in A$, $F(e)$ may be considered the set of e-approximate elements of the soft set (F,A) and if $e\notin A$, then $F(e)=\phi$ i.e $F_A=\{F(e):e\in A\subseteq E,\ F:A\rightarrow P(X)\}$. The family of all these soft sets over X denoted by $SS(X)_A$.

Definition 2.2.[31] Let $F_A,\ G_B\in SS(X)_E$. Then F_A is soft subset of G_B, denoted by $F_A\tilde{\subseteq}G_B$, if

(1)$A\subseteq B$, and
(2)$F(e)\subseteq G(e)$, $\forall e\in A$.

In this case, F_A is said to be a soft subset of G_B and G_B is said to be a soft superset of F_A, $G_B\tilde{\supseteq}F_A$.

Definition 2.3.[31] Two soft subset F_A and G_B over a common universe set X are said to be soft equal if F_A is a soft subset of G_B and G_B is a soft subset of F_A.

Definition 2.4.[6] The complement of a soft set (F,A), denoted by $(F,A)^c$, is defined by $(F,A)^c=(F^c,A)$, $F^c:A\rightarrow P(X)$ is a mapping given by $F^c(e)=X-F(e)$, $\forall\, e\in A$ and F^c is called the soft complement function of F. Clearly, $(F^c)^c$ is the same as F and $((F,A)^c)^c=(F,A)$.

Definition 2.5.[40] The difference between two soft sets (F,E) and (G,E) over the common universe X, denoted by $(F,E)-(G,E)$ is the soft set (H,E) where for all $e\in E$, $H(e)=F(e)-G(e)$.

Definition 2.6.[40] Let (F,E) be a soft set over X and $x\in X$. We say that $x\in(F,E)$ read as x belongs to the soft set (F,E) whenever $x\in F(e)$ for all $e\in E$. The soft set (F,E) over X such that $F(e)=\{x\}\ \forall e\in E$ is called singleton soft point and denoted by x_E or (x,E).

Definition 2.7.[31] A soft set (F,A) over X is said to be a NULL soft set denoted by $\tilde{\phi}$ or ϕ_A if for all $e\in A$, $F(e)=\phi$ (null set).

Definition 2.8.[31] A soft set (F,A) over X is said to be an absolute soft set denoted by $\tilde{A}$ or X_A if for all $e\in A$, $F(e)=X$. Clearly, we have $X_A^c=\phi_A$ and $\phi_A^c=X_A$.

Definition 2.9.[40] Let (F,E) be a soft set over X and $x\in X$. We say that $x\in(F,E)$ read as x belongs to the soft set (F,E) whenever $x\in F(e)$ for all $e\in E$.

Definition 2.10.[31] The union of two soft sets (F,A) and (G,B) over the common universe X is the soft set (H,C), where $C=A\cup B$ and for all $e\in C$,

$$H(e)=\begin{cases}F(e),\ e\in A-B,\\ G(e),\ e\in B-A,\\ F(e)\cup G(e),\ e\in A\cap B\,.\end{cases}$$

Definition 2.11.[31] The intersection of two soft sets (F,A) and (G,B) over the common universe X is the soft set (H,C), where $C=A\cap B$ and for all $e\in C$, $H(e)=F(e)\cap G(e)$. Note that, in order to efficiently discuss, we consider only soft sets (F,E) over a universe X in which all the parameter set E are same. We denote the family of these soft sets by $SS(X)_E$.

Definition 2.12.[48] Let I be an arbitrary indexed set and $L=\{(F_i,E),i\in I\}$ be a subfamily of $SS(X)_E$.

(1)The union of L is the soft set (H,E), where $H(e)=\bigcup_{i\in I}F_i(e)$ for each $e\in E$. We write $\tilde{\bigcup}_{i\in I}(F_i,E)=(H,E)$.

(2)The intersection of L is the soft set (M,E), where $M(e) = \bigcap_{i\in I} F_i(e)$ for each $e \in E$. We write $\tilde{\bigcap}_{i\in I}(F_i,E) = (M,E)$.

Definition 2.13.[40] Let τ be a collection of soft sets over a universe X with a fixed set of parameters E, then $\tau \subseteq SS(X)_E$ is called a soft topology on X if

(1)$\tilde{X}, \tilde{\phi} \in \tau$, where $\tilde{\phi}(e) = \phi$ and $\tilde{X}(e) = X$, $\forall e \in E$,
(2)the union of any number of soft sets in τ belongs to τ,
(3)the intersection of any two soft sets in τ belongs to τ.

The triplet (X,τ,E) is called a soft topological space over X. A soft set (F,A) over X is said to be closed soft set in X, if its relative complement $(F,A)^c$ is an open soft set.

Definition 2.14.[14] Let(X,τ,E) be a soft topological space. The members of τ are said to be open soft sets in X. We denote the set of all open soft sets over X by $OS(X,\tau,E)$, or when there can be no confusion by $OS(X)$ and the set of all closed soft sets by $CS(X,\tau,E)$, or $CS(X)$.

Definition 2.15.[40] Let (X,τ,E) be a soft topological space and $(F,E) \in SS(X)_E$. The soft closure of (F,E), denoted by $cl(F,E)$ is the intersection of all closed soft super sets of (F,E) i.e $cl(F,E) = \tilde{\cap}\{(H,E) : (H,E) \text{ is closed soft set and } (F,E)\tilde{\subseteq}(H,E)\})$.

Definition 2.16.[48] Let (X,τ,E) be a soft topological space and $(F,E) \in SS(X)_E$. The soft interior of (G,E), denoted by $int(G,E)$ is the union of all open soft subsets of (G,E) i.e $int(G,E) = \tilde{\cup}\{(H,E) : (H,E) \text{ is an open soft set and } (H,E)\tilde{\subseteq}(G,E)\})$.

Definition 2.17.[48] The soft set $(F,E) \in SS(X)_E$ is called a soft point in X_E if there exist $x \in X$ and $e \in E$ such that $F(e) = \{x\}$ and $F(e^C) = \phi$ for each $e^C \in E - \{e\}$, and the soft point (F,E) is denoted by x_e.

