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# Modified Linear Regression Estimators Using quartiles and Standard Deviation

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**Abstract:** In this paper, we proposed some regression estimators for estimating the population means using linear combination of first quartile, third quartile and standard deviation of the auxiliary variable which are known. Mean square error (MSE) of the proposed estimator is obtained and compared with that of simple random sampling without replacement (SRSWOR) sample mean, classical ratio estimator and existing ratio estimator's .Theoretical result is supported by a numerical illustration.

**Keywords:** First quartile, Third quartile, standard quartile deviation, Simple random sampling, Standard deviation.

## 1 Introduction

Consider a finite population $U = \{U_1, U_2, U_3, ......U_N\}$ of *N* distinct and identifiable units. Let *Y* be a study variable with value $Y_i$ measured on $Ui$=1,2,3……N giving a vector y={$y_1$,$y_2$………$y_N$}.The problem is to estimate the population mean on the basis of a random sample selected from the population U. The SRSWOR sample mean is the simplest estimator for estimating the population mean.If an auxiliary variable X,closely related to the study variable Y is available then one can improve the performance of the estimators of the study variable by using the known values of the population parameters of the auxiliary variable.That is ,when the population parameter of the auxiliary variable X such as population mean,coefficient of variation ,kurtosis ,skewness ,median are known ,a no of estimators such as ratio,product and linear regression estimators and their modifications are suggested in literature and are performing better than the SRSWOR sample mean under certain conditions. Among these estimators the ratio estimators and its modifications are widely attracted many researchers for the estimation of the mean of the study variable(see for example Murthy[1],Singh and Chaudhary[2],Al-Omari, et al.[3],Kadilar and Cingi[4,5],Yan and Tian[6],Subramanian[7] andcingi and Kadilar[8])Before discussing further about the existing modified ratio estimators and the proposed modified ratio estimators, the notations to be used in this paper are described below:

- N-Population size
- f=n\N, Sampling fraction
- Y-Study variable
- X-Auxiliary variable
- $\bar{X}, \bar{Y}$ − Population means
- *X*,*y*-sample means
- $S_X$,$S_Y$-Population standard deviations
- $S_{XY}$-Population covariance between X and Y
- $C_X$,$C_Y$-Coefficient of variations
- $\rho$ -coefficient of correlation between X and Y

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- $\mu_r = \frac{1}{N}\sum_{i=1}^{N}(X_i - \bar{X})^r$, rth order central moment
- $B_1 = \frac{\mu_3^2}{\mu_2^3}$ ,Coefficient of skewness of the auxiliary variable
- $B_1 = \frac{\mu_4}{\mu_2^2}$, Coefficient of kurtosis of the auxiliary variable
- b= $\beta = \frac{s_{xy}}{s_x^2}$ ,Regression coefficient of Y on X
- $Q_1$-First (lower )quartile of auxiliary variable
- $Q_3$-Third (upper)quartile of auxiliary variable
- $Q_r$=($Q_3$-$Q_1$)-Inter-quartile range of auxiliary variable
- $Q_d = \frac{(Q_3 - Q_1)}{2}$ -Semi-quartile range of auxiliary
- $Q_d = \frac{(Q_3 + Q_1)}{2}$ -Semi –quartile average of auxiliary variable
- B(.)-Bias of the estimator
- MSE(.)-Mean Squared error of the estimator
- $\hat{\bar{Y}}_i(\hat{\bar{Y}}_{Pj}) - i^{th}$ Existing($j^{th}$ proposed)modified ratio estimator of $\bar{Y}$

In the case of simple random sampling without replacement (SRSWOR), the sample $\bar{y}_{srs}$ is used to estimate population mean $\bar{Y}$ which is an unbiased estimator and its variance is

$$V(\bar{y}_{srs}) = \frac{(1-f)}{n} S_y^2 \quad (1)$$

The ratio estimator for estimating the population mean $\bar{Y}$ of the study variable $Y$ is defined as

$$\hat{\bar{Y}}_R = \frac{\bar{y}}{\bar{x}} \bar{X} \quad (2)$$

The bias and mean squared error of $\hat{\bar{Y}}_R$ to the first order of approximation are given below:

$$B(\hat{\bar{Y}}_R) = \frac{1-f}{n} \bar{Y}(C_x^2 - \rho C_X C_Y) \quad (3)$$

$$MSE(\hat{\bar{Y}}_R) = \frac{1-f}{n} \bar{Y}^2 (C_y^2 + C_x^2 - 2\rho C_x C_y) \quad (4)$$

