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Fixed Point Theorem in Intuitionistic Fuzzy Metric Spaces Using Compatible Mappings of Type (A)

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Abstract: In this paper, we prove common fixed point theorem in intuitionistic fuzzy metric space using compatible mappings of type (A).

Keywords: Intuitionistic Fuzzy metric space; Compatible mappings of type (A); Common fixed point.

1 Introduction

Atanassove [2] introduced and studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. In 2004, Park [25] defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norms and continuous t-conorms. Recently, in 2006, Alaca et al. [1] using the idea of In-tuitionistic fuzzy sets, defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norm and continuous t-conorms as a generalization of fuzzy metric space due to Kramosil and Michalek [3]. Subsequently, several authors [4]-[23] derived fixed point theorems in intuitionistic fuzzy metric space. In view of the considerations given by various authors, the principal motivation of this paper is to relate some results in the literature by discussing the existence and uniqueness of fixed points for new classes of mappings defined on a complete metric space. In particular, we prove common fixed point theorem in intuitionistic fuzzy metric space using compatible mappings of type (A).

2 Preliminaries

The concepts of triangular norms (t-norms) and triangular conorms (t-conorms) are known as the axiomatic skelton that we use are characterization fuzzy intersections and union respectively. These concepts were originally introduced by Menger [24] in study of statistical metric spaces.

Definition 2.1.[26] A binary operation $*:[0,1]\times[0,1]\to[0,1]$ is continuous t-norm if * satisfies the following conditions:for all $a,b,c,d\in[0,1]$,

(i) * is commutative and associative;
(ii) * is continuous;
(iii) $a*1=a$;
(iv) $a*b\leq c*d$ whenever $a\leq c$ and $b\leq d$.

Definition 2.2. [26] A binary operation $\diamondsuit:[0,1]\times[0,1]\to[0,1]$ is continuous t-conorm if $\diamondsuit$ satisfies the following conditions: for all $a,b,c,d\in[0,1]$,

(i) $\diamondsuit$ is commutative and associative;
(ii) $\diamondsuit$ is continuous;
(iii) $a\diamondsuit 0=a$;
(iv) $a\diamondsuit b\geq c\diamondsuit d$ whenever $a\leq c$ and $b\leq d$.

Alaca et al. [1] using the idea of Intuitionistic fuzzy sets, defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norm and continuous t-conorms as a generalization of fuzzy metric space due to Kramosil and Michalek [3] as:

Definition 2.3.[1] A 5-tuple $(X,M,N,*,\diamondsuit)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm, $\diamondsuit$ is a continuous t-conorm and M,N are fuzzy sets on $X^2\times[0,\infty)$ satisfying the following conditions:

(i) $M(x,y,t)+N(x,y,t)\leq 1$ for all $x,y\in X$ and $t>0$;
(ii) $M(x,y,0)=0$ for all $x,y\in X$;

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(iii) $M(x,y,t) = 1$ for all $x,y \in X$ and $t > 0$ if and only if $x = y$;
(iv) $M(x,y,t) = M(y,x,t)$ for all $x,y \in X$ and $t > 0$;
(v) $M(x,y,t) * M(y,z,s) \leq M(x,z,t+s)$ for all $x,y,z \in X$ and $s,t > 0$;
(vi) for all $x,y \in X$, $M(x,y,.) : [0,\infty) \to [0,1]$ is left continuous;
(vii) $lim_{t\to\infty} M(x,y,t) = 1$ for all and $t > 0$;
(viii) $N(x,y,0) = 1$ for all $x,y \in X$;
(ix) $N(x,y,t) = 0$ for all $x,y \in X$ and $t > 0$ if and only if $x = y$;
(x) $N(x,y,t) = N(y,x,t)$ for all $x,y \in X$ and $t > 0$;
(xi) $N(x,y,t) \diamondsuit N(y,z,s) \geq N(x,z,t+s)$ for all $x,y,z \in X$ and $s,t > 0$;
(xii) for all $x,y \in X$, $N(x,y,.) : [0,\infty) \to [0,1]$ is right continuous;
(xiii) $lim_{t\to\infty} N(x,y,t) = 0$ for all $x,y \in X$.

Then (M,N) is called an intuitionistic fuzzy metric space on X. The functions $M(x,y,t)$ and $N(x,y,t)$ denote the degree of nearness and the degree of non-nearness between x and y w.r.t. t respectively.

