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Application Stochastic Approximation Procedure in the Presence of Delayed Groups of Clients

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Abstract: In the presence of delayed groups of delayed customers, the Robbins-Monro stochastic approximation technique is modified to be functional. To get an accurate explanation of the suggested treatment, two loss systems are added. Each client is reliable after fixed time intervals with the process of the following client according to the result of the previous one, where a client 's serving time is considered to be a discrete random variable. Where the analysis of their results is obtained, taking into account that it was considered a special case is that the maximum time of delay is the unit. The analysis shows that the efficiencies of the procedure can be improved by controlling on some parameters such as maximize number of servers or minimize group's capacity, and the best result can be obtained when we take both in consideration. Efficiency depends on the client's overall service time and the group's number of clients. The most significant consequence is that the efficiencies of the process are enhanced by increasing the allocation of operating time and client service times. In general, in many fields, such as medicine, computer science, business, and applied sciences, our proposal can be used for other stochastic approximation processes to increase efficiency.

Keywords: Analysis of results; Clients; Groups; Efficiency; Loss systems; Modified procedure; Service time; Stochastic approximation.

1 Introduction

The Robbins-Monro stochastic approximation method Robbins et al. (1951) is an iterative algorithm for finding the root of an equation or the solution of an equation structure where it can not be explicitly computed, but only calculated using a random error-prone calculation. Dupač et al. (1985) used the Robbins-Monro stochastic approximation method for delayed measurements for a geometric delay distribution by allocating experiments into parallel S series and building a global approximation by averaging the individual S series. In the cited article, the time loss incurred by delayed observations or its complementation was analyzed study to compute the efficiency of the procedure. There are major literatures and we can use the review papers (see, Blum, 1954; Brain 1992; Cheung et al., 2010; Combes, 2013; Jonckheere et al., 2007 Joseph et al.,2007; Mahmoud, 1988; Nevel'son et al., 1972) as sources for a number of papers on the subject. In stochastic operations, after set time periods, clients follow up each other where the next client's point being corrected according to the outcome of the previous one. In our previous study, Mahmoud et al. (2011), in the case of compound delayed measurements, we used the Robbins-Monro Stochastic approximation method. To achieve the approximate utility of the method, the random time delay distribution of compound observations was calculated. Recently, in the paper Mahmoud et al. (2018), in the case of classes of delayed findings, we extended the Robbins-Monro method by examining two loss systems. A group of delayed observations are served by a portion of one of the two loss systems cans, where the number of observations served may be raised.
Here, in the presence of delayed groups of delayed multiservice clients and studying two loss systems as an implementation of the Robbins-Monro technique, we can portraying the Robbins-Monro stochastic approximation procedure to be functional. The two loss systems may be defined as follows:
A first loss system, a group of delayed clients arrive at the service system every time unit where the service time is an integer-valued random variable, servers are parallel, and if all servers are busy, there are no waiting areas,. A second loss system, a server serves a group according to this new way, where if the j^{th} customer is delayed, then the next client will

