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A Robust Fractional Model Predictive Control Design

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Abstract: In this paper, an efficient fractional model predictive controller (FMPC) is proposed to control an underactuated system, and its robustness is analysed by varying the system parameters. The performance merit of proposed design method of FMPC controller is compared to the MPC and various other existing controller designs. It is noticed using simulation that the transient response of the considered system improved by using FMPC control.

Keywords: Robustness, fractional calculus, Crane system, fractional model predictive control.

1 Introduction, preliminaries and motivation

The fractional order (FO) calculus have generated considerable recent research interest and has found various applications, especially in control systems. K. B. Oldhamand et al. [1] have given an extensive background study, and many researchers have been used the fractional order calculus to improve the performance of dynamical systems [2,3,4,5,6,7,8,9]. Many real-time dynamical systems possess fractional order behavior; therefore, fractional order calculus can provide a better response as it can attain the more accurate values of the required parameters. Designing an optimized controller for a systems is required and it is active fields o research currently, and fractional order controllers have been demonstrated commendable in this field [10,11,12,13,14,15]. Recently, several studies have been done on model predictive control (MPC) [16,17,18] in which the system parameters are controlled with information of the system's model which anticipate the system's behavior for further action. An efficient design of fractional model predictive controller is proposed in this paper for an underactuated system, i.e. 2 D Gantry Crane. The controller design proposed has less error, and it can stabilize the underactuated system in minimum time compared to the existing system.Various existing studies in MPC [19] and other controllers [20,21,22,23,24,25,26,27] designed for underactuated system have motivated this study. The Euler-Lagrange formulation is used to get the integer order (IO) modeling of the considered underactuated system. To design FMPC controller, the FO model of the system is derived from the IO model. To get real world significance Oustaloup-recursive-approximation is use to get approximated model of the considered system. The simulation experiment shows that the out response of FMPC is good when compared to MPC. Moreover, the robustness of a system is analyzed by varying the system parameters. After discussing a brief background in Section 1, the system modeling and its fractional embedding are discussed in Section 2. Section 3 provides the robustness analysis followed by conclusion.

2 Modeling of the system

The aim of this paper is, modeling and control of an underactuated system. The considered system is a 2-D gantry crane system shown in Fig. 1. It is used generally in industries, platforms, depots, etc. to carry load. This system is efficient and can handle heavy loads as well.

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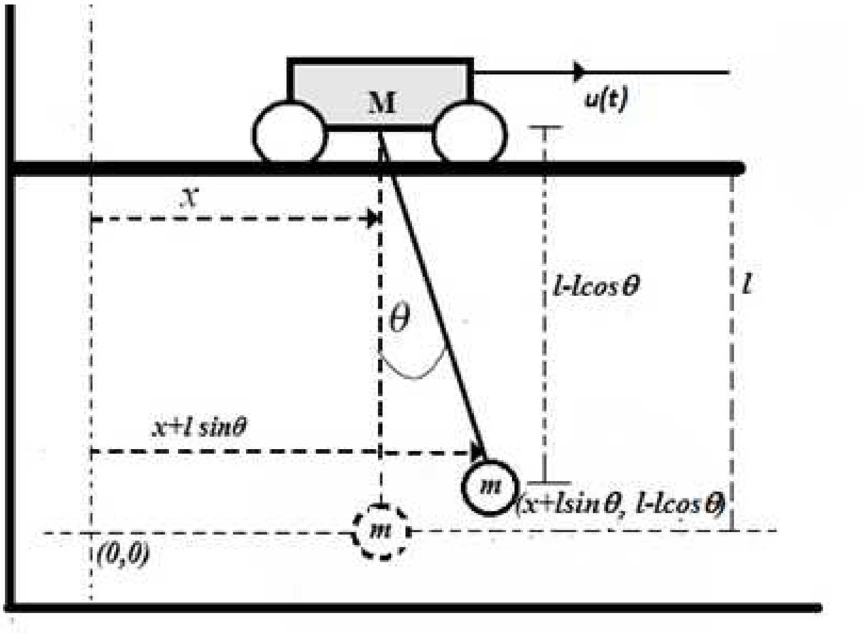

Fig. 1: Schematic diagram of 2-D Gantry crane

System's model equations can be achieved by taking into account the Euler-Lagrange(EL) formulation, and the Lagrangian equation for a system is given by

$$L = T - V \tag{1}$$

i.e., necessary calculations of kinetic and potential energy to get Lagrangian of a system is required. The various parameters to obtain the Lagrangian of the system are given as

