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Non-Classical Properties of Two Mode Dissipative Cavity

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Abstract: In this paper, we investigate the entropy squeezing phenomenon. The considerable system consists of noninteracting two two-level atoms, moving in the Kerr-like medium, interacting with two mode quantized Electromagnetic field through two photon process. We will expose the effects of decay rates of the atoms and the field in addition to the coupling variation parameter on the atomic inversion, the squeezing in the components of the atomic dipole moment, quadrature squeezing and the subpoissonian distribution. It is shown that the suggested system contains some new parameters that can be used to control the system dynamics.

Keywords: Atomic squeezing, normal squeezing, sub-Poissonian distribution.

1 Introduction

The interaction of two-level atom with the cavity field is one of the fundamental problem that engaged many of authors. The (JCM), which is exactly solvable model, is the simplest model describing this interaction [1], which explains important nonclassical properties as collapse and revival phenomenon in the atomic population inversion [2,3]. Recently, open quantum systems, which described mathematically by non-Hermitian Hamiltonian, attract attention of many authors. The non-Hermitian Hamiltonian has been used to describe dissipative process [4,5,6] such as the radioactive phenomenon [7]. Over the last few years, entropy squeezing has attracted more attention of many authors because of its main enforcement in high resolution spectroscopy [8], high precision spin polarization measurement [9] and we recently may observe that squeezed light is applied into quantum information theory, for example, in cryptograph, dense coding and teleporation. Authors in [8,10] established the concept of squeezed components of atom, Agarwal and Puri [11] pretested squeezing N two -level atoms system damped by a broadband squeezed vacuum space. It's serious to note that all this studies of atomic squeezing are based on Heisenberg uncertainty relation (HUR).

Recently, and because of the failing of (HUR) for supporting us by sufficient information about squeezing in some cases, researchers oriented to another form in terms of entropy, which so-called entropy uncertainty relation (EUR)[12].

Authors in [13] studied entropy squeezing of an atom with k-photons in the JC model. Entropy squeezing of a two-level atom moving in Kerr medium is also discussed [15]. In [14] authors studied the effect of detuning parameter Δ and the coupling parameter λ on entropic uncertainty for two two-level atoms.

In this article, we will study some nonclassical properties of the physical state such as the squeezing phenomena and the sub-Poissonian distribution, we will discuss and compare between variance and entropy squeezing of the atom components, so we subdivided the paper as follow : in section 2 we introduce the physical model and its solution, sec.3 exhibits the atomic inversion for a single atom, sec.4 is appropriated to exhibit the squeezing phenomenon and discussion to effects of decay parameters on squeezing, in sec.5 we introduce the sub-Poissonian distribution. Finally, we present the conclusion in sec.6.

2 The physical model and its solution

We suppose a system of two two-level atoms moving in the Kerr medium and interacting with two mode electromagnetic field through two photon process in a

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dissipative cavity, taking into account the time dependent coupling function. The effective Hamiltonian of the considered system under the rotating wave approximation (RWA) can be read as($\hbar = 1$)

$$\hat{H}_{eff} = \hat{H}_{A+F} + \hat{H}_{kerr} + \hat{H}_{damp} + \hat{H}_I, \tag{1}$$

where

$$\hat{H}_{A+F} = \sum_{j=1}^{2} \frac{1}{2}\Omega_j \hat{\sigma_z^j} + \omega_j \hat{a}_j^{\dagger}\hat{a}_j,$$
$$\hat{H}_{kerr} = \sum_{j=1}^{2} \chi_j \hat{a}_j^{\dagger 2}\hat{a}_j^{2},$$
$$\hat{H}_{damp} = -\frac{i}{2}\sum_{j=1}^{2}(\Gamma_j \hat{a}_j^{\dagger}\hat{a}_j + \gamma_j \hat{\sigma}_z^j)$$

and

$$\hat{H}_I = \sum_{j=1}^{2} \lambda_j(t)(\hat{a}_j^{2}\sigma_+^j + \hat{a}_j^{\dagger 2}\sigma_-^j),$$

where $\omega_j(\Omega_j)$; $(j=1,2)$ is the frequency of the j^{th} mode (the atomic transition frequency for the j^{th} atom) which is dissipating with the rate $\Gamma_j(\gamma_j)$, χ_j is the kerr-like medium parameter, $\hat{a}_j(\hat{a}_j^{\dagger})$ is the annihilation (creation) operator and $\lambda_j(t) = \lambda_j \cos(\varepsilon t)$ is the coupling function between the single atom and the field (where λ_j is an arbitrary constant and ε is the coupling variation parameter). The operators $\hat{\sigma_+}^j(\hat{\sigma_-}^j)$ and $\hat{\sigma_z}^j$ are the raising (lowering) and the pauli inversion operators.

