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# Hermite-Hadamard Type Mean Square Integral Inequalities for Stochastic Processes whose Twice Mean Square Derivative are Generalized $\eta-$convex.

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**Abstract:** In the present work, a new concept of generalized convexity (i.e. generalized $\eta-$convexity ) is established and applied to stochastic process.Using the aforementioned concept, some new Hermite - Hadamard type inequalities for stochastic processes are found. From these results, some other inequalities for convex stochastic processes and s-convex stochastic processes in the first sense are deduced. Some Lemmas are introduced and the classical Hölder and power mean inequalities are used as tools for the development of the main results.

**Keywords:** Hermite-Hadamard inequality, generalized $\eta$-convex stochastic processes, mean square integral inequalities

## 1 Introduction

Let $f : I \subset \mathbb{R} \to \mathbb{R}$ a convex function defined on the interval $I$ of real numbers and $a,b \in I$ with $a < b$, then the following inequality

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_0^1 f(t)dt \leq \frac{f(a)+f(b)}{2} \quad (1)$$

holds for all $t \in [0,1]$. This double inequality (1) is known in the literature as Hermite-Hadamard integral inequality for convex functions from the works of Jacques Hadamard and Charles Hermite [1,2]. Both inequalities hold in the reversed direction if $f$ is concave.

Kazimierz Nikodem in [3] makes an analogy of this inequality by defining the convex stochastic processes as follows. Let $(\Omega,(A),\mu)$ be a probability space and $I \subset \mathbb{R}$ be an interval. It is said that a stochastic process $X : I \times \Omega \to \mathbb{R}$ is convex if the following inequality holds almost everywhere

$$X(tu+(1-t)v,\cdot) \leq tX(u,\cdot)+(1-t)X(v,\cdot) \quad (2)$$

for all $u,v \in I$ and $t \in [0,1]$.

With this definition, in 2012, David Kotrys established in [4] the Hermite-Hadamard inequality version for convex stochastic processes as follows:

**Theorem 1.** *If $X : I \times \Omega \to \mathbb{R}$ is Jensen-convex, mean square continuous in the interval I, then for any $u,v \in I$ we have*

$$X\left(\frac{u+v}{2},\cdot\right) \leq \frac{1}{v-u}\int_u^v X(t,\cdot)dt \leq \frac{X(u,\cdot)+X(v,\cdot)}{2}. \quad (3)$$

*almost everywhere.*

The evolution of the concept of convexity has had a great impact on the community of investigators. Recently, generalized concepts, such as log-convexity, s-convexity,

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P-convexity, $\eta$-convexity, quasi convexity, MT-convexity, h-convexity, as well as combinations of these new concepts have been introduced. The following references comprises much relevant information [5,6,7,8,9,10,11,12,13,14,15].

In a direct relation, the results found for generalized convex functions have had a counterpart with stochastic processes, as illustrated in [16,17,18,19,20,21,22].

Motivated by the works of M.J. Vivas and Y. Rangel in [23], we have established some Hermite-Hadamard type mean square integral inequalities using generalized convex stochastic processes.

## 2 Preliminaries

The following notions can be found in some text books and articles. The reader can review then in [4,24,21].

Let $(\Omega,\mathscr{A},\mu)$ be an arbitrary probability space. A function $X:\Omega\to\mathbb{R}$ is called a random variable if it is $\mathscr{A}$-measurable. Let $I\subset\mathbb{R}$ be time. A function $X:I\times\Omega\to\mathbb{R}$ is called stochastic process if for all $u\in I$ the function $X(u,\cdot)$ is a random variable.

A stochastic process $X:I\times\Omega\to\mathbb{R}$ is called continuous in probability in the interval $I$ if for all $t_0\in I$. Then

$$\mu-\lim_{t\to t_0} X(t,\cdot)=X(t_0,\cdot),$$

where $\mu-\lim$ denotes the limit in probability, and it is called mean-square continuous in the interval $I$ if for all $t_0\in I$

$$\mu-\lim_{t\to t_0} \mathbb{E}(X(t,\cdot)-X(t_0,\cdot))=0,$$

where $\mathbb{E}(X(t,\cdot))$ denotes the expectation value of the random variable $X(t,\cdot)$.

In addition, the monotony property is attained. A stochastic process is called increasing (decreasing) if for all $u,v\in I$ such that $t<s$,

$$X(u,\cdot)\leq X(v,\cdot),\qquad (X(u,\cdot)\geq X(v,\cdot))\qquad (a.e.)$$

respectively, and, it called monotonic if it is increasing or decreasing.

In terms of differentiability, stochastic processes are differentiable at a point $t\in I$ if there is a random variable $X'(t,\cdot)$ such that

$$X'(t,\cdot)=\mu-\lim_{t\to t_0}\frac{X(t,\cdot)-X(t_0,\cdot)}{t-t_0}.$$

Let $[a,b]\subset I, a=t_0<t_1<...<t_n=b$ be a partition of $[a,b]$ and $\theta_k\in[t_{k-1},t_k]$ for $k=1,2,\ldots,n$. Let $X$ be a stochastic process such that $\mathbb{E}(X(u,\cdot)^2)<\infty$. A random variable $Y:\Omega\to\mathbb{R}$ is called mean-square integral of the process $X(t,\cdot)$ on $[a,b]$ if the following identity holds:

$$\lim_{n\to\infty} E[X(\theta_k,\cdot)(t_k-t_{k-1})-Y(\cdot)]^2=0$$

then

$$\int_a^b X(t,\cdot)dt=Y(\cdot)\qquad (a.e.).$$

The book of K. Sobczyk [25] involves substantial properties of mean-square integral.