$u(t)$ = force on the trolley in x-direction
M= mass of the cart
g = acceleration due to gravity
l = length of the cable
$x(t)$ = cart position
$\theta(t)$ = tilt angle in the vertical direction
m = the mass of the load

After various mathematical manipulations, linearization and choosing the values of values of $m = 1$ kg, $g = 9.8m/sec^2$, $l = 1$ m, $M = 2.5$ kg, we get a linear equation of the system as

$$\frac{d}{dt}(\delta \underline{z}) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 3.92 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -13.72 & 0 \end{pmatrix} \delta \underline{z} + \begin{pmatrix} 0 \\ 0.4 \\ 0 \\ -0.4 \end{pmatrix} \delta u \tag{2}$$

and,

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta}. \end{pmatrix} \tag{3}$$

After finding the Lagrangian, the FO model is incorporated in this system. To get physical interpretation, this model is then approximated by Oustaloup recursive method [28], then it is used to design the controller. Many real-time

Table 1: Output Response Comparison Table

Specifications and α	Settling Time (Sec)		Overshoot		Oscillations	
	x(t)	$\theta(t)$	x(t)	$\theta(t)$	x(t)	$\theta(t)$
0.3	0.8	16	No	No	No	No
0.5	0.8	3.5	No	No	No	No
0.8	0.8	5	Yes	Yes	No	Yes
1	3.8	4	Yes	Yes	Yes	Yes

dynamical systems possess fractional order behavior [29,30,31]; therefore, fractional order modeling will improve system performance. The Integer Order modeling is popular amongst the researchers because of the lack of the solution for fractional models [32]; however, different methodologies are present now for approximation to fractional equivalent [29,33,34] and used in control theory [35], circuital analysis [36], system models [37], mechanical systems analysis [38], etc. Bys using (2) and (3), a transfer function can be obtained as (4).

$$H(s) = \begin{pmatrix} \frac{0.4s^2+4}{s^4+13.7s^2} \\ \frac{-0.4s^2}{s^4+13.7s^2}. \end{pmatrix} \tag{4}$$

The fractional transfer function (TF) for this system can be obtained by introducing the fractional term in the TF [4,3, 40],

$$H(s) = \begin{pmatrix} \frac{0.4s^{2\alpha}+4}{s^{4\alpha}+13.7s^{2\alpha}} \\ \frac{-0.4s^{2\alpha}}{s^{4\alpha}+13.7s^{2\alpha}}. \end{pmatrix} \tag{5}$$

Selecting different values of $\alpha = 1$ in Eq. (5), will give different equivalent fractional dynamical models of the system, and the value of α is between $0 < \alpha < 1$. Substituting $\alpha = 1$ in (5) will result in the same transfer function represented in (4). The selection of α value is critical and important. FMPC can bedesigned by validating the model using the simulation for different values of α.

3 Robustness of the FMPC to the variation of the system parameters

An MPC toolbox of MATLAB [41] is used opted to design controller which can also be seen in [42,43]. The structural diagram of toolbox shown in Fig.2. The system has one input and two outputs.

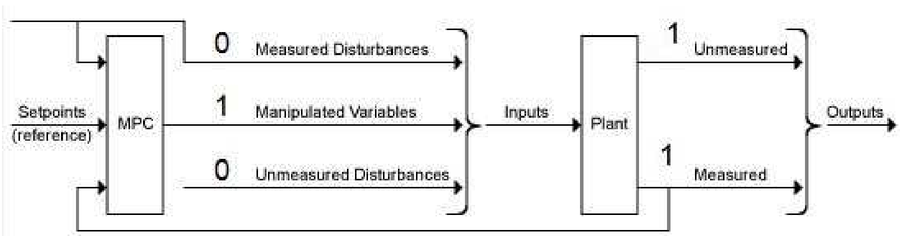

Fig. 2: MPC toolbox structure

The input applied to the system, as shown in Fig. 2 and has two outputs which are cart positions and swing angle. The designed MPC has prediction horizon 30, sampling interval 0.1 sec, and control horizon 6. To control the motion $x(t)$ and to control the swing $\theta(t)$ is the main objective with minimum oscillations and minimum overshoot for different values of $\alpha = 0.3$, 0.5, 0.8, and 1. Fig. 3 shows the output response and a comparison Table can be drawn from Fig. 3,

It can be observed from Table 1 that the FMPC gives a better response. Substituting $\alpha = 0.5$ gives the best response in terms of settling time, overshoot, and oscillations. Therefore, for further analysis, this paper will consider $\alpha = 0.5$ as best FO model and robustness is checked for this particular model of the system.