2.1 The solution

We devoted this subsection to introduce an approximate solution of the evolution equation for the wave state, which enable us to determine the density matrix, from which we can study some statistical properties of the considerable system. The wave state, represent the system under study, can be written as:

$$|\Psi(t)\rangle = \sum_{n_1=0}^{+\infty}\sum_{n_2=0}^{+\infty} q_{n_1}q_{n_2}\Big[B_1(n_1,n_2,t)e^{-i\alpha_1 t}|1,1,n_1,n_2\rangle + B_2(n_1,n_2,t)e^{-i\alpha_2 t}|1,0,n_1,n_2+2\rangle + B_3(n_1,n_2,t)e^{-i\alpha_3 t}|0,1,n_1+2,n_2\rangle + B_4(n_1,n_2,t)e^{-i\alpha_4 t}|0,0,n_1+2,n_2+2\rangle\Big], \tag{2}$$

where $|n_j\rangle;(j=1,2)$ is the fock state, the functions B_1, B_2, B_3 and B_4 are the probability amplitudes and the variables $\alpha_m(m=1,2,3,4)$ are given by :

$$\alpha_1 = \frac{\Omega_1}{2} + \frac{\Omega_2}{2} + \omega_1 n_1 + \omega_2 n_2,$$
$$\alpha_2 = \frac{\Omega_1}{2} - \frac{\Omega_2}{2} + \omega_1 n_1 + \omega_2(n_2+2),$$
$$\alpha_3 = -\frac{\Omega_1}{2} + \frac{\Omega_2}{2} + \omega_1(n_1+2) + \omega_2 n_2,$$
$$\alpha_4 = -\frac{\Omega_1}{2} - \frac{\Omega_2}{2} + \omega_1(n_1+2) + \omega_2(n_2+2).$$

We assume that the atomic system is initially in the superposition state and the field in the coherent state, so the initial state of the system takes the form:

$$|\Psi(0)\rangle = (\cos(\theta)|1,1\rangle + e^{-i\phi}\sin(\theta)|0,0\rangle) \bigotimes \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty} q_{n_1}q_{n_2}|n_1,n_2\rangle, \tag{3}$$

where $|1\rangle(|0\rangle)$ is the excited (ground) state for a single atom and

$$q_{n_j} = Exp[-\frac{|\alpha_j|^2}{2}]\sum_{n_j}\frac{(\alpha_j)^{n_j}}{\sqrt{n_j!}}, \quad j=1,2.$$

To achieve our aim, we apply the time-dependent Schrödinger equation $\hat{H}|\Psi(t)\rangle = i\hbar\frac{\partial}{\partial t}|\Psi(t)\rangle$, then we obtain the following system of differential equations:

$$i\dot{B_1} = \eta_1 B_1 + \lambda_2(t)\nu_2 e^{i\Delta t}B_2 + \lambda_1(t)\nu_1 e^{i\Delta t}B_3,$$
$$i\dot{B_2} = \eta_2 B_2 + \lambda_2(t)\nu_2 e^{-i\Delta t}B_1 + \lambda_1(t)\nu_1 e^{i\Delta t}B_4,$$
$$i\dot{B_3} = \eta_3 B_3 + \lambda_1(t)\nu_1 e^{-i\Delta t}B_1 + \lambda_2(t)\nu_2 e^{i\Delta t}B_4,$$
$$i\dot{B_4} = \eta_4 B_4 + \lambda_1(t)\nu_1 e^{-i\Delta t}B_2 + \lambda_2(t)\nu_2 e^{-i\Delta t}B_3,$$

where

$$\eta_1 = \sum_{j=0}^{2}\chi_j n_j(n_j-1) - \frac{i}{2}(\gamma_j + \Gamma_j n_j),$$
$$\eta_2 = \chi_1 n_1(n_1-1) + \chi_2(n_2+1)(n_2+2) - \frac{i}{2}(\gamma_1-\gamma_2) + \Gamma_1 n_1 + \Gamma_2(n_2+2),$$
$$\eta_3 = \chi_1(n_1+1)(n_1+2) + \chi_2 n_2(n_2-1) - \frac{i}{2}(\gamma_2-\gamma_1) + \Gamma_1(n_1+1) + \Gamma_2 n_2,$$
$$\eta_4 = \sum_{j=1}^{2}\chi_j(n_j+1)(n_j+2) - \frac{i}{2}(-\gamma_j + \Gamma_j(n_j+2)),$$
$$\nu_j = \sqrt{(n_j+1)(n_j+2)}, \qquad j=1,2$$