Definition 2.18.[48] The soft point x_e is said to be belonging to the soft set (G,A), denoted by $x_e \tilde{\in} (G,A)$, if for the element $e \in A$, $F(e) \subseteq G(e)$.

Theorem 2.1.[42] Let (X,τ,E) be a soft topological space. A soft point $x_e \tilde{\in} cl(F,E)$ if and only if each soft neighborhood of x_e intersects (F,E).

Definition 2.19.[40] Let (X,τ,E) be a soft topological space and Y be a non null subset of X. Then $\tilde{Y}$ denotes the soft set (Y,E) over X for which $Y(e) = Y$ $\forall e \in E$.

Definition 2.20.[40] Let (X,τ,E) be a soft topological space, $(F,E) \in SS(X)_E$ and Y be a non null subset of X. Then the sub soft set of (F,E) over Y denoted by (F_Y,E), is defined as follows:

$$F_Y(e) = Y \cap F(e)\ \forall e \in E.$$

In other words $(F_Y,E) = \tilde{Y} \tilde{\cap} (F,E)$.

Definition 2.21.[40] Let (X,τ,E) be a soft topological space and Y be a non null subset of X. Then

$$\tau_Y = \{(F_Y,E) : (F,E) \in \tau\}$$

is said to be the soft relative topology on Y and (Y,τ_Y,E) is called a soft subspace of (X,τ,E).

Theorem 2.2.[40] Let (Y,τ_Y,E) be a soft subspace of a soft topological space (X,τ,E) and $(F,E) \in SS(X)_E$. Then

(1)If (F,E) is an open soft set in Y and $\tilde{Y} \in \tau$, then $(F,E) \in \tau$.
(2)(F,E) is an open soft set in Y if and only if $(F,E) = \tilde{Y} \tilde{\cap} (G,E)$ for some $(G,E) \in \tau$.
(3)(F,E) is a closed soft set in Y if and only if $(F,E) = \tilde{Y} \tilde{\cap} (H,E)$ for some (H,E) is τ-closed soft set.

Definition 2.22.[12] Let (X,τ,E) be a soft topological space and $(F,E) \in SS(X)_E$. Then (F,E) is called a b-open soft set if $(F,E)\tilde{\subseteq} cl(int(F,E)) \tilde{\cup} int(cl(F,E))$. The set of all b-open soft sets is denoted by $BOS(X,\tau,E)$, or $BOS(X)$ and the set of all b-closed soft sets is denoted by $BCS(X,\tau,E)$, or $BCS(X)$.

Definition 2.23.[12] Let (X,τ,E) be a soft topological space and $(F,E) \in SS(X)_E$. Then, the b-soft interior of (F,E) is denoted by $bSint(F,E))$, where $bSint(F,E)) = \tilde{\bigcup}\{(G,E) : (G,E)\tilde{\subseteq}(F,E),\ (G,E) \in BOS(X)\}$.
Also, the b-soft closure of (F,E) is denoted by $bScl(F,E))$, where $bScl(F,E)) = \tilde{\bigcap}\{(H,E) : (H,E) \in BCS(X),\ (F,E)\tilde{\subseteq}(H,E)\}$.

Definition 2.24.[3] Let $SS(X)_A$ and $SS(Y)_B$ be families of soft sets, $u : X \to Y$ and $p : A \to B$ be mappings. Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a mapping. Then;

(1)If $(F,A) \in SS(X)_A$. Then the image of (F,A) under f_{pu}, written as $f_{pu}(F,A) = (f_{pu}(F), p(A))$, is a soft set in $SS(Y)_B$ such that

$$f_{pu}(F)(b) = \begin{cases} \cup_{x\in p^{-1}(b)\cap A}\ u(F(a)), & p^{-1}(b) \cap A \neq \phi, \\ \phi, & otherwise. \end{cases}$$

for all $b \in B$.

(2)If $(G,B) \in SS(Y)_B$. Then the inverse image of (G,B) under f_{pu}, written as $f^{-1}_{pu}(G,B) = (f^{-1}_{pu}(G), p^{-1}(B))$, is a soft set in $SS(X)_A$ such that

$$f^{-1}_{pu}(G)(a) = \begin{cases} u^{-1}(G(p(a))), & p(a) \in B, \\ \phi, & otherwise. \end{cases}$$

for all $a \in A$.

The soft function f_{pu} is called surjective if p and u are surjective, also is said to be injective if p and u are injective.

Definition 2.25.[17,28,48]
Let (X,τ_1,A) and (Y,τ_2,B) be soft topological spaces and $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. Then, the function f_{pu} is said to be

(1)The function f_{pu} is said to be continuous soft (cts-soft) if $f^{-1}_{pu}(G,B) \in \tau_1$ $\forall$ $(G,B) \in \tau_2$.
(2)The function f_{pu} is said to be open soft if $f_{pu}(G,A) \in \tau_2 \forall$ $(G,A) \in \tau_1$.
(3)The function f_{pu} is said to be b-irresolute soft if $f^{-1}_{pu}(G,B) \in BOS(X)[f^{-1}_{pu}(F,B) \in BCS(X)] \forall$ $(G,B) \in BOS(Y)[(F,B) \in BCS(Y)]$.

(6)The function f_{pu} is said to be irresolute b-open (closed) soft if $f_{pu}(G,A) \in BOS(Y)[f_{pu}(F,A) \in BCS(Y)] \forall\ (G,A) \in BOS(X)[(F,A) \in BCS(Y)]$.