In [26], S. Maden et al. established the following definition:

**Definition 1.** *Let $0<s\leq 1$. A stochastic process $X:I\times\Omega\to\mathbb{R}$ is said to be $s-$convex in the first sense if the inequality*

$$X(tu+(1-t)v,\cdot)\leq t^sX(u,\cdot)+(1-t^s)X(v,\cdot)\qquad(4)$$

*holds almost everywhere for all $u,v\in I$ and all $t\in[0,1]$.*

In this work, the authors introduce the following definitions:

**Definition 2.** *Let $\eta:\mathbb{R}\times\mathbb{R}\to\mathbb{R}$ be a bifunction. A stochastic process $X:I\times\Omega\to\mathbb{R}$ is called convex with respect to $\eta$, or briefly $\eta-$convex, if the inequality*

$$X(tu+(1-t)v,\cdot)\leq X(v,\cdot)+t\eta(X(u,\cdot),X(v,\cdot)\qquad(5)$$

*holds almost everywhere for all $u,v\in I$ and all $t\in[0,1]$.*

**Definition 3.** *Let $\eta:\mathbb{R}\times\mathbb{R}\to\mathbb{R}$ be a bifunction and $0<s\leq 1$. A stochastic process $X:I\times\Omega\to\mathbb{R}$ is called $s-$convex in the first sense with respect to $\eta$, or briefly $(s,\eta)-$convex in the first sense, if the inequality*

$$X(tu+(1-t)v,\cdot)\leq X(v,\cdot)+t^s\eta(X(u,\cdot),X(v,\cdot)\qquad(6)$$

*holds almost everywhere for all $u,v\in I$ and all $t\in[0,1]$.*

*Example 1.* Let $X(t,\cdot)$ be an stochastic process defined by $X(t,\cdot)=t^2A(\cdot)$, where $A(\cdot)$ is a random variable, then $X(t,\cdot)$ is a convex stochastic process and $(1/2,\eta)-$convex in the first sense with respect to the bifunction $\eta(u,v)=2u+v$.

*Example 2.* Let $X(t,\cdot)$ be an stochastic process defined by $X(t,\cdot)=t^nA(\cdot)$, where $A(\cdot)$ is a random variable, then $X(t,\cdot)$ is a convex stochastic process and $(s,\eta)-$ convex in the first sense for $0<s\leq 1$, with respect to the bifunction

$$\eta(u,v)=\sum_{k=1}^{n}\binom{n}{k}v^{1-\frac{k}{n}}\left(u^{1/n}-v^{1/n}\right)^n.$$

In some cases, the functions $B(x,y)$ and $B_\alpha(x,y)$ defined as the Beta and incomplete Beta functions are used, respectively.They are defined as follows:

$$B(x,y)=\int_0^1 t^{x-1}(1-t)^{y-1}dt$$

and

$$B_\alpha(x,y)=\int_0^\alpha t^{x-1}(1-t)^{y-1}dt$$

for $x,y>0$ and $0<\alpha<1$.

## 3 Main Results

First we establish the following Lemma as an auxiliary result.

**Lemma 1.** *Let $X : I\times\Omega\to\mathbb{R}$ be a mean square differentiable stochastic process where $a,b\in I$ and $a<b$. If $X''$ is mean square integrable. Then, the following equality holds*

$$\frac{1}{b-a}\int_a^b X(t,\cdot)dt - X\left(\frac{a+b}{2},\cdot\right) = \frac{(b-a)^2}{16}\left[\int_0^1 t^2 X''\left(t\frac{a+b}{2}+(1-t)a,\cdot\right)dt + \int_0^1 (t-1)^2 X''\left(tb+(1-t)\frac{a+b}{2},\cdot\right)dt\right]. \quad (7)$$

*Proof.* Using integration by parts, we found that

$$\begin{aligned}
&\int_0^1 t^2 X''\left(t\frac{a+b}{2}+(1-t)a,\cdot\right)dt\\
&= t^2\frac{2}{b-a}X'\left(t\frac{a+b}{2}+(1-t)a,\cdot\right)\Big|_0^1 - \frac{4}{b-a}\int_0^1 tX'\left(t\frac{a+b}{2}+(1-t)a,\cdot\right)dt\\
&= \frac{2}{b-a}X'\left(\frac{a+b}{2},\cdot\right) - \frac{4}{b-a}\left[t\frac{2}{b-a}X\left(t\frac{a+b}{2}+(1-t)a,\cdot\right)\Big|_0^1 - \frac{2}{b-a}\int_0^1 X\left(t\frac{a+b}{2}+(1-t)a,\cdot\right)dt\right]\\
&= \frac{2}{b-a}X'\left(\frac{a+b}{2}.\cdot\right) - \frac{8}{(b-a)^2}X\left(\frac{a+b}{2},\cdot\right) + \frac{8}{(b-a)^2}\int_0^1 X\left(t\frac{a+b}{2}+(1-t)a,\cdot\right)dt. \quad (8)
\end{aligned}$$

Now, using the change of variable $x=t\left((a+b)/2\right)+(1-t)a$ for $t\in[0,1]$ and multiplying the equality (8) by $(b-a)^2/16$, it follows that

$$\begin{aligned}
&\frac{(b-a)^2}{16}\int_0^1 t^2 X''\left(t\frac{a+b}{2}+(1-t)a,\cdot\right)dt\\
&= \frac{b-a}{8}X'\left(\frac{a+b}{2}.\cdot\right) - \frac{1}{2}X\left(\frac{a+b}{2},\cdot\right) + \frac{1}{b-a}\int_a^{a+b/2} X(t,\cdot)dt
\end{aligned} \quad (9)$$

Similarly, we have

$$\begin{aligned}
&\frac{(b-a)^2}{16}\int_0^1 (t-1)^2 X''\left(tb+(1-t)\frac{a+b}{2},\cdot\right)dt\\
&= -\frac{b-a}{8}X'\left(\frac{a+b}{2}.\cdot\right) - \frac{1}{2}X\left(\frac{a+b}{2},\cdot\right) + \frac{1}{b-a}\int_{a+b/2}^{b} X(t,\cdot)dt
\end{aligned} \quad (10)$$

Adding the equations (9) and (10), then the desired result (7) follows.

