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# Fault-Tolerant Control for Uncertain Stochastic Singular Time-Delayed Systems under Non-Linear Fault Inputs and Actuator Failures

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**Abstract:** This paper reports the novel fault-tolerant control issues for a class of uncertain stochastic singular time-delyed systems with respect to nonlinear fault inputs and actuator failures. More precisely, the delay factor is assumed to be time-varying which belongs to a given interval and parameter uncertainties which are assumed to be time-varying but norm-bounded. By using linear matrix inequality approach together with Lyapunov technique, a delay-dependent condition is derived, which ensures the uncertain stochastic singular system to be regular, impulse-free and asymptotically stable. A robust fault-tolerant control design has been derived to achieve the robust asymptotic stability for all admissible parameter uncertainties, independent of the time-delay and in the presence of certain actuator failures. A set of sufficient conditions is proposed for the existence of state feedback fault-tolerant control subject to mixed actuator failures in terms of LMIs, which can be efficiently solved via MATLAB LMI toolbox. Further, numerical example with simulation result is provided to demonstrate the applicability and effectiveness of the obtained results.

**Keywords:** Stochastic singular system; Fault-tolerant control; Time-delay; Linear matrix inequality (LMI)

## 1 Introduction

In recent years, the study of singular systems becomes an important topic due to their extensive applications in real-world problems [1,2]. Moreover, singular systems are also named as implicit systems, descriptor systems, or generalized differential-difference equations, which frequently occur in wide variety of practical engineering systems, including aircraft attitude control, flexible arm control of robots, large-scale electric network control, chemical engineering systems, and lossless transmission lines [3]-[5]. It is well known that time delays are frequently encountered in practical control systems including in networked control systems, hydraulic and chemical systems, mechanics systems, biological systems, nuclear reactors and so on [6]-[8]. Since time delays are frequently the source of instability and poor performance, the stability analysis and controller synthesis of dynamical systems with time delay have received remarkable attention in recent years [9]-[11]. It should be mentioned that the study of time-delay phenomena for singular systems is more complicated than the state space time delay systems since it requires to consider not only stability, but also regularity and absence of impulses for continuous-time singular systems and causality for discrete-time singular systems at the same time. Lu et al [12] addressed the problem of robust $H_\infty$ control for a class of uncertain singular time-delay systems with Markovian jumping parameters using the Lyapunov technique together with LMI-based approach. A delay dependent criterion is established to guarantee the dissipativity for singular systems with time-varying delays in [13]. On the other hand, stochastic systems have been playing an important role in various fields of science and engineering. Therefore, the study on stochastic systems has attracted a lot of research attention, and many useful results can be found in ([14]-[16]).

It is well know that system faults such as actuator and sensor failures may drastically change system dynamics and also produce undesirable performance degradation or even instability. Further, it should be noted that the

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conventional control methods may not attain the satisfactory performance on controlling a real plant with failures of control components ([17]-[19]). In order to tackle such a situation, fault-tolerant control techniques are proposed as a kind of effective control approaches to improve the system reliability [20]. The key idea of fault-tolerant control is to synthesis the controller to ensure the reliability of the resulting closed-loop systems under fault conditions [21]. Because of this excellent quality, in recent years, the fault-tolerant control strategy has attracted great attention and many important results have been reported (see [22]-[27] and references therein). Li et al. [22] studied the stabilization problem of Markovian stochastic jump systems subject to sensor faults, actuator faults and input disturbances by using the augmented sliding mode observer approach. In [28], a sliding mode controller design for a class of uncertain switched systems subject to actuator faults has been discussed, where sufficient criteria on the exponential stability of sliding mode dynamics are obtained via the average dwell-time method. The authors in [29] designed a new state-feedback controller to ensure the global stabilization of uncertain switched cascade nonlinear systems against actuator faults with the existence of structural uncertainties. The robust reliable stabilization for uncertain switched systems with random delays and norm-bounded uncertainties has been investigated in [14], where the involved delays are assumed to be randomly time-varying which obey certain mutually-correlated Bernoulli distributed white noise sequences. However, most of the existing fault-tolerant control results are mainly concerned with the actuator fault which is a linear multiplicative fault matrix. But the actuator faults don't always need to be a linear multiplicative fault, sometimes it may couple with nonlinearity due to some mechanical reason, for instance, the dead zone or relay, etc. Therefore, it is necessary and important to consider the nonlinearity in the actuator fault, which motivates our present research.

It should be noted that so far in the literature, there are only few researches on fault-tolerant control for uncertain stochastic singular systems with time-varying delay. Moreover, previously no work has been reported on fault-tolerant control problem of uncertain stochastic singular systems with uncertainties and nonlinear actuator failures. Motivated by the above discussions, in this paper, we consider the problems of uncertain stochastic singular time-delayed systems with uncertainties and mixed actuator failures via fault-tolerant control. The main contributions of this paper are summarized as follows:

(i)A more generalized actuator fault model which consists of both linear and nonlinear faults is proposed for the considered uncertain singular stochastic systems with time-delay.
(ii)Based on Lyapunov stability theory, a new set of sufficient criteria is obtained to ensure that the closed-loop system is asymptotically stable.
(iii)Developed controller design gain parameters can be obtained by solving a family of LMIs and the developed results are more general which unify the control performances in a single framework.

. **Notations:** Throughout this paper, unless otherwise specified, the superscripts "$T$" and "$(-1)$" stand for matrix transposition and matrix inverse respectively; $R^n$ and $R^{n\times m}$ denote the $n-$dimensional Euclidean space and the set of all $n\times m$ real matrices; $X\geq Y$(respectively $X>Y$), where $X$ and $Y$ are symmetric matrices, which means, that $X-Y$ is a positive semi-definite (respectively positive definite); $I$ is the identity matrix of appropriate dimensions; $\mathscr{L}_2[0,\infty)$ is the space of square integrable function over $[0,\infty)$; $\|\cdot\|$ refer to the Euclidean vector norm; $(\Omega,\mathscr{F},(\mathscr{F}_t)_{t>0},\mathscr{P})$ is a complete probability space with filtration $(\mathscr{F}_t)_{t>0}$, where $\Omega$ is the sample space, $\mathscr{F}$ is the $\sigma$-algebra of subsets of the sample space and $\mathscr{P}$ is the probability measure on $\mathscr{F}$. The notation $\mathbb{E}[\cdot]$ stands for the expectation operator; "$*$" is used as an ellipsis for terms that are induced by symmetry.