Theorem 2.3.[3] Let $SS(X)_A$ and $SS(Y)_B$ be families of soft sets. For the soft function $f_{pu} : SS(X)_A \rightarrow SS(Y)_B$, the following statements hold,

(a)$f_{pu}^{-1}((G,B)^c) = (f_{pu}^{-1}(G,B))^c \forall\ (G,B) \in SS(Y)_B$.
(b)$f_{pu}(f_{pu}^{-1}((G,B)))\tilde{\subseteq}(G,B) \forall\ (G,B) \in SS(Y)_B$. If f_{pu} is surjective, then the equality holds.
(c)$(F,A)\tilde{\subseteq} f_{pu}^{-1}(f_{pu}((F,A))) \forall\ (F,A) \in SS(X)_A$. If f_{pu} is injective, then the equality holds.
(d)$f_{pu}(\tilde{X})\tilde{\subseteq}\tilde{Y}$. If f_{pu} is surjective, then the equality holds.
(e)$f_{pu}^{-1}(\tilde{Y}) = \tilde{X}$ and $f_{pu}(\tilde{\phi}_A) = \tilde{\phi}_B$.
(f)If $(F,A)\tilde{\subseteq}(G,A)$, then $f_{pu}(F,A)\tilde{\subseteq} f_{pu}(G,A)$.
(g)If $(F,B)\tilde{\subseteq}(G,B)$, then $f_{pu}^{-1}(F,B)\tilde{\subseteq} f_{pu}^{-1}(G,B)\ \forall\ (F,B),(G,B) \in SS(Y)_B$.
(h)$f_{pu}^{-1}[(F,B)\tilde{\cup}(G,B)] = f_{pu}^{-1}(F,B)\tilde{\cup} f_{pu}^{-1}(G,B)$ and $f_{pu}^{-1}[(F,B)\tilde{\cap}(G,B)] = f_{pu}^{-1}(F,B)\tilde{\cap} f_{pu}^{-1}(G,B)$ $\forall\ (F,B),(G,B) \in SS(Y)_B$.
(I)$f_{pu}[(F,A)\tilde{\cup}(G,A)] = f_{pu}(F,A)\tilde{\cup} f_{pu}(G,A)$ and $f_{pu}[(F,A)\tilde{\cap}(G,A)]\tilde{\subseteq} f_{pu}(F,A)\tilde{\cap} f_{pu}(G,A)$ $\forall\ (F,A),(G,A) \in SS(X)_A$. If f_{pu} is injective, then the equality holds.

3 Soft b-separation axioms

Definition 3.1. Let (X,τ,E) be a soft topological space and $x,y \in X$ such that $x \neq y$. Then, (X,τ,E) is called a soft b-T_o-space if there exist b-open soft sets (F,E) and (G,E) such that either $x \in (F,E)$ and $y \notin (F,E)$ or $y \in (G,E)$ and $x \notin (G,E)$.

Proposition 3.1. Let (X,τ,E) be a soft topological space and $x,y \in X$ such that $x \neq y$. If there exist b-open soft sets (F,E) and (G,E) such that either $x \in (F,E)$ and $y \in (F,E)^c$ or $y \in (G,E)$ and $x \in (G,E)^c$. Then, (X,τ,E) is soft b-T_o-space. **Proof.** Let $x,y \in X$ such that $x \neq y$. Let (F,E) and (G,E) be b-open soft sets such that either $x \in (F,E)$ and $y \in (F,E)^c$ or $y \in (G,E)$ and $x \in (G,E)^c$. If $x \in (F,E)$ and $y \in (F,E)^c$. Then $y \in (F(e))^c$ for all $e \in E$, This implies that, $y \notin F(e)$ for all $e \in E$. Therefore, $y \notin (F,E)$. Similarly, if $y \in (G,E)$ and $x \in (G,E)^c$, then $x \notin (G,E)$. Hence, (X,τ,E) is soft b-T_o-space.

Theorem 3.1. A soft subspace (Y,τ_Y,E) of a soft b-T_o-space (X,τ,E) is soft b-T_o.

Proof. Let $x,y \in Y$ such that $x \neq y$. Then $x,y \in X$ such that $x \neq y$. Hence, there exist b-open soft sets (F,E) and (G,E) in X such that either $x \in (F,E)$ and $y \notin (F,E)$ or $y \in (G,E)$ and $x \notin (G,E)$. Since $x \in Y$. Then $x \in \tilde{Y}$. Hence, $x \in \tilde{Y}\tilde{\cap}(F,E) = (F_Y,E)$, (F,E) is b-open soft set. Consider $y \notin (F,E)$, This implies that, $y \notin F(e)$ for some $e \in E$. Therefore, $y \notin \tilde{Y}\tilde{\cap}(F,E) = (F_Y,E)$. Similarly, if $y \in (G,E)$ and $x \notin (G,E)$, then $y \in (G_Y,E)$ and $x \notin (G_Y,E)$. Thus, (Y,τ_Y,E) is soft b-T_o

Definition 3.2. Let (X,τ,E) be a soft topological space and $x,y \in X$ such that $x \neq y$. Then, (X,τ,E) is called a soft b-T_1-space if there exist b-open soft sets (F,E) and (G,E) such that $x \in (F,E)$ and $y \notin (F,E)$ and $y \in (G,E)$ and $x \notin (G,E)$.

Proposition 3.2. Let (X,τ,E) be a soft topological space and $x,y \in X$ such that $x \neq y$. If there exist b-open soft sets (F,E) and (G,E) such that $x \in (F,E)$ and $y \in (F,E)^c$ and $y \in (G,E)$ and $x \in (G,E)^c$. Then (X,τ,E) is soft b-T_1-space.

Proof. It is similar to the proof of Proposition 3.1.

Theorem 3.2. A soft subspace (Y,τ_Y,E) of a soft b-T_1-space (X,τ,E) is soft b-T_1.

Proof. It is similar to the proof of Theorem 3.

Theorem 3.3 Let (X,τ,E) be a soft topological space. If x_E is b-closed soft set in τ for all $x \in X$, then (X,τ,E) is soft b-T_1-space.

Proof. Suppose that $x \in X$ and x_E is b-closed soft set in τ. Then x_E^c is b-open soft set in τ. Let $x,y \in X$ such that $x \neq y$. For $x \in X$ and x_E^c is b-open soft set such that $x \notin x_E^c$ and $y \in x_E^c$. Similarly y_E^c is b-open soft set in τ such that $y \notin y_E^c$ and $x \in y_E^c$. Thus, (X,τ,E) is soft b-T_1-space over X.

Definition 3.3. Let (X,τ,E) be a soft topological space and $x,y \in X$ such that $x \neq y$. Then (X,τ,E) is called a soft b-Hausdorff space or soft b-T_2-space if there exist b-open soft sets (F,E) and (G,E) such that $x \in (F,E)$, $y \in (G,E)$ and $(F,E)\tilde{\cap}(G,E) = \tilde{\phi}$.

