Quantum Physics Letters
An International Journal

http://dx.doi.org/10.18576/qpl/070201

Momentum Space Treatment of the Two-Body Problem $A+B\rightarrow C+D$ and Its Application to the πN Scattering with Realistic Interaction

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Received: 2 Mar. 2018, Revised: 25 Apr. 2018, Accepted: 28 Apr. 2018
Published online: 1 Aug. 2018

Abstract: Based on a recently developed formalism for the two-body scattering problem $A+B\rightarrow C+D$ successfully applied to the nucleon-nucleon (NN) scattering problem [Int. J. New Horz. Phys. **5**, No. 2, 1-8 (2018)], an explicit momentum space solution for the pion-nucleon (πN) scattering equation is given. The partial wave T-matrix for πN scattering is calculated with a realistic and high-precision πN interaction. In addition, the πN scattering phase shifts for the important πN partial waves are predicted. These phase shifts are also compared with the data points form the Virginia Polytechnic Institute (VPI) partial wave analysis and a good description below 500 MeV is obtained.

Keywords: Scattering theory, Matrix theory, Partial wave analysis, pion-nucleon system, phase shifts

1 Introduction

During the last few years, many experimental and theoretical studies have been devoted to investigate meson photo- and electroproduction on few-body nuclei (see, for example, Refs. [1,2,3] and references therein). New perspectives for the study of these processes have been opened by the possibility of performing experiments using linearly or circularly polarized beams and polarized targets, e.g. at MAMI in Mainz and ELSA in Bonn (Germany), at JLab in Newport News and LEGS in Brookhaven (USA), at MAX-Lab in Lund (Sweden), at SPring-8 in Osaka and ELPH in Tohoku University (Japan), at GRAAL in Grenoble (France), and at VEPP-2 and VEPP-3 electron storage rings in Novosibirsk (Russia).

Most of the important properties of the many-body nuclei are already well described by the two-body interactions [4]. In nuclear physics, the fundamental two-body problem involving the force between two nucleons has not been completely solved as yet, for although interaction potentials exist for which the Schrödinger equation has a simple analytic solution, these potentials do not in practice yield a true value for the two-body force. It is also true that the neutron or proton (nucleon states) considered as one body is not nearly such an elementary particle as was once believed and in some respects may represent a many-body system of its own.

In our previous work [5], the formal expressions of the two-body scattering amplitudes which determine the differential cross section for two-body scattering process have been given and successfully applied to the nucleon-nucleon (NN) scattering process. In order to gain valuable information on the pion-nucleon (πN) scattering as well as the on-shell properties of the πN amplitudes, we investigate in the present work the two-body πN scattering. We present the kinematics and formulas for calculating the differential cross section of the two-body πN scattering and give the solution of the πN scattering equation, which is described by the Lippmann-Schwinger (LS) equation [6], with a realistic and high-precision πN potential model. In addition, the on-shell properties of the πN scattering, πN phase shifts, are calculated and compared to the data points from the Virginia Polytechnic Institute (VPI) partial wave analysis [7].

The organization of this paper is as follows. In Sec. 2, we give the kinematics and formulas for calculating the differential cross section of the two-body scattering. The

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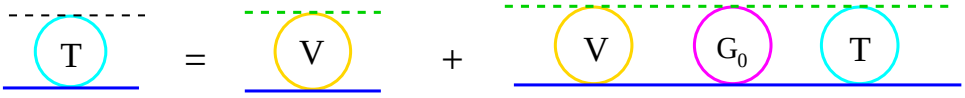

Fig. 1: The graphical diagram of the Lippmann-Schwinger equation for πN interaction.

solution of πN scattering equation, which is described by the LS equation [6], is given in Sec. 3. In Sec. 4, we present an example of the realistic and high-precision πN potential model which can be used to solve the LS equation. The on-shell properties of the πN scattering, πN phase shifts, are calculated and compared to the data points from the VPI analysis [7] in Sec. 5. Finally, we provide a short summary in Sec. 6.

