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On Non Conformable Fractional Laplace Transform

Miguel Vivas-Cortez[1,*], *Juan E. Nápoles Valdés*[2], *Jorge Eliecer Hernández Hernández*[3], *Jeaneth Velasco Velasco*[4] and *Oswaldo Larreal*[5]

[1] Faculty of Natural and Exact Sciences, School of Physics and Mathematics, Pontifical Catholic University of Ecuador , Quito, Ecuador
[2] Faculty of Exact, Natural and Surveying Sciences (UNNE, FaCENA), National University of the Northeast, Ave. Libertad 5450, Corrientes 3400, Argentina
[3] Economic and Business Sciences, Quantitative Techniques Department, Lisandro Alvarado Westcentral University (UCLA), Barquisimeto, Venezuela
[4] Exact Sciencies Department, Escuela Superior Politécnica del Ejército (ESPE), Quito, Ecuador
[5] Department of Mathematics and Statistics, Technical University of Manabí, Institute of Basic Sciences, 130105, Portoviejo, Ecuador

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Abstract: In the present paper, the main theorems of the classical Laplace transform are generalized in the non-conforming Laplace transform with nucleus $e^{t^{-\alpha}}$. We calculate the Laplace transform of non-conforming agreement of this kernel from some elementary functions and establish the non-conforming version of the transform of the successive derivative, the integral of a function and the convolution of fractional functions. In addition, the bounded and the existence of the non-conforming Laplace transform is presented. Finally, we show the application of N_1- Transform to solving fractional differential equations.

Keywords: Laplace fractional transform, fractional calculus

1 Introduction

What we know today as Fractional Calculus is the compendium of results found as a product of research and application developed from the answer given by L'Hopital to a question asked by Leibinitz [1–3]. In this context, derivatives and integrals of arbitrary order are defined. In some goodbooks and important research articles the contributions of the fractional calculus to science, engineering, applied mathematics, economics and biomechanics are explored [4–10].

The Riemann–Liouville, Caputo, Hadamard, Caputo–Hadamard, Erdélyi–Kober, Weyl, Marchaud and Riesz fractional derivative operator have been introduced to date [11]. All of them satisfy the property of linearity, but unfortunately the product rule of two functions, the chain rule and other properties are dissatisfying. Khalil et al. [12] proposed the so-called conformable fractional derivative of order $\alpha, 0<\alpha<1$, to generalize classical properties of integer–order calculus and proved the conformable fractional Leibniz rule. Furthermore, Abdeljawad in [13] generalized the conformable operators to higher orders, presented for instance the chain rule, integration by parts and Taylor series expansion. Consequently, the conformable derivative satisfies almost all the classical properties that the derivative holds. Some other works make refer to this conformable fractional derivative.

2 Preliminaries

Now, we present the definition of the non conformable derivative with its important properties which are useful for obtaining our main results, explained in the following definition [14, 15]:

Definition 1. *Given a function $h:[0,\infty)\to\mathbb{R}$. Then, the non conformable fractional derivative $N_1^{\alpha}h(t)$ of order $\alpha\in(0,1)$ of h at $t\in[0,\infty]$ is defined by*

$$N_1^{\alpha}h(t)=\lim_{\varepsilon\to 0}\frac{h\left(t+\varepsilon e^{t^{-\alpha}}\right)-h(t)}{\varepsilon}. \qquad (1)$$

We will say that h is $N_1^{\alpha}-$differentiable in a point t of $(0,\infty)$ if the limit (1) exists. Also, h is $N_1^{\alpha}-$differentiable

* Corresponding author e-mail: mjvivas@puce.edu.ec

on A, where $A \subset (0,\infty)$ *if h is* $N_1^{\alpha}-$*differentiable in every point of A. If h is* $N_1^{\alpha}-$*differentiable in some* $(0,t)$*, and* $\lim_{t\to 0^+} N_1^{\alpha}h(t)$ *exists, then define*

$$N_1^{\alpha}h(0) = \lim_{t\to 0^+} N_1^{\alpha}h(t).$$

Remark. In addition, note that if h is differentiable, then

$$N_1^{\alpha}h(t) = e^{t^{-\alpha}}h'(t), \tag{2}$$

where h' is the ordinary derivative.