Theorem 3.4. For a soft topological space (X,τ,E) we have:
soft b-T_2-space $\Rightarrow$ soft b-T_1-space $\Rightarrow$ soft b-T_o-space.
Proof.

(1)Let (X,τ,E) be a soft b-T_2-space and $x,y \in X$ such that $x \neq y$. Then, there exist b-open soft sets (F,E) and (G,E) such that $x \in (F,E)$, $y \in (G,E)$ and $(F,E)\tilde{\cap}(G,E) = \tilde{\phi}$. Since $(F,E)\tilde{\cap}(G,E) = \tilde{\phi}$. Then, $x \notin (G,E)$, $y \notin (F,E)$. Therefore, there exist b-open soft sets (F,E) and (G,E) such that $x \in (F,E)$ and $y \notin (F,E)$ and $y \in (G,E)$ and $x \notin (G,E)$. Thus, (X,τ,E) is soft b-T_1-space.
(2)Let (X,τ,E) be a soft b-T_1-space and $x,y \in X$ such that $x \neq y$. Then, there exist b-open soft sets (F,E) and (G,E) such that $x \in (F,E)$ and $y \notin (F,E)$ and $y \in (G,E)$ and $x \notin (G,E)$. Obviously then we have, either $x \in (F,E)$ and $y \notin (F,E)$ or $y \in (G,E)$ and $x \notin (G,E)$. Thus, (X,τ,E) is soft b-T_o-space.

Remark 3.1. The converse of Theorem 3.4 is not true in general, as shown in the following examples.

Examples 3.1.

(1)Let $X = \{a,b\}$, $E = \{e_1,e_2\}$ and $\tau = \{\tilde{X},\tilde{\phi},(F_1,E),(F_2,E),(F_3,E)\}$ where $(F_1,E),(F_2,E),(F_3,E)$ are soft sets over X defined as follows:

$F_1(e_1)=X$, $F_1(e_2)=\{b\}$,
$F_2(e_1)=\{a\}$, $F_2(e_2)=X$,
$F_3(e_1)=\{a\}$, $F_3(e_2)=\{b\}$.
Then, τ defines a soft topology on X. Also, (X,τ,E) is soft b-T_1-space, but it is not a soft b-T_2-space, for $a,b\in X$ and $a\neq b$, but there is no b-open soft sets (F,E) and (G,E) such that $a\in(F,E)$, $b\in(G,E)$ and $(F,E)\tilde{\cap}(G,E)=\tilde{\phi}$.

(2)Let $X=\{a,b\}$, $E=\{e_1,e_2\}$ and $\tau=\{\tilde{X},\tilde{\phi},(F_1,E)\}$ where (F_1,E) is soft set over X defined as follows by $F_1(e_1)=X$, $F_1(e_2)=\{b\}$.
Then τ defines a soft topology on X. Also (X,τ,E) is soft b-T_o-space but not a soft b-T_1-space, since $a,b\in X$, $a\neq b$, but all the b-open soft sets which contains a also contains b.

Theorem 3.5 A soft subspace (Y,τ_Y,E) of a soft b-T_2-space (X,τ,E) is soft b-T_2.

Proof. Let $x,y\in Y$ such that $x\neq y$. Then $x,y\in X$ such that $x\neq y$. Hence, there exist b-open soft sets (F,E) and (G,E) in X such that $x\in(F,E)$, $y\in(G,E)$ and $(F,E)\tilde{\cap}(G,E)=\tilde{\phi}$. It follows that, $x\in F(e)$, $y\in G(e)$ and $F(e)\cap G(e)=\phi$ for all $e\in E$. This implies that, $x\in Y\cap F(e)$, $y\in Y\cap G(e)$ and $F(e)\cap G(e)=\phi$ for all $e\in E$. Thus, $x\in\tilde{Y}\tilde{\cap}(F,E)=(F_Y,E)$, $y\in\tilde{Y}\tilde{\cap}(G,E)=(G_Y,E)$ and $(F_Y,E)\tilde{\cap}(G_Y,E)=\tilde{\phi}$, where $(F_Y,E),(G_Y,E)$ are b-open soft sets in Y. Therefore, (Y,τ_Y,E) is soft b-T_2-space.

Definition 3.4. Let (X,τ,E) be a soft topological space, (G,E) be a b-closed soft set in X and $x\in X$ such that $x\notin(G,E)$. If there exist b-open soft sets (F_1,E) and (F_2,E) such that $x\in(F_1,E)$, $(G,E)\tilde{\subseteq}(F_2,E)$ and $(F_1,E)\tilde{\cap}(F_2,E)=\tilde{\phi}$, then (X,τ,E) is called a soft b-regular space. A soft b-regular T_1-space is called a soft b-T_3-space.

Proposition 3.3. Let (X,τ,E) be a soft topological space, (G,E) be a b-closed soft set in X and $x\in X$ such that $x\notin(G,E)$. If (X,τ,E) is soft b-regular space, then there exists a b-open soft set (F,E) such that $x\in(F,E)$ and $(F,E)\tilde{\cap}(G,E)=\tilde{\phi}$.

Proof. Obvious from Definition 3.4.

Proposition 3.4. Let (X,τ,E) be a soft topological space, $(F,E)\in SS(X)_E$ and $x\in X$. Then:

(1)$x\in(F,E)$ if and only if $x_E\tilde{\subseteq}(F,E)$.
(2)If $x_E\tilde{\cap}(F,E)=\tilde{\phi}$, then $x\notin(F,E)$.

Proof. Obvious.

Theorem 3.6. Let (X,τ,E) be a soft topological space and $x\in X$. If (X,τ,E) is soft b-regular space, then:

(1)$x\notin(F,E)$ if and only if $x_E\tilde{\cap}(F,E)=\tilde{\phi}$ for every b-closed soft set (F,E).
(2)$x\notin(G,E)$ if and only if $x_E\tilde{\cap}(G,E)=\tilde{\phi}$ for every b-open soft set (G,E).

Proof.