## 2 Problem Formulation and Preliminaries

Consider the stochastic uncertain singular system with time-varying delay in the following form

$$\begin{aligned} Edx(t) &= [(A+\Delta A(t))x(t)+(A_d+\Delta A_d(t))x(t-\tau(t))\\ &\quad +Bu^F(t)]dt\\ &\quad +[Cx(t)+Dx(t-\tau(t))]dw(t), \qquad (1)\\ x(t) &= \phi(t),\quad t\in[-\tau_2,0], \end{aligned}$$

where $x(t)\in\mathbb{R}^n$ is the state vector; $u^F(t)\in\mathbb{R}^m$ is the control input; $A,A_d,B,C$ and $D$ are real matrices with appropriate dimensions; the matrix $E\in\mathbb{R}^{n\times n}$ may be singular with rank$(E)=r<n$; $\tau(t)$ denoting the time-varying delay satisfying $\tau_1\leq\tau(t)\leq\tau_2$, $\dot{\tau}(t)\leq\mu<1$, here $\tau_1$, $\tau_2$ and $\mu$ are positive constants. The initial function $\phi(t)$ is continuous function, defined on $[-\tau_2,0]$. Matrices $\Delta A(t)$ and $\Delta A_d(t)$ represent the parameter uncertainties with appropriate dimensions which are described as follows:

$$[\Delta A(t)\ \Delta A_d(t)] = MF(t)[N_1\ N_2], \qquad (2)$$

where $M$ and $N_i$, $i=1,2$ are known constant real matrices, $F(t)$ is an unknown time varying matrix with Lebesgue measurable elements bounded by $F^T(t)F(t)\leq I\ \forall\ t$. $w(t)$ is a Wiener process (Brownian Motion) defined on the probability space $(\Omega,\mathscr{F},(\mathscr{F}_t)_{t>0},\mathscr{P})$ and satisfying

$$\mathbb{E}[w(t)]=0,\ \ \mathbb{E}[w^2(t)]=t,\ \ \mathbb{E}[w(i)w(j)]=0\ \ (i\neq j).$$

The control input of actuator fault $u^F(t)$ is described as

$$u^F(t) = Gu(t)+g(u(t)), \qquad (3)$$

where $u(t) = Kx(t)$, $K$ is the feedback gain to be determined and $G$ is the actuator fault matrix defined as follows:

$$0 \le \underline{G} = \mathrm{diag}\{\underline{g}_1, \dots, \underline{g}_p\} \le G = \mathrm{diag}\{g_1, \dots, g_p\} \\ \le \overline{G} = \mathrm{diag}\{\overline{g}_1, \dots, \overline{g}_p\} \le I, \tag{4}$$

in which the variables $g_i (i = 1, \dots, p)$ quantify the failures of the actuators. Let us denote

$$G_0 = \mathrm{diag}\{g_{01}, \dots, g_{0p}\} = \frac{\underline{G} + \overline{G}}{2} \\ = \mathrm{diag}\left\{\frac{\underline{g}_1 + \overline{g}_1}{2}, \dots, \frac{\underline{g}_p + \overline{g}_p}{2}\right\}, \tag{5}$$

$$G_1 = \mathrm{diag}\{g_{11}, \dots, g_{1p}\} = \frac{\underline{G} - \overline{G}}{2} \\ = \mathrm{diag}\left\{\frac{\underline{g}_1 - \overline{g}_1}{2}, \dots, \frac{\underline{g}_p - \overline{g}_p}{2}\right\}. \tag{6}$$

The matrix $G$ can be written as follows

$$G = G_0 + \Delta = G_0 + \mathrm{diag}\{\theta_1, \dots, \theta_p\}, |\theta_i| \le g_{1i}. \tag{7}$$

Further, the vector-valued function $g(u(t)) = [g_1(u(t)), g_2(u(t)), \dots, g_m(u(t))]^T$ is assumed to obey the following relation:

$$|g_a(u(t))| \le \beta_a u_a(t), \quad a \in \{1, 2, \dots, m\}. \tag{8}$$

The above inequality (8) can be further written in the compact form as

$$g^T(u(t)) g(u(t)) \le u^T(t) \Xi_2 u(t), \tag{9}$$

where $\Xi_2 = \mathrm{diag}\{\beta_1, \beta_2, \dots, \beta_m\}$.

The following definitions and lemmas are used in the proof of main result.

**Definition 1.** *The pair $(E, A)$ is said to be*

*(i) regular, if $det(zE - A)$ is not identically zero.*
*(ii) impulse free, if it is regular and $deg\{det(zE - A)\} = rank\ E$.*
*where $z \in \mathbf{C}$, $Re(z) < 0$*

**Definition 2.** *The stochastic singular system (1) is said to be*

*(i) regular and impulse free, for given integers $\tau_1 > 0$, $\tau_2 > 0$, and any time delay $\tau(t)$ satisfying $\tau_1 \le \tau(t) \le \tau_2$, if the pair $(E, A)$ is regular and causal.*
*(ii) asymptotically stable, if for any $\varepsilon > 0$, there exists a scalar $\delta > 0$, such that for any compatible initial conditions $\phi(t)$ satisfying $\sup_{-\tau_2 \le t \le 0} ||\phi(t)|| \le \delta(\varepsilon)$, the solution $x(t)$ of (1) satisfies $||x(t)|| \le \varepsilon$ for any $t \ge 0$, furthermore, $\lim_{t \to \infty} x(t) = 0$.*
*(iii) admissible if it is regular, impulse free and asymptotically stable.*