Remark 2.1. Every fuzzy metric space $(X,M,*)$ is an intuitionistic fuzzy metric space of the form $(X,M,1-M,*,\diamondsuit)$ such that t-norm * and t-conorm $\diamondsuit$ are associated as $x \diamondsuit y = 1-((1-x)*(1-y))$ for all $x,y \in X$.

Remark 2.2. In intuitionistic fuzzy metric space $(X,M,N,*,\diamondsuit)$, $M(x,y,.)$ is non-decreasing and $N(x,y,.)$ is non-increasing, for all $x,y \in X$.

Alaca et al.[1] introduced the following notions:

Definition 2.4. Let $(X,M,N,*,\diamondsuit)$ be an intuitionistic fuzzy metric space. Then

(a) a sequence $\{x_n\}$ in X is said to be Cauchy sequence if, for all $t > 0$ and $p > 0$,

$lim_{n\to\infty} M(x_{n+p},x_n,t) = 1$ and $lim_{n\to\infty} N(x_{n+p},x_n,t) = 0$.

(b) a sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ if, for all $t > 0$,

$lim_{n\to\infty} M(x_n,x,t) = 1$ and $lim_{n\to\infty} N(x_n,x,t) = 0$.

Definition 2.5.[1] An intuitionistic fuzzy metric space $(X,M,N,*,\diamondsuit)$ is said to be complete if and only if every Cauchy sequence in X is convergent.

Example 2.1. Let $X = \{\frac{1}{n} : n = 1,2,3,..\} \cup \{0\}$ and let * be the continuous t-norm and $\diamondsuit$ be the continuous t-conorm defined by $a*b = ab$ and $a \diamondsuit b = \min\{1, a+b\}$ respectively, for all $a,b \in [0,1]$. For each $x,y \in X$ and $t > 0$, define (M,N) by $M(x,y,t) = \frac{t}{t+|x-y|}$ if $t > 0$, $M(x,y,0) = 0$ and $N(x,y,t) = \frac{|x-y|}{t+|x-y|}$ if $t > 0$, $N(x,y,0) = 1$. Clearly, $(X,M,N,*,\diamondsuit)$ is complete intuitionistic fuzzy metric space.

Definition 2.6. A pair of self mappings (A,B) on an intuitionistic fuzzy metric space $(X,M,N,*,\diamondsuit)$ is said to be compatible if $lim_{n\to\infty} M(ABx_n,BAx_n,t) = 1$ and $lim_{n\to\infty} N(ABx_n,BAx_n,t) = 0$ for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $lim_{n\to\infty} Ax_n = lim_{n\to\infty} Bx_n = u$ for some $u \in X$.

Definition 2.7. A pair of self mappings (A,B) on an intuitionistic fuzzy metric space $(X,M,N,*,\diamondsuit)$ is said to be compatible of type (A) if $lim_{n\to\infty} M(ABx_n,BBx_n,t) = 1$, $lim_{n\to\infty} N(ABx_n,BBx_n,t) = 0$ and $lim_{n\to\infty} M(BAx_n,AAx_n,t) = 1$, $lim_{n\to\infty} N(BAx_n,AAx_n,t) = 0$ for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $lim_{n\to\infty} Ax_n = lim_{n\to\infty} Bx_n = u$ for some $u \in X$.

Alaca [1] proved the following results:

Lemma 2.1. Let $(X,M,N,*,\diamondsuit)$ be intuitionistic fuzzy metric space and for all $x,y \in X$, $t > 0$ and if for a number $k > 1$ such that $M(x,y,kt) \geq M(x,y,t)$ and $N(x,y,kt) \leq N(x,y,t)$ then $x = y$.

Lemma 2.2. Let $(X,M,N,*,\diamondsuit)$ be intuitionistic fuzzy metric space and for all $x,y \in X$, $t > 0$ and if for a number $k > 1$ such that $M(y_{n+2},y_{n+1},t) \geq M(y_{n+1},y_n,kt)$, $N(y_{n+2},y_{n+1},t) \leq N(y_{n+1},y_n,kt)$, Then $\{y_n\}$ is a Cauchy sequence in X.

Lemma 2.3. Let A and B be compatible self mappings of type (A) on a complete intuitionistic fuzzy metric space $(X,M,N,*,\diamondsuit)$ with $a*b = \min\{a,b\}$ and $a \diamondsuit b = \max\{a,b\}$ for all $a,b \in [0,1]$. If $Au = Bu$ for some $u \in X$ then $ABu = BAu = AAu = BBu$.