(1)Let (F,E) be a b-closed soft set such that $x\notin(F,E)$. Since (X,τ,E) is soft b-regular space. Then, by Proposition 3.3, there exists a b-open soft set (G,E) such that $x\in(G,E)$ and $(F,E)\tilde{\cap}(G,E)=\tilde{\phi}$. It follows that, $x_E\tilde{\subseteq}(G,E)$ from Proposition 3.4 (1). Hence, $x_E\tilde{\cap}(F,E)=\tilde{\phi}$. Conversely, if $x_E\tilde{\cap}(F,E)=\tilde{\phi}$, then $x\notin(F,E)$ from Proposition 3.4 (2).
(2)Let (G,E) be a b-open soft set such that $x\notin(G,E)$. If $x\notin G(e)$ for all $e\in E$, then we get the proof. If $x\notin G(e_1)$ and $x\in G(e_2)$ for some $e_1,e_2\in E$, then $x\in G^c(e_1)$ and $x\notin G^c(e_2)$ for some $e_1,e_2\in E$. This means that, $x_E\tilde{\cap}(G,E)\neq\tilde{\phi}$. Hence, $(G,E)^c$ is b-closed soft set such that $x\notin(G,E)^c$. It follows by (1) $x_E\tilde{\cap}(G,E)^c=\tilde{\phi}$. This implies that, $x_E\tilde{\subseteq}(G,E)$ and so $x\in(G,E)$, which is contradiction with $x\notin G(e_1)$ for some $e_1\in E$. Therefore, $x_E\tilde{\cap}(G,E)=\tilde{\phi}$. Conversely, if $x_E\tilde{\cap}(G,E)=\tilde{\phi}$, then it obvious that $x\notin(G,E)$. This completes the proof.

Corollary 3.1. Let (X,τ,E) be a soft topological space and $x\in X$. If (X,τ,E) is soft b-regular space, then the following are equivalent:

(1)(X,τ,E) is soft b-T_1-space.
(2$\forall$ $x,y\in X$ such that $x\neq y$, there exist b-open soft sets (F,E) and (G,E) such that $x_E\tilde{\subseteq}(F,E)$ and $y_E\tilde{\cap}(F,E)=\tilde{\phi}$ and $y_E\tilde{\subseteq}(G,E)$ and $x_E\tilde{\cap}(G,E)=\tilde{\phi}$.

Proof. Obvious from Theorem 3.6.

Theorem 3.7. Let (X,τ,E) be a soft topological space and $x\in X$. Then the following are equivalent:

(1)(X,τ,E) is soft b-regular space.
(2)For every b-closed soft set (G,E) such that $x_E\tilde{\cap}(G,E)=\tilde{\phi}$, there exist b-open soft sets (F_1,E) and (F_2,E) such that $x_E\tilde{\subseteq}(F_1,E)$, $(G,E)\tilde{\subseteq}(F_2,E)$ and $(F_1,E)\tilde{\cap}(F_2,E)=\tilde{\phi}$.

Proof.

(1) $\Rightarrow$ (2) Let (G,E) be a b-closed soft set such that $x_E\tilde{\cap}(G,E)=\tilde{\phi}$. Then $x\notin(G,E)$ from Theorem 3.6 (1). It follows by (1), there exist b-open soft sets (F_1,E) and (F_2,E) such that $x\in(F_1,E)$, $(G,E)\tilde{\subseteq}(F_2,E)$ and $(F_1,E)\tilde{\cap}(F_2,E)=\tilde{\phi}$. This means that, $x_E\tilde{\subseteq}(F_1,E)$, $(G,E)\tilde{\subseteq}(F_2,E)$ and $(F_1,E)\tilde{\cap}(F_2,E)=\tilde{\phi}$.
(2) $\Rightarrow$ (1) Let (G,E) be a b-closed soft set such that $x\notin(G,E)$. Then $x_E\tilde{\cap}(G,E)=\tilde{\phi}$ from Theorem 3.6 (1). It follows by (2), there exist b-open soft sets (F_1,E) and (F_2,E) such that $x_E\tilde{\subseteq}(F_1,E)$, $(G,E)\tilde{\subseteq}(F_2,E)$ and $(F_1,E)\tilde{\cap}(F_2,E)=\tilde{\phi}$. Hence, $x\in(F_1,E)$, $(G,E)\tilde{\subseteq}(F_2,E)$ and $(F_1,E)\tilde{\cap}(F_2,E)=\tilde{\phi}$. Thus, (X,τ,E) is soft b-regular space.

Theorem 3.8. Let (X,τ,E) be a soft topological space. If (X,τ,E) is soft b-T_3-space, then $\forall$ $x\in X$, x_E is b- closed soft set.

Proof. We want to prove that x_E is b-closed soft set, which is sufficient to prove that x_E^c is b-open soft set for all $y \in \{x\}^c$. Since (X,τ,E) is soft b-T_3-space, then there exist b-open soft sets $(F,E)_y$ and (G,E) such that $y_E\tilde{\subseteq}(F,E)_y$ and $x_E\tilde{\cap}(F,E)_y = \tilde{\phi}$ and $x_E\tilde{\subseteq}(G,E)$ and $y_E\tilde{\cap}(G,E) = \tilde{\phi}$. It follows that, $\bigcup_{y\in\{x\}^c}(F,E)_y\tilde{\subseteq}x^c_E$. Now, we want to prove that $x^c_E\tilde{\subseteq}\bigcup_{y\in\{x\}^c}(F,E)_y$. Let $\bigcup_{y\in\{x\}^c}(F,E)_y = (H,E)$, where $H(e) = \bigcup_{y\in\{x\}^c}F(e)_y$ for all $e \in E$. Since $x^c_E(e) = \{x\}^c$ for all $e \in E$ from Definition 2.6. So, for all $y \in \{x\}^c$ and $e \in E$, $x^c_E(e) = \{x\}^c = \bigcup_{y\in\{x\}^c}\{y\} = \bigcup_{y\in\{x\}^c}y_E(e)\tilde{\subseteq}\bigcup_{y\in\{x\}^c}F(e)_y = H(e)$. Thus, $x^c_E\tilde{\subseteq}\bigcup_{y\in\{x\}^c}(F,E)_y$ from Definition 2.2, and so $x^c_E = \bigcup_{y\in\{x\}^c}(F,E)_y$. This means that, x^c_E is b-open soft set for all $y \in \{x\}^c$. Therefore, x_E is b-closed soft set.