**Lemma 1.** *[14] Let $H$, $F$ and $G$ be real matrices of appropriate dimensions with $F^T F \le I$, then for any scalar $\varepsilon > 0$, one has the following $HFG + G^T F^T H^T \le \varepsilon H H^T + \varepsilon^{-1} G^T G$.*

**Lemma 2.** *[14] For any constant matrix $M > 0$, any scalars a and b, and a vector function $x(t) : [a, b] \to R^n$ such that integrals concerned are also defined, then the following holds;*

$$\left[\int_a^b x(s) ds\right]^T M \left[\int_a^b x(s) ds\right] \le (b - a) \int_a^b x^T(s) M \\ \times x(s) ds.$$

**Lemma 3.** *[14] Given the constant matrices $\Sigma_1$, $\Sigma_2$, $\Sigma_3$; where $\Sigma_1 = \Sigma_1^T$ and $0 < \Sigma_2 = \Sigma_2^T$. Then $\Sigma_1 + \Sigma_3^T \Sigma_2^{-1} \Sigma_3 < 0$ if and only if $\begin{bmatrix} \Sigma_1 & \Sigma_3^T \\ \Sigma_3 & -\Sigma_2 \end{bmatrix} < 0$ or equivalently $\begin{bmatrix} -\Sigma_2 & \Sigma_3 \\ \Sigma_3^T & \Sigma_1 \end{bmatrix} < 0$.*

**Lemma 4.** *[25] Let $Y_0(\xi(t))$, $Y_1(\xi(t))$, $Y_2(\xi(t))$, $\dots$, $Y_p(\xi(t))$ be quadratic functions of $\xi(t) \in \mathbb{R}^n$*

$$Y_m(\xi(t)) = \xi(t)^T Z_m \xi(t), \quad m = 1, 2, \dots, p,$$

*with $Z_m = Z_m^T$, then the implication*

$$Y_1(\xi(t)) \le 0 \ \cdots \ Y_p(\xi(t)) \le 0 \Longrightarrow Y_0(\xi(t)) \le 0$$

*holds if there exist $\rho_1, \rho_2, \dots, \rho_p > 0$ such that*

$$Z_0 - \sum_{m=1}^{p} \rho_m^{-1} Z_m \le 0.$$

## 3 Robust Stability

In this section, we consider the problem of robust asymptotic stability for stochastic uncertain singular systems with time-varying delay via LMI approach. The stochastic uncertain singular system is described with time-varying delay and parameter uncertainties. Now, we consider the problem of robust asymptotic stability of the stochastic uncertain singular system in the absence of control input. Subsequently, the singular system (1) can be written as

$$E dx(t) = [(A + \Delta A(t)) x(t) + (A_d + \Delta A_d(t)) x(t - \tau(t))] dt \\ + [Cx(t) + Dx(t - \tau(t))] dw(t). \tag{10}$$

A new set of sufficient conditions will be derived in terms of LMI which guarantee regular, impulse free and stability of the system for all admissible uncertainties.

**Theorem 1.** *The uncertain stochastic singular system (10) is said to be admissible, if there exist positive symmetric definite matrices $P$, $Q_1$, $Q_2$, $Q_3$, $R_1$, $R_2$, any appropriately dimensioned matrices $T_i$, $V_i$, $i = 1, 2, 3$, $U_j$, $j = 1, 2, 3, 4$, positive scalar $\varepsilon_1$ and constant matrix*

*$R \in \mathbb{R}^{n\times(n-r)}$ satisfying $E^T R = 0$ with $rank(R) = (n-r)$, such that the following LMIs hold:*

$$\Omega = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} & \Sigma_{14} & 0 & 0 & 0 & 0 & 0 & M^T T_1^T & \varepsilon_1 N_a \\ * & \Sigma_{22} & \Sigma_{23} & \Sigma_{24} & 0 & 0 & 0 & 0 & 0 & M^T T_2^T & \varepsilon_1 N_a \\ * & * & \Sigma_{33} & \Sigma_{34} & 0 & 0 & 0 & 0 & 0 & M^T T_3^T & 0 \\ * & * & * & \Sigma_{44} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -Q_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -Q_3 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \frac{-1}{\tau_2}R_1 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \frac{-1}{\tau_2-\tau_1}(R_1+R_2) & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & \frac{-1}{\tau_2-\tau_1}R_2 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -\varepsilon_1 & 0 \\ * & * & * & * & * & * & * & * & * & * & -\varepsilon_1 \end{bmatrix} < 0, \tag{11}$$

*where* $\Sigma_{11} = T_1 A + A^T T_1^T + V_1 C + C^T V_1^T + Q_1 + Q_2 + Q_3 + (\tau_2 - \tau_1)R_2 + \tau_2 R_1$, $\Sigma_{12} = T_1 A_d + A^T T_2^T + V_1 D + C^T V_2^T +$, $\Sigma_{13} = E^T P - T_1 + U_1 R^T + A^T T_3^T$, $\Sigma_{14} = -V_1 + C^T V_3^T$, $\Sigma_{22} = T_2 A_d + A_d^T T_2^T + V_2 D + D^T V_2^T - (1-\mu)Q_1$, $\Sigma_{23} = -T_2 + U_2 R^T + A_d^T T_3^T$, $\Sigma_{24} = -V_2 + D^T V_3^T$, $\Sigma_{33} = -T_3 - T_3^T + U_3 R^T + R U_3^T$, $\Sigma_{34} = R U_4^T$, $\Sigma_{44} = E^T P - V_3 - V_3^T$.