Now for analysing the robustness of the controller, two scenarios are considered, i.e., variation in the mass of the load and variation in the mass of the cart(trolley).

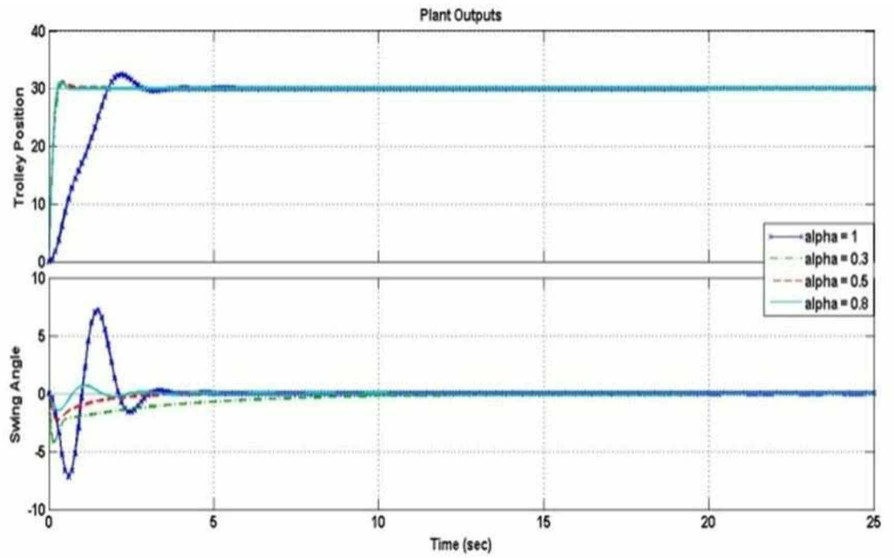

Fig. 3: Output Response Comparison for α = 0.3, 0.5, 0.8, 1

3.1 When the mass of cart changes

The FMPC model is analyzed for different values of cart mass by keeping α = 0.5. Two cases for cart mass M= 50kg and M=10kg are analyzed in this section. It can be concluded from the Fig. 4 that the controller corresponding to FMPC has very good settling time, and there is no oscillation with zero overshoot for both swing angle and position when trolley mass changes. However, the response corresponding to traditional MPC (figure 4 corresponding to α=1) concludes that the settling time is poor, it produces the oscillations, and it has an overshooting if the cart mass changes. For the higher value of trolley mass, traditional MPC has higher oscillation and overshooting.

