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Analytical Treatment and Convergence of Adomian decomposition Method for Fingero-Imbibition Phenomena Arising during Oil Recovery Process

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Abstract: Here we discusses fingero-imbibition phenomena arising in the double phase flow through porous media, modeled it with a nonlinear partial differential equation and studied the adomian decomposition scheme applied to a nonlinear partial differential equation that describes fingero-imbibition phenomena. An abstract result is proved the convergence of Adomian decomposition method for partial differential equations that modeled this fingero-imbibition phenomenon in a suitable Hilbert space.

Keywords: Miscible fluid, Fingero-Imbibition, Adomian decomposition method.

1 Introduction

Miscible viscous fingering is an interfacial fluid flow instability that occurs when a less viscous fluid displaces another more viscous one in a porous medium leading to the formation of finger-like patterns at the interface of both fluids. This instability impacts a variety of practical applications of environmental and industrial processes such as the recovery of crude oil from oil fields, filtration and hydrology, fixed-bed chemical processing and medical applications.
Fingero-Imbibition is the simultaneous occurrence of two special phenomena viz., fingering and imbibitions. It is known that if a porous medium filled with some fluid is brought in to contact with another fluid which preferentially wets the medium then there is a spontaneous flow of the wetting fluid in to the medium and a counter-flow of the resident fluid from the medium such phenomenon is called imbibitions simultaneously when a fluid contained in a porous medium is displaced by another of lesser viscosity instead of a regular displacement of the whole front. The perturberances may occur that shoot through the porous medium at relatively great speeds due to external forces that are called fingers. The fingero-imbibition phenomena have gained considerable current interest due to their frequent occurrence in the related problems of petroleum technology. This phenomenon occurs when water is injected in porous media during the secondary oil recovery process. The phenomenon of fingering and imbibition and the flow of two immiscible fluids through porous media have gained considerable current interest due to their frequent occurrence in problems of petroleum technology. Yildiz *et al.* [10] presented an experimental work on oil recovery process by spontaneous water imbibition.Verma [8] discussed Fingero-Imbibition in Artificial Replenishment of Ground Water through Cracked Porous Medium.
In this paper, we investigate the applicability of Adomian decomposition method to the nonlinear partial differential equation arising in fingero-imbibition phenomena in the double phase flow through porous media in order to obtain the approximate analytical solution:
The paper is organized as follows: Section 2 discusses the statement of the problem along with some relation. Section 3 discusses the Fundamental equation of fingero-imbibition phenomena and introduces the Adomian decomposition method in section 4. The novelty of this paper is in Section 5, where we developed and proved the convergence of the Adomian decomposition scheme, which leads to an abstract result, and analytical

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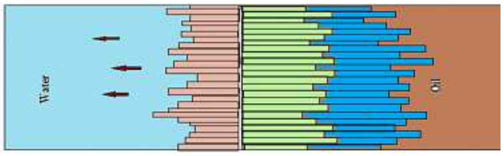

Fig. 1: Schematic diagram of Fingero-Imbibition phenomena

solution to the governing equation. Section 6 is devoted to simulation results of the governing equation for some interesting choices of initial data. We concluded by summarizing the results of the paper in Section 7.

2 Statement of the Problem

Water with constant velocity V is injected into a seam saturated with oil and consisting of homogenous porous medium It is assumed that the entire oil on the initial boundary of the seam, x = 0 (x is measured in the direction of displacement), is displaced through a small distance due to the impact of injecting water that is called imbibitions and due to some external force at the interface the displaced water takes the shape of fingers and extend up to distance L which is known as fingero-imbibition phenomena. It occurs in one direction i.e. in x-direction up to x=L due to some external force after imbibitions occurs at the interface in both direction up to small distance $-l<x<l$ but we consider here in one direction i.e. $0<x<l$.

3 Fundamental Equations

Assuming that Darcy's law is valid in the case being investigated, we may write the basic flow equations governing fingero-imbibitions phenomenon as

$$\mathrm{v_w} = -\frac{\mathrm{k_w}}{\mu_\mathrm{w}}\mathrm{K}\frac{\partial \mathrm{p_w}}{\partial \mathrm{x}} \tag{1}$$

$$\mathrm{v_o} = -\frac{\mathrm{k_o}}{\mu_\mathrm{o}}\mathrm{K}\frac{\partial \mathrm{p_o}}{\partial \mathrm{x}} \tag{2}$$

$$v_w = -v_o \tag{3}$$

$$\mathrm{p_c} = \mathrm{p_o} - \mathrm{p_w} \tag{4}$$

$$\varphi\frac{\partial \mathrm{S_w}}{\partial \mathrm{t}} + \frac{\partial\, \mathrm{v_w}}{\partial \mathrm{x}} = 0 \tag{5}$$