**Proof.**
In order to prove that the uncertain singular system (1) is admissible, we first prove that the system (1) is regular and impulse free for any time-varying delay $\tau(t)$ satisfying $\tau_1 \le \tau(t) \le \tau_2$. From (11), it follows that

$$\begin{bmatrix} \Theta_{11} & \Theta_{12} & \Theta_{13} \\ * & \Theta_{22} & \Theta_{23} \\ * & * & \Theta_{33} \end{bmatrix} < 0, \tag{12}$$

where $\Theta_{11} = T_1 A + A^T T_1^T + V_1 C + C^T V_1^T$, $\Theta_{12} = T_1 A_d + A^T T_2^T + V_1 D + C^T V_2^T$, $\Theta_{13} = E^T P - T_1 + U_1 R^T + A^T T_3^T$,, $\Theta_{22} = T_2 A_d + A_d^T T_2^T + V_2 D + D^T V_2^T$, $\Theta_{23} = -T_2 + U_2 R^T + A_d^T T_3^T$,, $\Theta_{33} = -T_3 - T_3^T + U_3 R^T + R U_3^T$.
Let

$$V = \begin{bmatrix} I & 0 & A \\ 0 & I & A_d \end{bmatrix}. \tag{13}$$

Pre and post multiplying (12) by $V$ and $V^T$ respectively yields

$$\Psi = \begin{bmatrix} \Psi_1 & \Psi_2 \\ * & \Psi_3 \end{bmatrix} < 0, \tag{14}$$

where

$$\Psi_1 = \Theta_{11} + A\Theta_{13}^T + \Theta_{13}A^T + A\Theta_{33}A^T,$$
$$\Psi_2 = \Theta_{12} + A\Theta_{23}^T + \Theta_{12}A_d^T + A\Theta_{33}A_d^T,$$
$$\Psi_3 = \Theta_{22} + A_d\Theta_{23}^T + \Theta_{23}A_d^T + A_d\Theta_{33}A_d^T.$$

Since $rank(E) = r < n$, there exist two non singular matrices $\hat{L}$ and $\hat{H} \in \mathbb{R}^{n\times n}$ such that

$$\hat{E} = \hat{L}E\hat{H}\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}.$$

Then R can be parameterized as $R = \hat{L}^T \begin{bmatrix} 0 \\ \phi \end{bmatrix}$, where $\phi \in \mathbb{R}^{(n-r)\times(n-r)}$ is any nonsingular matrix. Similarly, we define

$$\hat{A} = \hat{L}A\hat{H} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix},$$
$$\hat{A_d} = \hat{L}A_d\hat{H} = \begin{bmatrix} A_{d11} & A_{d12} \\ A_{d21} & A_{d22} \end{bmatrix},$$
$$\hat{C} = \hat{L}C\hat{H} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix},$$
$$\hat{D} = \hat{L}D\hat{H} = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix}, \quad \hat{U} = \hat{H}^T U = \begin{bmatrix} U_{11} \\ U_{21} \end{bmatrix},$$
$$\hat{P} = \hat{L}P\hat{H} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}.$$

Now, pre and post multiplying $\Psi_1$ by $\hat{H}^T$ and $\hat{H}$, we get

$$\hat{H}^T\Psi_1\hat{H} = \begin{bmatrix} \hat{\Psi}_{11} & \hat{\Psi}_{12} \\ * & \hat{A}_{22}\phi\hat{U}_{21}^T + \hat{U}_{21}\phi^T\hat{A}_{22} \end{bmatrix}. \tag{15}$$

From (15), it is easy to see that

$$\hat{A}_{22}\phi\hat{U}_{21}^T + \hat{U}_{21}\phi^T\hat{A}_{22} < 0. \tag{16}$$

Suppose that $\hat{A}_{22}$ is singular, there must exit a non-zero vector $\rho \in R^{n-r}$, which ensures that $\hat{A}_{22}\rho = 0$. Thus, we conclude that $\rho(\hat{A}_{22}\phi\hat{U}_{21}^T + \hat{U}_{21}\phi^T\hat{A}_{22})\rho = 0$, this contradicts (16). So $\hat{A}_{22}$ is non-singular. Also, it can be shown that

$$\begin{aligned} &\det(zE - A) \\ &= \det(\hat{L}^{-1})\det(z\hat{E} - \hat{A})\det(\hat{H}^{-1}) \\ &= \det(\hat{L}^{-1})\det\left[(zI_r - \hat{A}_{11})(-\hat{A}_{22}) - \hat{A}_{12}\hat{A}_{21}\right]\det(\hat{H}^{-1}) \\ &= \det(\hat{L}^{-1})\det(-\hat{A}_{22})\det\left[(zI_r - \hat{A}_{11}) - \hat{A}_{12}\hat{A}_{21}\right]\det(\hat{H}^{-1}) \end{aligned}$$

which implies that $\det(zE - A)$ is not identically zero and $\det(zE - A) = rank(E)$. Thus, the pair $(E, A)$ is regular and impulsive free.
Let $y(t) = (A + \Delta A(t))x(t) + (A_d + \Delta A_d(t))x(t - \tau(t))$ and $g(t) = Cx(t) + Dx(t - \tau(t))$. Then, the system (10) can be written as

$$Edx(t) = y(t)dt + g(t)dw(t). \tag{17}$$

Next, we prove the robust stability of the system (10). Let us define the following Lyapunov-Krasovskii functional

$$V(t, x(t)) = \sum_{i=1}^{3} V_i(t, x(t)), \tag{18}$$

where

$$V_1(t,x(t)) = x^T(t)E^TPx(t),$$
$$V_2(t,x(t)) = \int_{t-\tau(t)}^t x^T(s)Q_1x(s)ds + \int_{t-\tau_1}^t x^T(s)Q_2x(s)ds + \int_{t-\tau_2}^t x^T(s)Q_3x(s)ds,$$
$$V_3(t,x(t)) = \int_{-\tau_2}^0 \int_{t+\theta}^t x^T(s)R_1x(s)ds + \int_{-\tau_2}^{-\tau_1} \int_{t+\theta}^t x^T(s)R_2x(s)ds.$$

