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Performance Analysis of Cooperative Spectrum Sensing under Noise Uncertainty

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Abstract: Spectrum sensing is a critical technology for primary user detection in cognitive radio networks, in which energy detector is widely used for spectrum sensing due to its generality and low complexity. However, noise uncertainty degrades the sensing performance of energy detection severely because of its dependence on the noise power. In this paper, we take the average power as the decision statistic and derive the closed-formed expressions for the average detection and false alarm probabilities under noise uncertainty. We then demonstrate the performance of the three kinds of well known hard fusion rules (OR, AND and Majority) over AWGN and Rayleigh channels, and we identify the best rule in different conditions. Extensive simulations indicate that in AWGN channels, Majority rule is optimal at the higher SNR, while AND rule performs best for most situations at low SNR. Moreover, OR rule always exhibits the best performance in Rayleigh channels at low SNR. However, when SNR is higher, the optimal rule is changed with the variation of number of cognitive users. Our research is very helpful and meaningful for selecting the proper hard fusion rule in practical cognitive radio networks.

Keywords: Cognitive radio, Hard fusion, Fading channels, Fusion rules, Noise uncertainty, Spectrum sensing.

1 Introduction

Cognitive radio (CR) is seen as one of the crucial technologies for future spectrum utilization [1] [2]. In order to avoid interfering with the primary communication, it is required for the cognitive users to periodically detect if the primary signal is present in that region. So spectrum sensing is one of the key technologies in cognitive radio (CR) networks to use radio frequency resources more efficiently.

There have been many local spectrum sensing algorithms (spectrum sensing is carried out based on an individual user) including matched filtering [3], cyclostationary feature detection [4] and energy detection (power detection) [3]. Unlike other methods, energy detection is a major and basic approach due to its generality and simplicity. However, noise level variation with time (noise uncertainty) makes the energy detection difficult, even causes energy detection to encounter a SNR wall [5]. The noise uncertainty and interference are unavoidable in the real-world, hence robustness to them is a fundamental performance metric. A number of local spectrum sensing algorithms have been proposed to overcome the bad influence of noise uncertainty: multiantenna spectrum sensing [6], eigenvalue based detections [7], OFDM-based detection [8], et al. most of these detection methods have the huge computational burden, need a long collection time, and rely on the correlation of the primary signal. Recently, researchers discover that the cooperative spectrum sensing can be exploited as a feasible solution to tackle this problem [9] [10] [11]. Hard fusion is a simply and widely used scheme in cooperative sensing. However, up to our best knowledge, no performance comparison under noise uncertainty among various fusion rules has been reported. In this paper, we try to fill up this gap and study the spectrum sensing performance of hard schemes under noise uncertainty.

In this paper, we take the average power as the decision statistic and derive the closed-form expressions for detection and false alarm probabilities under noise uncertainty. Then we compare the sensing performance of the three kinds of hard fusion rules (OR, AND and Majority) over AWGN and Rayleigh channels. More importantly, we show that different hard fusion rules will apply to different networks and different sensing channels.

The rest of this paper is organized as follows: in section 2, the system model and power detection are briefly reviewed. In section 3, we discuss the local detection and the hard fusion rules of cooperative spectrum sensing under noise uncertainty and obtain the closed-form expressions for the detection and false alarm probabilities. Numerical simulation results are provided to evaluate the performance of the different fusion rules in section 4. Finally, the conclusions are drawn in section 5.

2 System Model

Spectrum sensing involves deciding whether the primary signal is present or not from the observed signal. The hypothesis testing problem of interest can be expressed as

$$r(n)=\begin{cases} w(n), & H_0 \\ hs(n)+w(n), & H_1 \end{cases} \tag{2.1}$$

where $r(n)$ is the received signal, and $s(n)$ is the primary user's signal to be detected. Likewise, $w(n)$ is the additive white Gaussian noise at the CR receiver, which is assumed to be complex Gaussian random variable with mean zero and the variance $2\sigma^2$, i.e., $w(n) \sim N(0,\ 2\sigma^2)$, and h is the amplitude gain of the wireless channel. Both $s(n)$ and $w(n)$ are modeled as mutually independent complex signals. Additionally, H_0 and H_1 denote hypothesis corresponding to the absence and presence of the primary signal, respectively.