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be lost while this delayed Ç client is in a service, and the rest services clients continue after its end. As a result of applying the two loss systems to the proposed procedure, the number of served clients can be increased, If the amount of clients served without interruption is maximum, this leads to an improvement in the procedure's success (efficiency). In the review paper Mahmoud et al.,(2005), applied Robbins Monro stochastic approximation procedure in the presence of compound delayed observations. We partitioned experiments into parts, where each part is treated as a sub-experiment; some of those parts are lost due to the delay of the preceding part during the time interval between any two consecutive parts. Also, Mahmoud et al.,(2018), this method is not used, since the topic was discussed in this paper by designing two particular loss schemes where the delayed client leads to lost all clients after him, while our solution is more generic, for a group of customers to receive server cans where this would maximize the number of customers served.
In the same direction, Rasha (2019) also, we used the case of loss system obey Negative Binomial distribution. The efficiency of the proposed procedure is calculated to determine the ways to improve the mentioned loss system, the results which are obtained show that our procedure can serve as a model of stochastic approximation with delayed observations. In the records (see, Mahmoud et al.,2011, Mahmoud et al.,2015, Mahmoud et al.,2018; Nevel son et al., 1972); The problem was investigated by adding some unique loss structures where servers do not extradite (serve) some observation during the period between some two sequential arrivals. However, we implemented a new condition in the suggested protocol by examining two loss systems, During the time between any two consecutive groups, the server will accept more than one client. In fact, specifically through this application, the analysis of the above two loss networks was carried out, where the number of lost clients would be minimized. The asymptotic efficiency of the loss system with time delay distribution of the compound observations and its approximation by the efficiency of the loss system with geometric time delay distribution was given in Tables 1-3. The results obtained show that the replacement of the procedure in the presence of delayed observations with geometrical time delay distribution by the procedure in the presence of compound delayed observations with independent random time delay distribution gives, as a rule, a satisfactory approximation to the efficiency of the procedure. Consequently, the procedure's success (efficiency) is this skill has been improved and used as an application of manufacturing to maximize the production of products from some industrial ventures. A group's service time is equal to the amount of client service times where it is independent of the number of clients, and this can be used to maximize the number of clients served during the same group service time.
A group's probability service time is equal to the amount of the product client service time which terminated with or without delay. If the clients' service time probabilities are without interruption and terminated with retard, so all future clients are destroyed. The technique investigated is new and we anticipate that it can be used for any stochastic approximation or recursive method of estimation.

2 The First & Second Loss Systems Interpretations

The first system of loss is formed as follows: Consider the service system *GIg/GIg/S/0* where symbols have the following meaning, respectively, positive integers are arrivals and service times; *S* the number of parallel servers; no queues. Assume that the distribution of arrival times is deterministic, that is, each time unit a group of customers come. When all servers are busy, groups are lost, in this case the service system is called a loss system with delayed customer groups and is denoted by *Dg/ GIg/S/0*. A group's time of service t is rounded down to $\varepsilon, \varepsilon = 0, 1, 2, ...$, If the task of the group that came at time ψ is done by time $\psi+\varepsilon+1$ but not before time $\psi+\varepsilon$ where a service time that does not skip one time is considered as a reference, ε is the delay and equals one time unit excess. Let the service time distribution of a set of clients and the state of the process at time ψ -0 be described by a *S*-duple of integers where a basic claim of the queuing principle proves that a time-homogeneous, finite or countable Markov chain is explained by the system of these states are $P_0, P_1, ..., P_T$.
The second loss system is designed as follows:
Consider the *GIs/GIs/1/0* service system for a group of Ç clients, where a group arrives per time unit and the clients' service time is equal to one time units, while the arrivals distributions and the clients' service time are not specified. The total number of servers is only equal to 1, where this server can support all the Ç clients. That is, we have Ç service times that are served by the same server for Ç group customers. 0 indicates that no waiting spaces are available. Such an organizational structure is a multiservice client loss system and will be denoted by *Ds/GIs/1/0*, Where each unit of a client arrives. Service time t is rounded down to 0, if the service of a client arriving at time ψ is completed at time ψ +1-0 (that is, before time ψ +1); rounded down to 1. Where, the service time of customer cannot exceed one time unite. In general, if the service of a client (who arrived at time ψ) is finished at time ψ +1+T-0, the service time t is rounded down to T.

3 Servicing Time and the Distribution of Group of Ç Clients

To find the service time Of group of Ç customers, keep track of those notes:

1. The compound service times of Ç clients are rounded down to Ç, the service of a client who arrived at $\psi + j - 1$ is completed by $\psi + j$, where the j^{th} client is served without delay; for all $j = 1, 2, \ldots$, Ç. In this case, the operation of a group of Ç clients that starts at time ψ is completed by time $\psi +$ Ç, and the group's service time will be rounded down to Ç where it equals the Ç customers' compound service times.
2. The compound service times of Ç clients are rounded down to Ç+1, the service of a client who arrived at $\psi + j - 1$ is completed by $\psi + j$, where the j^{th} client is served without delay; for all $j = 1, 2, \ldots$, Ç -1,the Çthclient served with a unite time of delay. In this case, the operation of a group of Ç clients that starts at time ψ is completed by time $\psi +$ Ç $+1$, and the group's service time will be rounded down to Ç+1 where it equals the Ç customers' compound service times.
3. The compound service times of Ç clients also are rounded down to Ç, the service of a client who arrived at $\psi + j - 1$ is completed by $\psi + j$, where the j^{th} client is served without delay; for all $j = 1, 2, \ldots$, Ç -2,the (Ç $-1)^{th}$client served with a unite time of delay, and the Çth is lost . In this case, the operation of a group of Ç clients that starts at time ψ is completed by time $\psi +$ Ç, and the group's service time will be rounded down to Ç where it equals the Ç customers' compound service times.

And so on....
Then we can deduce that: The distribution of compound service time of the two loss systems is treated in the following manner.
Denote by $p_0; p_1$, the distribution of the service time of a client. Let $D_i(t)$ be the event that the i clients are served without delay with probability p_0, and served with delay with probability p_1, let x is the number of served clients, z is the number of served clients without delay then, Ç $- x$ is the number of lost clients. Since the service times of the Ç clientss are independent random variables; it can be seen that the service time distribution of the group equals:

$$P\left(\cap_{i=1}^{r} D_i(t)\right) = p_0^z p_1^{x-z}, \quad ; \ z = no.\ of\ served\ clients\ without\ delay,\ x = 1, 2, \ldots, \text{Ç} \tag{1}$$

4 Interpretation of Suggested Applications.

In the situation that a group of two clients Ç $= 2$ enters the system each time unit, assume the service system of two servers $S = 2$. Claim that each client arrives at the server with service time distributions p_0, p_1, each time unit where the client is served without delay or with a unite time of delay. In this case, the group's service time distribution is P_1, P_2, where P_1 can be expressed as follows: each client is served with probability p_0 without delay or the first is served with probability p_1 after one time unit and the second is lost. In the other hand, if the first customer is served without interruption and the second is delayed one time unite, p_1 can be formulated. That is,

$$P_1 = p_0^2 + p_1, \qquad P_2 = p_0 p_1. \tag{2}$$

If Ç $= 3$, we have

$$P_2 = p_0^3 + 2p_0 p_1, \quad P_3 = p_0^2 p_1 + p_1^2. \tag{3}$$

In case Ç $= 4$, we have

$$P_3 = p_0^4 + 3p_0^2 p_1 + p_1^2, \quad P_4 = p_0^3 p_1 + 2p_0 p_1^2, \tag{4}$$

In general, for all Ç $= 2, 3, \ldots$.., the group of equations (2), (3), (4) , we deduce that the service time distribution of a group of Ç customers,Ç $= 2,\ 3,\ \ldots$.,can be obtained by the system:

$$P_{\text{Ç}+j} = \sum_{i=0}^{\tau} \binom{[\text{Ç} - (j+1) - i]}{i} p_0^{[\text{Ç}-(j+1)]-2i} p_1^{[i+(j+1)]}, \quad j = -1, 0, \quad \tau = the\ integer\ part\ of \left(\frac{\text{Ç}}{2}\right) \tag{5}$$

Now, we show that (5) is true. First, we show that (5) is true in case Ç $= 2$, that is, we show that The group of equation (2) achieve the axioms of probability:

1. $0 \leq P_l \leq 1, \quad l = 1, 2$
2. $\sum_{l=1}^{2} P_l = 1$

Also, in case Ç $= 3$, the group of equation (3) achieve the axioms of probability:

1. $0 \leq P_l \leq 1, \quad l = 2, 3$
2. $\sum_{l=2}^{3} P_l = 1$

In the same direction, in case $Ç = 4$, the group of equation (4) achieve the axioms of probability:

1. $0 \le P_l \le 1, \quad l = 3,4$
2. $\sum_{l=3}^{4} P_l = 1$

So, Let the system (5) is true for $Ç = U$ i.e:

$$P_{U+j} = \sum_{i=0}^{\tau} \binom{[U-(j+1)-i]}{i} p_0^{[U-(j+1)]-2i} p_1^{[i+(j+1)]}, \quad j = -1,0, \quad \tau = the\ integer\ part\ of\ \left(\frac{U}{2}\right), \tag{6}$$