In terms of Ito's formula, the time derivative along the solutions of (17) is given by

$$dV(t,x(t)) = \pounds V(t,x(t))dt + 2x^T(t)P[C(x(t)) \tag{19}$$
$$+D(x(t-\tau(t))]dw(t). \tag{20}$$

Now, we can calculate stochastic differential $\pounds V(t,x(t))$

$$\pounds V_1(t,x(t)) = 2x^T(t)E^TPy(t) + g^T(t)E^TPg(t) \tag{21}$$
$$\begin{aligned}\pounds V_2(t,x(t)) \leq{}& x^T(t)(Q_1+Q_2+Q_3)x(t) - (1-\mu)\\ &\times x^T(t-\tau(t))Q_1x(t-\tau(t)) - x^T(t-\tau_1)\\ &\times Q_2x(t-\tau_1) - x^T(t-\tau_2)Q_3x(t-\tau_2),\end{aligned} \tag{22}$$
$$\begin{aligned}\pounds V_3(t,x(t)) ={}& x^T(t)(\tau_2R_1 + (\tau_2-\tau_1)R_2)x(t)\\ &-\int_{t-\tau_2}^t x^T(s)R_1x(s)ds\\ &-\int_{t-\tau_2}^{t-\tau_1} x^T(s)R_2x(s)ds\end{aligned} \tag{23}$$
$$\begin{aligned}={}& x^T(t)(\tau_2R_1 + (\tau_2-\tau_1)R_2)x(t)\\ &-\int_{t-\tau_2}^{t-\tau(t)} x^T(s)R_1x(s)ds\\ &-\int_{t-\tau(t)}^{t} x^T(s)R_1x(s)ds\\ &-\int_{t-\tau_2}^{t-\tau(t)} x^T(s)R_2x(s)ds\\ &-\int_{t-\tau(t)}^{t-\tau_1} x^T(s)R_2x(s)ds,\end{aligned} \tag{24}$$

Using Lemma 2 in the above equation, we get

$$\begin{aligned}\pounds V_3(t,x(t)) \leq{}& x^T(t)(\tau_2R_1 + (\tau_2-\tau_1)R_2)x(t)\\ &-\frac{1}{\tau_2-\tau_1}\int_{t-\tau_2}^{t-\tau(t)} x^T(s)ds\,(R_1+R_2)\\ &\times\int_{t-\tau_2}^{t-\tau(t)} x(s)ds - \frac{1}{\tau_2}\int_{t-\tau(t)}^{t} x^T(s)ds\,R_1\\ &\times\int_{t-\tau(t)}^{t} x(s)ds - \frac{1}{\tau_2-\tau_1}\int_{t-\tau(t)}^{t-\tau_1} x^T(s)ds\\ &\times R_2\int_{t-\tau(t)}^{t-\tau_1} x(s)ds.\end{aligned} \tag{25}$$

Also for arbitrary matrices $T_i,\ V_i,\ i=1,\ 2,\ 3$ and $U_j,\ j=1,\ 2,\ 3,\ 4$, with compatible dimensions, we have

$$\begin{aligned}&2\left[x^T(t)T_1 + x^T(t-\tau(t))T_2 + y^T(t)T_3\right][(A+\Delta A(t))\\ &\times x(t) + (A_d+\Delta A_d(t))x(t-\tau(t)) - y(t)] = 0,\end{aligned} \tag{26}$$
$$\begin{aligned}&2\left[x^T(t)V_1 + x^T(t-\tau(t))V_2 + g^T(t)V_3\right]\\ &\times[Cx(t) + Dx(t-\tau(t)) - g(t)] = 0.\end{aligned} \tag{27}$$

Noting that $E^TR=0$, we can deduce that

$$\begin{aligned}&2\Big[x^T(t)U_1 + x^T(t-\tau(t))U_2 + y^T(t)U_3\\ &+g^T(t)U_4\Big]R^TEy(t) = 0.\end{aligned} \tag{28}$$

$$\begin{aligned}&2\left[x^T(t)T_1 + x^T(t-\tau(t))T_2 + y^T(t)T_3\right]\\ &[(\Delta A(t))x(t) + +\Delta A_d(t)x(t-\tau(t)]\\ &= 2\left[x^T(t)T_1M + x^T(t-\tau(t))T_2M + y^T(t)T_3M\right]F(t)\\ &\times[N_ax(t) + N_dx(t-\tau(t))]\\ &\leq \varepsilon_1^{-1}\left[x^T(t)T_1M + x^T(t-\tau(t))T_2M + y^T(t)T_3M\right]^T\\ &\left[x^T(t)T_1M + x^T(t-\tau(t))T_2M + y^T(t)T_3M\right]\\ &+\varepsilon_1[N_ax(t) + N_dx(t-\tau(t))][N_ax(t) + N_dx(t-\tau(t))]^T\end{aligned}$$

Combining (19)-(28) and taking the mathematical expectation on both sides, we get

$$\frac{d\,\mathbb{E}V(t,x(t))}{dt} \leq \mathbb{E}\big[\pounds V(t,x(t))\big] \leq \mathbb{E}\big[\zeta^t(t)\Omega\zeta(t)\big],$$

where $\zeta(t) = \Big[x^T(t)\ x^T(t-\tau(t))\ y^T(t)\ g^T(t)\ x^T(t-\tau_1)x^T(t-\tau_2)\ \int\limits_{t-\tau(t)}^{t} x^T(s)ds\ \int\limits_{t-\tau_2}^{t-\tau(t)} x^T(s)ds\ \int\limits_{t-\tau(t)}^{t-\tau_1} x^T(s)ds\Big]^T$ and $\Omega$ is the matrix given in (11). Hence, if the LMI (11) holds, then the stochastic singular system (10) is robustly asymptotically stable for all admissible parameter uncertainties. The proof is complete.