We can write $h^{(\alpha)}(t)$ for $D_{\alpha}(h)(t)$ or $\frac{d_{\alpha}}{d_{\alpha}t}(h(t))$ to denote the non conformable derivatives of h of order α at t. In addition, if the non conformable derivative N_1^{α} of h of order α exists, then we simply say h is N-differentiable.

We list some basic properties related to the N_1^{α} derivative [15, Theorem 2.3].

Theorem 1.*Let* $\alpha \in (0,1]$ *and* f,g *be* $N_1^{\alpha}-$*differentiable at a point* $t > 0$*. Then,*

1. $N_1^{\alpha}(uf+vg) = uN_1^{\alpha}(f) + vN_1^{\alpha}(g)$ *for all* $u,v \in \mathbb{R}$,
2. $N_1^{\alpha}(fg) = fN_1^{\alpha}(g) + gN_1^{\alpha}(h)$,
3. $N_1^{\alpha}\left(\frac{f}{g}\right) = \frac{fN_1^{\alpha}(g) - gN_1^{\alpha}(f)}{g^2}$,
4. $N_1^{\alpha}(\lambda) = 0\,,\ \lambda \in \mathbb{R}$,
5. $N_1^{\alpha}(t^p) = e^{t^{-\alpha}}pt^{p-1}$,
6. *If f is differentiable, then* $N_1^{\alpha}f(t) = e^{t^{-\alpha}}f'(t)$

Also some results for classical functions are found using the property 6 in the previous theorem. [15, Theorem 2.7].

Theorem 2. *We have*

1. $N_1^{\alpha}(1) = 0$
2. $N_1^{\alpha}(e^{ct}) = ce^{ct}e^{t^{-\alpha}}$
3. $N_1^{\alpha}(\sin(bt)) = be^{t^{-\alpha}}\cos(bt)$
4. $N_1^{\alpha}(\cos(bt) = -be^{t^{-\alpha}}\sin(bt)$

Some important properties with respect to this definition of non-conformable fractional derivative have been proved in [14, 15]. Among them are the following: the determination of the monotony of the function from the sign of the fractional derivative [14, Theorem 2.2], the Racetrack type principle [14, Theorem 2.3], Rolle's theorem [14, Theorem 2.6], the mean value theorem [14, Theorem 2.7], the determination of the uniform continuity of a function from the boundedness of the fractional derivative [14, Theorem 2.10], the representation of a function by Taylor series [14, Theorem 2.13], the chain rule [15, Theorem 3.1]

The following function will play an important role in our work.

Definition 2. *Let* $\alpha \in (0,1)$ *and c a real number. We define the fractional exponential in the following way*

$$E^{n_1}_{\alpha,c}(t) = e^{c\int_0^t e^{-u^{-\alpha}}du}.$$

Now, we give the definition of non conformable fractional integral:

Definition 3. *Let* $\alpha \in (0,1]$ *and* $0 \le u \le v$*. We say that a function* $h : [u,v] \to \mathbb{R}$ *is* α*-fractional integrable on* $[u,v]$*, if the integral*

$$_{N_1}J_u^{\alpha}h(x) = \int_u^x e^{-t^{-\alpha}}h(t)dt$$

exists and is finite.

The following statement is analogous to the one known from the Ordinary Calculus (see [17]).

Theorem 3. *Let f be N-differentiable function in* (t_0,∞) *with* $\alpha \in (0,1]$*. Then for all* $t > t_0$ *we have*

a)If f is differentiable $_{N_1}J_{t_0}^{\alpha}(N_1^{\alpha}f(t)) = f(t) - f(t_0)$.
b)$N_1^{\alpha}\left(_{N_1}J_{t_0}^{\alpha}f(t)\right) = f(t)$.