The resulting equations are:

$$P_{U-1} = p_0^U + (U-1)\,p_0^{U-1} p_1 + \dots\dots\dots\dots + \binom{U-\tau}{\tau} p_0^{U-2\tau} p_1^{\tau}, \tag{7}$$

$$P_U = p_0^{U-1} p_1 + (U-2)\,p_0^{U-3} {p_1}^2 + \dots\dots\dots\dots + \binom{U-1-\tau}{\tau} p_0^{U-2\tau-1} p_1^{\tau+1}, \tag{8}$$

Then, the group of equation (5) achieve the axioms of probability:

1. $0 \le ?_l \le 1, \quad l = U-1, U$
2. $\sum_{l=U-1}^{U} P_l = 1$

Now, we will prove that (5) is true for $M = U+1$, we can show that: From (5) we can obtain the following system of equations

$$P_{U+1+j} = \sum_{i=0}^{\tau} \binom{[U+1-(j+1)-i]}{i} p_0^{[U-(j+1)]-2i} p_1^{[i+(j+1)]}, \qquad j = -1,0, \tag{9}$$

Which tends to the following equations:

$$P_U = p_0^{U+1} + U p_0^{U-1} p_1 + \dots\dots\dots\dots + \binom{U+1-\tau}{\tau} p_0^{U+1-2\tau} p_1^{\tau}, \tag{10}$$

$$P_{U+1} = p_0^U p_1 + (U-1)\,p_0^{U-2} {p_1}^2 + \dots\dots\dots\dots + \binom{U-\tau}{\tau} p_0^{U-2\tau} p_1^{\tau+1}, \tag{11}$$

By taking the summation of first term from equations (10) and (11) we will obtain the following term: $p_0^U + p_0^U p_1 = p_0^U\,(p_0 + p_1) = p_0^U$ which is equal to the first term in (7), Also, by taking the summation of second term from equations (10), (11) we will obtain the following term: $U p_0^{U-1} p_1 + (U-1)\,p_0^{U-2} {p_1}^2 = (U-1) p_0^{U-2}\left(p_0 + p_1 + \left(\frac{p_0}{U-1}\right)\right) = p_0^{U-1} p_1$ which is equal to the first term in (8) And by taking the summation of the third term from equations (**??**), (11) we will obtain the second term in (8) And so on... The summation resulting probabilities will be equal to the summation of equations (10), (11) Which achieve the probability axioms according to the previous assignment. That is leads to prove that equation (5) is correct for all values of Ç.

5 Technical Tools

1. The analysis of statistical representations of two special structures of service in special cases when the maximum time of delay for a client equal to a unite and for all $Ç = 2,3,\dots.$.
2. As the structures considered can be seen as compound Markov chains, the principle of Considered when Markov chains is advantaged by countable (discrete-time) compounds.
3. The compound states of the ready chain were defined from the elementary zero state.
4. Important equations for stationary distribution have been calculated.
5. The S set of compound states that can be reached under the compound states 00...0 from the original assumption $?_i > 0$, for all $i = Ç-1,\ Ç$; in our procedure, jointly with the identical matrix of transition probabilities P wil be found. The set S is called the basal compound Markov chain.
6. It is clear to note that, using the assignment $P_i > 0$, for all $i = Ç-1,\ Ç$, the principle compound Markov chain is irreducible, ergodic. Hence, according to the ergodic theorem there is a unique stationary distribution π determined by

$$P^T \pi = \pi, T denotes transpose \tag{12}$$

7. A certain matrix algebra and techniques were used to solve linear equation structures.
8. Asymptotic efficiencies of the approximate stochastic procedure under review were calculated with parallel server numbers. If S>1, then every α state contains at least one 0, and in any situation, no client category would be lost. For $S \leq 1$, the unknowns π_α with α containing no 0's can be eliminated from the system (12), as successive transitions from these states to states containing 0's occur deterministically, with probability 1. One of the resting equations can always be deleted as unnecessary; another one is to be joined namely the equation