## 4 Robust reliable controller design

In this section, our main aim is to design a novel fault-tolerant controller (3) with gain $K$ such that for all possible actuator failures, time varying delays and parameter uncertainties, the closed-loop stochastic singular system (1) is robustly asymptotically stable. First, we study the robust stabilization problem when the actuator fault matrix $G$ is known.

**Theorem 2.***The uncertain stochastic singular system* (1) *with the known actuator failure parameter matrix $G$ is said to be robustly admissible, if there exist positive symmetric definite matrices $P,\ Q_1,\ Q_2,\ Q_3,\ R_1,\ R_2$, any appropriately-dimensioned matrices $T_i,\ V_i,\ i=1,\ 2,\ 3,\ U_j,\ j=1,\ 2,\ 3,\ 4$, positive scalars $\varepsilon_1,\ \varepsilon_2$ and constant matrix $R\in\mathbb{R}^{n\times(n-r)}$ satisfying $E^TR=0$ with*

*$rank(R) = (n-r)$, such that the following LMIs hold:*

$$\Pi = \begin{bmatrix} \bar{\Sigma}_{11} & \bar{\Sigma}_{12} & \bar{\Sigma}_{13} & \bar{\Sigma}_{14} & 0 & 0 & 0 & 0 & 0 & \lambda_1 TN_a & \lambda_1 TN_d \\ * & \bar{\Sigma}_{22} & \bar{\Sigma}_{23} & \bar{\Sigma}_{24} & 0 & 0 & 0 & 0 & 0 & \lambda_2 TN_a & \lambda_2 TN_d \\ * & * & \bar{\Sigma}_{33} & \bar{\Sigma}_{34} & 0 & 0 & 0 & 0 & 0 & \lambda_3 TN_a & \lambda_3 TN_d \\ * & * & * & \bar{\Sigma}_{44} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -Q_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -Q_3 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \frac{-1}{\tau_2}R_1 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \frac{-1}{\tau_2-\tau_1}(R_1+R_2) & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & \frac{-1}{\tau_2-\tau_1}R_2 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -\varepsilon_1 & 0 \\ * & * & * & * & * & * & * & * & * & * & -\varepsilon_2 \end{bmatrix} < 0, \quad (29)$$

where $\bar{\Sigma}_{11} = \lambda_1 T^T A^T + \lambda_1 AT + \lambda_1 Y^T G^T B^T + \lambda_1 BGY + V_1 C + C^T V_1^T + Q_1 + Q_2 + Q_3 + (\tau_2 - \tau_1)R_2 + \tau_2 R_1 + \varepsilon_1 N_a^T N_a$, $\bar{\Sigma}_{12} = \lambda_1 T^T A_d^T + \lambda_2 AT + V_1 D + C^T V_2^T + \varepsilon_1 Na^T N_d + \lambda_2 BGY$, $\bar{\Sigma}_{13} = E^T P - \lambda_1 T^T + U_1 R^T + \lambda_3 AT + \lambda_3 BGY$, $\bar{\Sigma}_{14} = -V_1 + C^T V_3^T$, $\bar{\Sigma}_{22} = \lambda_2 T^T A_d^T + \lambda_2 A_d T + V_2 D + D^T V_2^T - (1-\mu)Q_1 + \varepsilon_1 N_d^T N_d$, $\bar{\Sigma}_{23} = -\lambda_2 T^T + U_2 R^T + \lambda_3 A_d T$, $\bar{\Sigma}_{24} = -V_2 + D^T V_3^T$, $\bar{\Sigma}_{33} = -\lambda_3 T - \lambda_3 T^T + U_3 R^T + RU_3^T$, $\bar{\Sigma}_{34} = RU_4^T$, $\bar{\Sigma}_{44} = E^T P - V_3 - V_3^T$. Further, if the LMIs (29) are solvable, the estimator gain matrix of the reliable feedback controller (3) is given by $K = YT^{-1}$.

**Proof:** Applying the control term (3) to the stochastic uncertain singular system (1), we have

$$\begin{aligned} & Edx(t) \\ &= [(A+\Delta A(t)+BGK)x(t) + (A_d+\Delta A_d(t)) \\ &\quad \times x(t-\tau(t))]dt + [Cx(t)+Dx(t-\tau(t))]dw(t). \end{aligned} \quad (30)$$

Since $\det(sE-(A+BK)) = \det(sE^T-(A+BK)^T)$, the pair $(E,(A+BK))$ is regular and impulsive free if and only if $(E^T,(A+BK)^T)$ is regular and impulsive free. Therefore the system (30) is equivalent to the system

$$\begin{aligned} E^T dx(t) = {} & [(A+\Delta A(t)+BGK)^T x(t) \\ & +(A_d+\Delta A_d(t))^T x(t-\tau(t))]dt \\ & +[C^T x(t)+D^T x(t-\tau(t))]dw(t). \end{aligned} \quad (31)$$

Then by replacing $E$, $(A+\Delta A+BGK)$, $(A_d+\Delta A_d)$, $C$ $D$ in LMI (11) with $E^T$, $(A+\Delta A+BGK)^T$, $(A_d+\Delta A_d)^T$, $C^T$, $D^T$ respectively and setting $T_1 = \lambda_1 T$, $T_2 = \lambda_2 T$, $T_3 = \lambda_3 T$, we can obtain the LMI (29). The proof is complete.

Next, when the actuator failure parameter matrix $G$ is unknown but satisfying the constraints (4)-(7), the reliable state feedback controller is designed through the following Theorem by using the conditions obtained in Theorems 1 and Theorem 2.