Proof.

a) From definition we have

$$\begin{aligned} _{N_1}J_{t_0}^{\alpha}(N_1^{\alpha}f(t)) &= \int_{t_0}^t e^{-s^{-\alpha}}N_1^{\alpha}f(s)ds \\ &= \int_{t_0}^t e^{-s^{-\alpha}}e^{s^{-\alpha}}f'(s)ds \\ &= f(t) - f(t_0). \end{aligned}$$

b) Analogously we have

$$N_1^{\alpha}\left(_{N_1}J_{t_0}^{\alpha}f(t)\right) = e^{t^{-\alpha}}\frac{d}{dt}\left[\int_{t_0}^t e^{-s^{-\alpha}}f(s)ds\right] = f(t).$$

The proof is complete.

An important property in our work is established in the following result:

Theorem 4. *[Integration by parts] Be the functions u,v N-differentiable functions in* (t_0,∞) *with* $\alpha \in (0,1]$*. Then for all* $t > t_0$ *we have*

$$_{N_1}J_{t_0}^{\alpha}((uN_1^{\alpha}v)(t)) = [uv(t) - uv(t_0)] - _{N_1}J_{t_0}^{\alpha}((vN_1^{\alpha}u)(t))$$

Proof. It is sufficient to use Theorem 1 and Theorem 3.

In this paper we establish the first results to formalize a new version of a Laplace Transform which will allow its application to a wide class of fractional differential equations. In the conformable case, there are some attempts that can be consulted in [18–23].

3 Main Results

Definition 4. *(Exponential order) A function f is said to be of generalized exponential order if there exist constants M and a such that* $|f(t)| \le ME^{n_1}_{\alpha,a}(t)$ *for sufficiently large t.*

We are now in a position to define the non conformable fractional Laplace Transform.

Definition 5. *Let $\alpha \in (0,1)$ and c be a real number. Let f be a real function defined for $t \geq 0$ and consider $s \in \mathbb{C}$. If the integral*

$$_{N_1}J^{\alpha}_{0,+\infty}\left(E^{n_1}_{\alpha,-s}f\right) = \int_0^{+\infty} e^{-t^{-\alpha}} E^{n_1}_{\alpha,-s}(t)f(t)dt$$

converge for the given value of s, you can define the function F given by the expression

$$F(s) =_{N_1} J^{\alpha}_{0,+\infty}\left(E^{n_1}_{\alpha,-s}f\right), \tag{3}$$

and we will write $F = \mathscr{L}_{N_1}(f)$.

To the operator $\mathscr{L}_{N_1}$ we will call it the N_1-Transformed of Laplace and we will say that F is the N_1-Transformed of f. In turn, f is the N_1-Inverse transform function of F and we will write it as $f = \mathscr{L}^{-1}_{N_1}\{F\}$, where $\mathscr{L}^{-1}_{N_1}$ is the N_1-transformed inverse Laplace operator.

As in the classic case, we must impose conditions to (3), so the previous definition makes sense. If f satisfies the following two conditions:

1. f is a piecewise continuous in the interval $(0,T]$ for any $T \in (0,+\infty)$.
2. f is of generalized exponential order; that is, there are positive constants M and a, satisfying Definition 4 with $Re(a-c) < 0$ and $|f(t)| \leq ME^{n_1}_{\alpha,a}(t)$ for all t and $\alpha \in (0,1]$,

then the N_1-Transformed of Laplace $F(s)$ of f exists for $s > a$. Indeed, since f is of generalized exponential order, there exists constants $T > 0$, $K > 0$ and $a \in \mathbb{R}$ such that $|f(t)| \leq KE^{n_1}_{\alpha,a}(t)$ for all $t \geq T$ and $\alpha \in (0,1]$. Now we write

$$\begin{aligned} I &=_{N_1} J^{\alpha}_{0,+\infty}\left(E^{n_1}_{\alpha,-s}f\right) \\ &=_{N_1} J^{\alpha}_{0}\left(E^{n_1}_{\alpha,-s}f\right)(T) +_{N_1} J^{\alpha}_{T,+\infty}\left(E^{n_1}_{\alpha,-s}f\right) \\ &= I_1 + I_2. \end{aligned}$$

Since f is a piecewise continuous, I_1 exists. For the second integral I_1, we note that for $t \geq T$ we have $\left|E^{n_4}_{\alpha}(-s,t)f(t)\right| \leq KE^{n_1}_{\alpha,-(s-a)}(t)$. Thus

$$\begin{aligned} {}_{N_1}J^{\alpha}_{T,+\infty}\left(E^{n_1}_{\alpha,-s}f\right) &\leq K_{N_1}J^{\alpha}_{T,+\infty}\left(E^{n_1}_{\alpha,-(s-a)}\right) \\ &= \frac{K}{s-a}, \quad s > a. \end{aligned}$$

Since the integral I_2 converges absolutely for $s > a$, I_2 converges for $s > a$. Thus, both I_1 and I_2 exist and so I exists for $s > a$. Then we will say that f is an N_1-transformable function.