and the detuning parameter Δ given by

$$\Delta = \Omega_1 - 2\omega_1 = \Omega_2 - 2\omega_2.$$

We can represent the Cosine function in terms of exponential function and substituting in $\lambda_j(t)$. After substituting we obtain the exponential functions $e^{\pm\imath(\Delta+\varepsilon)t}$ and $e^{\pm\imath(\Delta-\varepsilon)t}$ in the system of differential equations. Approximately, the counter oscillating parts $e^{\pm\imath(\Delta+\varepsilon)t}$ can be defaulted (which accepted physically for aplenty models [19]) as the approximation in the (RWA). Thus the differential equations will become

$$
\begin{aligned}
i\dot{B_1} &= \eta_1 B_1 + f_2 e^{i(\Delta-\varepsilon)t} B_2 + f_1 e^{i(\Delta-\varepsilon)t} B_3,\\
i\dot{B_2} &= \eta_2 B_2 + f_2 e^{-i(\Delta-\varepsilon)t} B_1 + f_1 e^{i(\Delta-\varepsilon)t} B_4,\\
i\dot{B_3} &= \eta_3 B_3 + f_1 e^{-i(\Delta-\varepsilon)t} B_1 + f_2 e^{i(\Delta-\varepsilon)t} B_4,\\
i\dot{B_4} &= \eta_4 B_4 + f_1 e^{-i(\Delta-\varepsilon)t} B_2 + f_2 e^{-i(\Delta-\varepsilon)t} B_3,
\end{aligned}
$$

where

$$\bar{f}_j = \frac{\lambda_j}{2}\nu_j, \qquad j = 1,2.$$

Assuming

$$
\begin{aligned}
\bar{B}_1(t) &= B_1(t) e^{-i(\Delta-\varepsilon)t},\\
\bar{B}_4(t) &= B_4(t) e^{+i(\Delta-\varepsilon)t}.
\end{aligned}
$$

By using the method in [20], we obtain

$$
i\frac{d}{dt}\begin{bmatrix}\bar{B_1}(t)\\ B_2(t)\\ B_3(t)\\ \bar{B_4}(t)\end{bmatrix} = \begin{bmatrix}\eta_1+k & f_2 & f_1 & 0\\ f_2 & \eta_2 & 0 & f_1\\ f_1 & 0 & \eta_3 & f_2\\ 0 & f_1 & f_2 & \eta_4-k\end{bmatrix}\begin{bmatrix}\bar{B_1}(t)\\ B_2(t)\\ B_3(t)\\ \bar{B_4}(t)\end{bmatrix}. \quad (4)
$$

The solution of this system is given by

$$M(n_1,n_2,t) = M(n_1,n_2,0)e^{-iC(n_1,n_2)t},$$

where

$$
C(n_1,n_2) = \begin{bmatrix}\eta_1+k_1 & f_2 & f_1 & 0\\ f_2 & \eta_2 & 0 & f_1\\ f_1 & 0 & \eta_3 & f_2\\ 0 & f_1 & f_2 & \eta_4-k_2\end{bmatrix}.
$$

The matrix exponential in eq.(4) calculated by many analytical ways[21,22].

2.2 The density matrix

After obtaining the wave vector, it becomes easy to obtain the time dependent density matrix which determined by

$$
\begin{aligned}
\hat{\rho}(t) &= |\Psi(t)\rangle\langle\Psi(t)|\\
&= \sum_{n_1=0}^{+\infty}\sum_{n_2=0}^{+\infty} q_{n_1}q_{n_2}[B_1(n_1,n_2,t)e^{-i\alpha_1 t}|1,1,n_1,n_2\rangle\\
&+ B_2(n_1,n_2,t)e^{-i\alpha_2 t}|1,0,n_1,n_2+2\rangle\\
&+ B_3(n_1,n_2,t)e^{-i\alpha_3 t}|0,1,n_1+2,n_2\rangle\\
&+ B_4(n_1,n_2,t)e^{-i\alpha_4 t}|0,0,n_1+2,n_2+2\rangle]\bigotimes \qquad (5)\\
&\sum_{m_1=0}^{+\infty}\sum_{m_2=0}^{+\infty} q^*_{m_1}q^*_{m_2}[B^*_1(m_1,m_2,t)e^{i\alpha_1 t}\langle 1,1,m_1,m_2|\\
&+ B^*_2(m_1,m_2,t)e^{i\alpha_2 t}\langle 1,0,m_1,m_2+2|\\
&+ B^*_3(m_1,m_2,t)e^{i\alpha_3 t}\langle 0,1,m_1+2,m_2|\\
&+ B^*_4(m_1,m_2,t)e^{i\alpha_4 t}\langle 0,0,m_1+2,m_2+2|].
\end{aligned}
$$

We can calculate the atomic reduced density operator by taking the trace over the field.

$$\hat{\rho}_{A_1A_2}(t) = Tr_{F_1F_2}|\Psi(t)\rangle\langle\Psi(t)|. \quad (6)$$