3 Main Results

Now we prove our main result.

Theorem 3.1. Let $(X,M,N,*,\diamondsuit)$ be a complete intuitionistic fuzzy metric space with $a*b = \min\{a,b\}$ and $a \diamondsuit b = \max\{a,b\}$ for all $a,b \in [0,1]$. Let A,B,S,T,P and Q be six self-mappings on X satisfying the following conditions:

(3.1) $P(X) \subseteq ST(X)$, $Q(X) \subseteq AB(X)$;
(3.2) $AB = BA, ST = TS, PB = BP, QT = TQ$;
(3.3) P or AB is continuous;
(3.4) (P,AB) and (Q,ST) are pairs of compatible mappings of type (A);
(3.5) there exist $k \in (0,1)$ such that for every $x,y \in X$ and $t > 0$,

$M(Px,Qy,kt) \geq M(ABx,STy,t) * M(Px,ABx,t)$
$M(Qy,STy,t) * M(Px,STy,t)$
$N(Px,Qy,kt) \leq N(ABx,STy,t) \diamondsuit N(Px,ABx,t)$
$\diamondsuit N(Qy,STy,t) \diamondsuit N(Px,STy,t)$.

Then A,B,S,T,P and Q have a unique common fixed point in X.

Proof: Let $x_0 \in X$, from (3.1), there exist $x_1,x_2 \in X$ such that $Px_0 = STx_1$, $Qx_1 = ABx_2$. Inductively, we construct sequences $\{x_n\}$ and $\{y_n\}$ in X such that $Px_{2n-2} = STx_{2n-1} = y_{2n-1}$ and $Qx_{2n-1} = ABx_{2n} = y_{2n}$ for n = 1, 2, 3,. . .. Take $x = x_{2n}$ and $y = x_{2n+1}$ in (3.5), we get

$M(Px_{2n},Qx_{2n+1},kt) \geq$
$M(ABx_{2n},STx_{2n+1},t) * M(Px_{2n},ABx_{2n},t)$
$* M(Qx_{2n+1},STx_{2n+1},t) * M(Px_{2n},STx_{2n+1},t)$
$M(y_{2n+1},y_{2n+2},kt) \geq$
$M(y_{2n},y_{2n+1},t) * M(y_{2n+1},y_{2n},t)$
$* M(y_{2n+2},y_{2n+1},t) * M(y_{2n+1},y_{2n+2},t)$
$M(y_{2n+1},y_{2n+2},kt) \geq M(y_{2n},y_{2n+1},t) * M(y_{2n+1},y_{2n+2},t)$
$M(y_{2n+1},y_{2n+2},kt) \geq M(y_{2n},y_{2n+1},t)$

and

$N(Px_{2n},Qx_{2n+1},kt) \leq$

$N(ABx_{2n},STx_{2n+1},t)\diamondsuit N(Px_{2n},ABx_{2n},t)$
$\diamondsuit N(Qx_{2n+1},STx_{2n+1},t)\diamondsuit N(Px_{2n},STx_{2n+1},t)$
$N(y_{2n+1},y_{2n+2},kt)\leq$
$N(y_{2n},y_{2n+1},t)\diamondsuit N(y_{2n+1},y_{2n},t)$
$\diamondsuit N(y_{2n+2},y_{2n+1},t)\diamondsuit N(y_{2n+1},y_{2n+2},t)$
$N(y_{2n+1},y_{2n+2},kt)\leq N(y_{2n},y_{2n+1},t)\diamondsuit M(y_{2n+1},y_{2n+2},t)$
$N(y_{2n+1},y_{2n+2},kt)\leq N(y_{2n},y_{2n+1},t)$.
Similarly,
$M(y_{2n+2},y_{2n+3},kt)\geq M(y_{2n+1},y_{2n+2},t)$
and
$N(y_{2n+2},y_{2n+3},kt)\leq N(y_{2n+1},y_{2n+2},t)$.
Thus, we have
$M(y_{n+1},y_{n+2},kt)\geq M(y_n,y_{n+1},t)$
and
$N(y_{n+1},y_{n+2},kt)\leq N(y_n,y_{n+1},t)$
for n = 1,2,3,....
Therefore, we have
$M(y_n,y_{n+1},t)\geq M(y_n,y_{n+1},\frac{t}{q})\geq M(y_{n-1},y_n,\frac{t}{q^2})$
$\geq ...\geq M(y_1,y_2,\frac{t}{q^n})\rightarrow 1,$
and
$N(y_n,y_{n+1},t)\leq N(y_n,y_{n+1},\frac{t}{q})\leq N(y_{n-1},y_n,\frac{t}{q^2})$
$\leq ...\leq N(y_1,y_2,\frac{t}{q^n})\rightarrow 0$
when $n\rightarrow\infty$.
For each $\varepsilon>0$ and $t>0$, we can choose $n_0\in\mathbb{N}$ such that $M(y_n,y_{n+1},t)>1-\varepsilon$ and $N(y_n,y_{n+1},t)<\varepsilon$ for each $n\geq n_0$.