Theorem 5. *Let $\alpha \in (0,1]$ so we have*

1. *$\mathscr{L}_{N_1}(1) = \frac{1}{s}$, from here we have $\mathscr{L}_{N_1}(c) = c\mathscr{L}_{N_1}(1)$ for any $c \in \mathbb{R}$.*
2. *$\mathscr{L}_{N_1}(E^{n_1}_{\alpha,c}(t)) = \frac{1}{s-c}$, c any real number and $s-c >$ 0.*
3. *$\mathscr{L}_{N_1}(f(t)E^{n_1}_{\alpha,c}(t)) = F(s-c)$, with $\mathscr{L}_{N_1}(f(t)) = F(s)$, c any real number and $s-c > 0$.*
4. *$\mathscr{L}_{N_1}\left(\sin\left(c\int_0^t e^{-u^{-\alpha}}du\right)\right) = \frac{c}{s^2+c^2}$.*
5. *$\mathscr{L}_{N_1}\left(\cos\left(c\int_0^t e^{-u^{-\alpha}}du\right)\right) = \frac{s}{s^2+c^2}$.*
6. *$\mathscr{L}_{N_1}\left(\sinh\left(c\int_0^t e^{-u^{-\alpha}}du\right)\right) = \frac{c}{s^2-c^2}$.*
7. *$\mathscr{L}_{N_1}\left(\cosh\left(c\int_0^t e^{-u^{-\alpha}}du\right)\right) = \frac{s}{s^2-c^2}$.*

Proof. (1) From definition directly.

(2) Consider $f(t) = E^{n_1}_{\alpha,c}(t)$ with $c \in \mathbb{R}$ then

$$\begin{aligned} {}_{N_1}J^{\alpha}_{0,+\infty}\left(E^{n_1}_{\alpha,-s}(t)E^{n_1}_{\alpha,c}(t)\right) &=_{N_3} J^{\alpha,\infty}_{0}E^{n_1}_{\alpha,-(s-c)}(t) \\ &= \frac{1}{s-c}. \end{aligned}$$

(3) Suppose $\mathscr{L}_N f(t) = F(s)$ for $s > k$. Hence we have

$$\begin{aligned} &{}_{N_1}J^{\alpha}_{0,+\infty}\left(E^{n_1}_{\alpha,-s}(t)E^{n_1}_{\alpha,c}(c,t)f(t)\right) \\ &=_{N_1} J^{\alpha}_{0,+\infty}\left(E^{n_1}_{\alpha,-(s-c)}(t)f(t)\right) = F(s-c), \quad s-c > k. \end{aligned}$$

(4) Using the definition of the N_1-transform we have

$$\begin{aligned} &\int_0^{+\infty} e^{-t^{-\alpha}} E^{n_1}_{\alpha,-s}(t)\sin\left(c\int_0^t e^{-u^{-\alpha}}du\right)dt \\ &= \int_0^{+\infty} e^{-t^{-\alpha}} e^{-s\int_0^t e^{-u^{-\alpha}}du}\sin\left(c\int_0^t e^{-u^{-\alpha}}du\right)dt \\ &= \int_0^{+\infty} e^{-su}\sin(cu)du, \end{aligned}$$

using integration by parts we obtain the desired result.
(5) Similar to previous one.
(6) and (7). Using the definition of sinh and cosh; the Definition 2 and the linearity of the N_1-transform we obtain the desired results.
The proof is complete.

Analogously, the following propositions can be proved from the definition of N-Transformed and the non-conformable integral.