$$\sum_\beta \pi_\beta = 1.$$

Solving the reduced system of equations using Matlab Program (2018b), and summing the coordinates of the solution, we get the loss probability l, where

$$l = \sum_\gamma \pi_\gamma;$$

γ containing no 0, or complementarily, the efficiency e of the two service systems, where

$$e = \sum_\alpha \pi_\alpha; \alpha contains at least one 0, \tag{13}$$

$$l + e = 1.$$

9. Investigating the modified method of stochastic approximation with delayed groups of groups delayed clients and parallel series for the assignment of groups.
10. A comparison between the exact values of the efficiency of the proposed procedure and those obtained by an approximation based on geometric time delay distribution are calculated, where the geometric parameter p is determined using the moment method.
11. The results obtained for the above-mentioned service structures became the principal method of the Evaluation.
12. Results on almost sure convergence and asymptotic normality, known for the stochastic approximation technique was efficient without restriction.

6 Applictions

Two applications will be given here to show that the investigated procedure can be utilized to other stochastic approximation procedure. Application 1 Consider the case = 2; Ç= 2 , and assume that $p_0 = 0.9$, $p_1 = 0.1$; where

$$p_1 = (1 - p_0)$$

Substitute the service time distribution of the customer (p_0, p_1) into (2) we get the service time distribution of the group of two customers as P_1, P_2 , where

$P_0 = 0$; $P_1 = 0.91$; $P_2 = 0.09$.

Substitute the service time distribution (P_1, P_2) into (12) and solve the resulting system of equations with respect to π; insert the stationary distribution π into (13) to get the efficiency e of the investigated stochastic procedure as e = 0.9237. Application 2 Consider the case K= 3; Ç= 4 , and assume that $p_0 = 0.3$; where $p_1 = 1 - p_0$. Substitute the service time distribution of the customer as (P_3, P_4) into (4) , where

$P_0 = P_1 = P_2 = 0$; $P_3 = 0.6871$; $P_4 = 0.3129$.

Substitute the service time distribution (P_3, P_4) into (12) and solve the resulting system of equations with respect to $\pi\pi$ insert the stationary distribution π into (13) to get the efficiency e of the investigated stochastic procedure as e = 0.5942. The results obtained here show that the modified Robbins-Monro stochastic approximation procedure can serve as a model of stochastic approximation with delayed groups of Ç customers with efficiency e. That is, the new approach can be investigated by modifying the Robbins-Monro stochastic approximation procedure to be applicable in the presence of the described two loss systems with groups of delayed multiservice Ç customers.

Table 1: Percentage asymptotic efficiency e of the proposed procedure with $K = 2$ servers. delay group of $Ç = 2$ clients and its approximation by efficiency of a procedure with geometrically distribution e_g with parameter p_g.

p_0	P_1	p_g	e	e_g
0.1	0.91	0.4785	0.9237	0.811
0.2	0.84	0.463	0.8788	0.794
0.3	0.79	0.4525	0.8521	0.7818
0.4	0.76	0.4464	0.8378	0.7747
0.5	0.75	0.444	0.8333	0.7723
0.6	0.76	0.4464	0.8378	0.7747
0.7	0.79	0.4525	0.8521	0.7818
0.8	0.84	0.463	0.8788	0.794
0.9	0.91	0.4785	0.9237	0.811

Note that: $p_1 = 1 - p_0$. $P_2 = 1 - P_1$

Table 2: Percentage asymptotic efficiency e of the proposed procedure with $K = 2,3$ servers. delay group of $Ç = 3$ clients and its approximation by efficiency of a procedure with geometrically distribution e_g with parameter p_g.