**Theorem 2.** *Suppose that $X : I\times\Omega\to\mathbb{R}$ be a twice mean square differentiable stochastic process and mean square integrable on $I$. If $|X''|$ is $(s,\eta)-$convex on $[a,b]$, where $a,b\in I$ with $a<b$, and $0<s\le 1$, the following inequality holds almost everywhere*

$$\begin{aligned}
&\left|X\left(\frac{a+b}{2},\cdot\right) - \frac{1}{b-a}\int_a^b X(t,\cdot)dt\right|\\
&\le \frac{(b-a)^2}{16}\left(\frac{|X''(a,\cdot)|+\left|X''\left(\frac{a+b}{2},\cdot\right)\right|}{3} + \frac{\eta\left(\left|X''\left(\frac{a+b}{2},\cdot\right)\right|,|X''(a,\cdot)|\right)}{s+3} + \frac{2\eta\left(|X''(b,\cdot)|,\left|X''\left(\frac{a+b}{2},\cdot\right)\right|\right)}{(s+1)(s+2)(s+3)}\right). \quad (11)
\end{aligned}$$

*Proof.* Using Lemma 1 is immediate that

$$\begin{aligned}
&\left|X\left(\frac{a+b}{2},\cdot\right) - \frac{1}{b-a}\int_a^b X(t,\cdot)dt\right|\\
&\le \frac{(b-a)^2}{16}\int_0^1 t^2\left|X''\left(t\frac{a+b}{2}+(1-t)a,\cdot\right)\right|dt + \frac{(b-a)^2}{16}\int_0^1 (t-1)^2\left|X''\left(tb+(1-t)\frac{a+b}{2},\cdot\right)\right|dt.
\end{aligned} \quad (12)$$

Now, since $|X''|$ is $(s,\eta)-$convex on $[a,b]$, it is easy to see that

$$\left|X''\left(t\frac{a+b}{2}+(1-t)a,\cdot\right)\right| \le |X''(a,\cdot)| + t^s\eta\left(\left|X''\left(\frac{a+b}{2},\cdot\right)\right|,|X''(a,\cdot)|\right) \quad (13)$$

and

$$\left|X''\left(tb+(1-t)\frac{a+b}{2},\cdot\right)\right| \le \left|X''\left(\frac{a+b}{2},\cdot\right)\right| + t^s\eta\left(|X''(b,\cdot)|,\left|X''\left(\frac{a+b}{2},\cdot\right)\right|\right). \quad (14)$$

Replacing the inequalities (13) and (14) in (12), we have

$$\left|X\left(\frac{a+b}{2},\cdot\right)-\frac{1}{b-a}\int_a^b X(t,\cdot)dt\right|$$

$$\leq \frac{(b-a)^2}{16}\left[\int_0^1 t^2\left|X''(a,\cdot)\right|dt + \int_0^1 t^{2+s}\eta\left(\left|X''\left(\frac{a+b}{2},\cdot\right)\right|,\left|X''(a,\cdot)\right|\right)dt\right]$$

$$+\frac{(b-a)^2}{16}\left[\int_0^1 (t-1)^2\left|X''\left(\frac{a+b}{2},\cdot\right)\right|dt + \int_0^1 t^s\eta\left(\left|X''(b,\cdot)\right|,\left|X''\left(\frac{a+b}{2},\cdot\right)\right|\right)\right]dt.$$

$$=\frac{(b-a)^2}{16}\left(\frac{|X''(a,\cdot)|}{3}+\frac{\eta\left(\left|X''\left(\frac{a+b}{2},\cdot\right)\right|,|X''(a,\cdot)|\right)}{s+3}\right)$$

$$+\frac{(b-a)^2}{16}\left(\frac{\left|X''\left(\frac{a+b}{2},\cdot\right)\right|}{3}+\frac{2\eta\left(|X''(b,\cdot)|,\left|X''\left(\frac{a+b}{2},\cdot\right)\right|\right)}{(s+1)(s+2)(s+3)}\right),$$

that is because

$$\int_0^1 t^{s+2}dt=\frac{1}{s+3}$$

and

$$\int_0^1 (t-1)^2t^s dt=\frac{2}{(s+1)(s+2)(s+3)}.$$

The proof is complete. ■

*Remark.* If in Theorem 2 we choose $\eta(x,y)=x-y$, the inequality (11) can be restated as

$$\left|X\left(\frac{a+b}{2},\cdot\right)-\frac{1}{b-a}\int_a^b X(t,\cdot)dt\right|$$

$$\leq\frac{(b-a)^2}{16(s+3)}\left(\frac{s|X''(a,\cdot)|}{3}+\frac{2|X''(b,\cdot)|}{(s+1)(s+2)}+\frac{(s+1)(s+2)(s+6)-6}{3(s+1)(s+2)}\left|X''\left(\frac{a+b}{2},\cdot\right)\right|\right),$$

almost everywhere, which corresponds to $s-$convex stochastic process in the first sense; and if $s=1$, we obtain

$$\left|X\left(\frac{a+b}{2},\cdot\right)-\frac{1}{b-a}\int_a^b X(t,\cdot)dt\right|$$

$$\leq\frac{(b-a)^2}{64}\left(\frac{|X''(a,\cdot)|+|X''(b,\cdot)|}{3}+2\left|X''\left(\frac{a+b}{2},\cdot\right)\right|\right),$$

almost everywhere, for a convex stochastic process.

**Theorem 3.** *Suppose that $X:I\times\Omega\to\mathbb{R}$ be a twice mean square differentiable stochastic process and mean square integrable on $I$. If $|X''|^q$ is $(s,\eta)-$convex on $[a,b]$, where $a,b$ with $a<b$, and $0<s\leq 1$, the following inequality holds almost everywhere*

$$\left|X\left(\frac{a+b}{2},\cdot\right)-\frac{1}{b-a}\int_a^b X(t,\cdot)dt\right|$$

$$\leq\frac{(b-a)^2}{16}\left(\frac{1}{2p+1}\right)^{1/p}\times$$

$$\left[\left(|X''(a,\cdot)|^q+\frac{\eta\left(\left|X''\left(\frac{a+b}{2},\cdot\right)\right|^q,|X''(a,\cdot)|^q\right)}{s+1}\right)^{1/q}\right.$$

$$\left.+\left(\left|X''\left(\frac{a+b}{2},\cdot\right)\right|^q+\frac{\eta\left(|X''(b,\cdot)|^q,\left|X''\left(\frac{a+b}{2},\cdot\right)\right|^q\right)}{s+1}\right)^{1/q}\right],$$