The usual linear regression estimator and its variance are given b

$$\hat{\bar{Y}}_{lr} = \bar{y} + \beta(\bar{X} - x) \quad (5)$$

$$V(\hat{\bar{Y}}_{lr}) = \frac{1-f}{n} S_y^2 (1-\rho^2) \quad (6)$$

The ratio and the linear regression estimators are used for improving the precision of estimate of the population mean based on SRSWOR when thereexistan auxiliary variable X and positively correlated with Y. It has been established; in general that the linear regression estimator is more efficient than the ratio estimator whenever the regression line of the study variable on the auxiliary variable does not passes through the neighborhood of the origin. Thus,the linear regression estimator is more precise than the ratio estimator unless $\beta = R$

Kadilarandcingi[4]have suggested a class of modified ratio estimators in the form of linear regression estimator as given below:

$$\hat{Y}_1 = [\bar{y} + b(\bar{X} - x)]\left[\frac{\bar{X}}{x}\right] \qquad (7)$$

Kadilar and Cingi[4] have shown that the modified linear regression type ratio estimator given in (7) perform well compare to the usual ratio estimator under certain conditions.Further estimators are proposed by modifying the estimators given in (7) by using the known first quartile,third quartile and the standard deviation etc, together with some other modified ratio estimator in kadilarandcingi [4,5] and Yian and Tian [6].Khare BB, Srivastava U, Kumar K [10] A generalized chain ratio in regression estimator for population mean using two auxiliary characters in sample survey. S. Kumar, P. Maheshwari& P. Chhaparwal [11]An improved regression type estimator to estimate population mean under non-normality in simple random sampling.The list of existing modified linear regression type ratio estimators together with the constant,the bias and the mean squared error are given below in table 1:

**Table 1**: Existing modified ratio estimators class 1 together with the constant, bias and mean squared error.

| Estimators | Constant | MSE (.) |
|---|---|---|
| $\bar{Y}_1 = \left[\bar{y} + b(\bar{X} - \bar{x})\right]\left[\frac{\bar{X} + \beta_1}{\bar{x} + \beta_1}\right]$ | $R_1 = \frac{\bar{Y}}{\bar{X} + \beta_1}$ | $\frac{1-f}{n}(R_2^2 S_x^2 + S_y^2(1-\rho^2)$ |
| $\hat{\bar{Y}}_2 = \left[\bar{y} + b(\bar{X} - \bar{x})\right]\left[\frac{\beta_1\bar{X} + \beta_2}{\beta_1\bar{x} + \beta_2}\right]$ | $R_2 = \frac{\beta_1\bar{Y}}{\beta_1\bar{X} + \beta_2}$ | $\frac{1-f}{n}(R_3^2 S_x^2 + S_y^2(1-\rho^2)$ |

It is to be noted that "the existing modified ratio estimators "means the list of modified ratio estimators to be considered in this paper unless otherwise stated. It does not mean to the entire list of modified ratio estimator available in the literature.
The list of modified linear regression type ratio estimator given in table 1 uses the known values of the parameters like and their linear combinations to improve the ratio estimator in estimation of population mean. In this paper an attempt is made to use the first quartile, third quartile and standard deviation of the auxiliary variables to introduced modified linear regression type ratio estimators for estimating population mean in line with kadilar and cingi [4].

## 2 ProposedModified Linear Regression Type Ratio Estimators

In this section, a class of modified linear regression type ratio estimators is proposed for estimating the finite population mean and also derived the bias and the mean squared error of the proposed estimators (see Appendix A).The proposed estimators is defined as $\hat{\bar{Y}}_{pj}; j = 1,2$ for estimating the population mean $\bar{Y}$ together with the constant, the bias and the mean squared error are presented in the following table 2:

**Table 2:** Proposed class of estimators.

| Estimators | Constant $R_{pj}$ | MSE (.) |
|---|---|---|
| $\bar{Y}_{p1} = \left[\bar{y} + b(\bar{X} - \bar{x})\right]\left[\frac{\beta_1\bar{X} + (Q_1 + S_x)}{\beta_1\bar{x} + (Q_1 + S_x)}\right]$ | $R_{p1} = \frac{\beta_1\bar{Y}}{\beta_1\bar{X} + (Q_1 + S_x)}$ | $\frac{1-f}{n}(R_{p1}^2 S_x^2 + S_y^2(1-\rho^2)$ |
| $\bar{Y}_{p2} = \left[\bar{y} + b(\bar{X} - \bar{x})\right]\left[\frac{\beta_1\bar{X} + (Q_1 + Q_3)}{\beta_1\bar{x} + (Q_1 + Q_3)}\right]$ | $R_{p2} = \frac{\beta_1\bar{Y}}{\beta_1\bar{X} + (Q_1 + Q_3)}$ | $\frac{1-f}{n}(R_{p2}^2 S_x^2 + S_y^2(1-\rho^2)$ |

## 3 Efficiencyofthe ProposedEstimators

The variance of SRSWOR sample mean $\bar{y}_{srs}$ is given below:

$$V(\bar{y}_{srs}) = \frac{1-f}{n} S_y^2 \quad (8)$$

The mean squared error of the usual ratio estimator $\hat{\bar{Y}}$ to the first degree approximation is given below:

$$MSE(\hat{\bar{Y}}) = \frac{1-f}{n} \bar{Y}^2 (C_y^2 + C_x^2 - 2\rho C_x C_y) \quad (9)$$

The modified ratio estimators given in table 1 and the proposed modified ratio estimators given in table 2 are represented in three classes as given below:

**Class 1 :**The mean squared error and the constant of the existing modified linear regression type ratio estimator $\hat{\bar{Y}}_1$ to $\hat{\bar{Y}}_2$ listed in the Table 1 are represented in a single class(say,class1),which will be very much useful for comparing with that of proposed modified linear regression type ratio estimators and are given below:

$$MSE(\hat{\bar{Y}}_{pj}) = \frac{1-f}{n} (R_{pj}^2 S_x^2 + S_y^2 (1-\rho^2); \ j = 1,2 \quad (10)$$

**Class 2:** The mean squared error and the constant of the proposed modified linear regression estimator $\hat{\bar{Y}}_{p1}$ to $\hat{\bar{Y}}_{p2}$ listed in table 2 are represented in a single class, which will be very much useful for comparing with that of existing modified linear regression type ratio estimators

$$MSE(\hat{\bar{Y}}_{pj}) = \frac{1-f}{n} (R_{pj}^2 S_x^2 + S_y^2 (1-\rho^2); \ j = 1,2 \quad (11)$$

From the expression given in (8)and (11) we have derived the conditions (see Appendix –B)for which the proposed estimator $\hat{\bar{Y}}_{pj}$ are more efficient than the simple random sampling without replacement (SRSWOR) sample mean $\bar{y}_{srs}$ and are given below

$$MSE(\hat{\bar{Y}}_{pj}) < V(\bar{y}_{srs}) if \ R_{pj} \le \rho \frac{s_y}{s_x} \quad (12)$$

From the expression given in (9) and (11)we have derived the conditions(see Appendix –C) for which the proposed estimators $\hat{\bar{Y}}_{pj}$ are more efficient than the usual ratio estimators and are given below:

$$MSE(\hat{\bar{Y}}_{pj}) \le MSE(\hat{\bar{Y}}) \text{ if } \begin{array}{l} \bar{Y}(\frac{C_x - \rho C_y}{S_x}) \le R_{pj} < \bar{Y}(\frac{\rho C_y - C_x}{S_x}) \\ \bar{Y}(\frac{\rho C_y - C_x}{S_x}) \le R_{pj} < \bar{Y}(\frac{C_x - \rho C_y}{S_x}) \end{array} \quad (13)$$

From the expression given in (10) and (11) we have derived the conditions (see Appendix-D) for which the proposed estimators

$\hat{\bar{Y}}_{pj}$ ;j=1,2 and are given below:

$$MSE(\hat{\bar{Y}}_{pj}) < MSE(\hat{\bar{Y}}_i) if \ R_{pj} \le R_i; i = 1,2,.........12 \quad (14)$$

## 4 Empirical Studies

The performance of the proposed modified linear regression type ratio estimators are assessed with that of the SRSWOR sample mean, the usual ratio estimator and the existing modified ratio estimators for a natural population considered by kadilar and cingi[4].The population consists of data on apple production amount (as study variable) and no of apple trees (as auxiliary variable) in 106 villages of Aegean region in 1999.the parameters computed from the above population are given below:

**Table 3:** Data Statistics.

| N=106 | n=40 |
|---|---|
| $\bar{Y} = 2212.5943$ | $\bar{X} = 27421.6981$ |
| $S_y = 11496.9102$ | $C_y = 5.1961$ |
| $C_x = 2.0855$ | $S_x = 57188.9320$ |
| $\beta_2(x) = 34.5723$ | $\rho = 0.8560$ |
| $b = 0.1721$ | $\beta_1(x) = 5.1238$ |
| $Q_3 = 26700$ | $Q_1 = 2387.5$ |

**Table 4**: The constant, the bias and the mean squared error of the existing and proposed modified ratio estimators are given.

| **Estimators** | **Constant** | **B(.)** | **MSE(.)** |
|---|---|---|---|
| SRSWOR sample mean | $\bar{y}_{srs}$ - | - | 2057484.9094 |
| Ratio estimator | $\hat{\bar{Y}}_R$ 0.0807 | 169.6823 | 975171.1057 |
| Linear Regression Estimator | $\hat{\bar{Y}}_{lr}$ - | - | 549896.3301 |
| Existing Modified linear regression type ratio estimators(class 1) | $\hat{\bar{Y}}_1$ 0.081<br>$\hat{\bar{Y}}_2$ 0.074 | 72.58<br>66.31 | 851646.74<br>798418.15 |
| Proposed Modified linear regression type ratio estimators(class 2) | $\hat{\bar{Y}}_{p1}$ 0.067<br>$\hat{\bar{Y}}_{p2}$ 0.054 | 60.03<br>48.38 | 749997.08<br>672828.112 |

From the values of table 4,it is observed that mean squared error of the proposed modified linear regression type ratio estimator are less than the variance of the SRSWOR sample mean ,mean squared error of the usual ratio type estimator and existing modified linear regression estimators.

## 5Conclusions

In this paper, we have proposed a class of modified ratio estimators using the quartiles and standard deviation of the auxiliary variable. The bias and mean squared error of the proposed estimators are obtained and compared with that of the existing estimators. We have derived the conditions for which the proposed estimators are more efficient than the SRSWOR sample mean, the usual ratio estimator and the existing modified linear regression type ratio estimators. Hence we recommend the proposed estimators for the practical use of application.

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**Appendix –A**

We have derived the bias and mean squared error of the proposed modified linear regression type ratio estimator $\hat{\hat{Y}}_{pj};\ j=1,2,3$ to first order of approximation and are given below:

Let $e_o=\frac{\bar{y}-\bar{Y}}{\bar{Y}}$ and $e_o=\frac{\bar{x}+\bar{X}}{\bar{X}}$ .Further we can write $\bar{y}=\bar{Y}(1+e_o)$ and $\bar{x}=\bar{X}(1+e_1)$ and from the definition of $e_o$ and $e_1$ we obtain:

E[ $e_o$ ]=E[ $e_1$ ]=0

$$E[e_o^2]=\frac{(1-f)}{n}C_y^2;\ E[e_1^2]=\frac{(1-f)}{n}C_x^2;\ E[e_0e_1]=\frac{(1-f)}{n}\rho C_yC_x$$

The proposed estimators $\hat{\hat{Y}}_{pj}$ in the form of $e_o$ and $e_1$ is given below:

$$\hat{\hat{Y}}_{pj}=\bar{Y}(1+e_0)+b(\bar{X}+\bar{x})\left[\frac{(\bar{X}+\lambda_j)}{(\bar{X}(1+e_1)+\lambda_j)}\right];\ j=1,2$$

Where $\lambda_1=\frac{Q_1+S_X}{\beta_1}$ and $\lambda_2=\frac{Q_1+Q_3}{\beta_1}$

$$\hat{\hat{Y}}_{pj}=\bar{Y}(1+e_0)+b(\bar{X}+\bar{x})\left[\frac{(\bar{X}+\lambda_j)}{(\bar{X}+\lambda_j)(1+\frac{e_1\bar{X}}{\bar{X}+\lambda_j})}\right]$$

$$\hat{\hat{Y}}_{pj}=\frac{\bar{Y}(1+e_0)+b(\bar{X}+\bar{x})}{(1+\theta_{pj}e_1)}\ \text{Where}\ \theta_{pj}=\frac{\bar{X}}{\bar{X}+\lambda_j};\ j=1,2$$

$$\hat{\hat{Y}}_{pj}=\bar{Y}(1+e_0)+b(\bar{X}+\bar{x})(\ (1+\theta_{pj}e_1)^{-1}$$

$$\hat{\hat{Y}}_{pj}=\bar{Y}(1+e_0)+b(\bar{X}+\bar{x})(\ (1-\theta_{pj}e_1+\theta^2{}_{pj}e^2{}_1-\theta^3{}_{pj}e^3{}_1+\ldots\ldots\ldots)^{-1}$$