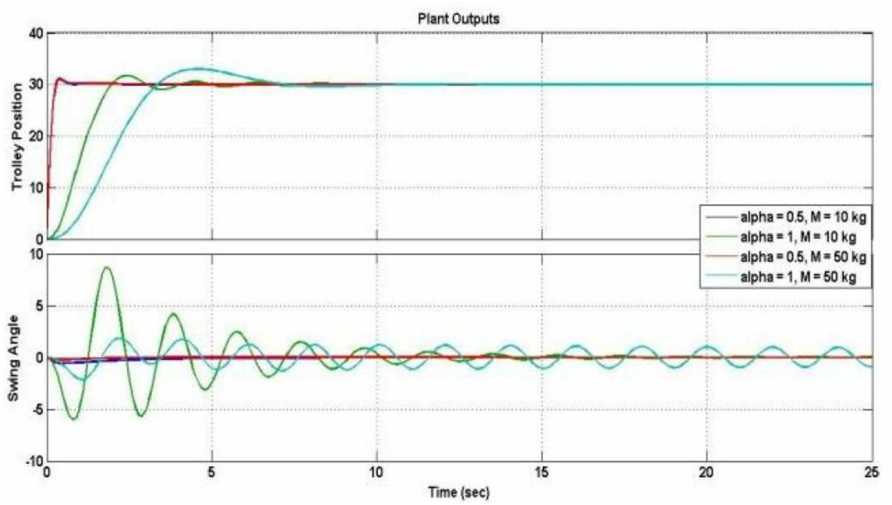

Fig. 4: Output Response of FMPC and MPC controllers when trolley mass M changes

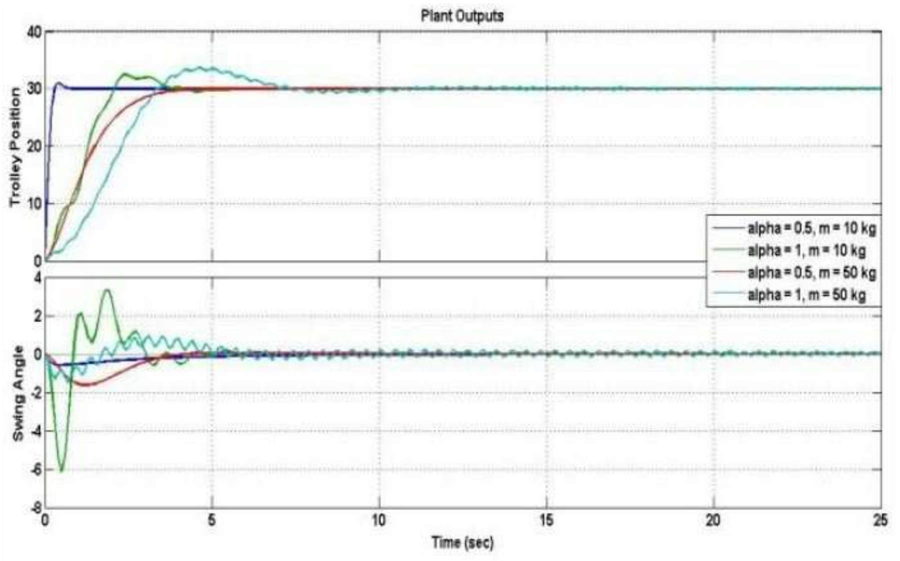

Fig. 5: Output Response of FMPC and MPC controllers when load mass m changes

Table 2: Summary table when the trolley mass changes

Specifications and α	Mass (kg) M	Settling Time (Sec) x(t)	Settling Time (Sec) $\theta(t)$	Overshoot x(t)	Overshoot $\theta(t)$	Oscillations x(t)	Oscillations $\theta(t)$
0.5	10	0.66	14.4	No	No	No	No
0.5	20	0.75	9.2	No	No	No	No
1	10	8.9	17.8	Yes	Yes	Yes	Yes
1	20	9.5	90	Yes	Yes	Yes	Yes

Table 3: Summary table when the load mass changes

Specifications and α	Mass (kg) M	Settling Time (Sec) x(t)	Settling Time (Sec) $\theta(t)$	Overshoot x(t)	Overshoot $\theta(t)$	Oscillations x(t)	Oscillations $\theta(t)$
0.5	10	0.66	17.4	No	No	Yes	No
0.5	20	4.03	7.18	No	No	No	No
1	10	5.65	7.75	Yes	Yes	Yes	Yes
1	20	28.5	70	Yes	Yes	Yes	Yes

3.2 When the mass of load changes

The FMPC model is analysed for different values of load mass by keeping α = 0.5. Two cases for cart mass m= 50kg and m=10kg are analysed in this section. It can be concluded from Fig. 5 that the controller corresponding to FMPC has very good settling time, and there is no oscillation with zero overshoot for both swing angle and position when load mass changes. However, the response corresponding to traditional MPC (figure 5 corresponding to α=1) concludes that the settling time is poor, it produces the oscillations, and it has an overshooting if the load mass changes. For the higher value of load mass, traditional MPC has higher oscillation and overshooting.

Therefore, it can be concluded from Table 2 and Table 3 that the FMPC designed for 2-D Gantry crane system is robust, and corresponding performance is improved in all aspects compared to the traditional MPC of this system.

The results obtained in this paper is compared with the existing controllers for the same system in Table 4. The comparison Table 4 tells us that FMPC controller gives a better response than all the existing controllers.