Proposition 1. *If the functions f and g are N_1-transformable, then there exists the transform of the sum and is equal to the sum of the transforms, that is*

$$\mathscr{L}_{N_1}(f+g) = \mathscr{L}_{N_1}(f) + \mathscr{L}_{N_1}(g).$$

Proposition 2. *If the function f is N_1-transformable and λ is a real number, then there exists the N_1-transform of product of λ by f and is equal to the product of λ by the transform of f, i.e.*

$$\mathscr{L}_{N_1}(\lambda f) = \lambda\mathscr{L}_{N_1}(f).$$

Remark. Considering the two previous propositions, we say that $\mathscr{L}_N$ is a linear operator.

Proposition 3. *If f is a N_1-transformable function, then so is its N_1-derivative and you have*

$$\mathscr{L}_{N_1}(N_1^\alpha f) = s\mathscr{L}_{N_1}(f) - f(0). \tag{4}$$

Proof. $\mathscr{L}_{N_1}(N_1^\alpha f)$ exists because f is of non conformable exponential order and continuous. On an arbitrary interval $[a,b]$ where $N_1^\alpha f$ is continuous, integrating by parts in (4) gives

$$\begin{aligned}\int_a^b e^{-t^{-\alpha}} E_{\alpha,-s}^{N_1}(t) N_1^\alpha f(t)dt \\ = f(b)E_{\alpha,-s}^{N_1}(b) - f(a)E_{\alpha,-s}^{N_1}(a) \\ + s\int_a^b e^{-t^{-\alpha}} E_{\alpha,-s}^{N_1}(t) N_1^\alpha f(t)dt.\end{aligned}$$

Without loss of generality, we can consider an interval of the type $[0,K]$ where $N_1^\alpha f$ is continuous. So

$$\begin{aligned}\int_0^K e^{-t^{-\alpha}} E_{\alpha,-s}^{N_1}(t) N_1^\alpha f(t)dt \\ = f(K)E_{\alpha,-s}^{N_1}(b) - f(0) \\ + s\int_0^K e^{-t^{-\alpha}} E_{\alpha,-s}^{N_1}(t) N_1^\alpha f(t)dt.\end{aligned}$$

Taking the limit $K \to +\infty$ across this equality, we obtain the desired result.
The proof is complete.

Analogously we have

Proposition 4. *If the k consecutive derivatives $N_1^\alpha(N_1^\alpha(\cdots(N_1^\alpha f)))$ are N-transformable, then we have*

$$\begin{aligned}&\mathscr{L}_{N_1}[N_1^\alpha(N_1^\alpha(\cdots(N_1^\alpha f)))] \\ &= s^k\mathscr{L}_{N_1}(f) - s^{k-1}f(0) - s^{k-2}N_1^\alpha f(0) \\ &- s^{k-3}N_1^\alpha(N_1^\alpha f(0)) - \cdots - N_1^\alpha(N_1^\alpha(\cdots(N_1^\alpha f(0)))).\end{aligned}$$

Proposition 5. *Let $g(t)$ be of non conformable exponential order and continuous for $t \geq 0$. Then*

$$\mathscr{L}_{N_1}\left(\int_0^x e^{-t^{-\alpha}} g(t)dt\right) = \frac{1}{s}\mathscr{L}_{N_1}\{g(x)\}.$$

Proof. Let $f(x) = \left(\int_0^x e^{-t^{-\alpha}} g(t)dt\right)$. Then f is of exponential order and continuous. So using (4) and considering $f(0) = 0$, we obtain the desired result.
The proof is complete.

The following result establishes the relationship between the classic Laplace Transform and the N_1-transform defined above.

Theorem 6. *Let $\alpha \in (0,1)$ and f be a N-transformable function, then we have*

$$\mathscr{L}_{N_1}(f) = \mathscr{L}\left[f\left(\ln^{-1/\alpha}(z)\right)\right],$$

where $\mathscr{L}$ is the classical Laplace transform defined by $\mathscr{L}(g) = \int_0^{+\infty} e^{-st} g(t)dt$.

Proof. With the change of the variables $z = e^{t^{-\alpha}}$ the proof follows.