For $m,n\in\mathbb{N}$, we suppose $m\geq n$. Then, we have
$M(y_n,y_m,t)\geq M(y_n,y_{n+1},\frac{t}{m-n})$
$M(y_{n+1},y_{n+2},\frac{t}{m-n})*...*M(y_{m-1},y_m,\frac{t}{m-n})$
$>((1-\varepsilon)*(1-\varepsilon)*...(m-n)times...*(1-\varepsilon))$
$\geq(1-\varepsilon)$,
and
$N(y_n,y_m,t)\leq N(y_n,y_{n+1},\frac{t}{m-n})\diamondsuit$
$N(y_{n+1},y_{n+2},\frac{t}{m-n})\diamondsuit...\diamondsuit N(y_{m-1},y_m,\frac{t}{m-n})$
$<((\varepsilon)\diamondsuit(\varepsilon)\diamondsuit...(m-n)times...\diamondsuit(\varepsilon))$
$\leq(\varepsilon)$.
$M(y_n,y_m,t)>(1-\varepsilon),N(y_n,y_m,t)<\varepsilon$.
Hence $\{y_n\}$ is a Cauchy sequence in X. As X is complete, $\{y_n\}$ converges to some point $z\in X$. Also, its subsequences converges to this point $z\in X$, i.e.

$\{Qx_{2n+1}\}\rightarrow z$, $\{STx_{2n+1}\}\rightarrow z$, $\{Px_{2n}\}\rightarrow z$, $\{ABx_{2n}\}\rightarrow z$.

Suppose AB is continuous, therefore, we have $\{ABABx_{2n}\}\rightarrow ABz$, $\{ABPx_{2n}\}\rightarrow ABz$. As (P,AB) is compatible pair of type (A), we have $\{PABx_{2n}\}\rightarrow ABz$.