*where $q>1$ and $1/p+1/q=1$.*

*Proof.* Using Lemma 1 and the Hölder inequality, we have

$$\left|X\left(\frac{a+b}{2},\cdot\right)-\frac{1}{b-a}\int_a^b X(t,\cdot)dt\right|$$

$$\leq\frac{(b-a)^2}{16}\int_0^1 t^2\left|X''\left(t\frac{a+b}{2}+(1-t)a,\cdot\right)\right|dt+\frac{(b-a)^2}{16}\int_0^1(t-1)^2\left|X''\left(tb+(1-t)\frac{a+b}{2},\cdot\right)\right|dt$$

$$\leq\frac{(b-a)^2}{16}\times\left[\left(\int_0^1 t^{2p}dt\right)^{1/p}\left(\int_0^1\left|X''\left(t\frac{a+b}{2}+(1-t)a,\cdot\right)\right|^q dt\right)^{1/q}\right.$$

$$\left.+\left(\int_0^1(t-1)^{2p}dt\right)^{1/p}\times\left(\int_0^1\left|X''\left(tb+(1-t)\frac{a+b}{2},\cdot\right)\right|^q dt\right)^{1/q}\right] \tag{15}$$

Using the $(s,\eta)-$convexity of $|X''|^q$ we observe that

$$\int_0^1\left|X''\left(t\frac{a+b}{2}+(1-t)a,\cdot\right)\right|^q dt$$

$$=|X''(a,\cdot)|^q+\frac{1}{s+1}\eta\left(\left|X''\left(\frac{a+b}{2},\cdot\right)\right|^q,|X''(a,\cdot)|^q\right) \tag{16}$$

and

$$\int_0^1\left|X''\left(tb+(1-t)\frac{a+b}{2},\cdot\right)\right|^q dt$$

$$=\left|X''\left(\frac{a+b}{2},\cdot\right)\right|^q+\frac{1}{s+1}\eta\left(|X''(a,\cdot)|^q,\left|X''\left(\frac{a+b}{2},\cdot\right)\right|^q\right). \tag{17}$$

Replacing (16) and (17) in (15), we obtain

$$\left|X\left(\frac{a+b}{2},\cdot\right)-\frac{1}{b-a}\int_a^b X(t,\cdot)dt\right|$$

$$\leq \frac{(b-a)^2}{16}\left(\frac{1}{2p+1}\right)^{1/p}\times$$

$$\left[\left(|X''(a,\cdot)|^q+\frac{1}{s+1}\eta\left(\left|X''\left(\frac{a+b}{2},\cdot\right)\right|^q,|X''(a,\cdot)|^q\right)\right)^{1/q}\right.$$

$$+\left(\left|X''\left(\frac{a+b}{2},\cdot\right)\right|^q\right.$$

$$\left.\left.+\frac{1}{s+1}\eta\left(|X''(b,\cdot)|^q,\left|X''\left(\frac{a+b}{2},\cdot\right)\right|^q\right)\right)^{1/q}\right]$$

because

$$\left(\int_0^1 t^{2p}dt\right)^{1/p}=\left(\int_0^1 (t-1)^{2p}dt\right)^{1/p}=\left(\frac{1}{2p+1}\right)^{1/p}.$$

The proof is complete. ■

*Remark.* If in Theorem 3 we choose $\eta(x,y)=x-y$, it follows that

$$\left|X\left(\frac{a+b}{2},\cdot\right)-\frac{1}{b-a}\int_a^b X(t,\cdot)dt\right|$$
$$\leq \frac{(b-a)^2}{16(s+1)^{1/q}}\left(\frac{1}{2p+1}\right)^{1/p}\times$$
$$\left[\left(s|X''(a,\cdot)|^q+\left|X''\left(\frac{a+b}{2},\cdot\right)\right|^q\right)^{1/q}\right.$$
$$\left.+\left(s\left|X''\left(\frac{a+b}{2},\cdot\right)\right|^q+|X''(b,\cdot)|^q\right)^{1/q}\right]\quad \text{(a.e.)}\ ,$$

which corresponds to $s-$convex stochastic process in the first sense, and if $s=1$, we obtain

$$\left|X\left(\frac{a+b}{2},\cdot\right)-\frac{1}{b-a}\int_a^b X(t,\cdot)dt\right|$$
$$\leq \frac{(b-a)^2}{2^{4+1/q}}\left(\frac{1}{2p+1}\right)^{1/p}\times$$
$$\left[\left(|X''(a,\cdot)|^q+\left|X''\left(\frac{a+b}{2},\cdot\right)\right|^q\right)^{1/q}\right.$$
$$\left.+\left(\left|X''\left(\frac{a+b}{2},\cdot\right)\right|^q+|X''(b,\cdot)|^q\right)^{1/q}\right]$$

almost everywhere, for a convex stochastic process.

**Theorem 4.** *Suppose that $X:I\times\Omega\to\mathbb{R}$ be a twice mean square differentiable stochastic process and mean square integrable on $I$. If $|X''|^q$ is $(s,\eta)-$convex on $[a,b]$, where $a,b\in I$ with $a<b$, and $0<s\leq 1$, the following inequality holds almost everywhere*

$$\left|X\left(\frac{a+b}{2},\cdot\right)-\frac{1}{b-a}\int_a^b X(t,\cdot)dt\right|$$
$$\leq \frac{(b-a)^2}{16(3)^{1/p}}\times$$
$$\left[\left(\frac{|X''(a,\cdot)|^q}{3}+\frac{\eta\left(\left|X''\left(\frac{a+b}{2},\cdot\right)\right|^q,|X''(a,\cdot)|^q\right)}{s+3}\right)^{1/q}\right.$$
$$\left.+\left(\frac{\left|X''\left(\frac{a+b}{2},\cdot\right)\right|^q}{3}+\frac{2\eta\left(|X''(b,\cdot)|^q,\left|X''\left(\frac{a+b}{2},\cdot\right)\right|^q\right)}{(s+1)(s+2)(s+3)}\right)^{1/q}\right],\tag{18}$$