One of the most important results of the classic Laplace Transform is the Convolution Product of two $\mathscr{L}$-transformable functions. We are already in a position to provide an analogous result for the N_1-transform defined in (3).

Definition 6. *Let f and g be two functions which are piecewise continuous at each interval $[0;T]$ and of generalized exponential order. We define the N_1-convolution of f and g by*

$$(f * g)_{N_1}(t) = \int_0^T f(u)g\left(\ln^{1/\alpha}(e^{t^{-\alpha}} - e^{u^{-\alpha}})\right)du$$

Proposition 6. *Let f and g be two functions which are piecewise continuous at each interval $[0;T]$ and of generalized exponential order, then*

$$(f * g)_{N_1} = (g * f)_{N_1}$$

Proof. From definition 6 and the change $e^{r^{-\alpha}} = e^{t^{-\alpha}} - e^{u^{-\alpha}}$ we establish the desired result.

Theorem 7. *Let f and g be two functions which are piecewise continuous at each interval $[0;T]$ and of generalized exponential order, then*

$$\mathscr{L}_N(f * g)(s) = \mathscr{L}_{N_1}(f).\mathscr{L}_{N_1}(g).$$

Proof. It is sufficient to change the variables $e^{r^{-\alpha}} = e^{t^{-\alpha}} - e^{u^{-\alpha}}$ and apply the properties of the $\mathscr{L}_{N_1}$ operator.

3.1 Existence of non Conformable Laplace Transform

In this subsection, the bounded and existence of non conformable Laplace transform are presented.

Theorem 8.*Let f be piecewise continuous on $[0,\infty)$ and non conformable exponentially bounded, then*

$$\lim_{s\to\infty} \mathscr{L}_{N_1}(f)(s) = 0.$$

Proof. Since f is generalized order exponential, there exists t_0, M_1, c such that $|f(t)| \leq M_1 E^{n_1}_{\alpha,c}(t)$ for $t \geq t_0$. Also, f is piecewise continuous on $[0,t_0]$ and so f is bounded. Accordingly, there exists M_2 such that $|f(t)| \leq M_2$ for $t \in [0,t_0]$. Choosing $M = \max\{M_1, M_2\}$, so $|f(t)| \leq M E^{n_1}_{\alpha,c}(t)$ for $t \geq 0$. Now we have‘

$$\begin{aligned}\left|\int_0^\tau E^{n_1}_{\alpha,-s}(t)f(t)d_\alpha t\right| &\leq \int_0^\tau \left|E^{n_1}_{\alpha,-s}(t)f(t)\right| dt\\ &\leq M\int_0^\tau E^{n_1}_{\alpha,-(s-c)}(,t)dt\\ &= \frac{M}{s-c} - \frac{E^{n_1}_{\alpha,-(s-c)}(t)}{s-c}.\end{aligned}$$

This gives

$$\lim_{\tau\to\infty}\left|\int_0^\tau E^{n_1}_{\alpha,-s}(t)f(t)dt\right| \leq \frac{M}{s-c}.$$

This completes the proof.

4 Examples and applications

*Example 1.*Consider the nonconformable differential equation:

$$N_1^\alpha x(t) = \lambda x(t), \quad x(0) = x_0, \alpha \in (0,1]. \tag{5}$$

Clearly, if $\alpha = 1$ the equation above is just one of the simplest classical ordinary differential equations which are defined by the hypothesis that the rate of growth of a given function $x(t)$ is proportional to the current value (e.g. Maltius's population model), i.e. $x'(t) = \lambda x(t)$, $x(0) = x_0$. The exact solution of this is $x(t) = x_0 e^{\lambda t}$.