2 Kinematics and cross section

In this section, we consider the two-body scattering process

$$A(p_A)+B(p_B)\to C(p_C)+D(p_D)\,, \tag{1}$$

where $p_i=(E_i,\mathbf{p}_i)$ denotes the four-momentum of particle "i" with $i\in\{A,B,C,D\}$. Conservation of total four-momentum gives $p_A+p_B=p_C+p_D$. Diagrammatic representation of this process is shown in Fig. 1. The Mandelstam variables for this two-body process are defined by the following equations

$$\begin{aligned} s&=(P_A+P_B)^2\,,\\ t&=(P_A-P_C)^2\,,\\ u&=(P_A-P_D)^2\,. \end{aligned} \tag{2}$$

The absolute value of the initial and final center-of-mass (c.m.) three-momenta can be expressed in terms of the Mandelstam variable s and the particle masses, respectively, as follows

$$|\,\mathbf{p}_A\,|^2=\frac{[s-(m_A+m_B)^2]\,[s-(m_A-m_B)^2]}{4s} \tag{3}$$

and

$$|\,\mathbf{p}_C\,|^2=\frac{[s-(m_C+m_D)^2]\,[s-(m_C-m_D)^2]}{4s}\,. \tag{4}$$

Following the conventions of Bjorken and Drell [8], the two-body differential cross section is given in the c.m. system by

$$\frac{d\sigma}{d\Omega_C}=\frac{1}{(2\pi W)^2}\,\frac{p_C}{p_A}\,\frac{E_AE_BE_CE_D}{F_AF_BF_CF_D} \times\frac{1}{s}\sum_{\mu_D\mu_C\mu_B\mu_A}\left|\mathscr{M}_{\mu_D\mu_C\mu_B\mu_A}(\mathbf{p}_D,\mathbf{p}_C,\mathbf{p}_B,\mathbf{p}_A)\right|^2 \tag{5}$$

with $\mathscr{M}_{\mu_D\mu_C\mu_B\mu_A}$ the reaction matrix which has been discussed in detail in Ref. [5], μ_i denotes the spin projection of particle "i" on some quantization axis, and F_i is a factor arising from the covariant normalization of the states and its form depends on whether the particle is a boson ($F_i=2E_i$) or a fermion ($F_i=E_i/m_i$), where E_i and m_i are its energy and mass, respectively. The factor $s=(2s_A+1)(2s_B+1)$ takes into account the averaging over the initial spin states, where s_A and s_B denote the spins of the incoming particles A and B, respectively.

In case of the πN scattering process one obtains

$$F_i=2E_\pi;\qquad i\in\{A,C\}\,, \tag{6}$$

$$F_j=E_N/M_N;\qquad j\in\{B,D\}\,, \tag{7}$$

and therefore one finds $s=1$ and $F=M_N^2/4$. The differential cross section of a πN reaction in the c.m. system is then given by

$$\frac{d\sigma}{d\Omega_C}=\frac{1}{(2\pi W)^2}\,\frac{M_N^2\,p_C}{4\,p_A} \times\sum_{\mu_D\mu_C\mu_B\mu_A}\left|\mathscr{M}_{\mu_D\mu_C\mu_B\mu_A}(\mathbf{p}_D,\mathbf{p}_C,\mathbf{p}_B,\mathbf{p}_A)\right|^2\,. \tag{8}$$

3 Solution of the πN-scattering equation

Analogous to the case of NN-scattering [5], the formalism to obtain the full $\mathscr{T}$-matrix and the scattering phase shifts for πN-scattering is given in this section, but not in more details since we give enough details in case of the NN-scattering [5]. The scattering equation and its three-dimensional reduction are given. A partial wave decomposition of the separable πN potential is performed in the next section.

We discuss here, analogous to the case of NN-scattering [5], how the $\mathscr{T}$-matrix of the πN scattering is calculated. For a given πN potential V, the $\mathscr{T}$-matrix for πN scattering is obtained from the LS equation which in partial wave decomposition, specified by the orbital angular momentum ℓ, total angular momentum J and isospin t, reads[1]

$$\mathscr{T}(p',p;E)=V(p',p)+\int_0^\infty dk k^2\,V(p',k) \times\frac{1}{E-E_\pi(k)-E_N(k)+i\varepsilon}\,\mathscr{T}(k,p;E)\,, \tag{9}$$

where $\mathbf{p}$, $\mathbf{k}$ and $\mathbf{p}'$ are the πN relative momentum in the initial, intermediate and final state, respectively; and $p\equiv|\mathbf{p}|$, $k\equiv|\mathbf{k}|$ and $p'\equiv|\mathbf{p}'|$. $E=\sqrt{M_N^2+p_0^2}+\sqrt{m_\pi^2+p_0^2}$ denotes the total collision energy with on-shell momentum p_0 in