*where $q>1$ and $1/p+1/q=1$.*

*Proof.* Using Lemma 1 and the power mean inequality, it holds that

$$\left|X\left(\frac{a+b}{2},\cdot\right)-\frac{1}{b-a}\int_a^b X(t,\cdot)dt\right|$$
$$\leq \frac{(b-a)^2}{16}\int_0^1 t^2\left|X''\left(t\frac{a+b}{2}+(1-t)a,\cdot\right)\right|dt$$
$$+\frac{(b-a)^2}{16}\int_0^1 (t-1)^2\left|X''\left(tb+(1-t)\frac{a+b}{2},\cdot\right)\right|dt$$
$$\leq \frac{(b-a)^2}{16}\times$$
$$\left[\left(\int_0^1 t^2dt\right)^{1/p}\left(\int_0^1 t^2\left|X''\left(t\frac{a+b}{2}+(1-t)a,\cdot\right)\right|^q dt\right)^{1/q}\right.$$
$$+\left(\int_0^1 (t-1)^2dt\right)^{1/p}\times$$
$$\left.\left(\int_0^1 (t-1)^2\left|X''\left(tb+(1-t)\frac{a+b}{2},\cdot\right)\right|^q dt\right)^{1/q}\right].\tag{19}$$

Now, using the $(s,\eta)-$convexity of $|X''|^q$, it is easy to verify that

$$\int_0^1 t^2\left|X''\left(t\frac{a+b}{2}+(1-t)a,\cdot\right)\right|^q dt$$
$$\leq \frac{|X''(a,\cdot)|^q}{3}+\frac{\eta\left(\left|X''\left(\frac{a+b}{2},\cdot\right)\right|^q,|X''(a,\cdot)|^q\right)}{s+3}$$

and

$$\int_0^1 (t-1)^2\left|X''\left(tb+(1-t)\frac{a+b}{2},\cdot\right)\right|^q dt$$
$$\leq \frac{\left|X''\left(\frac{a+b}{2},\cdot\right)\right|^q}{3}+\frac{2\eta\left(|X''(b,\cdot)|^q,\left|X''\left(\frac{a+b}{2},\cdot\right)\right|^q\right)}{(s+1)(s+2)(s+3)}$$

Thus, it is attained that

$$\left|X\left(\frac{a+b}{2},\cdot\right)-\frac{1}{b-a}\int_a^b X(t,\cdot)dt\right|$$
$$\leq \frac{(b-a)^2}{16(3)^{1/p}}\times$$
$$\left[\left(\frac{|X''(a,\cdot)|^q}{3}+\frac{\eta\left(\left|X''\left(\frac{a+b}{2},\cdot\right)\right|^q,|X''(a,\cdot)|^q\right)}{s+3}\right)^{1/q}\right.$$
$$\left.+\left(\frac{\left|X''\left(\frac{a+b}{2},\cdot\right)\right|^q}{3}+\frac{2\eta\left(|X''(b,\cdot)|^q,\left|X''\left(\frac{a+b}{2},\cdot\right)\right|^q\right)}{(s+1)(s+2)(s+3)}\right)^{1/q}\right].$$

The proof is complete.

*Remark.* If in Theorem 4, we select $\eta(x,y)=x-y$. Then the inequality (18) can be restated as

$$\left|X\left(\frac{a+b}{2},\cdot\right)-\frac{1}{b-a}\int_a^b X(t,\cdot)dt\right|$$
$$\leq \frac{(b-a)^2}{16(3)^{1/p}(s+3)^{1/q}}\left[\left(\frac{s|X''(a,\cdot)|^q}{3}+\left|X''\left(\frac{a+b}{2},\cdot\right)\right|^q\right)^{1/q}\right.$$
$$+\left(\frac{((s+1)(s+2)(s+3)-6)\left|X''\left(\frac{a+b}{2},\cdot\right)\right|^q}{3(s+1)(s+2)}\right.$$
$$\left.\left.+\frac{2|X''(b,\cdot)|^q}{(s+1)(s+2)}\right)^{1/q}\right],$$

which corresponds to $s-$convex stochastic process in the first sense, and if $s=1$, we obtain

$$\left|X\left(\frac{a+b}{2},\cdot\right)-\frac{1}{b-a}\int_a^b X(t,\cdot)dt\right|$$
$$\leq \frac{(b-a)^2}{(3)^{1/p}(4)^{2+1/q}}\left[\left(\frac{|X''(a,\cdot)|^q}{3}+\left|X''\left(\frac{a+b}{2},\cdot\right)\right|^q\right)^{1/q}\right.$$
$$\left.+\left(\left|X''\left(\frac{a+b}{2},\cdot\right)\right|^q+\frac{|X''(b,\cdot)|^q}{3}\right)^{1/q}\right],$$

which corresponds to convex stochastic process.

The next results are obtained throughout the following Lemma.

**Lemma 2.** *Let $X:I\times\Omega\to\mathbb{R}$ be a mean square differentiable stochastic process where $a,b\in I$ and $a<b$. If $X''$ is mean square integrable, the following equality holds*

$$\frac{1}{b-a}\int_a^b X(t,\cdot)dt-X\left(\frac{a+b}{2},\cdot\right)$$
$$=\frac{(b-a)^2}{4}\times$$
$$\int_0^1 m(t)\left(X''(ta+(1-t)b,\cdot)+X''(tb+(1-t)a,\cdot)\right)dt, \quad (20)$$

*where*

$$m(t)=\begin{cases} t^2, & if\ t\in[0,1/2) \\ (1-t)^2, & if\ t\in[1/2,1]. \end{cases}$$

*Proof.* In a similar way to the Proof of Lemma 1 , easily is achieved the desired result .