Where "$\mathrm{v_w}$" and "$\mathrm{v_0}$" are velocities, "$\mathrm{k_w}$" and "$\mathrm{k_o}$" are relative permeability's; "$\mathrm{p_w}$" and "$\mathrm{p_o}$" are pressures of water and oil respectively; "μ_w" and "μ_o" are kinematic viscosities (which are constants) of the wetting phase and non-wetting phase respectively; "ϕ" and 'K' are the porosity and permeability of homogeneous medium. The coordinate x is measured along the axis of the cylindrical medium, the origin being located at the imbibition face x = 0.

It may be mentioned that the statistical treatment of fingers is formally identical to the Buckley Leveret description of two immiscible fluids flow and the average cross-sectional area occupied by fingers defines the displaced phase saturation.

Combining equation (1)-(5), we get

$$\varphi\frac{\partial S_w}{\partial t} + \frac{\partial}{\partial x}\left[K\frac{k_w k_0}{k_w\mu_0 + k_0\mu_w}\frac{dp_c}{dS_w}\frac{\partial S_w}{\partial x}\right] = 0 \tag{6}$$

An analytical expression due to [8] for relationship between the relative permeability, phase saturation and capillary pressure for Fingero-imbibition phenomena is given by

$$\mathrm{k_w} = S_w^3 \;\; and \;\; P_c = -\beta S_w \tag{7}$$

Negative sign shows the direction of saturation is opposite to capillary pressure.

Simplifying equation (6) and substituting the value of P_c and k_w from equation (7), we get

$$\varphi\frac{\partial S_w}{\partial t} - \frac{\beta K}{\mu_w}\frac{\partial}{\partial x}\left[S_w^3\frac{\partial S_w}{\partial x}\right] = 0 \tag{8}$$

Where k_w is a function of S_w only [7] and the negative sign of the second term is due to the fact that p_c is a decreasing function of S_w.

Equation (8) can be rewritten in the dimensionless form by considering

$$X = \frac{x}{L}, \qquad T = \frac{\beta K}{\varphi L^2 \mu_w}t$$

The dimensionless form of equation (8) can be written as

$$\frac{\partial S_w}{\partial T} - \frac{\partial}{\partial X}\left[S_w^3\frac{\partial S_w}{\partial X}\right] = 0 \tag{9}$$

We choose appropriate initial and Dirichlit's boundary condition due to the behavior of saturation of displaced water at the interface in fingero-imbibition phenomena i.e. length of saturation fingers increases with continuous external force which implies saturation of displaced water increases with distance X from the interface X=0 as the length of fingers increases and by [6] as

$$S_w(X,0) = S_{w,0}(X) = X^{2/3} \text{ and } Dirchlit's\, boundary\;\; conditions$$
$$S_w(0,T) = S_1(T) \;\;, S_w(1,T) = S_2(T) \tag{10}$$

Where S_1 and S_2 are saturation of water at common interface X=0 and saturation of water at end of matrix of length X=1(i.e. x=L). Here during fingero-imbibition phenomena saturation fingers may takes place up to the end of matrix i.e. up to x=L. To stabilize or to find the behavior of the saturation fingers it is necessary to discuss the behavior of saturation of displace water by solving equation (9) together with equation (10).

The equation (9) together with (10) is the desired governing non-linear partial differential equation describing the Fingero-imbibition phenomenon with capillary pressure.

4 Analysis of the Adomian Method

In the early 1980s, a new numerical method was developed by George Adomian [1] in order to solve non-linear functional equations of the form

$$L\,S_w + RS_w + NS_w = g \tag{11}$$

Using an iterative decomposition scheme that led to elegant computation of closed-form analytical solutions or analytical approximations to solutions. In (11), L represents the linear part, N represents the non-linear part, R represents the remainder or lower order terms, and g is the non-homogeneous right hand side. The solution S_w and the non-linearity N are assumed to have the following analytic expansions, respectively

$$S_w = \sum_{n=0}^{\infty} S_{wn} \,\text{and} NS_w = \sum_{n=0}^{\infty} A_n \tag{12}$$

Where the A_n's are the Adomian polynomials that depend only on $S_{w0}, S_{w1}, S_{w2}, ...S_{wn}$ and are given by the following formula:

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{k=0}^{\infty}\lambda^k S_{wk}\right)\right]_{\lambda=0}, n \ge 0 \tag{13}$$