The reduced density operator for one atom obtained by taking the trace over the other one and given by

$$\hat{\rho}_{A_1} = Tr_{A_2}\hat{\rho}_{A_1A_2}(t) = \begin{bmatrix}\rho_{11} & \rho_{12}\\ \rho_{21} & \rho_{22}\end{bmatrix}, \quad (7)$$

where

$$\rho_{11} = \sum_{n_1,n_2}|q_{n_1}|^2|q_{n_2}|^2(|B_1(n_1,n_2,t)|^2 + |B_2(n_1,n_2,t)|^2),$$

$$
\begin{aligned}
\rho_{12} &= \sum_{n_1,n_2} q_{n_1+2}\,q^*_{n_1}|q_{n_2}|^2\Big(B_1(n_1+2,n_2,t)B^*_3(n_1,n_2,t)\\
&\times e^{-i\alpha_1(n_1+2,n_2)t}e^{i\alpha_3(n_1,n_2)t}\\
&+ B_2(n_1+2,n_2,t)B^*_4(n_1,n_2,t)\times e^{-i\alpha_2(n_1+2,n_2)t}e^{i\alpha_4(n_1,n_2)t}\Big),
\end{aligned}
$$

$$\rho_{21} = \rho^*_{12}$$

and

$$\rho_{22} = \sum_{n_1,n_2}|q_{n_1}|^2|q_{n_2}|^2(|B_3(n_1,n_2,t)|^2 + |B_4(n_1,n_2,t)|^2).$$

The reduced density matrix for the field can be obtained from eq.(5) by taking the trace over the atoms.

$$\hat{\rho}_{F_1F_2}(t) = Tr_{A_1A_2}|\Psi(t)\rangle\langle\Psi(t)|. \quad (8)$$

And thus, the reduced density matrix for the first mode can be obtained after taking the trace over the second mode.

$$
\begin{aligned}
&\hat{\rho}_{F_1}(t) = Tr_{F_2}\hat{\rho}_{F_1F_2}(t) =\\
&|D\rangle\langle D| + |T\rangle\langle T| + |G\rangle\langle G| + |R\rangle\langle R|,
\end{aligned} \quad (9)
$$

where

$$|D\rangle = \sum_{n_1,n_2} q_{n_1} q_{n_2} B_1(n_1,n_2,t) e^{-i\alpha_1(n_1,n_2)t} |n_1\rangle,$$
$$|T\rangle = \sum_{n_1,n_2} q_{n_1} q_{n_2} B_2(n_1,n_2,t) e^{-i\alpha_2(n_1,n_2)t} |n_1\rangle,$$
$$|G\rangle = \sum_{n_1,n_2} q_{n_1} q_{n_2} B_3(n_1,n_2,t) e^{-i\alpha_3(n_1,n_2)t} |n_1+2\rangle,$$
$$|R\rangle = \sum_{n_1,n_2} q_{n_1} q_{n_2} B_4(n_1,n_2,t) e^{-i\alpha_4(n_1,n_2)t} |n_1+2\rangle.$$

In the following we employ this results to discuss some statistical properties of our system.

3 The atomic inversion of a single atom

The atomic inversion is defined as the difference between the population of the excited and ground atomic states, it can be used to determine if the atom is in its excited, ground or maximum state (the atom reaches the maximum state if the probabilities of finding it in the excited and ground states are equal). And for the considerable system, the atomic inversion of a single atom is determined from eq.(7) by

$$w(t) = \rho_{11} - \rho_{22}.$$

Now, we will discuss the time evolution of the atomic inversion behavior related to the present system and study the effect of the damping parameters of the atoms and the field on it. We have prepared the field initially in the coherent state with the average photon number $\bar{n} = 25$ and the atoms in the superposition state given by (eq.3) where we chose $\theta = 0$ and $\phi = \frac{\pi}{2}$.

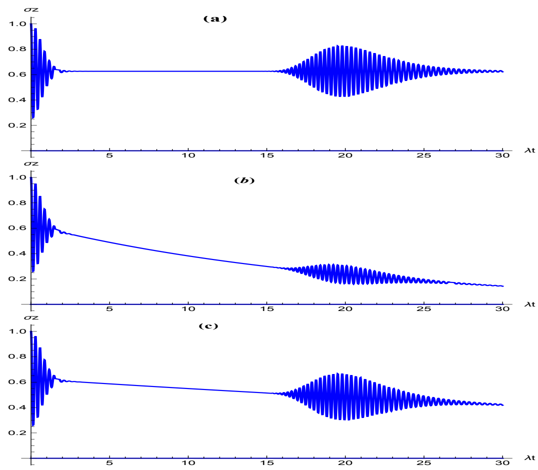

Fig. 1: the time evolution of the atomic inversion at fixed parameters $\Delta = 5$,$\chi = 0.0001$ and (a) $\Gamma = \gamma = 0$ and $\varepsilon = 7\pi$ (b) $\Gamma = 0.001, \gamma = 0$ and $\varepsilon = 7\pi$ (c) $\Gamma = 0, \gamma = 0.008$ and $\varepsilon = 7\pi$