[1] The indices ℓJt are dropped for simplicity.

the c. m. frame. For our calculations we use relativistic kinematics for both pion and nucleon, thus

$$E_\pi(k) = \sqrt{m_\pi^2 + k^2}\ ,$$
$$E_N(k) = \sqrt{M_N^2 + k^2}\ . \qquad (10)$$

The relativistic πN propagator is given by

$$\mathscr{G}_{\pi N}(E) = \frac{1}{E - E_\pi(k) - E_N(k) + i\varepsilon} = G(k)\ \frac{1}{p_0^2 - k^2 + i\varepsilon}\ , \qquad (11)$$

where

$$G(k) = \frac{\left[\sqrt{M_N^2 + p_0^2} + \sqrt{M_N^2 + k^2}\right]\left[\sqrt{m_\pi^2 + p_0^2} + \sqrt{m_\pi^2 + k^2}\right]}{\left[\sqrt{M_N^2 + p_0^2} + \sqrt{M_N^2 + k^2} + \sqrt{m_\pi^2 + p_0^2} + \sqrt{m_\pi^2 + k^2}\right]}\ . \qquad (12)$$

The partial wave $\mathscr{T}$-matrix for πN scattering is then given by

$$\mathscr{T}(p', p; E) = V(p', p)\ V(p', k) + \int_0^\infty dk k^2 \frac{G(k)}{p_0^2 - k^2 + i\varepsilon} \mathscr{T}(k, p; E)\ . \qquad (13)$$

To transform this equation into a principle value integral equation, one uses the identity

$$\frac{1}{x^2 - x_0^2 + i\varepsilon} = \mathscr{P}\frac{1}{x^2 - x_0^2} - i\pi\delta(x^2 - x_0^2)\ , \qquad (14)$$

to get the matrix elements in partial wave decomposition as

$$\mathscr{T}(p', p; E) = V(p', p) + \mathscr{P}\int_0^\infty dk k^2\ V(p', k) \frac{G(k)}{p_0^2 - k^2} \mathscr{T}(k, p; E) - \frac{1}{2} i\pi p_0 G(p_0) V(p', p_0) \mathscr{T}(p_0, p; E)\ . \qquad (15)$$

This one-dimensional integral equation can be solved numerically for a given πN potential model $V(p', p)$ by using the matrix inversion method which is explained in details in case of NN-scattering in Ref. [5]. In this work we solved this equation using a separable πN potential model. This model is given in more details in the coming section.

4 Separable πN Potential Model

In the case of the πN scattering a large number of dynamical models have been developed over the past few years (see for example Refs. [9,10,11]). Most begin with a separable potential which is iterated in a LS equation to give the scattering amplitude, from which phase shifts and observables are obtained. In this work, we used the model of Nozawa *et al.* [11] in order to study the πN interaction. This model is consistent with the existing unitary description of the πNN system and treats the πN interaction dynamically, with all S-, P- and D-wave πN phase shifts being well reproduced below 500 MeV.

For partial wave specified by quantum numbers ℓJt, the πN scattering equation takes the form given in Eq. (9). Our goal now is to determine the πN potential $V(p', p)$ which we used in our calculation. According to [11] this potential is given due to both a vertex interaction $f^0_{\pi N,B}$ and a "background" potential $\mathscr{V}$ (which is assumed to be of a separable form) by

$$V(p', p; E) = \sum_{B=N,\Delta} f^0_{\pi N,B}\ g^0_B(E)\ f^0_{B,\pi N} + \mathscr{V}\ , \qquad (16)$$

where $g^0_B(E)$ is the free propagator in subspace B and takes the form

$$g^0_B = \frac{1}{E - m_{0B}}\ , \qquad (17)$$

with m_{0B} is the bare mass of the baryon.

For P_{11} and P_{33} channels, both terms in the right-hand-side of Eq. (16) are taken into account. Thus, the total potential matrix element can be written in terms of *baryon* pole and non-pole parts as follows

$$V(p', p; E) = f_0(p')\ g^0_B(E)\ f_0(p) + h_0(p')\ \lambda_0\ h_0(p)\ , \qquad (18)$$

where λ_0 is a phase parameter and it is given in Table 1 [11]. The form factors $h_0(k)$ and $f_0(k)$ are parameterized as follows

$$h_0(k) = \frac{a_1\ k^{m_1}}{(k^2 + b_1^2)^{n_1}}\ k^\ell\ , \qquad (19)$$

$$f_0(k) = \frac{a_2\ k^{m_2}}{(k^2 + b_2^2)^{n_2}}\ k^\ell\ , \qquad (20)$$

The values of the parameters n_1, n_2, m_1, m_2, a_1, a_2, b_1 and b_2 are given in Table 1 for each partial wave [11].