Take $x=ABx_{2n},y=x_{2n+1}$ in (3.5), we get
$M(PABx_{2n},Qx_{2n+1},kt)\geq$
$M(ABABx_{2n},STx_{2n+1},t)*M(PABx_{2n},ABABx_{2n},t)$
$*M(Qx_{2n+1},STx_{2n+1},t)*M(PABx_{2n},STx_{2n+1},t)$,
as $n\rightarrow\infty$
$M(ABz,z,kt)\geq$
$M(ABz,z,t)*M(ABz,ABz,t)*M(z,z,t)*M(ABz,z,t)$
$M(ABz,z,kt)\geq M(ABz,z,t)$ and
$N(PABx_{2n},Qx_{2n+1},kt)\leq$
$N(ABABx_{2n},STx_{2n+1},t)\diamondsuit N(PABx_{2n},ABABx_{2n},t)$
$\diamondsuit N(Qx_{2n+1},STx_{2n+1},t)\diamondsuit N(PABx_{2n},STx_{2n+1},t)$
as $n\rightarrow\infty$
$N(ABz,z,kt)\leq$
$N(ABz,z,t)\diamondsuit N(ABz,ABz,t)\diamondsuit$
$N(z,z,t)\diamondsuit N(ABz,z,t)$
$N(ABz,z,kt)\leq N(ABz,z,t)$. By Lemma 2.1, $ABz=z$.
Next, we show that $Pz=z$. Put $x=z$ and $y=x_{2n}$ in (3.5), we get
$M(Pz,Qx_{2n},kt)\geq M(ABz,STx_{2n},t)$
$*M(Pz,ABz,t)*M(Qx_{2n},STx_{2n},t)*M(Pz,STx_{2n},t)$
as $n\rightarrow\infty$,
$M(Pz,z,kt)\geq M(z,z,t)*$
$M(Pz,z,t)*M(z,z,t)*M(Pz,z,t)$
$M(Pz,z,kt)\geq M(Pz,z,t)$
and
$N(Pz,Qx_{2n},kt)\leq$
$N(ABz,STx_{2n},t)\diamondsuit N(Pz,ABz,t)$
$\diamondsuit N(Qx_{2n},STx_{2n},t)\diamondsuit N(Pz,STx_{2n},t)$
as $n\rightarrow\infty$,
$N(Pz,z,kt)\leq N(z,z,t)$
$\diamondsuit N(Pz,z,t)\diamondsuit N(z,z,t)\diamondsuit N(Pz,z,t)$
$N(Pz,z,kt)\leq N(Pz,z,t)$.
Therefore, $ABz=z=Pz$. Now, we show that $Bz=z$. Put $x=Bz$ and $y=x_{2n-1}$ in (3.5), we get
$M(PBz,Qx_{2n-1},kt)\geq$
$M(ABBz,STx_{2n-1},t)*M(PBz,ABBz,t)$
$*M(Qx_{2n-1},STx_{2n-1},t)*M(PBz,STx_{2n-1},t)$
and
$N(PBz,Qx_{2n-1},kt)\leq$
$N(ABBz,STx_{2n-1},t)\diamondsuit N(PBz,ABBz,t)$
$\diamondsuit N(Qx_{2n-1},STx_{2n-1},t)\diamondsuit N(PBz,STx_{2n-1},t)$.
As $BP=PB$ and $AB=BA$, so that $P(Bz)=(PB)z=BPz=Bz$ and $(AB)(Bz)=(BA)(Bz)=B(AB)z=Bz$. Taking, $n\rightarrow\infty$, we get
$M(Bz,z,kt)\geq$
$M(Bz,z,t)*M(Bz,Bz,t)*M(z,z,t)*M(Bz,z,t)$
$M(Bz,z,kt)\geq M(Bz,z,t)$
and
$N(Bz,z,kt)\leq$
$N(Bz,z,t)\diamondsuit N(Bz,Bz,t)\diamondsuit$
$N(z,z,t)\diamondsuit N(Bz,z,t)$
$N(Bz,z,kt)\leq N(Bz,z,t)$.
Therefore, by using Lemma 2.1, we get $Bz=z$ and also we have, $ABz=z$. Therefore, $Az=Bz=Pz=z$. As $P(X)\subseteq ST(X)$, there exist $u\in X$ such that $z=Pz=STu$.
Putting, $x=x_{2n},y=u$ in (3.5), we get
$M(Px_{2n},Qu,kt)\geq$
$M(ABx_{2n},STu,t)*M(Px_{2n},ABx_{2n},t)*$
$M(Qu,STu,t)*M(Px_{2n},STu,t)$
taking, $n\rightarrow\infty$,
$M(z,Qu,kt)\geq$
$M(z,z,t)*M(z,z,t)*M(Qu,z,t)*M(z,z,t)$
$M(z,Qu,kt)\geq M(z,Qu,t)$
and $N(Px_{2n},Qu,kt)\leq$
$N(ABx_{2n},STu,t)\diamondsuit N(Px_{2n},ABx_{2n},t)\diamondsuit$
$N(Qu,STu,t)\diamondsuit N(Px_{2n},STu,t)$
taking, $n\rightarrow\infty$,
$N(z,Qu,kt)\leq$

$N(z,z,t) \diamondsuit N(z,z,t) \diamondsuit N(Qu,z,t)$
$\diamondsuit N(z,z,t)$
$N(z,Qu,kt) \leq N(z,Qu,t)$.
By using Lemma 2.1, we get $Qu = z$. Hence, $STu = z = Qu$. Since (Q,ST) is compatible pair of type (A), therefore, by Lemma, we have $QSTu = STQu$. Therefore, $Qz = STz$. Now, we show that $Qz = z$. Take $x = x_{2n}, y = z$ in (3.5), we get
$M(Px_{2n},Qz,kt) \geq M(ABx_{2n},STz,t) * M(Px_{2n},ABx_{2n},t)$
$M(Qz,STz,t) * M(Px_{2n},STz,t)$
taking, $n \to \infty$,
$M(z,Qz,kt) \geq$
$M(z,Qz,t) * M(z,z,t) * M(Qz,Qz,t) * M(z,Qz,t)$
$M(z,Qz,kt) \geq M(z,Qz,t)$
and
$N(Px_{2n},Qz,kt) \leq N(ABx_{2n},STz,t) \diamondsuit N(Px_{2n},ABx_{2n},t)$
$\diamondsuit N(Qz,STz,t) \diamondsuit N(Px_{2n},STz,t)$
taking, $n \to \infty$,
$N(z,Qz,kt) \geq N(z,Qz,t) \diamondsuit N(z,z,t)$
$\diamondsuit N(Qz,Qz,t) \diamondsuit M(z,Qz,t)$
$N(z,Qz,kt) \geq N(z,Qz,t)$.
Therefore, by using Lemma 2.1, $Qz = z$. As $QT = TQ, ST = TS$, we have $QTz = TQz = Tz$ and $STTz = TSTz = TQz = Tz$.