Fig.(1) shows the effect of the decay rates Γ,γ on the behavior of the atomic inversion where we considered $\Gamma_1 = \Gamma_2 = \Gamma, \gamma_1 = \gamma_2 = \gamma$ and $\chi_1 = \chi_2 = \chi$. In (a) we can see that the atom oscillates in the excited state around $w \simeq 0.67$, this excitation is due to the time variation parameter ε. The effect of the decay rate of the field is clear, by increasing the parameter Γ the function $w(t)$ shifts down and the amplitude of the revival period decreased (as expected), which means that the interaction is weak, see (1b). While the atomic decay parameter γ makes the oscillation base line of $w(t)$ to shift down with the same amplitude, see (1c).

4 The squeezing phenomenon

More attention has been paid for the squeezing phenomenon over the last few years, the squeezing in the atomic quadratures has been manipulated in the frame work of (HUR) which is stated in terms of the variance (which is containing only second-order correlation function), but in some cases (HUR) cannot purvey sufficient information on squeezing, for instance: for a state in which $\langle\sigma_z\rangle = 0$ the definition of variance squeezing becomes trivial and has no information about atomic squeezing, while entropy squeezing relation [16, 17, 18] is not. Besides the atomic quadrature, we consider another type of squeezing that depends on the field quadratures.

4.1 Normal squeezing

It is well known that the coherent and vacuum states are the minimum uncertainty states of the E.M field amplitudes i.e. the quantum fluctuations of the field in the coherent or vacuum states are equal to the zero point fluctuations and are randomly distributed in phase. The zero point fluctuations is considered the standard quantum limit to the reduction of noise in a signal. The squeezing occurs if one of the field quadratures has less noise than the coherent or the vacuum states and the noise in the other quadrature gets increase such that *the Hiesenberg uncertainty principle* is not violated. Therefore, we define two quadratures $\hat{E}_x^1$ and $\hat{E}_y^1$ for the first mode where $\hat{E}_x^1 = \frac{\hat{a}_1 + \hat{a}_1^{\dagger}}{2}$ and $\hat{E}_y^1 = \frac{\hat{a}_1 - \hat{a}_1^{\dagger}}{2i}$. The quadrature $(\Delta\hat{E}_\alpha^1)^2$; $(\alpha = x,y)$ is said to be squeezed if $(\Delta\hat{E}_\alpha^1)^2 - \frac{1}{4} < 0$. In Fig.2 we show the quadrature squeezing against the scaled time λt. In Fig.2a, we note that the squeezing occurs in the E_X^1 component, but no squeezing in E_y^1 component. When we take into account the time variation parameter ε to be non zero, the squeezing still in the same direction, but the interval of squeezing decreases, see (2b). Fig.2c shows the effect of the damping parameter Γ on the quadrature squeezing. We can see that the squeezing in E_X^1 component is

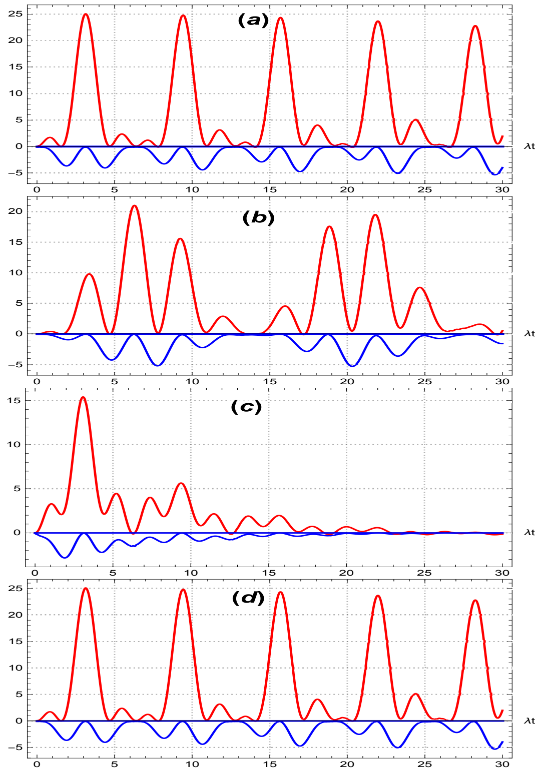

Fig. 2: the time evolution of the normal squeezing in E_x and E_y component *vs* the scaled time λt where the blue curve represent E_x and the red represent E_y, at fixed parameters $\Delta = 5\lambda$,$\chi = 0.0001\lambda$ and (a) $\Gamma = \gamma = 0$ and $\varepsilon = 0$ (b) $\Gamma = 0, \gamma = 0$ and $\varepsilon = 7\pi\lambda$ (c) $\Gamma = 0.003\lambda$ and $\gamma = \varepsilon = 0$ (d)$\Gamma = 0, \gamma = 0.003\lambda$ and $\varepsilon = 0$