By inserting Eq. (18) into Eq. (9), the πN amplitude can be written as

$$\mathscr{T}(p', p; E) = T^{NP}(p', p; E) + f(p')\ g_B(E)\ f(p)\ , \qquad (21)$$

where the non-pole T^{NP}-matrix is given by

$$T^{NP}(p', p; E) = h_0(p')\ \tau_0(E)\ h_0(p)\ , \qquad (22)$$

with

$$\tau_0(E) = \frac{\lambda_0}{1 - \lambda_0 \int dk k^2\ |h_0(k)|^2\ \mathscr{G}_{\pi N}(k, E)}\ . \qquad (23)$$

The pole term in Eq. (21) consists of a dressed form factor $f(k_\alpha)$ $(\alpha = i, f)$ and a dressed propagator $g_B(E)$ defined by

$$f(p_\alpha) = f_0(p_\alpha) + \tau_0(E)\ h_0(p_\alpha) \int dk k^2 h_0(k) f_0(k) \mathscr{G}_{\pi N}(k, E)\ , \qquad (24)$$

Table 1: Parameters of the πN separable potential of Nozawa *et al.* [11] for the πN partial waves. These parameters are determined by fitting the phase shift data [12] up to 500 MeV pion laboratory kinetic energy.

$L_{2t,2J}$	ℓ	m_1	n_1	$a_1^{\dagger}$	$b_1^{\S}$	m_2	n_2	$a_2^{\ddagger}$	$b_2^{\S}$	λ_0
S_{11}	0	0	3	100.00	2.598	2	2	4.9520	2.877	–1
S_{31}	0	0	2	3.0850	1.806	2	2	1.9250	1.275	+1
P_{11}	1	2	3	31.623	2.665	0	2	0.5793	1.185	–1
P_{13}	1	0	2	0.4269	1.181	2	3	3.9700	1.721	+1
P_{31}	1	0	2	1.4730	1.542	2	3	8.0530	1.861	+1
P_{33}	1	0	2	2.7700	1.415	0	2	1.7780	1.218	+1
D_{13}	2	0	2	1.6390	2.165¶	2	3	9.3120	3.263	–1
D_{15}	2	0	2	0.2172	1.175	2	3	1.0110	1.461	–1
D_{33}	2	0	2	0.1306	1.128	2	3	1.0810	1.972	–1
D_{35}	2	0	2	0.2270	1.168	2	3	1.1510	1.780	+1

† a_1 is given in units of $(\text{fm})^{-2n_1+m_1+\ell+1}$.
‡ a_2 is given in units of $(\text{fm})^{-2n_2+m_2+\ell+\frac{1}{2}}$ for P_{11} and P_{33} partial waves, otherwise it is given in units of $(\text{fm})^{-2n_2+m_2+\ell+1}$.
§ b_1 and b_2 are given in units of $(\text{fm})^{-1}$.
¶ This value is a misprint in Table 2 of Ref. [11]. The correct value is obtained from [13].

$$g_B(E) = \frac{1}{E - m_{0B} - \int dk k^2 \; f(k) \; f_0(k) \, \mathscr{G}_{\pi N}(k,E)} \; . \tag{25}$$

For channels other than P_{11} and P_{33}, the vertex interaction of Eq. (16) does not contribute. The πN potential is then assumed to be of rank 2 separable form

$$\mathscr{V}(p',p) = h_1(p') \; \lambda_1 \; h_1(p) + h_2(p') \; \lambda_2 \; h_2(p) \; . \tag{26}$$

The form factors are parameterized as

$$h_1(k) = \frac{a_1 \; k^{m_1}}{(k^2 + b_1^2)^{n_1}} \; k^{\ell} \; , \tag{27}$$

$$h_2(k) = \frac{a_2 \; k^{m_2}}{(k^2 + b_2^2)^{n_2}} \; k^{\ell} \; . \tag{28}$$

As before, the parameters n_1, n_2, m_1, m_2, a_1, a_2, b_1 and b_2 are given in Table 1 for each partial wave [11].