In order to better explain the method, we first assume the convergence of the series in (12) and deal with the rigorous convergence issues later. The parameter k is a dummy variable introduced for ease of computation. There are several different versions of (13) that can be found in the literature that leads to easier computation of the A_n's. It should be noted that the A_n's are the terms of analytic expansion of NS_w , where $S_w = \sum_{n=0}^{\infty} S_{w\,n}\lambda^n$. In [1] Adomian has shown that the expansion for NS_w in (12) is a rearrangement of the Taylor series expansion of Nu about the initial function S_{w0} in a suitable Hilbert or Banach space. Substitution of (12) in (11) results in the following:

$$L\left(\sum_{n=0}^{\infty} S_{wn}\right) = -R\left(\sum_{n=0}^{\infty} S_{wn}\right) - \sum_{n=0}^{\infty} A_n + g \tag{14}$$

The above equation can be rewritten in a recursive fashion, yielding iterates S_{wn}, the sum of which converges to the solution S_w satisfying (14) if it exists

$$\begin{aligned}\sum_{n=0}^{\infty} S_{wn} &= -L^{-1}R\left(\sum_{n=0}^{\infty} S_{wn}\right) - L^{-1}\sum_{n=0}^{\infty} A_n + L^{-1}(g)\\ S_{w0} &= L^{-1}(g)\\ S_{w,\,n+1} &= -L^{-1}R\,(S_{wn}) - L^{-1}(A_n)\end{aligned} \tag{15}$$

Typically, the symbol L^{-1} represents a formal inverse of the linear operator L. In the case of partial differential equations, L is the highest order partial derivative operator for which the formal inverse can be computed using integrations.

A general theory of decomposition schemes for non-linear functional equations was developed by Gabet [4]. Convergence results as applied to ordinary differential equations and non-linear functional equations can be found in [2,**?**]. Mavoungou [5] has proved a convergence result for the Adomian scheme as applied to partial differential equations. A compendium of interesting examples of partial differential equations for which the Adomian method was utilized can be found in [9]. In general, the iterates in the Adomian decomposition scheme (15) converge very rapidly to the unique solution of the functional equation (11) provided the scheme satisfies the property of strong convergence as discussed in [2].

5 Convergence Analysis of the Adomian decomposition method

We recall the following theorem from [2] which guarantees the convergence of Adomian's method for the general operator equation given by $L\,S_w + RS_w + NS_w = g$.

Consider the Hilbert space $H = L^2\left((\alpha,\beta)\times[0,T]\right)$ defined by the set of applications:

$$\begin{aligned} S_W &: (\alpha,\beta)\times[0,T] \to R \\ \text{with} \int\limits_{(\alpha,\beta)\times[0,T]} & S_W^2(\eta,\xi)\;d\eta\,d\xi < +\infty \end{aligned} \tag{16}$$

Let us denote $LS_W = \frac{\partial S_W}{\partial T}$, $NS_W = -S_w^3\frac{\partial S_W}{\partial X}$ and $TS_W = RS_W + NS_W = -\frac{\partial}{\partial X}\left(S_w^3\frac{\partial S_W}{\partial X}\right)$.

Theorem 1: Let $TS_w = -RS_w - NS_w$ be a hemi-continuous operator in a Hilbert space H and satisfy the following hypothesis (H_1) : $(TS_w - TS_w^*, S_w - S_w^*) \ge k\|S_w - S_w^*\|^2,\ k > 0,\ \forall S_w, S_w^* \in H$

(H_2) : Whatever may be $M > 0$, there exist constant $C(M) > 0$ such that for $S_w, S_w^* \in H$ with $\|S_w\| \le M,\ \|S_w^*\| \le M,$ we have $(TS_w - TS_w^*, w) \le C(M)\,\|S_w - S_w^*\|\,\|w\|$ for every $w \in H$.

Then, for every $g \in H$, the nonlinear functional equation $LS_w + RS_w + NS_w = g$ admits a unique solution $S_w \in H$. Furthermore, if the solution S_w can be represented in a series form given by $S_w = \sum\limits_{n=0}^{\infty} S_{wn}\lambda^n$, then the Adomian decomposition scheme corresponding to the functional equation under consideration converges strongly to $S_w \in H$, which is the unique solution to the functional equation.

