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Estimation of Stress-Strength Parameter for Rayleigh Distribution based on Progressive Type-II Censoring

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Abstract: In this paper, the estimation of stress-strength parameter $R = P(Y < X)$ is considered when X, Y the strength and stress respectively are two independent random variables of Rayleigh distribution. The samples taken for X and Y are progressively censoring of type II. Maximum likelihood estimator (MLE), uniformly minimum variance unbiased estimator (UMVUE)and Bayes estimator of $R = P(Y < X)$ are obtained. The exact confidence interval of R based on MLE is obtained. The performance of the proposed estimators is compared using computer simulation.

Keywords: Rayleigh distribution; progressive type-II censoring; stress-strength model; unbiased estimator; maximum-likelihood estimator.

1 Introduction

Rayleigh distribution was first considered by Lord Rayleigh [1]. It used in modeling lifetime data and reliability. The Rayleigh distribution has the following distribution function for $X > 0$:

$$F(x) = 1 - e^{-x^2/2\sigma^2}; \quad \sigma > 0, x \geq 0 \tag{1.1}$$

where, the density function of Rayleigh for $X > 0$ denoted by Ray (σ) is

$$f(x; \sigma) = \frac{x}{\sigma^2} e^{-x^2/2\sigma^2}; \quad \sigma > 0 \tag{1.2}$$

Rayleigh distribution is related to other distributions such as Weibull distribution where Rayleigh (σ) =Weibull $(2, \sigma\sqrt{2})$. In life-testing experiments, one often encounters situations where it takes a substantial amount of time to obtain a reasonable number of failures necessary to carry out reliable inference, so censored samples are used for analyzing lifetime data. Among various censoring schemes, the Type-II progressive censoring scheme has become very popular one in the last decade. It can be described as follows: let n units be placed on test at time zero with m failures to be observed. At the first failure a number r_1 of the surviving units $(n-1)$ are randomly selected and removed from the experiment. At the second observed failure, r_2 of the surviving units $(n - r_1 - 2)$ are randomly selected and removed from the experiment, and so on until the m-th failure is observed in which all remaining surviving units $r_m = n - m - r_1 - r_2 - \ldots - r_{m-1}$ are removed.We denote to progressively Type-II censoring with scheme $(n, m, r_1, r_2, \ldots, r_m)$. Traditional Type-II censoring scheme is included when $(r_1 = r_2 = \ldots = r_{m-1} = 0)$ and $(r_m = n - m)$ and complete sampling scheme when $(n = m)$ and $(r_1 = \ldots = r_{m-1} = r_m = 0)$. Balakrishnan and Aggarwala [2] and Balakrishnan [3] presented a study on different features of progressive censoring schemes.

In stress-strength model, the stress (Y) and the strength (X) are treated as random variables and the reliability of a component during a given period is taken to be the probability that its strength exceeds the stress during the entire interval, i.e. the reliability R of a component is $R = P(Y < X)$. For a particular situation, if we consider Y as the pressure of a chamber generated by ignition of a solid propellant and X as the strength of the chamber. Then R represents the probability of successful firing of the engine. Stress-strength model can be used as a general measure of the difference between two populations and has applications in many areas. For example, comparing two treatments X and Y, then $R = P(Y < X)$ is the measure of the response of treatment X. For other applications see Kotz et al. [4]. Many authors considered the problem

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of estimating the stress-strength parameter based on complete samples, it first considered by Birnbaum [5]. Johnson [6] present a good review on stress-strength model in reliability. Awad and Charraf [7] studied the case when X and Y are independent Burr random variables of type XII, they obtained maximum likelihood, uniformly minimum variance unbiased estimator (UMVUE) and Bayesian estimates of R. Fathipour et al. [8] obtained the estimation of R in the generalized Rayleigh distribution with different scale parameters. Ahmed et al. [9] consider this problem when X and Y are two independent random variables have Burr Type X distribution. Based on censored samples, Saraçoǧlu et al. [10] obtained the estimation for R based on exponential distribution with type II progressive censoring. Abd-Elfattah et al. [11] get the estimation of R based on Burr type XII distribution with type II progressive censoring, they discussed two cases the first when X and Y have common shape parameter and different scale parameters while the second case when X and Y have common scale parameter and different shape parameters. For some of the recent references, the readers may refer to [12-14].

In the present paper, the study the estimation of $R = P(Y < X)$ when X and Y are two independents but not identically random variables belonging to Rayleigh distribution with parameters σ_1 and σ_2 respectively. In Section (2), maximum likelihood estimator of R and exact confidence interval are obtained. Also, UMVUE of R is obtained in section (3). In section (4) Bayes estimator for R is obtained based on conjugate priors. Numerical results and simulations are presented in section (5). Finally, some concluding remarks are given in section (6).

2 MLE of *R*

In this section the MLE of R is obtained. Let X, Y are two independent random variables such that $X\sim Ray(\sigma_1)$ and $Y\sim Ray(\sigma_2)$ then R is:

$$R = P(Y < X) = \int_0^\infty \int_0^x f(x)f(y)dydx$$
$$= \int_0^\infty \int_0^x \frac{x}{\sigma_1^2} e^{-x^2/2\sigma_1^2} . \frac{y}{\sigma_2^2} e^{-y^2/2\sigma_2^2} dydx$$
$$= \frac{\sigma_1^2}{\sigma_1^2+\sigma_2^2} \tag{2.1}$$

Now to get MLE of R we first get the MLE of parameters σ_1 and σ_2.