By inserting Eq. (26) into Eq. (16), one obtains the following analytic solution

$$\begin{aligned} \mathscr{T}(p',p;E) = & \; h_1(p') \; \tau_{11}(E) \; h_1(p) \\ & + h_1(p') \; \tau_{12}(E) \; h_2(p) \\ & + h_2(p') \; \tau_{21}(E) \; h_1(p) \\ & + h_2(p') \; \tau_{22}(E) \; h_2(p) \, , \end{aligned} \tag{29}$$

where

$$\tau_{11}(E) = \frac{\lambda_1 (1 - \lambda_2 H_2)}{(1 - \lambda_1 H_1)(1 - \lambda_2 H_2) - \lambda_1 \lambda_2 H_{12}^2} \; , \tag{30}$$

$$\begin{aligned} \tau_{12}(E) &= \tau_{21}(E) \\ &= \frac{\lambda_1 \lambda_2 H_{12}}{(1 - \lambda_1 H_1)(1 - \lambda_2 H_2) - \lambda_1 \lambda_2 H_{12}^2} \; , \end{aligned} \tag{31}$$

$$\tau_{22}(E) = \frac{\lambda_2 (1 - \lambda_1 H_1)}{(1 - \lambda_1 H_1)(1 - \lambda_2 H_2) - \lambda_1 \lambda_2 H_{12}^2} \; , \tag{32}$$

with

$$H_1 = \int dk k^2 \; |h_1(k)|^2 \, \mathscr{G}_{\pi N}(k,E) \; , \tag{33}$$

$$H_2 = \int dk k^2 \; |h_2(k)|^2 \, \mathscr{G}_{\pi N}(k,E) \; , \tag{34}$$

$$H_{12} = \int dk k^2 \; h_1(k) \; h_2(k) \, \mathscr{G}_{\pi N}(k,E) \; . \tag{35}$$

5 Calculation of πN Phase Shifts

Once the scattering equation given in Eq. (15) is solved for the on-shell $\mathscr{T}$-matrix, one can obtains the scattering phase shifts. The information about the scattering process is commonly represented by phase shifts $\delta(E)$ with the on-shell S-matrix in each channel defined by

$$S(E) = e^{2i\delta(E)} \; , \tag{36}$$

or

$$S(E) = 1 + 2i \, e^{i\delta(E)} \; \sin(\delta(E)) \; , \tag{37}$$

where $E(p) = \sqrt{M_N^2 + p^2} + \sqrt{m_\pi^2 + p^2}$ is the invariant total energy of the two interacting particles. The partial wave on-shell S-matrix is related to the partial wave on-shell $\mathscr{T}$-matrix by

$$S(E) = 1 - 2i\pi \; \rho(p) \; \mathscr{T}(E) \; , \tag{38}$$

where the density of states $\rho(p)$ is given by

$$\rho(p) = \frac{p^2}{\frac{dE(p)}{dp}} = \frac{p E_N(p) E_\pi(p)}{E_N(p) + E_\pi(p)} \; . \tag{39}$$

Combining Eqs. (36) and (38) gives

$$\tan(2\delta(E)) = \frac{-2\pi\,\rho(p)\,\Re e(\mathscr{T}(E))}{1+2\pi\,\rho(p)\,\Im m(\mathscr{T}(E))}\,. \qquad (40)$$

Alternatively, we can express the on-shell $\mathscr{T}$-matrix in terms of the phase shifts as

$$\mathscr{T}(E) = \frac{-1}{\pi\rho(p)}\,e^{i\delta(E)}\,\sin(\delta(E))\,, \qquad (41)$$

whence

$$\tan(\delta(E)) = \frac{\Im m(\mathscr{T}(E))}{\Re e(\mathscr{T}(E))}\,. \qquad (42)$$

The phase shifts calculated from the dynamical model of Nozawa *et al.* [11] for the more important πN scattering partial waves are shown in Figs. 2-4 with the corresponding parameters being given in Table 1. In these figures we also compare the calculated phase shifts with the data points from the VPI partial wave analysis [7]. We see that the LS equation gives a good description of πN phase shifts below 500 MeV. We note in particular the perfect resonance shape of the P_{33} phase shift corresponding to the $\Delta(1232)$ resonance of the nucleon (δ=90^0 at $T_{lab} \simeq$ 180 MeV). The steep rise of the P_{11} phase shift for $T_{lab} \to$ 500 MeV is an indication of the Roper resonance N(1440). In addition there is some background scattering due to interactions in the S_{11} and S_{31} partial waves.