Theorem 3.9. Every soft b-T_3-space is soft b-T_2-space.

Proof. Let (X,τ,E) be a soft b-T_3-space and $x,y \in X$ such that $x \neq y$. By Theorem 3.9, y_E is b-closed soft set and $x \notin y_E$. It follows from the soft b-regularity, there exist b-open soft sets (F_1,E) and (F_2,E) such that $x \in (F_1,E)$, $y_E\tilde{\subseteq}(F_2,E)$ and $(F_1,E)\tilde{\cap}(F_2,E) = \tilde{\phi}$. Thus, $x \in (F_1,E)$, $y \in y_E\tilde{\subseteq}(F_2,E)$ and $(F_1,E)\tilde{\cap}(F_2,E) = \tilde{\phi}$. Therefore, (X,τ,E) is soft b-T_2-space.

Theorem 3.10. A soft subspace (Y,τ_Y,E) of a soft b-T_3-space (X,τ,E) is soft b-T_3.

Proof. By Theorem 3.2, (Y,τ_Y,E) is soft b-T_1-space. Now, we want to prove that (Y,τ_Y,E) is soft b-regular space. Let $y \in Y$ and (G,E) be a b-closed soft set in Y such that $y \notin (G,E)$. Then, $(G,E) = (Y,E)\tilde{\cap}(F,E)$ for some b-closed soft set (F,E) in X from Theorem 2.2. Hence, $y \notin (Y,E)\tilde{\cap}(F,E)$. But $y \in (Y,E)$, so $y \notin (F,E)$. Since (X,τ,E) is soft b-T_3-space, so there exist b-open soft sets (F_1,E) and (F_2,E) in X such that $y \in (F_1,E)$, $(F,E)\tilde{\subseteq}(F_2,E)$ and $(F_1,E)\tilde{\cap}(F_2,E) = \tilde{\phi}$. Take $(G_1,E) = (Y,E)\tilde{\cap}(F_1,E)$ and $(G_2,E) = (Y,E)\tilde{\cap}(F_2,E)$, then $(G_1,E),(G_2,E)$ are b-open soft sets in Y such that $y \in (G_1,E)$, $(G,E)\tilde{\subseteq}(Y,E)\tilde{\cap}(F_2,E) = (G_2,E)$ and $(G_1,E)\tilde{\cap}(G_2,E)\tilde{\subseteq}(F_1,E)\tilde{\cap}(F_2,E) = \tilde{\phi}$. Thus, (Y,τ_Y,E) is soft b-T_3-space.

Definition 3.5 Let (X,τ,E) be a soft topological space, $(F,E),(G,E)$ be b-closed soft sets in X such that $(F,E)\tilde{\cap}(G,E) = \tilde{\phi}$. If there exist b-open soft sets (F_1,E) and (F_2,E) such that $(F,E)\tilde{\subseteq}(F_1,E)$, $(G,E)\tilde{\subseteq}(F_2,E)$ and $(F_1,E)\tilde{\cap}(F_2,E) = \tilde{\phi}$, then (X,τ,E) is called a soft b-normal space. A soft b-normal T_1-space is called a soft b-T_4-space.

Theorem 3.11. Let (X,τ,E) be a soft topological space and $x \in X$. Then the following are equivalent:

(1)(X,τ,E) is soft b-normal space.
(2)For every b-closed soft set (F,E) and b-open soft set (G,E) such that $F,E)\tilde{\subseteq}(G,E)$, there exists a b-open soft set (F_1,E) such that $(F,E)\tilde{\subseteq}(F_1,E)$, $bScl(F_1,E)\tilde{\subseteq}(G,E)$.

Proof.

(1) $\Rightarrow$ (2) Let (F,E) be a b-closed soft set and (G,E) be a b-open soft set such that $(F,E)\tilde{\subseteq}(G,E)$. Then $(F,E),(G,E)^c$ are b-closed soft sets such that $(F,E)\tilde{\cap}(G,E)^c = \tilde{\phi}$. It follows by (1), there exist b-open soft sets (F_1,E) and (F_2,E) such that $(F,E)\tilde{\subseteq}(F_1,E)$, $(G,E)^c\tilde{\subseteq}(F_2,E)$ and $(F_1,E)\tilde{\cap}(F_2,E) = \tilde{\phi}$. Now, $(F_1,E)\tilde{\subseteq}(F_2,E)^c$, so $bScl(F_1,E)\tilde{\subseteq}bScl(F_2,E)^c = (F_2,E)^c$, where (G,E) is b-open soft set. Also $(F_2,E)^c\tilde{\subseteq}(G,E)$. Hence, $bScl(F_1,E)\tilde{\subseteq}(F_2,E)^c\tilde{\subseteq}(G,E)$. Thus, $F,E)\tilde{\subseteq}(F_1,E)$, $bScl(F_1,E)$ $\tilde{\subseteq}(G,E)$.

(2) $\Rightarrow$ (1) Let $(G_1,E),(G_2,E)$ be b-closed soft sets such that $(G_1,E)\tilde{\cap}(G_2,E) = \tilde{\phi}$. Then $(G_1,E)\tilde{\subseteq}(G_2,E)^c$, then by hypothesis, there exists a b-open soft set (F_1,E) such that $G_1,E)\tilde{\subseteq}(F_1,E)$, $bScl(F_1,E)\tilde{\subseteq}(G_2,E)^c$. So $(G_2,E)\tilde{\subseteq}[bScl(F_1,E)]^c$, $G_1,E)\tilde{\subseteq}(F_1,E)$ and $[bScl(F_1,E)]^c\tilde{\cap}(F_1,E) = \tilde{\phi}$, where (F_1,E) and $[bScl(F_1,E)]^c$ are b-open soft sets. Thus, (X,τ,E) is soft b-normal space.

Theorem 3.12. A b-closed soft subspace (Y,τ_Y,E) of a soft b-normal space (X,τ,E) is soft b-normal.