			$K = 2$		$K = 3$	
p_0	P_2	p_g	e	e_g	e	e_g
0.1	0.181	0.2618	0.5166	0.5014	0.7631	0.7053
0.2	0.328	0.2723	0.5327	0.5195	0.7772	0.7261
0.3	0.447	0.2815	0.548	0.535	0.792	0.7435
0.4	0.544	0.2894	0.5623	0.5482	0.8074	0.758
0.5	0.625	0.2963	0.5758	0.5597	0.8235	0.7703
0.6	0.696	0.3027	0.589	0.5702	0.841	0.7812
0.7	0.763	0.3089	0.6028	0.5804	0.8615	0.7917
0.8	0.832	0.3157	0.618	0.5912	0.8883	0.8026
0.9	0.909	0.3235	0.6389	0.6037	0.9285	0.8148

Note that: $p_1 = 1 - p_0$. $P_3 = 1 - P_2$

Table 3: Percentage asymptotic efficiency e of the proposed procedure with $K = 2,3$ servers. delay group of $Ç = 4$ clients and its approximation by efficiency of a procedure with geometrically distribution e_g with parameter p_g.

			$K = 2$		$K = 3$	
p_0	$Ƥ_3$	p_g	e	e_g	e	e_g
0.1	0.8371	0.2229	0.443	0.4324	0.6523	0.6208
0.2	0.7376	0.2095	0.4142	0.4079	0.6102	0.5893
0.3	0.6871	0.204	0.4028	0.3979	0.5942	0.5762
0.4	0.6736	0.204	0.4026	0.3978	0.5939	0.5761
0.5	0.6875	0.2078	0.4105	0.4048	0.6047	0.5853
0.6	0.7216	0.2145	0.4246	0.4171	0.624	0.6012
0.7	0.7711	0.2232	0.4431	0.4329	0.6498	0.6215
0.8	0.8336	0.2329	0.4639	0.4503	0.6803	0.6435
0.9	0.9091	0.2423	0.4841	0.4672	0.714	0.6644

Note that: $p_1 = 1 - p_0$. $Ƥ_4 = 1 - Ƥ_3$

Table 4: Percentage asymptotic efficiency e of the proposed procedure with $K = 2,3$ servers. delay group of $Ç = 5$ clients and its approximation by efficiency of a procedure with geometrically distribution e_g with parameter p_g.

			$K = 2$		$K = 3$	
p_0	$Ƥ_4$	p_g	e	e_g	e	e_g
0.1	0.2466	0.1738	0.3516	0.3415	0.5141	0.5
0.2	0.4099	0.1789	0.366	0.3511	0.5258	0.5132
0.3	0.519	0.1824	0.377	0.3578	0.535	0.5223
0.4	0.5958	0.185	0.3855	0.3626	0.5424	0.529
0.5	0.6563	0.1871	0.3927	0.3665	0.5488	0.5343
0.6	0.7114	0.1891	0.3996	0.3702	0.5525	0.5392
0.7	0.7687	0.1912	0.4073	0.374	0.5624	0.5444
0.8	0.8333	0.1935	0.4167	0.3785	0.5714	0.5504
0.9	0.9091	0.1964	0.4286	0.3839	0.58333	0.5575

Note that: $p_1 = 1 - p_0$. $Ƥ_5 = 1 - Ƥ_4$

7 Results and Conclusion

From Table 1 to Table 4 the calculates values refer to the following notes:

1. From Table 1, by increasing the values of p_0 the resulting efficiencies increase until the value $p_0 = 0.6$, we can note that the values of efficiencies are repeated again that is because from (2) $P_1 = 1 - p_0 p_1, P_2 = p_0 p_1$, then their values are repeated that is tended to the same efficiencies.

2. By increasing the values of Ç in Tables 2 to Table 4 with the same number of servers S the efficiencies are decreased.

3. On the other hand, by increasing the number of servers for the same values of Ç the efficiencies are increased so, that is referred to the hint for choice the number of servers to improve the proposed loss system.

4. The results obtained show that the replacement of the procedure in the presence of delayed groups of delayed clients with geometrical time delay distribution by the procedure gives, as a rule, a satisfactory approximation to the efficiency of the procedure.

5. Also, The results obtained show that the proposed procedure can be applied to the investigated loss system which can serve as a model of the modified Robbins-Monro procedure with compound delayed observations.

Conflicts of interest The authors declare that there is no conflict of interest regarding the publication of this article.

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