**Theorem 5.** *Suppose that $X:I\times\Omega\to\mathbb{R}$ is a twice mean square differentiable stochastic process and mean square integrable on $I$. If $|X''|$ is $(s,\eta)-$convex on $[a,b]$, where $a,b\in I$ with $a<b$, and $0<s\leq 1$, the following inequality holds almost everywhere*

$$\left|X\left(\frac{a+b}{2},\cdot\right)-\frac{1}{b-a}\int_a^b X(t,\cdot)dt\right|$$
$$\leq\frac{(b-a)^2}{4}\left(\frac{|X''(a,\cdot)|+|X''(b,\cdot)|}{24}\right.$$
$$\left.+C\left(\eta\left(|X''(a,\cdot)|,|X''(b,\cdot)|\right)+\eta\left(|X''(b,\cdot)|,|X''(a,\cdot)|\right)\right)\right)$$

*where*

$$C=\frac{1}{2^{s+3}(s+3)}+B(s+1,3)-B_{1/2}(s+1,3).$$

*Proof.* Using Lemma 2, we obtain

$$\left|X\left(\frac{a+b}{2},\cdot\right)-\frac{1}{b-a}\int_a^b X(t,\cdot)dt\right|$$
$$\leq\frac{(b-a)^2}{4}\times$$
$$\int_0^1 m(t)\left|\left(X''(ta+(1-t)b,\cdot)+X''(tb+(1-t)a,\cdot)\right|dt\right)$$
$$\leq\frac{(b-a)^2}{4}\times \quad (21)$$
$$\int_0^1 m(t)\left(\left|X''(ta+(1-t)b,\cdot)\right|+\left|X''(tb+(1-t)a,\cdot)\right|\right)dt$$

Now, using the definition of the function $m(t)$ and the $(s,\eta)-$convexity of $|X''|$, one can observe that

$$\int_0^1 m(t)\left|X''(ta+(1-t)b,\cdot)\right|dt$$
$$\leq\int_0^{1/2} t^2\left(|X''(b,\cdot)|+t^s\eta\left(|X''(a,\cdot)|,|X''(b,\cdot)|\right)\right)dt$$
$$+\int_{1/2}^1 (1-t)^2\left(|X''(b,\cdot)|+t^s\eta\left(|X''(a,\cdot)|,|X''(b,\cdot)|\right)\right)dt$$
$$=\frac{|X''(b,\cdot)|}{24}+\frac{\eta\left(|X''(a,\cdot)|,|X''(b,\cdot)|\right)}{2^{s+3}(s+3)}$$
$$+(B(s+1,3)-B_{1/2}(s+1,3))\eta\left(|X''(a,\cdot)|,|X''(b,\cdot)|\right), \quad (22)$$

and similarly

$$\int_0^1 m(t)\left|X''(tb+(1-t)a,\cdot)\right|dt$$

$$\leq \frac{|X''(a,\cdot)|}{24} + \frac{\eta\left(|X''(b,\cdot)|, |X''(a,\cdot)|\right)}{2^{s+3}(s+3)}$$

$$+(B(s+1,3) - B_{1/2}(s+1,3))\eta\left(|X''(b,\cdot)|, |X''(a,\cdot)|\right) \tag{23}$$

Replacing (22) and (23) in (21), we get the desired result. The proof is complete.

*Remark.* If in Theorem 5 we select $\eta(x,y) = x - y$, we obtain

$$\left|X\left(\frac{a+b}{2},\cdot\right) - \frac{1}{b-a}\int_a^b X(t,\cdot)dt\right| \leq \frac{(b-a)^2}{24}\left(|X''(a,\cdot)| + |X''(b,\cdot)|\right)$$

for convex stochastic process and $s-$convex stochastic process in the first sense. This makes a coincidence with the result by Hernández Hernández,J. E. and Gómez, J.F. in [27].

*Remark.* If the bifunction $\eta$ has the following property: $\eta(x,y) = \eta(y,x)$, the inequality in Theorem 5 can be written as

$$\left|X\left(\frac{a+b}{2},\cdot\right) - \frac{1}{b-a}\int_a^b X(t,\cdot)dt\right|$$

$$\leq \frac{(b-a)^2}{4} \times \left(\frac{|X''(a,\cdot)| + |X''(b,\cdot)|}{24} + 2C\left(\eta\left(|X''(a,\cdot)|, |X''(b,\cdot)|\right)\right)\right)$$

where

$$C = \frac{1}{2^{s+3}(s+3)} + B(s+1,3) - B_{1/2}(s+1,3).$$

**Theorem 6.** *Let $q > 1$ and $1/p + 1/q = 1$. Suppose that $X : I \times \Omega \to \mathbb{R}$ is a twice mean square differentiable stochastic process and mean square integrable on $I$. If $|X''|^q$ is $(s,\eta)-$convex on $[a,b]$, where $a,b \in I$ with $a < b$, and $0 < s \leq 1$, the following inequality holds almost everywhere*

$$\left|X\left(\frac{a+b}{2},\cdot\right) - \frac{1}{b-a}\int_a^b X(t,\cdot)dt\right| \leq \frac{(b-a)^2}{16}\left(\frac{1}{2p+1}\right)^{1/p} \times \left[\left(|X''(b,\cdot)|^q + \frac{\eta\left(|X''(a,\cdot)|^q, |X''(b,\cdot)|^q\right)}{s+1}\right)^{1/q} + \left(|X''(a,\cdot)|^q + \frac{\eta\left(|X''(b,\cdot)|^q, |X''(a,\cdot)|^q\right)}{s+1}\right)^{1/q}\right].$$

*Proof.* Using Lemma (2) and the Hölder inequality, it follows that

$$\left|X\left(\frac{a+b}{2},\cdot\right) - \frac{1}{b-a}\int_a^b X(t,\cdot)dt\right|$$

$$\leq \frac{(b-a)^2}{4}\left(\int_0^1 |m(t)|^p dt\right)^{1/p} \times \left[\left(\int_0^1 |X''(ta+(1-t)b,\cdot)|^q dt\right)^{1/q} + \left(\int_0^1 |X''(tb+(1-t)a,\cdot)|^q dt\right)^{1/q}\right] \tag{24}$$

Now, it is easy to see that

$$\int_0^1 |m(t)|^p dt = \frac{1}{4^p(2p+1)}, \tag{25}$$

and using the $(s,\eta)-$convexity of $|X''|^q$,it follows that

$$\int_0^1 |X''(ta+(1-t)b,\cdot)|^q dt \leq |X''(b,\cdot)|^q + \frac{\eta\left(|X''(a,\cdot)|^q, |X''(b,\cdot)|^q\right)}{s+1} \tag{26}$$

and

$$\int_0^1 |X''(tb+(1-t)a,\cdot)|^q dt \leq |X''(a,\cdot)|^q + \frac{\eta\left(|X''(b,\cdot)|^q, |X''(a,\cdot)|^q\right)}{s+1}. \tag{27}$$

Replacing (25), (26) and (27) in (24), it follows the desired result.