Proof: Verification of hypothesis H_1

$$TS_W - TS_W^* = -\frac{1}{4}\frac{\partial^2}{\partial X^2}\left(S_W^4 - S_W^{4\,*}\right)$$

$$\begin{aligned}&\left(TS_W - TS_W^*\,,\,S_W - S_W^*\right)\\ &= \left(-\frac{1}{4}\frac{\partial^2}{\partial X^2}\left(S_W^4 - S_W^{4*}\right)\,,S_W - S_W^*\right)\end{aligned} \tag{17}$$

Since $\frac{\partial^2}{\partial X^2}$ is a differential operator in H, then there exist constant'δ' such that:
According to Schwartz inequality, we get

$$\begin{aligned}&\left(\frac{1}{4}\frac{\partial^2}{\partial X^2}\left(S_W^4 - S_W^{4*}\right)\,,S_W - S_W^*\right)\\ &\leq \frac{\delta}{4}\left\|S_W^4 - S_W^{4*}\right\|\,\left\|S_W - S_W^*\right\|\end{aligned}$$

Now we use mean value theorem ,then we have

$$\begin{aligned}&\left(\frac{1}{4}\frac{\partial^2}{\partial X^2}\left(S_W^4 - S_W^{4*}\right)\,,S_W - S_W^*\right)\\ &\leq \frac{\delta}{4}\left\|S_W^4 - S_W^{4*}\right\|\,\left\|S_W - S_W^*\right\|\\ &\leq M^3\delta\|S_W - S_W^*\|^2\end{aligned}$$

For $\|S_W\| \leq M$ and $\|S_W^*\| \leq M$.
Therefore

$$\left(-\frac{1}{4}\frac{\partial^2}{\partial X^2}\left(S_W^4 - S_W^{4*}\right)\,,S_W - S_W^*\right) \geq M^3\delta\|S_W - S_W^*\|^2 \tag{18}$$

Substituting equation (20) into (19) yields

$$\left(TS_W - TS_W^*\,,\,S_W - S_W^*\right) \geq k\|S_W - S_W^*\|^2 \tag{19}$$

where $k = M^3\delta$, hence we fine the hypothesis H_1.
For hypothesis H_2:

$$\begin{aligned}\left(TS_W - TS_W^*\,,\,V\right) &= \left(-\frac{\partial^2}{\partial X^2}\left(S_W^4 - S_W^{4*}\right)\,,V\right)\\ &\leq M^3\left\|S_W - S_W^*\right\|\;\;\|V\|\\ &= C(M)\left\|S_W - S_W^{*'}\right\|\;\;\|V\|\end{aligned} \tag{20}$$

Where $C(M) = M^3$ and therefore, (H_2) Hold. The proof is complete.
Remark 2: We note that the constant $C(M)$is function of M and the linearity of T allows us to prove (H_2). Furthermore, since every linear continuous operator is hemi-continuous, the operator T is hemi-continuous.

6 Simulation Results

Using the analysis of Adomian Decomposition Method, Equation (9) can be written in operator form $L_T S_w$ as:

$$L_T S_w(X,T) = L_X\left(NS_w(X,T)\right) \tag{21}$$

Operating the inverse operators on both sides of equation (21), it gives

$$S_w(X,T) = S_{w0}(X) + L_T^{-1}\left[L_X\left(NS_w(X,T)\right]\right. \tag{22}$$

Where $NS_w(X,T) = S_w\frac{\partial S_w}{\partial X}$ and $S_{w0}(X)$ can be solved subject to the corresponding initial condition (10).
It is well known from equation (12), the solution of equation (9) can be written in series form as

$$S_w(X,T) = \sum_{n=0}^{\infty} S_{wn}(X,T) \tag{23}$$

Where $S_{w0}, S_{w1}, S_{w2}, \cdots$ are the saturations of different fingers at any distance X and at any time T>0 and the nonlinear term can be represented as $NS_w(X,T) = \sum_{n=0}^{\infty} A_n$. where$A_n$'s are the Adomian's special polynomials to be determined and defined by (13). Following the analysis of Adomian decomposition method as discussed in [2,**?**] for the determination of the components $S_{w,n}(X,T)$ of$S_w(X,T)$, we set the recursive relation as

$$\sum_{n=0}^{\infty} S_{w,n}(X,T) = (X^2+1)^{-1/2} + L_T{}^{-1}\left[L_X(\sum_{n=0}^{\infty} A_n)\right]$$