Applying the non-conformable Laplace Transform to both sides of the equation (5), we get

$$\begin{aligned}&\mathscr{L}_{N_1}\left(N_1^\alpha x(t)\right) = \lambda \mathscr{L}_{N_1}\left(x(t)\right),\\ &sX_\alpha(s) - x_0 = \lambda X_\alpha(s).\end{aligned}$$

Simplifying this we get

$$X_\alpha(s) = \frac{x_0}{s-\lambda}. \tag{6}$$

Taking the inverse non conformable Laplace transform to (6), we get

$$x(t) = x_0 E^{N_1}_{\alpha,\lambda}(t) = x_0 e^{\lambda\int_0^t e^{-u^{-\alpha}}du}.$$

*Example 2.*Consider the non-conformable fractional Bertalanffy-logistic differential equation

$$N_1^\alpha x(t) = x^{\frac{2}{3}}(t) - x(t), \quad x(0) = x_0,\ \alpha \in (0,1). \tag{7}$$

The solution of the classic Bertalanffy-logistic differential equation $x'(t) = x^{\frac{2}{3}}(t) - x(t)$, $x(0) = x_0$ is $x(t) = \left[1 + \left(x_0^{\frac{2}{3}} - 1\right)e^{-\frac{t}{3}}\right]^3$. Using the change variable $z = 3x^{\frac{1}{3}}$ in equation (7), we find

$$N_1^\alpha z(t) = 1 - \frac{2}{3}z(t), z_0 = 3x_0^{\frac{1}{3}}. \tag{8}$$

Applying the non conformable Laplace Transform $\mathscr{L}$ to both sides of the equation (8) we obtain

$$\mathscr{L}_N(z(t)) = \frac{3}{s} + \frac{z_0 - 3}{s + \frac{1}{3}}.$$

Finally, applying the inverse Laplace Transform we have the solution of (7) in the form $x(t) = \left[1 + \left(x_0^{\frac{2}{3}} - 1\right)e^{-\frac{t^{1+\alpha}}{3(1+\alpha)}}\right]^3$.

Example 3. Consider the non-conformable fractional differential equation

$$N_1^\alpha(N_1^\alpha x(t)) + cx(t) = 0, \quad \alpha \in (0,1], \tag{9}$$

with the initial conditions $x(0) = x_0$, $N_1^\alpha x(0) = 0$. Clearly, if $\alpha = 1$ the previous differential equation approximates the characterization of small oscillations of a pendulum, i.e. $x''(t) + cx(t) = 0$, $x(0) = x_0$, $x'(0) = 0$ where $c = \frac{g}{L}$ with g the gravity acceleration an L the length of the pendulum rod. The exact solution to this problem is $x(t) = x_0 cos\sqrt{c}t = x_0 cos\sqrt{\frac{g}{L}}t$. Applying the non-conformable Laplace Transform to both sides of the equation (9) we get $(s^2 + c)X(s) - sx_0 = 0$. Thus $X(s) = \frac{sx_0}{(s^2+c)}$. Taking the inverse non-conformable Laplace transform we obtain $x(t) = x_0 cos\left(\sqrt{\frac{g}{L}}\frac{t^{\alpha+1}}{\alpha+1}\right)$.

Example 4. Now consider the circuit consisting of a voltage source v(t) in series with a resistor (R), a capacitor (C) and an inductor (L), as well as a switch that can be in the open or closed position. The circuit equation in the time domain is $Rx(t) + \frac{1}{c}\int_0^t x(u)du + v_C(0) + Lx'(t) = v(t)$. We assume that $x(0) = 0$ (i.e. the switch is open until $t = 0$, allowing the capacitor to maintain its initial condition $v_C(t)$ before that moment), and $v(t) = A$. The corresponding non-conformable fractional differential equation is

$$Rx(t) + \frac{1}{c}\ {}_NJ_0^\alpha(x)(t) + v_C(0) + LN_1^\alpha x(t) = A, \alpha \in (0,1].$$

Applying the non-conformable Laplace Transform to both sides of above equation, we get $X(s) = \frac{A - v_C(0)}{L(s^2 + \frac{R}{L}s + \frac{1}{LC})}$. The poles of the characteristic equation can be obtained as $s = -\frac{R}{2L} \pm i\sqrt{\frac{1}{LC} - \left(\frac{R}{2C}\right)^2} = -\sigma \pm iw$. Assuming the radicand is positive we have $X(s) = \frac{A - v_C(0)}{L((s+\sigma)^2+w^2)}$. Taking inverse N_1-transform and reorder we get $x(t) = \frac{A-v_C(0)}{wL} \quad E^{N_1}_{\alpha,-\sigma}(t) sin\left(w\frac{t^{\alpha+1}}{\alpha+1}\right)$.