Next, we claim that $Tz = z$.

For this, take $x = x_{2n}, y = Tz$ in (3.5), we get
$M(Px_{2n},QTz,kt) \geq$
$M(ABx_{2n},STTz,t) * M(Px_{2n},ABx_{2n},t)$
$* M(QTz,STTz,t) * M(Px_{2n},STTz,t)$ as $n \to \infty$,
$M(z,Tz,kt) \geq M(z,Tz,t) * M(z,z,t)$
$M(Tz,Tz,t) * M(z,Tz,t)$
$M(z,Tz,kt) \geq M(z,Tz,t)$
and
$N(Px_{2n},QTz,kt) \leq N(ABx_{2n},STTz,t) \diamondsuit N(Px_{2n},ABx_{2n},t)$
$\diamondsuit N(QTz,STTz,t) \diamondsuit N(Px_{2n},STTz,t)$
as $n \to \infty$,
$N(z,Tz,kt) \leq N(z,Tz,t) \diamondsuit N(z,z,t)$
$\diamondsuit N(Tz,Tz,t) \diamondsuit N(z,Tz,t)$
$N(z,Tz,kt) \leq N(z,Tz,t)$
therefore, by Lemma 2.1, we get $Tz = z$. As $STz = Qz = z = Tz$. This gives $Sz = z$. Hence, $Az = Bz = Pz = Qz = Sz = Tz = z$. Hence, z is a common fixed point of A,B,S,T,P and Q. The proof is similar P is continuous.

For uniqueness: Let u is another fixed point of A,B,S,T,P and Q. Therefore, take $x = z$ and $y = u$ in (3.5), we get
$M(Pz,Qu,kt) \geq M(ABz,STu,t) * M(Pz,ABz,t)$
$* M(Qu,STu,t) * M(Pz,STu,t)$
$M(z,u,kt) \geq M(z,u,t) * M(z,z,t)$
$* M(u,u,t) * M(z,u,t)$
$M(z,u,kt) \geq M(z,u,t)$ and
$N(Pz,Qu,kt) \leq N(ABz,STu,t) \diamondsuit N(Pz,ABz,t)$
$\diamondsuit N(Qu,STu,t) \diamondsuit N(Pz,STu,t)$
$N(z,u,kt) \leq N(z,u,t) \diamondsuit N(z,z,t)$
$\diamondsuit N(u,u,t) \diamondsuit N(z,u,t)$
$N(z,u,kt) \leq N(z,u,t)$.
By Lemma 2.1, we get $z = u$. Hence, z is a unique common fixed point of A,B,S,T,P and Q.

Take $B = T = I$ (Identity map), then Theorem 3.1 becomes:
Corollary 3.1. Let $(X,M,N,*,\diamondsuit)$ be a complete intuitionistic fuzzy metric space with $a * b = \min\{a,b\}$ and $a \diamondsuit b = \max\{a,b\}$ for all $a,b \in [0,1]$. Let A,S,P and Q be four self-mappings on X satisfying following conditions:

(3.6) $P(X) \subseteq S(X)$, $Q(X) \subseteq A(X)$;

(3.7) P or A is continuous;

(3.8) (P,A) and (Q,S) are pairs of compatible mappings of type (A);

(3.9) there exist $k \in (0,1)$ such that for every $x,y \in X$ and $t > 0$,
$M(Px,Qy,kt) \geq M(Ax,Sy,t) * M(Px,Ax,t)$
$* M(Qy,Sy,t) * M(Px,Sy,t)$
$N(Px,Qy,kt) \leq N(Ax,Sy,t) \diamondsuit N(Px,Ax,t)$
$\diamondsuit N(Qy,Sy,t) \diamondsuit N(Px,Sy,t)$.

Then A,S,P and Q have a unique common fixed point in X.

4 Conclusion

The present paper extended and generalized various known fixed point theorems in the literature in the setting of fuzzy and intuitionistic fuzzy metric spaces.

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