Neglecting the terms higher than third order, we will get:

$$\hat{\bar{Y}}_{pj} = \bar{Y} + \bar{Y}e_0 - b\bar{X}e_1 - \bar{Y}\theta_{pj}e_1 - \bar{Y}\theta_{pj}e_0e_1 + b\bar{X}\theta_{pj}e_1^2 + \bar{Y}\theta_{pj}^2 e_1^2$$

$$\hat{\bar{Y}}_{pj} - \bar{Y} = \bar{Y}e_0 - b\bar{X}e_1 - \bar{Y}\theta_{pj}e_1 - \bar{Y}\theta_{pj}e_0e_1 + b\bar{X}\theta_{pj}e_1^2 + \bar{Y}\theta_{pj}^2 e_1^2 \quad \text{(A)}$$

Taking expectations on both sides of (A),we get:

$$E(\hat{\bar{Y}}_{pj} - \bar{Y}) = \bar{Y}E(e_0) - b\bar{X}E(e_1) - \bar{Y}\theta_{pj}E(e_1) - \bar{Y}\theta_{pj}E(e_0e_1) + b\bar{X}\theta_{pj}E(e_1^2) + \bar{Y}\theta_{pj}^2 E(e_1^2)$$

$$Bias(\hat{\bar{Y}}_{pj}) = \frac{(1-f)}{n}\left(\bar{Y}\theta_{pj}{}^2 C_x^2 - \theta_{pj}\frac{S_{xy}}{S_x}C_x + \theta_{pj}\frac{S_{xy}}{S_x}C_x\right)$$

$$Bias(\hat{\bar{Y}}_{pj}) = \frac{(1-f)}{n}\left(\bar{Y}\theta_{pj}{}^2 C_x^2\right); \; j = 1,2 \quad \text{(B)}$$

Substitute the value of $\theta_{pj}$ in (B),we get the bias of the proposed estimator

$(\hat{\bar{Y}}_{pj}); \; j = 1,2$ as given below:

$$Bias(\hat{\bar{Y}}_{pj}) = \frac{(1-f)}{n}\left(\bar{Y}\frac{\bar{X}^2}{\bar{X}+\lambda_j}C_x^2\right) \quad \text{(C)}$$

Multiply and divide by $\bar{Y}^2$ in (3.19), we get

$$Bias(\hat{\bar{Y}}_{pj}) = \frac{(1-f)}{n}\left(\frac{S_x^2}{\bar{Y}}R_{pj}^2\right) \text{ where } R_{pj} = \frac{\bar{Y}}{\bar{X}+\lambda_j}; \; j = 1,2 \quad \text{(D)}$$

Squaring both sides of (A),neglecting the terms more than 2nd order and taking expectation on both sides we get

$$E(\hat{\bar{Y}}_{pj} - \bar{Y})^2 = E(\bar{Y}^2 e_o^2) + b^2\bar{X}^2 E(e_1^2) + \bar{Y}^2\theta_{pj}^2 E(e_1^2) - 2b\bar{Y}\bar{X}E(e_0e_1) - 2\bar{Y}^2\theta_{pj}E(e_oe_1) + 2b\bar{X}\bar{Y}\theta_{pj}E(e_1^2)$$

$$MSE(\hat{\bar{Y}}_{pj}) = \frac{(1-f)}{n}(\bar{Y}^2C_y^2 + b^2\bar{X}^2C_x^2 + \bar{Y}^2\theta_{pj}^2C_x^2 - 2b\bar{Y}\bar{X}\rho C_yC_x - 2\bar{Y}^2\theta_{pj}\rho C_yC_x + 2b\bar{X}\bar{Y}\theta_j C_x^2)$$