negatively effected, also we can see that the direction of squeezing changes in the end of the considered interval and begins to appear in the E_y^1 component. The atomic damping parameter γ has no effect on squeezing phenomena in both quadratures for the considered system, see (2d).

4.2 Squeezing in the atomic quadratures

In this subsection, we study and compare between variance and entropy squeezing of the atom components for the system under discussion.
It is well known for any two-level atom, the information entropy satisfies the inequality:

$$0 \leq H(\sigma_i) \leq \ln 2, \qquad i = x,\ y\ or\ z, \tag{10}$$

where

$$H(\sigma_i) = -\sum_{j=1}^{2} p_j(\sigma_i) \ln[p_j(\sigma_i)], \tag{11}$$

where $p_j(\sigma_i) = \langle \psi_{ij}|\rho|\psi_{ij}\rangle$; $(j = 1,2)$ represent the probability distribution for the two possible outcomes of measurements of the pauli operators σ_i, and $|\psi_{ij}\rangle$ is an eigenvector of σ_i. And hence

$$H(\sigma_x) + H(\sigma_y) + H(\sigma_z) \geq 2\ln 2. \tag{12}$$

If we consider $\delta H(\sigma_i) = Exp[H(\sigma_i)]$, eq.(12) can be written as

$$\delta H(\sigma_x)\delta H(\sigma_y) \geq \frac{4}{\delta H(\sigma_z)}. \tag{13}$$

The fluctuations in the pauli component $\sigma_i (i = x,y,z)$ of a two-level atom are said to be entropy squeezed if the Shannon information entropy $H(\sigma_i)$ satisfies the condition:

$$E(\sigma_i) = \delta H(\sigma_i) - \frac{2}{\sqrt{\delta H(\sigma_z)}} < 0, \qquad i = x\ or\ y \tag{14}$$

and said to be squeezed in variance if σ_i satisfies the condition :

$$V(\sigma_i) = \Delta\sigma_i - \sqrt{|\langle\sigma_z\rangle|} < 0, \qquad i = x\ or\ y,$$

where

$$\Delta\sigma_i = \sqrt{\langle\sigma_i^2\rangle - \langle\sigma_i\rangle^2}.$$

The information entropy of the operators σ_x, σ_y and σ_z can be obtained by using the reduced density matrix (7) as

$$H(\sigma_x) = -\frac{1}{2}[(\rho_{11} + \rho_{22} + 2Re[\rho_{12}])$$
$$\ln[\frac{1}{2}(\rho_{11} + \rho_{22} + 2Re[\rho_{12}])] - \frac{1}{2}(\rho_{11} + \rho_{22} - 2Re[\rho_{12}])$$
$$\ln[\frac{1}{2}(\rho_{11} + \rho_{22} - 2Re[\rho_{12}])],$$
$$H(\sigma_y) = -\frac{1}{2}[(\rho_{11} + \rho_{22} + 2Im[\rho_{12}])$$
$$\ln[\frac{1}{2}(\rho_{11} + \rho_{22} + 2Im[\rho_{12}])] - \frac{1}{2}(\rho_{11} + \rho_{22} - 2Im[\rho_{12}])$$
$$\ln[\frac{1}{2}(\rho_{11} + \rho_{22} - 2Im[\rho_{12}])],$$

and

$$H(\sigma_z) = -\rho_{11}\ln\rho_{11} - \rho_{22}\ln\rho_{22}.$$

Now, we are in a position to discuss the variance and entropy squeezing of the atomic dipole quadratures for the first atome.
Fig.(3) illustrates the variance and entropy squeezing in σ_x component. In (3a) we can see that the squeezing occurs at several short intervals during the considered period of time, the amount of squeezing begins small and increases as the time developed, also one can see that the squeezing in variance is almost non-exist. In (3b), where we put $\varepsilon = \frac{2\pi}{3}\lambda$, we observe that the amount of squeezing increases with more oscillation and higher maximum. The effect of the parameter Γ on squeezing is negative, the curves shifted upwardes to display no squeezing niether