We use the power detector to sense the primary signal, and the test statistic, T, can be written as

$$T=\frac{1}{M}\sum_{n=1}^{M}|r(n)|^2 \tag{2.2}$$

where M denotes the number of samples for the received signal, under both hypotheses H_0 and H_1, the test statistic T follows chi-square distribution. Hence, its Probability Distribution Function (PDF), $f(y)$, can be written as

$$f(y)=\begin{cases} \dfrac{1}{(\frac{\sigma^2}{M})^2 2^M \Gamma(M)} y^{M-1} e^{-\frac{My}{2\sigma^2}} & H_0 \\ \dfrac{M}{2\sigma^2}(\dfrac{y}{s^2})^{\frac{M-1}{2}} e^{-\frac{M(s^2+y)}{2\sigma^2}} \mathrm{I}_{M-1}(M\sqrt{y}\dfrac{s}{\sigma^2}) & H_1 \end{cases} \tag{2.3}$$

where $\Gamma(.)$ is the gamma function and $I_v(.)$ is the vth-order modified Bessel function of the first kind.

2.1 AWGN Channels

Based on the equation(0.1), the detection and false alarm probabilities over AWGN channels, P_d and P_f, can be generally computed as

$$P_d = P_r(y \ge \lambda | H_1) = Q_M(\sqrt{M\frac{P}{\sigma^2}}, \sqrt{M\frac{\lambda}{\sigma^2}}) \tag{2.4}$$

and

$$P_f = P_r(y \ge \lambda | H_0) = \frac{1}{\Gamma(M)}\Gamma(M, \frac{M\lambda}{2\sigma^2}) \tag{2.5}$$

where λ is the decision threshold, P is the signal power, $\Gamma(\cdot,\cdot)$ is the incomplete gamma function, and $Q_u(a,b)$ is the generalized Marcum Q -function [12].

2.2 Rayleigh Channels

If the signal amplitude follows a Rayleigh distribution, then the signal power is exponential distributed [13]

$$f_{Ray}(p)=\frac{1}{\bar{\gamma}}\exp\left(-\frac{p}{\bar{\gamma}}\right), p \ge 0 \tag{2.6}$$

where $\bar{\gamma}$ is the average SNR and $P=\frac{1}{M}|h^2|\left|\sum_{i=1}^{M}|s(n)|\right|^2$. By averaging the conditional P_d in AWGN case as given by the equation(0.2) over the SNR fading distribution (2.6), the detection probability in Rayleigh channels, $\overline{P}_{d\_Ray}$ can be given by

$$\begin{aligned} \overline{P}_{d\_Ray} &= e^{-\frac{M\lambda}{4\sigma^2}} \sum_{j}^{\frac{M}{2}-2} \frac{1}{j!}(\frac{M\lambda}{4\sigma^2})^j + (\frac{4\sigma^2+MP}{MP})^{\frac{M}{2}-1} \\ &\times [e^{-\frac{M\lambda}{4\sigma^2+MP}} - e^{-\frac{M\lambda}{4\sigma^2}} \sum_{j=0}^{\frac{M}{2}-2} \frac{1}{j!}(\frac{M^2\lambda P}{16\sigma^4+4MP\sigma^2})^j] \end{aligned} \tag{2.7}$$

3 Spectrum Sensing under Noise Uncertainty

The above discussion is based on the assumption that the noise is independent and identically-distributed complex Gaussian white noise, and the noise power is accurately known or can be estimated unbiasedly at the receiver. However, it is impossible to have an exact knowledge of the noise power in practical systems because the noise is an aggregation of various sources. We can denote the noise power uncertainty as [14]

$$\sigma'^2 \in [(2/\rho)\sigma_n{}^2, 2\rho\sigma_n{}^2] \qquad (3.1)$$

where $\sigma_n{}^2$ is the nominal noise power of the real part or imaginary part, ρ is a parameter that indicates the quantity of noise uncertainty in dB, and the PDF of noise power is $f(\sigma'^2)$,which follows the uniform distribution. Upon the above assumptions, we discuss the average effect of noise uncertainty in this session.