*Remark.* If in Theorem 6 we choose $\eta(x,y) = x - y$, it follows that

$$\left|X\left(\frac{a+b}{2},\cdot\right) - \frac{1}{b-a}\int_a^b X(t,\cdot)dt\right| \leq \frac{(b-a)^2}{16(s+1)^{1/q}}\left(\frac{1}{2p+1}\right)^{1/p} \times \left[\left(s|X''(b,\cdot)|^q + |X''(a,\cdot)|^q\right)^{1/q} + \left(s|X''(b,\cdot)|^q + |X''(a,\cdot)|^q\right)^{1/q}\right],$$

which corresponds to $s-$convex stochastic process in the first sense, and if $s = 1$, it follows that

$$\left|X\left(\frac{a+b}{2},\cdot\right) - \frac{1}{b-a}\int_a^b X(t,\cdot)dt\right| \leq \frac{(b-a)^2}{(2)^{3+1/q}}\left(\frac{1}{2p+1}\right)^{1/p}\left(|X''(a,\cdot)|^q + |X''(b,\cdot)|^q\right)^{1/q}$$

which corresponds to convex stochastic process.

**Theorem 7.** *Let $q > 1$ and $1/p + 1/q = 1$. Suppose that $X : I \times \Omega \to \mathbb{R}$ is a twice mean square differentiable stochastic process and mean square integrable on $I$. If $|X''|^q$ is $(s,\eta)-$convex on $[a,b]$, where $a,b \in I$ with $a < b$, and $0 < s \leq 1$, the following inequality holds almost everywhere*

$$\left|X\left(\frac{a+b}{2},\cdot\right) - \frac{1}{b-a}\int_a^b X(t,\cdot)dt\right|$$

$$\leq \frac{(b-a)^2}{4(12)^{1/p}}\left[\left(\frac{|X''(b,\cdot)|^q}{24}+C\eta\left(|X''(a,\cdot)|^q,|X''(b,\cdot)|^q\right)\right)^{1/q}\right.$$

$$\left.+\left(\frac{|X''(a,\cdot)|^q}{24}+C\eta\left(|X''(b,\cdot)|^q,|X''(a,\cdot)|^q\right)\right)^{1/q}\right]$$

*where*

$$C=\frac{1}{2^{s+3}(s+3)}+B(s+1,3)-B_{1/2}(s+1,3)).$$

*Proof.* Using the Lemma (2) and the power mean inequality it follows that

$$\left|X\left(\frac{a+b}{2},\cdot\right)-\frac{1}{b-a}\int_a^b X(t,\cdot)dt\right|$$

$$\leq \frac{(b-a)^2}{4}\left(\int_0^1|m(t)|\,dt\right)^{1/p}\times$$

$$\left[\left(\int_0^1|m(t)|\left|X''(ta+(1-t)b,\cdot)\right|^q dt\right)^{1/q}+\right.$$

$$\left.\left(\int_0^1|m(t)|\left|X''(tb+(1-t)a,\cdot)\right|^q dt\right)^{1/q}\right]. \tag{28}$$

Observing that

$$\int_0^1|m(t)|\,dt=1/12 \tag{29}$$

and

$$\int_0^1 m(t)\left|X''(ta+(1-t)b,\cdot)\right|^q dt$$

$$\leq \int_0^{1/2} t^2\left(|X''(b,\cdot)|^q+t^s\eta\left(|X''(a,\cdot)|^q,|X''(b,\cdot)|^q\right)\right)dt$$

$$+\int_{1/2}^1(1-t)^2\left(|X''(b,\cdot)|^q+t^s\eta\left(|X''(a,\cdot)|^q,|X''(b,\cdot)|^q\right)\right)dt$$

$$=\frac{|X''(b,\cdot)|^q}{24}+\frac{\eta\left(|X''(a,\cdot)|^q,|X''(b,\cdot)|^q\right)}{2^{s+3}(s+3)}$$

$$+B(3,s+1)(1-I_{1/2}(3,s+1))\eta\left(|X''(a,\cdot)|^q,|X''(b,\cdot)|^q\right), \tag{30}$$

and similarly

$$\int_0^1 m(t)\left|X''(tb+(1-t)a,\cdot)\right|dt$$

$$\leq \frac{|X''(a,\cdot)|^q}{24}+\frac{\eta\left(|X''(b,\cdot)|^q,|X''(a,\cdot)|^q\right)}{2^{s+3}(s+3)}$$

$$+B(s+1,3)-B_{1/2}(s+1,3))\eta\left(|X''(b,\cdot)|^q,|X''(a,\cdot)|^q\right). \tag{31}$$

Replacing (29),(30) and (31) in (28), we obtain the desired result.

The proof is complete.

*Remark.* If in Theorem 7 we choose $\eta(x,y)=x-y$, we have

$$\left|X\left(\frac{a+b}{2},\cdot\right)-\frac{1}{b-a}\int_a^b X(t,\cdot)dt\right|$$

$$\leq \frac{(b-a)^2}{4(12)^{1/p}}\left[\left(C\left|X''(a,\cdot)\right|^q+\left(\frac{1}{24}-C\right)\left|X''(b,\cdot)\right|^q\right)^{1/q}\right.$$

$$\left.+\left(C\left|X''(b,\cdot)\right|^q+\left(\frac{1}{24}-C\right)\left|X''(a,\cdot)\right|^q\right)^{1/q}\right]$$

where

$$C=\frac{1}{2^{s+3}(s+3)}+B(s+1,3)-B_{1/2}(s+1,3)),$$

which corresponds to $s-$convex stochastic process in the first sense, if $s=1$, then it follows that

$$C=\frac{1}{2^6}+B(2,3)-B_{1/2}(2,3))\cong 4.1666\times 10^{-2}$$

and

$$\left|X\left(\frac{a+b}{2},\cdot\right)-\frac{1}{b-a}\int_a^b X(t,\cdot)dt\right|$$

$$\leq \frac{(b-a)^2}{2^{2+2/p}(3)^{1/p}}\times$$

$$\left[\left(4.1666\times 10^{-2}\left|X''(a,\cdot)\right|^q+6.6667\times 10^{-7}\left|X''(b,\cdot)\right|^q\right)^{1/q}\right.$$

$$\left.+\left(4.1666\times 10^{-2}\left|X''(b,\cdot)\right|^q+6.6667\times 10^{-7}\left|X''(a,\cdot)\right|^q\right)^{1/q}\right]$$

which corresponds to convex stochastic process.