Table 4: Comparison with the existing control strategy

Publisher	Authors	Trolley settling time (sec)	Swing angle settling time (sec)	Methodology used
Acta Polytechnica Hungarica	Fetah Kolonic, Alen Poljugan, Ivan Petrovic (2006)	3	4	Tensor Product
Journal of Automation and Control Engineering	Le Anh Tuan (2016)	5	6	Sliding Mode Controller
Indian Journal of Engineering	Materials Sciences	Huseyin Arpaci and O Faruk Ozguvenb (2011)	6, 6, 8	6, 6, 6 Fractional PID, ANFIS Controller, PID Controller
International Journal of Advanced Robotic Systems	Wahyudi; Jamaludin Jalani, et al. (2007)	5.36	4	NCTF Control
Engineering, Scientific research	Shebel Asad, Maazouz Salahat, et al. (2011)	10	-	Fuzzy-PD controller
-	**Proposed Controller**	**0.8**	**3.5**	**Uses fractional model predictive control strategy**

4 Conclusion

It is observed in this paper that the overshooting is minimized and settling time is reduced for 2D Gantry crane system by using FMPC control. The proposed design of FMPC gives better results in all control aspects compared to traditional MPC and various other existing controller designs. Moreover, it is identified in this paper that FMPC is a robust controller to the variations of the system parameters of the considered crane system when compared to a traditional MPC controller.

References

[1] K. B. Oldham and J. Spanier, *The fractional calculus*, Academic Press, New York, 1974.
[2] I. Podlubny, Fractional-order systems and controllers, *IEEE Transact. Automat. Contr.* **44**(1), 208–214 (1999).
[3] H. Lubich, Discretized fractional calculus, *SIAM J. Math. Anal.* 704–719 (1986).
[4] T. Srivastava, A. P. Singh and H. Agarwal, Modeling the under-actuated mechanical system with fractional order derivative, *Progr. Fract. Differ. Appl.* **1** (1), 57–64 (2015).
[5] A. Oustaloup, Fractional order sinusoidal oscillators: optimization and their use in highly linear FM modulators, *IEEE Transact. Circ. Syst.*, 1007–1009 (1981).
[6] C. Ma and Y. Hori, Fractional order control and its application of PI/sup/spl alpha//D controller for robust two-inertia speed control, *Power Electronics and Motion Control Conference, IEEE* **3**, 1477–1482 (2004).
[7] D. Baleanu and O. P. Agrawal, Fractional Hamilton formalism within Caputo's derivative, *Czech. J. Phys.* **56**(10-11), 1087–1092 (2006).
[8] D. Baleanu, S. Muslih and K. Tas, Fractional Hamiltonian analysis of higher order derivatives systems, . *J. Math. Phys.* **55**(6), 633–642 (2006).
[9] S. I. Muslih and D. Baleanu, Formulation of Hamiltonian equations for fractional variational problems. *Czech. J. Phys. 55*(6), 633–642 (2005).
[10] A.P. Singh, F. S. Kazi, N. M. Singh and P. Srivastava, $PI\alpha D\beta$ controller design for underactuated mechanical systems, *Control Automation Robotics & Vision (ICARCV)*, 2012 12th International Conference, 1654–1658 (2012).
[11] P. Srivastava, A. P. Singh and N. M. Singh, Observer design for non-linear systems, *Int. J. Appl. Eng. Res. (IJAER) 8*(8), 957–967 (2013).
[12] K. S. Miller and B. Ross, *An introduction to the fractional calculus and fractional differential equations*,John Wiley and Sons, New York, 1993.
[13] A. P. Singh, H. Agarwal and P. Srivastava, Fractional order controller design for inverted pendulum on a cart system (POAC), *WSEAS Transact. Syst. Contr. 10*, 172–178 (2015).
[14] A. P. Singh, F. Kazi, N. M. Singh and V. Vyawahare, Fractional Order controller design for underactuated mechanical systems, *The 5th IFAC symposium on fractional differentiation and its applications-FDA*, 2012.
[15] P. Shah, S. D. Agashe and A. P. Singh, Design of fractional order controller for undamped control system, Engineering (NUiCONE), *Nirma University International Conference, IEEE*, 1–5 2013.
[16] J. Maciejowski, *Predictive control with constraints*, Prentice-Hall, Englewood Cliffs, NJ, 2002.
[17] E. F. Camacho and C. Bordons, *Model predictive control*, Springer-Verlag, Berlin, 2004.
[18] J. A. Rossiter, *Model-based predictive control: a practical approach*, CRC Press, Boca Raton, 2003.
[19] B. Kapernick and K. Graichen, Model predictive control of an overhead crane using constraint substitution, In *American Control Conference (ACC)*, IEEE, 3973–3978, 2013.
[20] H. Wei, S. Zhang and S. Sam Ge, Adaptive control of a flexible crane system with the boundary output constraint, *Ind. Electr. IEEE Transact.* **61**(8), 4126–4133 (2014).
[21] W. Mahmud and I. Solihinand, Sensorless anti-swing control for automatic gantry Crane system: model-based approach, *Int. J. Appl. Eng. Res.* **2**(1), 147–161 (2007).
[22] M. S. Kenneth and B. Ross, Fractional Green's functions, *Ind. J. Pure Appl. Math.* **22**(9), 763–767 (1991).
[23] R. Aymen and F. Bouani, Robust model predictive control of uncertain fractional systems: a thermal application, *IET Contr. Theor. Appl.* **8**(17), 1986–1994 (2014).