3.1 Local Spectrum Sensing

Under the above model of noise uncertainty, using the equations(0.3) and (2.5), we obtain,

$$P_{d\_AWGN\_un} = \frac{\rho}{\sigma_n{}^2(\rho^2-1)}\int_{\frac{1}{\rho}\sigma_n{}^2}^{\rho\sigma_n{}^2} Q_M(\sqrt{M\frac{P}{\sigma^2}}, \sqrt{M\frac{\lambda}{\sigma^2}})d\sigma^2 \qquad (3.2)$$

and

$$P_{f\_un} = \frac{\rho}{\sigma_n{}^2(\rho^2-1)}\int_{\frac{1}{\rho}\sigma_n{}^2}^{\rho\sigma_n{}^2} \frac{1}{\Gamma(M)}\Gamma(M, \frac{M\lambda}{2\sigma^2})d\sigma^2 \qquad (3.3)$$

where $P_{d\_AWGN\_un}$ denotes the average detection probability in AWGN Channels under noise uncertainty, and P_{f_un} is the false alarm probability under noise uncertainty over AWGN channels and Rayleigh channels.

Similar to the AWGN channels, based on the equations (2.7) and (3.1), the average detection probability in Rayleigh channels, $\overline{P}_{d\_Ray\_un}$, has the following form

$$\begin{aligned}\overline{P}_{d\_Ray\_un} &= \frac{\rho}{\sigma_n{}^2(\rho^2-1)}\int_{\frac{1}{\rho}\sigma_n{}^2}^{\rho\sigma_n{}^2} e^{-\frac{M\lambda}{4\sigma^2}} \sum_{j}^{\frac{M}{2}-2} \frac{1}{j!}(\frac{M\lambda}{4\sigma^2})^j \\ &+ (\frac{4\sigma^2+MP}{MP})^{\frac{M}{2}-1} \times [e^{-\frac{M\lambda}{4\sigma^2+MP}} - \qquad (3.4) \\ &e^{-\frac{M\lambda}{4\sigma^2}} \sum_{j=0}^{\frac{M}{2}-2} \frac{1}{j!}(\frac{M^2\lambda P}{16\sigma^4+4MP\sigma^2})^j]d\sigma^2\end{aligned}$$

3.2 Cooperative Spectrum Sensing

Here, we discuss the hard-information based cooperative sensing scheme since the hard fusion rules have minimum communication overhead (control channels) and are easy to be implemented. Moreover, we mainly focus on the performance comparison for signal detection under noise uncertainty for the three kinds of hard-fusion rules.

After finished the local spectrum sensing, each sensor makes a binary decision b_i , where $b_i = 0$ and $b_i = 1$, response to the primary signal is absent and present, respectively. Then all of the sensors send their local 1-bit decisions to the fusion center [15]

$$\delta = \begin{cases} 1 & if \quad \mathrm{D} \geq k \quad H_1 \\ 0 & if \quad \mathrm{D} < k \quad H_0 \end{cases} \qquad (3.5)$$

where H_1 and H_0 denote the primary signal is transmitted and not transmitted, respectively. In addition, $D = \sum_{i=1}^{N} b_i$, δ denotes the final decision in the fusion center, and k is the decision threshold in the fusion center. Hard fusion scheme is also called as the k rule decision, the OR rule corresponds to the case $k = 1$, the AND rule corresponds to the case $k = N$, and $k = \lceil N/2 \rceil$ is the Majority rule.

In the hard fusion scheme, the signal detection probability $Q_{d\_or\_un}$ and the false alarm probability $Q_{f\_or\_un}$ for the OR rule are given by [16]

$$Q_{f\_or\_un} = 1-\left(1-P_{f\_un}\right)^N \qquad (3.6)$$

$$Q_{d\_or\_un} = 1-\left(1-P_{d\_un}\right)^N \qquad (3.7)$$

In the AND rule, $Q_{d\_and\_un}$ and $Q_{f\_and\_un}$ can be expressed as [16]

$$Q_{f\_and\_un} = P_{f\_un}{}^N \qquad (3.8)$$

$$Q_{d\_and\_un} = P_{d\_un}{}^N \qquad (3.9)$$

and [16]