## 4 Conclusion

In the present paper, we established some new Hermite - Hadamard type mean square integral inequalities for stochastic processes whose mean square derivative is generalized $\eta-$convex. From these results, some other inequalities are deduced for convex stochastic processes and $s-$convex stochastic processes in the first sense.

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## References


[1] J. Hadamard , Etude sur les propriétés des fonctions entiéres et en particulier d'une fonction considéré par Riemann,*J. Math. Pures et Appliquées* ,**58**, 171-216 (1893).
[2] Ch. Hermite, *Sur deux limites dune integrale definie*, Mathesis **3**, 1-82 (1883).
[3] K. Nikodem , On convex stochastic processes,*Aequat. Math.*, **20(2)**, 184-197 (1980).
[4] D. Kotrys, Hermite-Hadamard inequality for convex stochastic processes, *Aequat. Math.*, **83(1)** , 143-151 (2012) .
[5] M. Alomari, M. Darus , On the hadamard's inequality for log-convex functions on the coordinates.*J. Ineq. Appl.*, **1**, 1-13 (2009).
[6] M. Alomari , M. Darus , S.S. Dragomir , P. Cerone , Ostrowski type inequalities for functions whose derivatives are s-convex in the second sense, *Appl. Math. Lett.* **23(9)** , 1071-1076 (2010)
[7] A. Barani , S. Barani , S.S. Dragomir, Refinements of Hermite-Hadamard inequalities for functions when a power of the absolute value of the second derivative is p-convex, *J. Appl. Math.* **2012**, 1-10 (2012).
[8] M. Eshagy Gordji , S.S. Dragomir, M.R. Delavar, An inequality related to $\eta-$convex functions (II), *Intern. J. Nonlin. Anal. Appl.* **6(2)**, 27-33 (2015).
[9] H. Greenberg, W. Pierskalla , A review of quasi convex functions, *Repr. Operat. Research*, **19(7)**, 1553-1569(1971) .
[10] W. Liu, W. Wen , J. Park , Hermite-Hadamard type inequalities for MT-convex functions via classical integrals and fractional integrals,*J. Nonlinear Sci. Appl.* ,**9(3)**, 766-777 (2016).
[11] S. Varošanec , On h-convexity, *J. Math. Anal. Appl.*, **326(1)**, 303-311 (2007).
[12] J. Medina Viloria , M. Vivas-Cortez, Hermite-Hadamard type inequalities for harmonically convex functions on n-coordinates, *Appl. Math. and Inf. Sci Lett.*, **5(2)**, 53-58 (2018).
[13] M. Vivas-Cortez, Relative strongly h-convex functions and integral inequalities, *Appl. Math. and Inf. Sci. lett.*, **4(2)**, 39-45 (2016).
[14] M. Vivas, J. Hernández, N. Merentes, New Hermite-Hadamard and Jensen type inequalities for h-convex functions on fractal sets, *Rev. Col. Mat* , **50(2)**, 145-164 (2016).
[15] M. Bracamonte , J. Gimenez , N. Merentes, M. Vivas, Fejer type inequalities for m-convex functions, *Publ. en Cs. y Tec.* **10(1)**, 10-15 (2016).
[16] J.E. Hernádez Hernández, Some fractional integral inequalities for stochastic processes whose first and second derivatives are quasi-convex, *Rev. Matua, Universidad del Atlantico, Colombia* **5(2)**, 1-13 (2018).
[17] J.E. Hernández Hernández , J.F. Gómez J.F, Hermite-Hadamard type inequalities, convex stochastic processes and Katugampola fractional integral.*Rev. Integración: temas de matemáticas (Universidad de Santander Col.)* **36(2)**, 133-149 (2019).
[18] D. Kotrys, K. Nikodem, Quasiconvex stochastic processes and a separation theorem, *Aequat. Math.*, **89(1)**, 41-48 (2015).
[19] E. Set, M. Tomar, S. Maden, Hermite-Hadamard type inequalities for s-convex stochastic processes in the second sense. *Turk. J. Anal. Numb. Theor.*, **2(6)**, 202–207 (2014).
[20] A. Skowronski, On some properties of J-convex stochastic processes,*Aequat. Math.*, **44(1)**, 249–258 (1922).
[21] A. Skowronski, On wright-convex stochastic processes,*Annal. Math. Silec.*, **9(1)**, 29–32 (1995).
[22] M.J. Vivas-Cortez , J.E. Hernández Hernández , On $(m,h_1,h_2)-$-GA-Convex stochastic processes, *Appl. Math. & Inf. Sci.*, **11(3)** 649-657 (2017).
[23] M.J. Vivas-Cortez , Y.C. Rangel Oliveros, On some Hermite-Hadamard type inequalities for functions whose second derivative are convex generalized, *Rev. Matua (Universidad del Atlántico, Colombia)*, **5(2)**, 21-31 (2018).
[24] T. Mikosh, , *Elementary Stochastic Calculus with Finance in View*,Series: Advan. Ser. Stat. Sci. Appl. Prob., World Scientific Publishing , Singapore,55-60, (1998).
[25] K. Sobczyk , *Stochastic Differential Equations with Applications to Physics and Engineering*. Kluwer Academic Publishers, Dordrecht , 125-140,(1991).
[26] S. Maden , M. Tomar, E. Set, Hermite-Hadamard type inequalities for s-convex stochastic processes in first sense.*Pure Appl. Math. Lett.* , **2015**(1) , 1-7 (2015).
[27] J.E. Hernández Hernández , J.F. and Gómez, Hermite hadamard type inequalities for stochastic processes whose second derivatives are $(m,h_1,h_2)-$-convex using Riemann-Liouville fractional integral, *Rev. Matua, Universidad del Atlantico ,Colombia*, **5(1)**, 13-26 (2018) .


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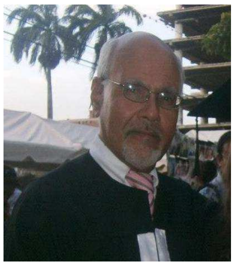

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