Proof. Let $(G_1,E),(G_2,E)$ be b-closed soft sets in Y such that $(G_1,E)\tilde{\cap}(G_2,E) = \tilde{\phi}$. Then $(G_1,E) = (Y,E)\tilde{\cap}(F_1,E)$ and $(G_2,E) = (Y,E)\tilde{\cap}(F_1,E)$ for some b-closed soft sets $(F_1,E),(F_2,E)$ in X from Theorem 2.2. Since Y is sa b-closed soft subset of X. Then $(G_1,E),(G_2,E)$ are b-closed soft sets in X such that $(G_1,E)\tilde{\cap}(G_2,E) = \tilde{\phi}$. Hence, by soft b-normality there exist b-open soft sets (H_1,E) and (H_2,E) such that $(G_1,E)\tilde{\subseteq}(H_1,E)$, $(G_2,E)\tilde{\subseteq}(H_2,E)$ and $(H_1,E)\tilde{\cap}(H_2,E) = \tilde{\phi}$. Since $(G_1,E),(G_2,E)\tilde{\subseteq}(Y,E)$, then $(G_1,E)\tilde{\subseteq}(Y,E)$ $\tilde{\cap}(H_1,E)$, $(G_2,E)\tilde{\subseteq}(Y,E)\tilde{\cap}(H_2,E)$ and $[(Y,E)\tilde{\cap}(H_1,E)]\tilde{\cap}[(Y,E)\tilde{\cap}(H_2,E)] = \tilde{\phi}$, where (Y,E) $\tilde{\cap}(H_1,E)$ and $(Y,E)\tilde{\cap}(H_2,E)$ are b-open soft sets in Y. Therefore, (Y,τ_Y,E) is a soft b-normal space.

Theorem 3.13. Let (X,τ,E) be a soft topological space. If (X,τ,E) is soft b-normal space and x_E is b-closed soft set in τ for all $x \in X$, then (X,τ,E) is soft b-T_3-space.

Proof. Since x_E is b-closed soft set for all $x \in X$, then (X,τ,E) is soft b-T_1-space from Theorem 3.3. Also (X,τ,E) is soft b-regular space from Theorem 3.7 and Definition 3.5. Hence, (X,τ,E) is soft b-T_3-space.

4 Irresolute b-open soft functions

Theorem 4.1. Let (X,τ_1,A) and (Y,τ_2,B) be soft topological spaces and $f_{pu} : SS(X)_A \to SS(Y)_B$ be a soft function which is bijective and irresolute b-open soft. If (X,τ_1,A) is soft b-T_o-space, then (Y,τ_2,B) is also a soft b-T_o-space.

Proof. Let $y_1,y_2 \in Y$ such that $y_1 \neq y_2$. Since f_{pu} is surjective, then $\exists x_1,x_2 \in X$ such that $u(x_1) = y_1$, $u(x_2) = y_2$ and $x_1 \neq x_2$. By hypothesis, there exist b-open soft sets (F,A) and (G,A) in X such that either $x_1 \in (F,A)$ and $x_2 \notin (F,A)$, or $x_2 \in (G,A)$ and $x_1 \notin (G,A)$. So, either $x_1 \in F_A(e)$ and $x_2 \notin F_A(e)$ or $x_2 \in G_A(e)$ and $x_1 \notin G_A(e)$

for all $e \in A$. This implies that, either $y_1 = u(x_1) \in u[F_A(e)]$ and $y_2 = u(x_2) \notin u[F_A(e)]$ or $y_2 = u(x_2) \in u[G_A(e)]$ and $y_1 = u(x_1) \notin u[G_A(e)]$ for all $e \in A$. Hence, either $y_1 \in f_{pu}(F,A)$ and $y_2 \notin f_{pu}(F,A)$ or $y_2 \in f_{pu}(G,A)$ and $y_1 \notin f_{pu}(G,A)$. Since f_{pu} is irresolute b-open soft function, then $f_{pu}(F,A), f_{pu}(G,A)$ are b-open soft sets in Y. Hence, (Y,τ_2,B) is also a soft b-T_o-space.

Theorem 4.2 Let (X,τ_1,A) and (Y,τ_2,B) be soft topological spaces and $f_{pu} : SS(X)_A \to SS(Y)_B$ be a soft function which is bijective and irresolute b-open soft. If (X,τ_1,A) is soft b-T_1-space, then (Y,τ_2,B) is also a soft b-T_1-space.

Proof. It is similar to the proof of Theorem 4.1. **Theorem 4.3.** Let (X,τ_1,A) and (Y,τ_2,B) be soft topological spaces and $f_{pu} : SS(X)_A \to SS(Y)_B$ be a soft function which is bijective and irresolute b-open soft. If (X,τ_1,A) is soft b-T_2-space, then (Y,τ_2,B) is also a soft b-T_2-space.

Proof. $y_1, y_2 \in Y$ such that $y_1 \neq y_2$. Since f_{pu} is surjective, then $\exists x_1, x_2 \in X$ such that $u(x_1) = y_1$, $u(x_2) = y_2$ and $x_1 \neq x_2$. By hypothesis, there exist b-open soft sets (F,A) and (G,A) in X such that $x_1 \in (F,A)$, $x_2 \in (G,A)$ and $(F,A)\tilde{\cap}(G,A) = \tilde{\phi}_A$. So $x_1 \in F_A(e)$, $x_2 \in G_A(e)$ and $F_A(e)\tilde{\cap}G_A(e) = \phi$ for all $e \in A$. This implies that, $y_1 = u(x_1) \in u[F_A(e)], y_2 = u(x_2) \in u[G_A(e)]$ for all $e \in A$. Hence, $y_1 \in f_{pu}(F,A)$, $y_2 \in f_{pu}(G,A)$ and $f_{pu}(F,A)\tilde{\cap}f_{pu}(G,A) = f_{pu}[(F,A)\tilde{\cap}(G,A)] = f_{pu}[\tilde{\phi}_A] = \tilde{\phi}_B$ from Theorem 2.3. Since f_{pu} is irresolute b-open soft function, then $f_{pu}(F,A), f_{pu}(G,A)$ are b-open soft sets in Y. Thus, (Y,τ_2,B) is also a soft b-T_2-space.

Theorem 4.4. Let (X,τ_1,A) and (Y,τ_2,B) be soft topological spaces and $f_{pu} : SS(X)_A \to SS(Y)_B$ be a soft function which is bijective, b-irresolute soft and irresolute b-open soft. If (X,τ_1,A) is soft b-regular space, then (Y,τ_2,B) is also a soft b-regular space.