$$Q_{f\_majority\_un} = \sum_{l=k}^{N}\binom{N}{l}P_{f\_un}{}^l\left(1-P_{f\_un}\right)^{N-l} \qquad (3.10)$$

$$Q_{d\_majority\_un} = \sum_{l=k}^{N}\binom{N}{l}P_{d\_un}{}^l\left(1-P_{d\_un}\right)^{N-l} \qquad (3.11)$$

where $Q_{d\_majority\_un}$ and $Q_{f\_majority\_un}$ denote the detection probability and the false alarm probability for the Majority rule, respectively. Also, it can be seen that the probability of missed detection $Q_m = 1 - Q_d$.

4 Performance Analysis of Cooperative Spectrum Sensing under Noise Uncertainty

In this section, we mainly compare the sensing performance of the above-mentioned three kinds of hard fusion rules over AWGN and Rayleigh channels in different cases. Without loss of generality, we assume the signals from the primary user to all users are independent and identically distributed.

To select a proper fusion rule in different situations, extensive experiments are done and typical results are shown in this section. Moreover, both theoretical simulations and Monte Carlo simulations are carried out.

4.1. AWGN Channels

Firstly, we discuss the performance of different hard fusion rules over the AWGN channels.

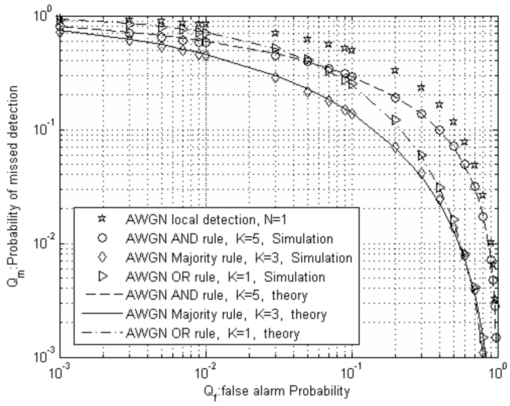

Figure 1: Q_m vs. Q_f (ROC) for various fusion rules over the AWGN channels with ρ =1dB, M=6, SNR=0 dB and N=5.

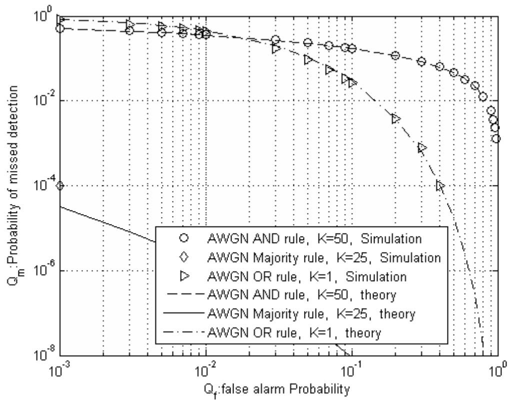

Figure 2: Q_m vs. Q_f (ROC) for various fusion rules over the AWGN channels with ρ =1dB, M=6, SNR=0 dB and N=50.

Fig 1 shows the complementary ROC curves of the three kinds of fusion rules in a cognitive radio network with a few users ($N \le 15$) when $SNR = 0$ dB, $\rho = 1$dB and $M = 6$. For a given false alarm probability Q_f, for example, $Q_f = 10^{-2}$, Majority rule has the lowest Q_m, and the local detection ($N = 1$ in Fig 1) has the highest Q_m in Fig 1. It can be readily seen that, comparing to the local detection under noise uncertainty, collaboration of sensors can significantly improve the performance of spectrum sensing. The Majority rule demonstrates the best detection probability among the three kinds of well known fusion rules. For a cognitive network with many users ($N \ge 50$),Fig 2 illustrates the performance comparison among the three hard fusion rules in AWGN channels when experimental parameters are the same with that in the Fig 1 except that the number of cognitive users is different. It also shows a superior performance for the Majority rule in a large network.