[24] M. M. Joshi, V. Vyawahare and A. V. Tare, Design of model predictive control for linear fractional-order systems, *In Intelligent Control and Automation (WCICA)*, 11th World Congress, IEEE, 4452–4457 2014.

[25] Z. Deng, H. Cao, X. Li, J. Jiang, J. Yang and Y. Qin, Generalized predictive control for fractional order dynamic model of solid oxide fuel cell output power, *J. Power Sour. 195*(24), 97–103 (2010).

[26] D. Boudjehem and B. Boudjehem, *A fractional model predictive control for fractional order systems*, In Fractional dynamics and control, Springer New York, 59–71 (2012).

[27] J. Yu, F. L. Lewis and T. Huang, Nonlinear feedback control of a gantry crane, In *American Control Conference, Proceedings*, **6**, IEEE, 4310–4315 (1995).

[28] D. Xue, C. Zhao and Y. Chen, A modified approximation method of fractional order system, In *Mechatronics and Automation, Proceedings of the 2006 IEEE* International Conference, 1043–1048 (2006).

[29] J. Sabatier, O. P. Agrawal and J. A Tenreiro Machado, *Advances in fractional calculus*, **4**(9), Dordrecht, Springer, 2007.

[30] T. Aleksei, E. Petlenkov, J. Belikov and M. Halas, Design and implementation of fractional-order PID controllers for a fluid tank system, In *American Control Conference (ACC)*, IEEE, 1777–1782 (2013).

[31] E. Kaslik and S. Sivasundaram, Nonlinear dynamics and chaos in fractional-order neural networks,*Neural Net.* **32**, 245–56 (2012).

[32] Y. Chen, I. Petráš and D. Xue, Fractional order control-a tutorial. In *American Control Conference*, 1397–1411 (2009).

[33] B. M. Vinagre, I. Podlubny, A. Hernandez and V. Feliu, Some approximations of fractional order operators used in control theory and applications, *Fract. Calc. Appl. Anal.* **3**(3), 231–48 (2000).

[34] M. S. Tavazoei and M. Haeri, Unreliability of frequency-domain approximation in recognising chaos in fractional-order systems, *IET Sign. Proc.* **1**(4), 171–181 (2007).

[35] M. L. Corradini, R. Giambò and S. Pettinari, On the adoption of a fractional-order sliding surface for the robust control of integer-order LTI plants, *Automatica 51*, 364–71 (2015).

[36] T. J. Freeborn, B. Maundy and A. S. Elwakil, Fractional-order models of supercapacitors, batteries and fuel cells: a survey, *Mat. Renew. Sust. Energ.* **4**(3), 1–7 (2015).

[37] B. Xu, D. Chen, H. Zhang and F. Wang, The modeling of the fractional-order shafting system for a water jet mixed-flow pump during the startup process, *Commun. Nonlin. Sci.* **29**(1), 12–24 (2015).

[38] C. K. Kwuimy, G. Litak and C. Nataraj, Nonlinear analysis of energy harvesting systems with fractional order physical properties, *Nonlin. Dynam.* **80**(1-2), 491–501 (2015).

[39] W. Krajewski and U. Viaro, A method for the integer-order approximation of fractional-order systems, *J. Frank. Inst.* **351**(1), 555–564 (2014).

[40] T. Hartley and C. Lorenzo. Fractional-order system identification based on continuous order-distributions, *Sign. Proc.* **83**(11), 2287–2300 (2003).

[41] A. Bemporad, M. Morari and N. L. Ricker, *Model predictive control toolbox 3 user's guide*, The Mathworks, 2010.

[42] A. P. Singh and H. Agrawal, A fractional model predictive control design for 2-D gantry Crane system, *J. Engin. Sci. Techn.* **13**(7), 2224–2235 (2018).

[43] A. P. Singh, H. Agrawal and P. Srivastava, Robust fractional model predictive controller (FMPC) design for under-actuated robotic systems, *Int. J. Contr. Automat.* **11**(7), 109–118 (2018).