**Theorem 3.** *The uncertain stochastic singular system (1) with the unknown actuator failure matrix $G$ is said to be robustly admissible, if there exist positive symmetric definite matrices $P$, $Q_1$, $Q_2$, $Q_3$, $R_1$, $R_2$, any appropriately-dimensioned matrices $T_i$, $V_i$, $i = 1, 2, 3$, $U_j, j = 1, 2, 3, 4$, positive scalars $\varepsilon_1$, $\varepsilon_2$ and constant matrix $R \in \mathbb{R}^{n\times(n-r)}$ satisfying $E^T R = 0$ with $rank(R) = (n-r)$, such that the following LMIs hold:*

$$\begin{bmatrix} \hat{\Pi} & \varepsilon_3 \hat{B}^T & \hat{Y} G_1 \\ * & -\varepsilon_3 I & 0 \\ * & * & -\varepsilon_3 I \end{bmatrix} < 0 \quad (32)$$

*where*

$$\hat{\Pi} = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & 0 & 0 & 0 & 0 & 0 & \lambda_1 TN_a & \lambda_1 TN_d \\ * & \Omega_{22} & \Omega_{23} & \Omega_{24} & 0 & 0 & 0 & 0 & 0 & \lambda_2 TN_a & \lambda_2 TN_d \\ * & * & \Omega_{33} & \Omega_{34} & 0 & 0 & 0 & 0 & 0 & \lambda_3 TN_a & \lambda_3 TN_d \\ * & * & * & \Omega_{44} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -Q_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -Q_3 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \frac{-1}{\tau_2}R_1 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \frac{-1}{\tau_2-\tau_1}(R_1+R_2) & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & \frac{-1}{\tau_2-\tau_1}R_2 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -\varepsilon_1 & 0 \\ * & * & * & * & * & * & * & * & * & * & -\varepsilon_2 \end{bmatrix},$$

*where* $\Omega_{11} = \lambda_1 T^T A^T + \lambda_1 AT + \lambda_1 Y^T G^T B^T + \lambda_1 BGY + V_1 C + C^T V_1^T + Q_1 + Q_2 + Q_3 + (\tau_2 - \tau_1)R_2 + \tau_2 R_1 + \varepsilon_1 N_a^T N_a$, $\Omega_{12} = \lambda_1 T^T A_d^T + \lambda_2 AT + V_1 D + C^T V_2^T + \varepsilon_1 Na^T N_d + \lambda_2 BGY$, $\Omega_{13} = E^T P - \lambda_1 T^T + U_1 R^T + \lambda_3 AT + \lambda_3 BGY$, $\Omega_{14} = -V_1 + C^T V_3^T$, $\Omega_{22} = \lambda_2 T^T A_d^T + \lambda_2 A_d T + V_2 D + D^T V_2^T - (1-\mu)Q_1 + \varepsilon_1 N_d^T N_d$, $\Omega_{23} = -\lambda_2 T^T + U_2 R^T + \lambda_3 A_d T$, $\Omega_{24} = -V_2 + D^T V_3^T$, $\Omega_{33} = -\lambda_3 T - \lambda_3 T^T + U_3 R^T + RU_3^T$, $\Omega_{34} = RU_4^T$, $\Omega_{44} = E^T P - V_3 - V_3^T$, $\hat{B} = [\lambda_1 B^T\ \lambda_2 B^T\ \lambda_3 B^T\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]$ *and* $\hat{Y} = [Y\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]$. *Further, if the LMIs (32) are solvable, the estimator gain matrix of the feedback controller (3) can be designed as $K = YT^{-1}$.*

**Proof:** If the actuator failure matrix $G$ is unknown, then from (7) the matrix $\Pi$ in Theorem 2 can be written as

$$\Pi = \hat{\Pi} + \lambda_1 \hat{B} \Delta \hat{Y} + \lambda_1 \hat{Y}^T \Delta \hat{B}^T. \tag{33}$$

From Lemma 1 and (7), the above inequality can be written as

$$\Pi = \hat{\Pi} + \varepsilon_3 \hat{B}^T \hat{B} + \varepsilon_3^{-1} \hat{Y} G_1^2 \hat{Y}^T. \tag{34}$$

Then by using Lemma 3, it is easy to see that (34) is equivalent to LMI (32). Hence the system (1) is robustly asymptotically stabilizable through the controller (3). The proof of this theorem is similar to that of Theorem 1 and 2 and hence it is omitted.

*Remark.*It is noted that all the above proposed results are based on Lyapunov stability theory where the number of variables of the obtained LMIs plays an important rule. Moreover, it handles the control problems which are very difficult and impossible to solve in an analytic manner. Particularly, in many control problems, LMIs occur as a function of matrix variables and more number of decision variables, consequently it yields to the computational burdens [**?**]. However, in this results of Theorems 4.1 and 4.2, no free weighting matrices are introduced. So the construction of the obtained LMIs is smaller and computational burden is also reduced significantly

*Remark.*It is worth mentioning that the considered actuator fault model (3) in this paper is more general than the previously existing fault models in [22,27] and [29]. This is because if we choose the nonlinear fault matrix $\Xi_2 = 0$, the proposed reliable controller (3) corresponds to the conventional reliable controller. On the other side, if we select $G = 1$ and $g(u(t)) = 0$, the actuator works in normal condition. Moreover, if $0 < G < 1$ and $g(u(t)) = 0$, then the actuator works in partial failure condition.

## 5 Numerical Example

In this section, we provide a numerical example to illustrate the effectiveness and applicability of the proposed results.