Proof. Let (G,B) be a b-closed soft set in Y and $y \in Y$ such that $y \notin (G,B)$. Since f_{pu} is surjective and b-irresolute soft, then $\exists x \in X$ such that $u(x) = y$ and $f_{pu}^{-1}(G,B)$ is b-closed soft set in X such that $x \notin f_{pu}^{-1}(G,B)$. By hypothesis, there exist b-open soft sets (F,A) and (H,A) in X such that $x \in (F,A)$, $f_{pu}^{-1}(G,B)\tilde{\subseteq}(H,A)$ and $(F,A)\tilde{\cap}(H,A) = \tilde{\phi}_A$. It follows that, $x \in F_A(e)$ for all $e \in A$ and $(G,B) = f_{pu}[f_{pu}^{-1}(G,B)]\tilde{\subseteq}f_{pu}(H,A)$ from Theorem 2.3. So, $y = u(x) \in u[F_A(e)]$ for all $e \in A$ and $(G,B)\tilde{\subseteq}f_{pu}(H,A)$. Hence, $y \in f_{pu}(F,A)$ and $(G,B)\tilde{\subseteq}f_{pu}(H,A)$ and $f_{pu}(F,A)\tilde{\cap}f_{pu}(H,A) = f_{pu}[(F,A)\tilde{\cap}(H,A)] = f_{pu}[\tilde{\phi}_A] = \tilde{\phi}_B$ from Theorem 2.3. Since f_{pu} is irresolute b-open soft function. Then, $f_{pu}(F,A), f_{pu}(H,A)$ are b-open soft sets in Y. Thus, (Y,τ_2,B) is also a soft b-regular space. **Theorem 4.5.** Let (X,τ_1,A) and (Y,τ_2,B) be soft topological spaces and $f_{pu} : SS(X)_A \to SS(Y)_B$ be a soft function which is bijective, b-irresolute soft and irresolute b-open soft. If (X,τ_1,A) is soft b-T_3-space, then (Y,τ_2,B) is also a soft b-T_3-space.

Proof. Since (X,τ_1,A) is soft b-T_3-space, then (X,τ_1,A) is soft b-regular T_1-space. It follows that, (Y,τ_2,B) is also a soft b-T_1-space from Theorem 4.2 and soft b-regular space from Theorem 4.4. Hence, (Y,τ_2,B) is also a soft b-T_3-space.

Theorem 4.6. Let (X,τ_1,A) and (Y,τ_2,B) be soft topological spaces and $f_{pu} : SS(X)_A \to SS(Y)_B$ be a soft function which is bijective, b-irresolute soft and irresolute b-open soft. If (X,τ_1,A) is soft b-normal space, then (Y,τ_2,B) is also a soft b-normal space.

Proof. Let $(F,B),(G,B)$ be b-closed soft sets in Y such that $(F,B)\tilde{\cap}(G,B) = \tilde{\phi}_B$. Since f_{pu} is b-irresolute soft, then $f_{pu}^{-1}(F,B)$ and $f_{pu}^{-1}(G,B)$ are b-closed soft set in X such that $f_{pu}^{-1}(F,B)\tilde{\cap}f_{pu}^{-1}(G,B) = f_{pu}^{-1}[(F,B)\tilde{\cap}(G,B)] = f_{pu}^{-1}[\tilde{\phi}_B] = \tilde{\phi}_A$ from Theorem 2.3. By hypothesis, there exist irresolute b-open soft sets (K,A) and (H,A) in X such that $f_{pu}^{-1}(F,B)\tilde{\subseteq}(K,A)$, $f_{pu}^{-1}(G,B)\tilde{\subseteq}(H,A)$ and $(F,A)\tilde{\cap}(H,A) = \tilde{\phi}_A$. It follows that, $(F,B) = f_{pu}[f_{pu}^{-1}(F,B)]\tilde{\subseteq}f_{pu}(K,A)$,$(G,B) = f_{pu}[f_{pu}^{-1}(G,B)]\tilde{\subseteq}f_{pu}(H,A)$ from Theorem 2.3 and $f_{pu}(K,A)\tilde{\cap}f_{pu}(H,A) = f_{pu}[(K,A)\tilde{\cap}(H,A)] = f_{pu}[\tilde{\phi}_A] = \tilde{\phi}_B$ from Theorem 2.3. Since f_{pu} is irresolute b-open soft function. Then $f_{pu}(K,A), f_{pu}(H,A)$ are b-open soft sets in Y. Thus, (Y,τ_2,B) is also a soft b-normal space.

Corollary 4.1. Let (X,τ_1,A) and (Y,τ_2,B) be soft topological spaces and $f_{pu} : SS(X)_A \to SS(Y)_B$ be a soft function which is bijective, b-irresolute soft and irresolute b-open soft. If (X,τ_1,A) is soft b-T_4-space, then (Y,τ_2,B) is also a soft b-T_4-space.

Proof. It is obvious from Theorem 4.2 and Theorem 4.6.

5 Conclusion

Recently, many scientists have studied the soft set theory, which is initiated by Molodtsov [33] and easily applied to many problems having uncertainties from social life. In the present work, we introduce the notion of soft b-separation axioms. In particular we study the properties of the soft b-regular spaces and soft b-normal spaces. We show that, if x_E is b-closed soft set for all $x \in X$ in a soft topological space (X,τ,E), then (X,τ,E) is soft b-T_1-space. Also, we show that if a soft topological space (X,τ,E) is soft b-T_3-space, then $\forall\, x \in X$, x_E is b-closed soft set. Also, we show that the property of being b-T_i-spaces $(i = 1,2)$ is soft topological property under a bijection and irresolute b-open soft mapping. Further, the properties of being soft b-regular and soft b-normal are soft topological properties under a bijection, b-irresolute soft and irresolute b-open soft functions. Finally, we show that the property of being b-T_i-spaces $(i = 1,2,3,4)$ is a hereditary property. We hope that, the results in this paper will help researcher enhance and promote the further study on soft topology to carry out a general framework for their applications in practical life.

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