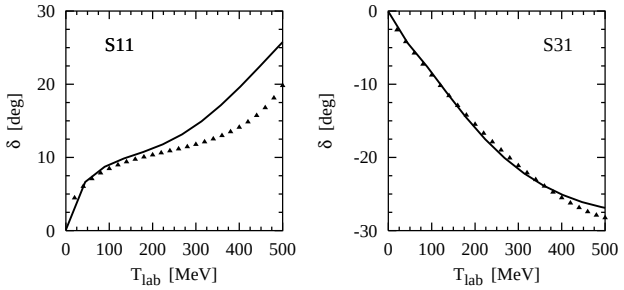

Fig. 2: The πN scattering phase shifts of the S partial waves obtained from the LS equation using the separable potential model of Nozawa *et al.* [11] shown versus the pion laboratory energy T_{lab} in MeV. The data points are from the VPI partial wave analysis [7].

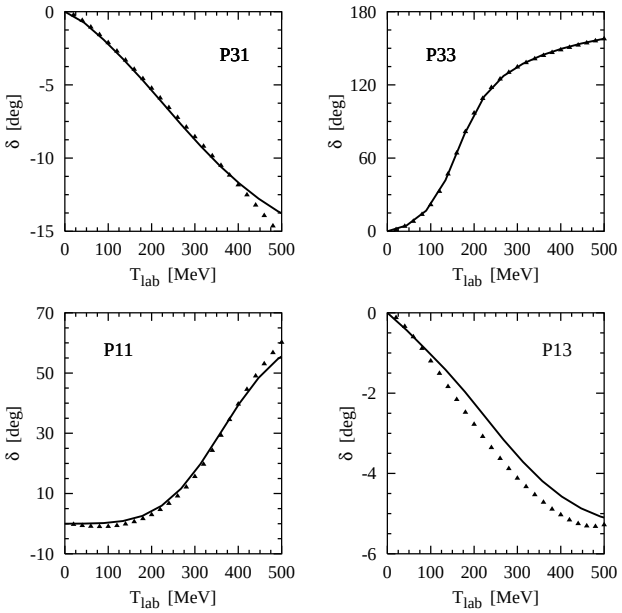

Fig. 3: Same as Fig. 2 but for the P partial waves.

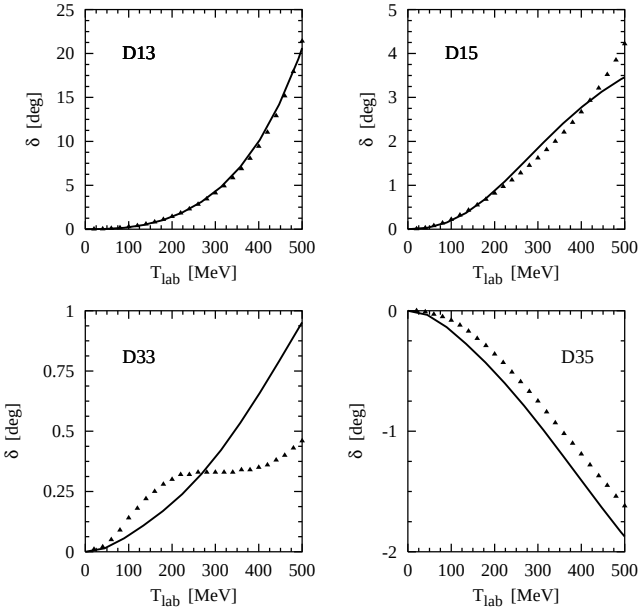

Fig. 4: Same as Fig. 2 but for the D partial waves.

6 Summary

In this work we have applied the theoretical formulation for the two-body scattering problem $A+B \to C+D$ [5] to the πN scattering problem. We have solved the πN scattering equation with a realistic and high-precision πN interaction and calculated the πN scattering phase shifts for all S, P, and D partial waves. The calculated phase shifts are compared with the data points form the VPI partial wave analysis [7]. We have obtained a good agreement between our prediction for the πN scattering phase shifts and the data points from Ref. [7]. The expressions given in this work are generally valid for studying the πN scattering in many-body problem.

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