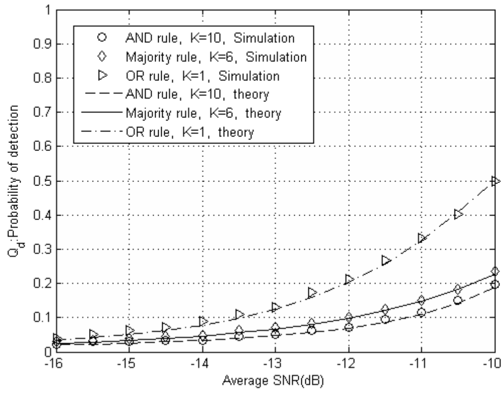

Figure 3: Q_d vs. SNR for various fusion rules over the AWGN channels with ρ =1dB, M=2000, Q_f =0.01 and N=10.

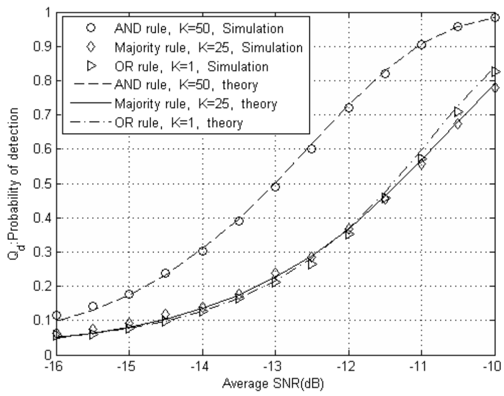

Figure 4: Q_d vs. SNR for various fusion rules over the AWGN channels with ρ =1dB, M=2000, Q_f =0.01 and N=50.

Here, we pay our attention to the low SNR scenarios, for example, $SNR = -15$ dB. When the number of second users are $N = 10$ and $N = 50$, the detection probability versus SNR curves for different hard fusion schemes are shown in Fig 3 and Fig 4, respectively. It is obvious that theoretical calculations accord with simulation results. When there are a few users in a cognitive network as shown in Fig 3, OR rule exhibits the best performance. However, with the number of cognitive users increasing, the sensing performance of AND rule is getting better. Moreover, AND rule performs best with further increasing the number of cognitive users, for example, when $N = 50$, the sensing performance of AND rule is much better than that of the other two fusion rules.

From all mentioned above, we note that, when SNR is low, AND rule is optimal in most cases except the number of users in a cognitive radio network is few. However, Majority rule is the optimal choice when the SNR is larger ($SNR \geq 0$ dB).

4.2. Rayleigh Channels

In the real-world, the primary signal often experiences shadowing and mulitpath fading. In this subsection, we mainly discuss the performance of the cooperative spectrum sensing (hard fusion) over the Rayleigh channels.

Firstly we compare the sensing performance among the three kinds of fusion rules when the SNR is higher. The complementary ROC curves for various hard fusion schemes with ρ =1dB, $M = 10$, $SNR = 0$ dB are shown in Fig 5 and Fig 6 when the number of second users are $N = 5$ and $N = 50$, respectively. From these figures, it can be seen that hard fusion based on the OR rule can give the best performance in the cognitive radio networks with a few users. However, Majority rule is optimal if the number of cognitive users is large.

We also discuss the case of low SNR, and the theoretical and simulation results for the average detection probability Q_d versus SNR curves under noise uncertainty are shown in Fig 7 and Fig 8 with ρ=1dB, $M = 1000$, and Q_f =0.01. Besides, we set the number of second users as $N = 10$ and 70 in Fig 7 and Fig 8, respectively. It can be noted that, at low SNR, OR rule provides the best performance not only in the network with a few sensors but also in the network with many users.

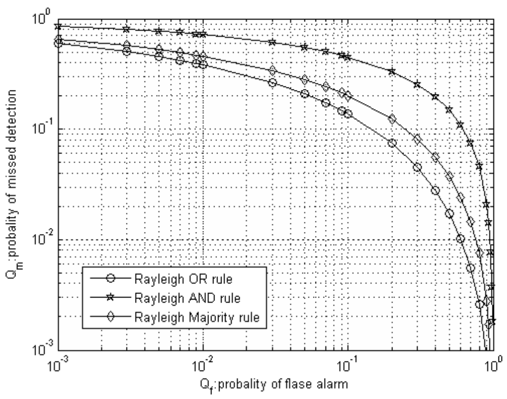

Figure 5: Q_m vs. Q_f (ROC) for various fusion rules over the Rayleigh channels with ρ =1dB, M=10, SNR= 0 dB and N=5.