We now solve equation (9) using ADM with the initial condition:

$$S_w(X,0) = X^{2/3} \tag{24}$$

For the solution of the equation, we use the recursive relation given by (15) to obtain the terms of the decomposition series as:

$$\begin{aligned}&S_{w,0} = X^{2/3},\;\; A_0 = \tfrac{2}{3}X^{5/3}\\ &S_{w,1} = \tfrac{10}{9}X^{2/3}T,\;\; A_1 = \tfrac{80}{27}X^{5/3}T\\ &S_{w,2} = \tfrac{200}{81}X^{2/3}T^2,\;\; A_2 = \tfrac{2800}{243}X^{5/3}T^2\\ &S_{w,3} = \tfrac{14000}{2187}X^{2/3}T^3,\;\; \text{and so on.}\end{aligned} \tag{25}$$

Substituting these individual terms in (23), we obtain

$$\begin{aligned}&\sum_{n=0}^{k} S_{wn}(X,T) = S_{w,0}(X,T) + S_{w,1}(X,T) + S_{w,2}(X,T) + \cdots\\ &= X^{2/3} + \frac{10}{9}X^{2/3}T + \frac{200}{81}X^{2/3}T^2 + \frac{14000}{2187}X^{2/3}T^3 + \ldots\end{aligned} \tag{26}$$

Which converges to $S_w(X,T)\ as\ \ k \to \infty$and the closed form solution of equation (9) $S_w(X,T) = \frac{X^{2/3}}{\left(1-\frac{10}{3}T\right)^{1/3}}$

$$S_w(X,T) = \frac{X^{2/3}}{\left(1-\frac{10}{3}T\right)^{1/3}} \tag{27}$$

This gives the exact solution in the closed form. This result can be verified through substitution.

Table 1: Table 1: Satuartion Vs Time Keeping Distance fixed

X=0.1 LTA	X=0.1 ADM	X=0.2 LTA	X=0.2 ADM	X=0.3 LTA	X=0.3 ADM
.21544	.21544	.22287	.21789	.23083	.22045
.34199	.34199	.35378	.34588	.36642	.34995
.44814	.44814	.46359	.45323	.48015	.45856
.54288	.54288	.56160	.54905	.58166	.55551
.62996	.62996	.65168	.63712	.67495	.64461

X=0.4 LTA	X=0.4 ADM	X=0.5 LTA	X=0.5 ADM
.23938	.22314	.24858	.22595
.37999	.35421	.39461	.35868
.49793	.46415	.51708	.47001
.60320	.56228	.62640	.56937
.69995	.65246	.72687	.66070

Table 2: Absolute Error(A.E) $|S_w(X,T)-\sigma_n(X,T)|$ considering appx. up to 4 terms

X=0.1	X=0.2	X=0.3	X=0.4	X=0.5
0.0	.00498	.01038	.01624	.02263
0.0	.0079	.01647	.02578	.03593
0.0	.01036	.02159	.03378	.04707
0.0	.01255	.02615	.04092	.05703
0.0	.01456	.03034	.04749	.06617

It is interesting to note that $S_w(X,T)$ in (27) has the following asymptotic behavior

$$\lim_{T\to\infty} S_w(X,T) = 0, \tag{28}$$

and the flux satisfies

$$\lim_{T\to\infty} S_w\left(S_w\right)_X = 0 \tag{29}$$

Implies Saturation of water increases with distance and decreases as time increases.
An explicit general exact solution of the form

$$S_w(X,T) = \frac{X^{2/3}}{\left(A-\frac{10}{3}aT\right)^{1/3}} \tag{30}$$

by using a Lie theoretic approach, where A and a >0 are arbitrary constants.
In the following section we demonstrate the absolute errors $|S_w(X,T)-\sigma_n(X,T)|$ in tables 1-2.
Equations (26) represents the approximate solution of Saturation of injected water for Fingero- imbibition phenomena during oil recovery process at any distance X and for any time T >0. we can use the boundary condition if we proceed in X-direction it gives constant solution provided S_1 and S_2 are constants and gives trivial solution if S_1 and S_2 are zero.

7 Conclusion

Here the convergence of the Adomian decomposition scheme has been proved for the case of Fingero-Imbibition phenomena. The solution (26) of equation (9) gives the saturation of water $S_w(X,T)$ as a function of distance and time in displaced oil zone causing instability at the interface up to distance X=1 due to an external force of injected water through injecting well during oil recovery process. Equation (26) is an approximate solution containing four terms of an infinite series. The solution contains positive power terms of X which shows that the saturation of displaced water i.e. saturation of fingers increases as the distance X from the interface increases and it is physical consistent with the real phenomena.
As seen from table the absolute error is pretty small and we do a very good approximation to the partial exact solution by using only three terms of the decomposition series. Here the results has been compared with Lie Theoretic Approach. As a result it can be noted that the speed of the convergence of this method is very fast and the overall errors can be made pretty small by adding more terms to the series.

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