$$MSE(\hat{\bar{Y}}_{pj}) = \frac{(1-f)}{n}(\bar{Y}^2C_y^2 + \frac{S_{xy}{}^2}{S_x^2}S_x^2 + \bar{Y}^2\theta_{pj}^2C_x^2 - 2\frac{S_{xy}}{S_x^2}\frac{S_{xy}}{S_xS_y}S_yS_x - 2\bar{Y}^2\theta_{pj}\frac{S_{xy}}{S_xS_y}S_yC_x + 2\frac{S_{xy}}{S_x^2}S_x\bar{Y}\theta_jC_x)$$

$$MSE(\hat{\bar{Y}}_{pj}) = \frac{(1-f)}{n}(\bar{Y}^2C_y^2 + \frac{S_{xy}{}^2}{S_x^2}S_x^2 + \bar{Y}^2\theta_{pj}^2C_x^2 - 2\frac{S_{xy}}{S_x^2}\frac{S_{xy}}{S_xS_y}S_yS_x - 2\bar{Y}^2\theta_{pj}\frac{S_{xy}}{S_xS_y}S_yC_x + 2\frac{S_{xy}}{S_x^2}S_x\bar{Y}\theta_jC_x)$$

$$MSE(\hat{\bar{Y}}_{pj}) = \frac{(1-f)}{n}(\bar{Y}^2C_y^2 + \bar{Y}^2\theta_{pj}^2C_x^2 + \frac{S_{xy}{}^2}{S_x^2}S_x^2 - 2\frac{S_{xy}}{S_x^2}S - 2\bar{Y}^2\theta_{pj}\frac{S_{xy}}{S_xS_y}C_x + 2\bar{Y}\theta_jC_x 2\frac{S_{xy}}{S_x^2}C_x)$$

$$MSE(\hat{\bar{Y}}_{pj}) = \frac{(1-f)}{n}(\bar{Y}^2\theta_{pj}^2C_x^2 + \bar{Y}^2C_y^2 - \frac{S_{xy}^2}{S_x^2})$$

$$MSE(\hat{\bar{Y}}_{pj}) = \frac{(1-f)}{n}(\bar{Y}^2\theta_{pj}^2C_x^2 + \bar{Y}^2C_y^2 - \frac{\rho S_x^2S_y^2}{S_x^2}) \quad \text{since} \quad \rho = \frac{S_{xy}}{S_xS_y}$$

$$MSE(\hat{\bar{Y}}_{pj}) = \frac{(1-f)}{n}(\bar{Y}^2\theta_{pj}^2C_x^2 + S_y^2 - \rho^2S_y^2)$$

$$MSE(\hat{\bar{Y}}_{pj}) = \frac{(1-f)}{n}(\bar{Y}^2\theta_{pj}^2C_x^2 + S_y^2(1-\rho^2))$$

$\theta_{pj}$ in the above expression, we will get:

Substitute the value of

$$MSE(\hat{\bar{Y}}_{pj}) = \frac{(1-f)}{n}(\bar{Y}^2\frac{\bar{X}^2}{(\bar{X}+\lambda_j)^2}C_x^2 + S_y^2(1-\rho^2))$$

$$MSE(\hat{\hat{Y}}_{pj}) = \frac{(1-f)}{n}(\bar{Y}^2 \frac{\bar{Y}^2}{(\bar{X}+\lambda_j)^2}\bar{X}^2 C_x^2 + S_y^2(1-\rho^2))$$

The mean squared error of the proposed modified linear regression type ratio estimator $\hat{\hat{Y}}_{pj}$

Is given below:

$$MSE(\hat{\hat{Y}}_{pj}) = \frac{(1-f)}{n}(R_{pj}^2 S_x^2 + S_y^2(1-\rho^2)) \quad \text{Where} \quad R_{pj} = \frac{\bar{Y}}{\bar{X}+\lambda_j};\ j=1,2 \quad \text{(E)}$$

**Appendix-(B)**

The condition for which the proposed modified linear regression type ratio estimators(class 2) perform better than the SRSWOR sample mean are derived and are given below:

For $MSE(\hat{\hat{Y}}_{pj}) \le V(\bar{y}_{srs})$

$$\frac{(1-f)}{n}(R_{pj}^2 S_x^2 + S_y^2(1-\rho^2)) \le \frac{1-f}{n} S_y^2$$

$$\Rightarrow (R_{pj}^2 S_x^2 + S_y^2(1-\rho^2)) \le S_y^2$$

$$\Rightarrow R_{pj}^2 S_x^2 + S_y^2 - S_y^2\rho^2 - S_y^2 \le 0$$

$$\Rightarrow R_{pj}^2 S_x^2 - S_y^2\rho^2 \le 0$$

$$\Rightarrow R_{pj}^2 S_x^2 \le S_y^2\rho^2$$

$$\Rightarrow R_{pj} S_x \le S_y\rho$$

$$\Rightarrow \rho \ge R_{pj}\frac{S_x}{S_y}$$

$$\Rightarrow R_{pj} \le \rho\frac{S_y}{S_x}$$

That is $MSE(\hat{\hat{Y}}_{pj}) \le V(\bar{y}_{srs})$ if $R_{pj} \le \rho\frac{S_y}{S_x}$

**Appendix-C**

The conditions for which the proposed modified ratio estimators (class 2) perform better than the usual ratio estimators are derived and are given below:

For $MSE(\hat{\hat{Y}}_{pj}) \le MSE(\hat{\hat{Y}}_R)$

$$\frac{1-f}{n}(R_{pj}^2 S_x^2 + S_y^2(1-\rho^2)) \le \frac{1-f}{n}\bar{Y}^2(C_y^2 + C_x^2 - 2\rho C_x C_y)$$

$$\Rightarrow \bar{Y}^2(R^{*2}_{pj} S_x^2 + S_y^2(1-\rho^2)) \le \bar{Y}^2(C_y^2 + C_x^2 - 2\rho C_x C_y) \quad \text{Where } R_{pj}^* = \frac{R_{pj}}{\bar{Y}}$$

$$\Rightarrow R^{*2}_{pj} S_x^2 + C_y^2(1-\rho^2) - C_y^2 - C_x^2 + 2\rho C_x C_y \le 0$$

$$\Rightarrow R^{*2}_{pj} S_x^2 + C_y^2 - \rho^2 C_y^2 - C_y^2 - C_x^2 + 2\rho C_x C_y \le 0$$

$$\Rightarrow R^{*2}_{pj} S_x^2 - \rho^2 C_y^2 - C_x^2 + 2\rho C_x C_y \le 0$$

$$\Rightarrow (C_x - \rho C_y)^2 - R^{*2}_{pj} S_x^2 \ge 0$$

$$\Rightarrow (C_x - \rho C_y + R^*{}_{pj} S_x)(C_x - \rho C_y + R^*{}_{pj} S_x) \geq 0$$

**Condition 1:** $(C_x - \rho C_y + R^*{}_{pj} S_x) \leq (C_x - \rho C_y + R^*{}_{pj} S_x) \leq 0$

$$C_x + R^*{}_{pj} S_x \leq \rho C_y and C_x + R^*{}_{pj} S_x \leq \rho C_y$$

$$\Rightarrow R^*_{pj} \leq \frac{\rho C_y - C_x}{S_x} and R^*_{pj} \geq \frac{\rho C_x - C_y}{S_x}$$

$$\Rightarrow \frac{C_x - \rho C_y}{S_x} \leq R^*_{pj} \leq \frac{\rho C_y - C_x}{S_x} \text{ where } R^*_{pj} = \frac{R_{pj}}{\bar{Y}}$$

$$\Rightarrow \bar{Y}(\frac{C_x - \rho C_y}{S_x}) \leq R^*_{pj} \leq \bar{Y}(\frac{\rho C_y - C_x}{S_x})$$

**Condition 2:** $(C_x - \rho C_y + R^*{}_{pj} S_x) \geq (C_x - \rho C_y + R^*{}_{pj} S_x) \geq 0$

$$C_x + R^*{}_{pj} S_x \geq \rho C_y and C_x + R^*{}_{pj} S_x \geq \rho C_y$$

$$\Rightarrow R^*_{pj} \geq \frac{\rho C_y - C_x}{S_x} and R^*_{pj} \leq \frac{\rho C_x - C_y}{S_x}$$

$$\Rightarrow \frac{C_x - \rho C_y}{S_x} \geq R^*_{pj} \geq \frac{\rho C_y - C_x}{S_x} \text{ where } R^*_{pj} = \frac{R_{pj}}{\bar{Y}}$$

$$\Rightarrow \bar{Y}(\frac{C_x - \rho C_y}{S_x}) \geq R^*_{pj} \geq \bar{Y}(\frac{\rho C_y - C_x}{S_x})$$

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