*Example 1.*Consider the uncertain stochastic singular system (1) with the following parameters

$$A = \begin{bmatrix} -2.1 & 1 \\ 2 & -1.2 \end{bmatrix}, B = \begin{bmatrix} 1.1 & 0.3 \\ 1.2 & 2.4 \end{bmatrix},$$

$$A_d = \begin{bmatrix} 1.3 & 0.1 \\ 0.1 & 0.3 \end{bmatrix}, C = \begin{bmatrix} -2.1 & 1 \\ 2 & -3.1 \end{bmatrix},$$

$$D = \begin{bmatrix} -4.3 & 0.1 \\ 0.2 & -1.2 \end{bmatrix}, E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},$$

$$S = \begin{bmatrix} -2 \\ 1 \end{bmatrix}, N_a = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix}, N_d = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix},$$

$$M = \text{diag}\{0.4\ 0.4\}.$$

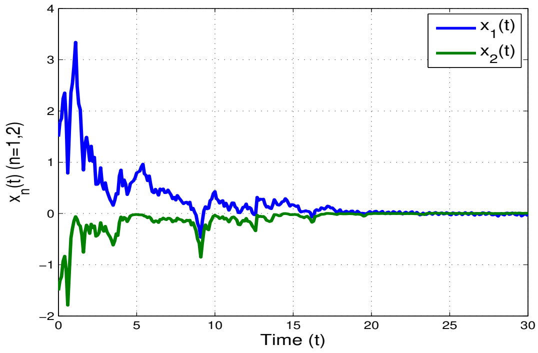


**Fig. 1:** State responses of the uncertain stochastic singular system (1)

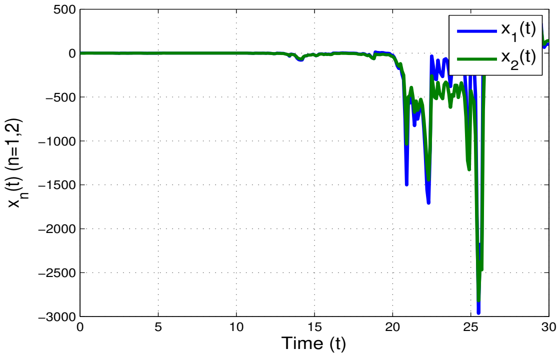


**Fig. 2:** State responses of the open-loop stochastic singular system (1)

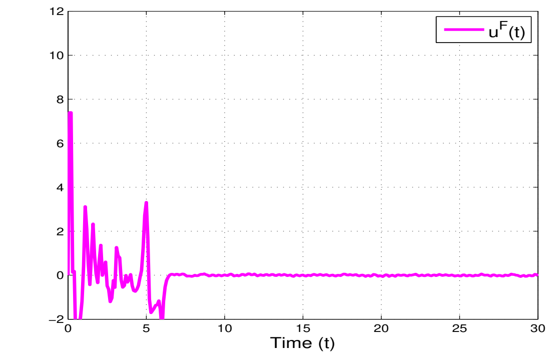


**Fig. 3:** Control response of the system (1)

The fault matrix $G$ is assumed to satisfy $\text{diag}\{0.1,\ 0.2\} \leq G \leq \text{diag}\{0.5,\ 0.4\}$. Then it follows from (7) that $G_0 = \text{diag}\{0.3,\ 0.3\}$ and

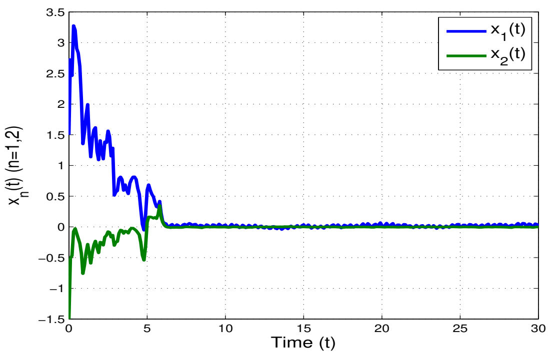


**Fig. 4:** State responses of the uncertain stochastic singular system (1) with $gu(t) = 0$

$G_1 = \text{diag}\{0.2, \quad 0.2\}$. Further, we set $\tau_1 = 0 \quad \tau_2 = 3 \quad \mu = 0.1$, and $\lambda_1 = -1.5$, $\lambda_2 = -0.5$, $\lambda_3 = -1.5$, $\Xi_2 = 0.01$, $g(u(t)) = 1.6u(t)\cos(u(t))$, $\rho = 0.02$, $F_1(t) = F_2(t) = 0.5\cos(t)$. In order to stabilize the concerned uncertain stochastic singular system (1), we design the more generalized reliable controller as mentioned in (3) with the above said parameters. By solving the LMIs obtained in Theorem 3 using Matlab LMI tool box, it is quite easy to get the feasible solutions and the feedback reliable gain matrix. Due to page limitation, only gain matrix is displayed as follows:

$$K = YT^{-1} = \begin{bmatrix} 19.5127 & -31.2835 \\ -15.5530 & 27.3503 \end{bmatrix}.$$

**Table 1:** Upper bound $\tau_2$ for different $\tau_1$

| $\tau_1$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 |
|---|---|---|---|---|---|---|
| $\tau_2$ | 2.9871 | 2.7682 | 2.5609 | 2.2872 | 1.7137 | 1.3953 |

Under the aforementioned gain matrix and the initial condition $x(0) = [1.5 \ \ -1.5]^T$, the simulation results are shown in Figs. 1, 2, 3 and 4. Precisely, Figs. 1 and 2 depict the response curves of the state of the uncertain stochastic singular closed-loop and open-loop system (1). Moreover, the control curve is given in Fig. 3. Further, $g(u(t)) = 0$ in the uncertain stochastic singular closed-loop system (1), the corresponding state responses are depicted in Fig. 4. In addition, the upper-bound values of $\tau_2$ for different values of $\tau_1$ are calculated and listed in Table 1. From these simulations, it can be strongly concluded that the considered system is asymptotically stable under fault-tolerant controller even in presence of nonlinear faults.

# 6 Conclusion

In this paper, linear matrix inequality optimization technique has been employed to study the reliable control issues for a class of uncertain stochastic singular system with time delays. A novel fault-tolerant control has been derived through the construction of an appropriate LyapunovKrasovskii functional in order to guarantee delay dependent robust stability of the uncertain stochastic singular system in the presence of actuator failures. The derived conditions are expressed in terms of linear matrix inequalities which can be efficiently solved via MATLAB LMI toolbox. Finally, numerical example has been given to illustrate the effectiveness of the proposed results. .


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