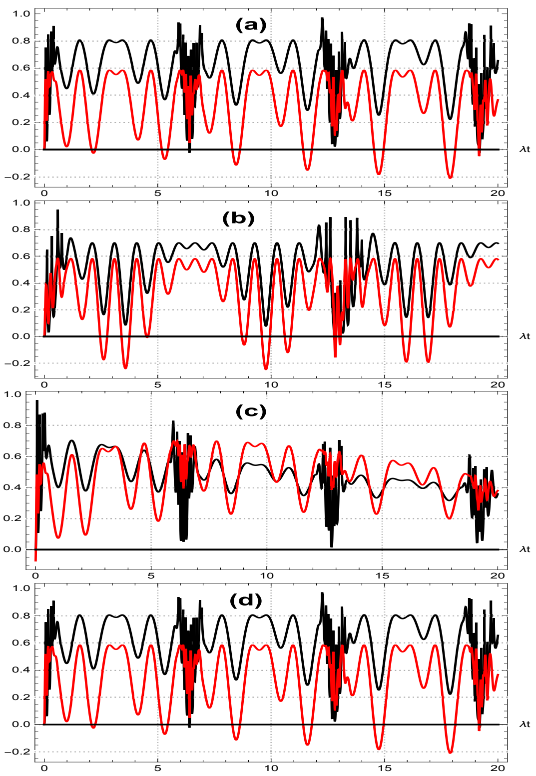

Fig. 3: The variance and entropy squeezing in σ_x component *vs* the scaled time λt, where the black curve represent the variance and the red represent entropy squeezing, at fixed parameters $\Delta = 5\lambda$,$\chi = 0.0001\lambda$ and (a) $\Gamma = \gamma = 0$ and $\varepsilon = 0$ (b) $\Gamma = 0, \gamma = 0$ and $\varepsilon = \frac{2\pi}{3}\lambda$ (c) $\Gamma = 0.001\lambda, \gamma = 0$ and $\varepsilon = 0$ (d)$\Gamma = 0, \gamma = 0.0001\lambda$ and $\varepsilon = 0$

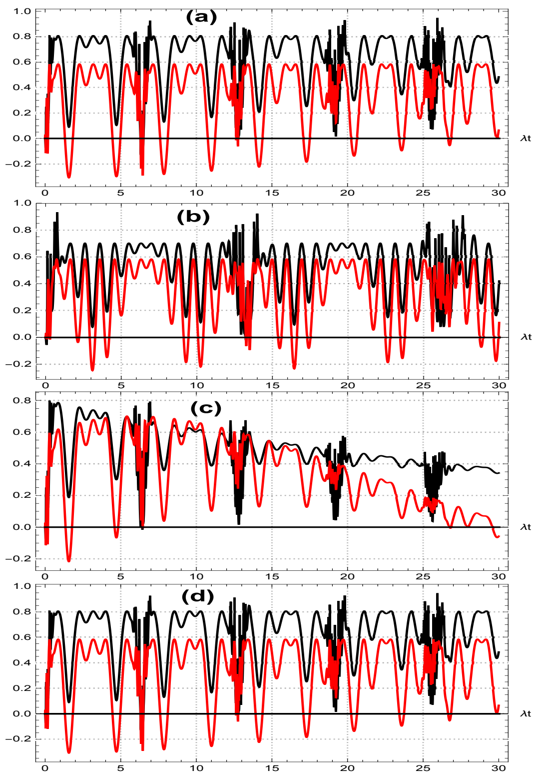

Fig. 4: The variance and entropy squeezing in σ_y component *vs* the scaled time λt with the same deta of Fig.(3)

in the entropy nor in the variance, see (3c). In Fig.(3d) we note the atomic damping parameter γ almost has no effect on the squeezing.

Fig.(4) is devoted to show the squeezing in the σ_y component, the situation is not more different. In Fig.(4a) we can see that the squeezing also occurs at several short intervals but the maximum of squeezing is begining greater and decreasing gradually over the time and then vanish in the end of the considered interval. For the case of $\varepsilon = \frac{2\pi}{3}\lambda$ the squeezing in entropy occurs apparently in many periods of the considered time; see Fig.(4b). After taking many values of the damping parameter Γ, we noted that it also effects negatively on the squeezing in the σ_y component while the effect of the atomic decay rate γ is almost not exist; see Fig.(4c) and (4d). Finally, we can note that there exist no squeezing in the variance at the considered cases.

5 Sub-Poissonian distribution

In this section, we discuss another nonclassical effect, that is the sub-Poissonian distribution. To discuss such a kind of distribution we have to study the second order correlation function g^2(t). A light field displays a sub-Poissonian distribution if $g^2(t) < 1$ which is a non classical effect, super-Poissonian distribution if $g^2(t) > 1$ which is a classical effect and Poissonian distribution if $g^2(t) = 1$ which means that the probability of detecting an incident pair of photons is equal to it would be for a coherent field. In the mean time the system has thermal statistical behavior if $g^2(t) = 2$ and super-thermal when $g^2(t) > 2$. The second order correlation function $g^2(t)$ is given by[23]:

$$g_j^2(t) = \frac{< \hat{a}_j^{\dagger 2}(t)\hat{a}_j^{2}(t) >}{< \hat{a}_j^{\dagger}(t)a_j\hat{(}t) >^2}, \quad j = 1,2, \qquad (15)$$

where the subscript j relates to j^{th} mode.