5 Conclusion

The present paper aimed to generalized the main theorems of the classical Laplace Transform into the non-conformable Laplace Transform. The goal has been achieved and the non-conformable derivative definition has been used to construct some of these theorems and relations. We calculated the non-conformable Laplace Transform from some elementary functions and established the non-conformable version of the transform of the successive derivative, the integral of a function and the convolution of the fractional functions. In addition, the bounded and the existence of the non-conformable Laplace Transform were presented. The findings of this study indicate that the results obtained in the fractional case are adjusted to the results obtained in the ordinary case. Finally, we showed the application of the N_1-Transform to the resolution of fractional differential equations.

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Conflict of Interest

The authors declare that they have no conflict of interest.

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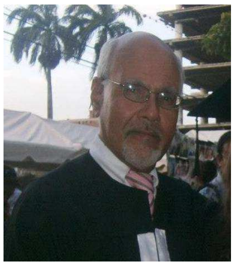
Miguel J. Vivas-Cortez. earned his Ph.D. degree from Universidad Central de Venezuela, Caracas, Distrito Capital (2014) in the field Pure Mathematics (Nonlinear Analysis), and earned his Master Degree in Pure Mathematics in the area of Differential Equations (Ecological Models). He has vast experience of teaching and research at university levels. It covers many areas of Mathematical such as Inequalities, Bounded Variation Functions and Ordinary Differential Equations. He has written and published several research articles in reputed international journals of mathematical and textbooks. He was Titular Professor in Decanato de Ciencias y Tecnología of Universidad Centroccidental Lisandro Alvarado (UCLA), Barquisimeto, Lara state, Venezuela, and invited Professor in Facultad de Ciencias Naturales y Matemáticas from Escuela Superior Politécnica del Litoral (ESPOL), Guayaquil, Ecuador, actually is Principal Professor and Researcher in Pontificia Universidad Católica del Ecuador. Sede Quito, Ecuador.

Juan E. Náapoles V. Graduated from Bachelor of Education, Specialist in Mathematics in 1983, studied two specialties and finished the Doctorate in Mathematical Sciences in 1994, in Universidad de Oriente (Santiago de Cuba). In 1997 he was elected President of the Cuban Society of Mathematics and Computing until 1998 when it established residence in the Argentine Republic. He has directed postgraduate careers in Cuba and Argentina and held management positions at various universities Cuban and Argentine. He has participated in various scientific conferences of Cuba, Argentina, Brazil, Colombia and published different works in magazines scientists specialized in the topics of qualitative theory of equations ordinary differentials, math education, problem solving, and history and philosophy of mathematics. For his teaching and research work he has received several awards and distinctions, both in Cuba and Argentina.



Jorge Eliecer Hernández H. earned his M.Sc. degree from Universidad Centroccidental Lisandro Alvarado, Barquisimeto, Estado Lara (2001) in the field Pure Mathematics (Harmonic Analysis). He has vast experience of teaching at university levels. It covers many areas of Mathematical such as Mathematics applied to Economy, Functional Analysis, Harmonical Analysis (Wavelets). He is currently Associated Professor in Decanato de Ciencias Económicas y Empresariales of Universidad Centroccidental Lisandro Alvarado (UCLA), Barquisimeto, Lara state, Venezuela.

Janneth Velasco received the Master degree in Mathematics for Faculty of Engineering, Mathematics and Physical Sciences at The Central University of Ecuador. She is Professor of Mathematical at Armed Forces University-ESPE. Her research interests are in the mathematical analysis. She has published research articles in fractional calculus

Oswaldo Larreal received the PhD degree at Universitat Politécnica de Catalunya under the supervision of Tere M-Seara. His main research interest are in the areas of dynamical systems, differential equations, numerical methods and applied mathematics. He has published research articles in reputed international journals of mathematics. He has also been a visiting professor at universitat Politécnica de Catalunya (UPC-Spain), Spain and Centre de Recerca Matemática (8CRM), Spain. Aditionally, he is referee of several international journals in Mathematics.