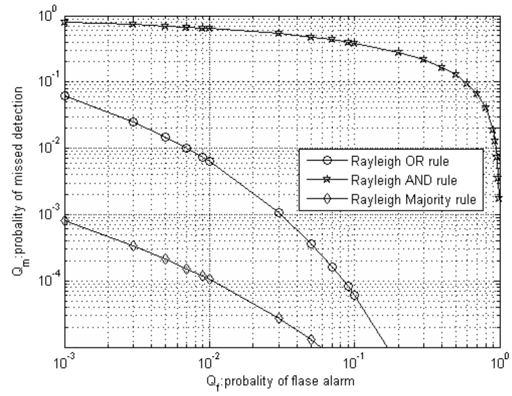

Figure 6: Q_m vs. Q_f (ROC) for various fusion rules over the Rayleigh channels with ρ =1dB, M=10, SNR= 0 dB and N=50.

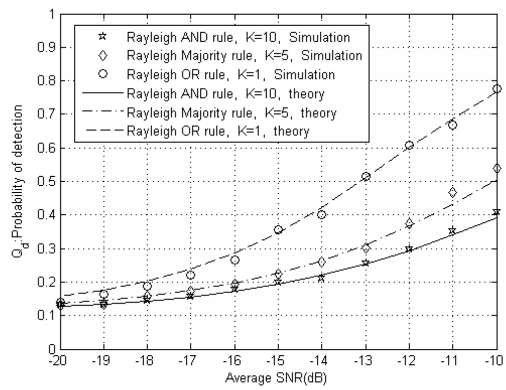

Figure 7: Q_d vs. SNR for various fusion rules over the Rayleigh channels with ρ =1dB, M=1000, Q_f =0.01 and N=10.

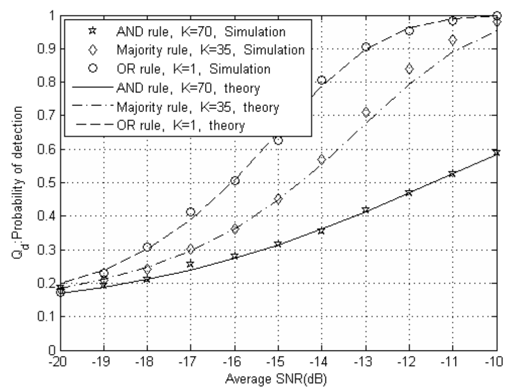

Figure 8: Q_d vs. SNR for various fusion rules over the Rayleigh channels with ρ =1dB, M=1000, Q_f =0.01 and N=70.

Finally, after discuss the extensive experiments in the Rayleigh channels, we can see that the simulations and theoretical results are meshed with each other well. Comparing the performance of the three hard fusion rules over Rayleigh channel, we can draw the conclusion that, OR rule is the optimal rule in most cases no matter the SNR is low or high except the case that there are many cognitive users at low SNR.

5 Conclusions

In this paper, we study the detection problem under noise uncertainty and mainly discuss collaborative spectrum sensing schemes based on the hard fusion rules. We adopt the power detector and obtain the closed-form expressions for the average detection probability over AWGN and Rayleigh channels, respectively. Then, in different channels, we analyze the performance of various hard fusion rules. Theoretical and simulation results illustrate that collaboration of sensors can significantly improve the spectrum sensing performance, though variational noise power level produce severe degradation in the detector's performance.

In the AWGN channels, Majority rule is optimal at higher SNR, while AND rule performs best for most situations at low SNR except for the case where the fewer users take part in decision fusion.

In the Rayleigh channels, when SNR is high, OR rule is a good choice in the cognitive radio networks with a few sensors, whereas Majority rule is the optimal fusion rule in the cognitive radio network with many users. However, at low SNR, OR rule always performs best no matter the users are a few or many.

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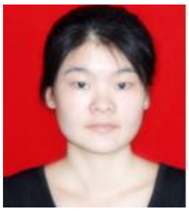

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