To study the sub-Poissonian distribution for the considered system, we plot the second order correlation function for the first mode $g_1^2(t)$. In Fig.(5) we exhibit the effect of the decay rates Γ and γ on the correlation function where we considered the atoms initially in the superposition coherent state such that $\theta = \frac{\pi}{4}$ and $\varphi = \frac{\pi}{2}$, the field in the coherent state where $\alpha = 5$ and we chose the Kerr parameter $\chi = 0.0001\lambda$, we devoted the sub-figures a, b and c to illustrate the effect of Γ parameter, we find that when $\Gamma = 0.00015\lambda$ and 0.0002λ the system displays a full sub-Poissonian distribution

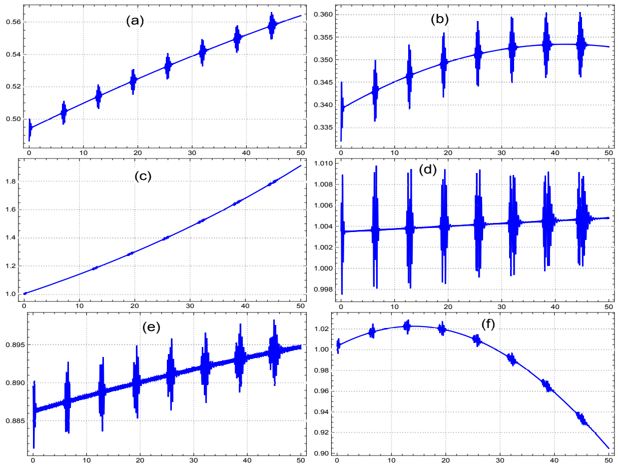

Fig. 5: The second order correlation function *vs* λt at fixed parameters $\Delta = 5\lambda$, $\varepsilon = 0$, (a) $\Gamma = 0.00015\lambda$, (b) $\Gamma = 0.0002\lambda$, (c) $\Gamma = 0.00025\lambda$, (d) $\gamma = 0.0005\lambda$, (e) $\gamma = 0.005\lambda$ and (f) $\gamma = 0.05\lambda$

($g_1^2 < 1$) which is a non classical behavior. At $\Gamma = 0.00025\lambda$ the function begins as a Poissonian distribution ($g_1^2(0) = 1$) then the function displays full super-Poissonian distribution and approximately reaches the thermal distribution in the end of the considered interval. After taking various values of γ exhibited in Fig.(5)d, e and f, we find that at $\gamma = 0.0005\lambda$ the oscillation base curve approximately become around $g_1^2 = 1.004$ which means that the function gives super-Poissonian distribution. For a higher value of γ ($\gamma = 0.005\lambda$) the oscillation base curve is shifted down to give sub-Poissonian distribution, as observed in (5e). At $\gamma = 0.05\lambda$ the function starts super-Poissonian distribution until $\lambda t = 30$ then the system displays a sub-Poissonian distribution; see (5f).

6 Conclusion

In this paper, we studied the problem of the interaction of two two-level atoms and two mode electromagnetic field in a dissipative cavity. Under some consideration, we studied the effect of the decay rates Γ and γ on some nonclassical aspects such as the atomic inversion, squeezing phenomenon and the sub-Poissonian distribution. We noted that by increasing the damping parameter Γ, the atomic inversion shifted down as the time developed, also the amplitude of the revival interval reduced. By taking various values for the atomic damping parameter γ we noted the oscillation base line shifted downwards with the same revival amplitude. We also considered the variance and the entropy squeezing of the atomic components σ_x and σ_y for the first atom. We noted that the effect of the damping parameter of the field on the entropy squeezing is negative, and the atomic damping parameter almost has no effect on squeezing in entropy. The variance displays no squeezing neither in σ_x nor in σ_y for the considered cases. Also we considered the quadrature squeezing for the first mode and we observed that the damping parameter of the field has negative effect on squeezing in E_X^1 component, we noted that the atomic damping parameter also has no effect on the quadrature squeezing. Finally, we discussed the sub-Poissonian distribution, we observed that the system displays sub-Poissonian distribution if the damping parameter of the field is less than 0.00022λ the higher the value of the damping parameter of the field, the more classic characteristics the system will display. And we noted that the atomic damping parameter has random effect on the correlation function.

Competing interests

The authors declare that they have no competing interests.

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