Let $X_{1:m_1:n_1}, \dots, X_{m_1:m_1:n_1}$ be a progressive censored sample from $Ray(\sigma_1)$ with progressive censoring scheme $(n_1, m_1, r_1, \dots, r_{m_1})$, and let $Y_{1:m_2:n_2}, \dots, Y_{m_2:m_2:n_2}$ be a progressive censored sample from $Ray(\sigma_2)$ with progressive censoring scheme $(n_2, m_2, s_1, \dots, s_{m_2})$, then the jointly likelihood function $L(\sigma_1, \sigma_2)$ is

$$L(\sigma_1,\sigma_2) = [k_1 \prod_{i=1}^{m_1} f(x_i)\,[1-F(x_i)]^{r_i}].\ [k_2 \prod_{j=1}^{m_2} f(y_j)\,[1-F(y_j)]^{s_j}]$$
$$= \frac{k_1 k_2}{\sigma_1^{2m_1}\sigma_2^{2m_2}} \left(\prod_{i=1}^{m_1} x_i\right)\left(\prod_{j=1}^{m_2} y_j\right) \times exp\left(\frac{-1}{2\sigma_1^2}\sum_{i=1}^{m_1}(1+0r_i)x_i^2\right) exp\left(\frac{-1}{2\sigma_2^2}\sum_{j=1}^{m_2}(1+s_j)y_j^2\right) \quad (2.2$$

Where k_1 and k_2 are:

$$k_1 = n_1(n_1 - 1 - r_1)(n_1 - 2 - r_1 - r_2) \dots (n_1 - m_1 + 1 - r_1 - \dots - r_{m_1-1})$$
$$k_2 = n_2(n_2 - 1 - s_1)(n_2 - 2 - s_1 - s_2)\dots(n_2 - m_2 + 1 - s_1 - \dots - s_{m_2-1}) \tag{2.3}$$

Now the jointly log-likelihood function ℓ is:

$$\ell = ln(k_1k_2) - 2m_1 ln\sigma_1 - 2m_2 ln\sigma_2 + \sum_{i=1}^{m_1} lnx_i + \sum_{j=1}^{m_2} lny_j - \frac{1}{2\sigma_1^2}\sum_{i=1}^{m_1}(1+r_i)x_i^2 - \frac{1}{2\sigma_2^2}\sum_{j=1}^{m_2}(1+s_j)y_j^2 \tag{2.4}$$

By differentiation on equation (2.4) with respect to σ_1 and σ_2. Then we get:

$$\frac{\partial \ell}{\partial \sigma_1} = -\frac{2m_1}{\sigma_1} + \frac{1}{\sigma_1^3}\sum_{i=1}^{m_1}(1+r_i)x_i^2 \tag{2.5}$$

$$\frac{\partial \ell}{\partial \sigma_2} = -\frac{2m_2}{\sigma_2} + \frac{1}{\sigma_2^3}\sum_{j=1}^{m_2}(1+s_j)y_j^2 \tag{2.6}$$

By putting equations (2.5) and (2.6) equal to zero then we get:

$$\hat{\sigma}_1 = \left[\frac{1}{2m_1}\sum_{i=1}^{m_1}(1+r_i)x_i^2\right]^{0.5} \tag{2.7}$$

$$\hat{\sigma}_2 = \left[\frac{1}{2m_2}\sum_{j=1}^{m_2}(1+s_j)y_j^2\right]^{0.5} \tag{2.8}$$

Then MLE of R is

$$\hat{R} = \frac{1}{1+\frac{m_1 E}{m_2 T}} = \frac{1}{1+\frac{\sigma_2^2}{\sigma_1^2}\lambda} \tag{2.9}$$

where $T = \sum_{i=1}^{m_1}(1+r_i)x_i^2$, $E = \sum_{j=1}^{m_2}(1+s_j)y_j^2$ and $\lambda = \frac{\sigma_1^2 m_1 E}{\sigma_2^2 m_2 T}$.

consider the following transformations:

$$Z_1 = n_1 X_1^2$$

$$Z_2 = (n_1 - r_1 - 1)[X_2^2 - X_1^2]$$

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$$Z_{m_1} = (n_1 - r_1 - r_2 - \ldots - r_{m_1-1} - m_1 + 1)[X_{m_1}^2 - X_{m_1-1}^2] \tag{2.10}$$

Balakrishnan and Aggarwala [2] show that $Z_i's$ are independent and identically distributed exponential random variables with mean σ_1, moreover

$$T = \sum_{i=1}^{m_1} Z_i = \sum_{i=1}^{m_1}(1+r_i)\, T_i = \sum_{i=1}^{m_1}(1+r_i)\, X_i^2 \sim Gamma(m_1, \sigma_1). \tag{2.11}$$

Similarly, E has a gamma distribution with the shape parameter m_1 and scale parameter σ_1. Hence, λ follow F-distribution with degrees of freedom $2m_2$ and $2m_1$. Using the fact That:

$$\frac{R}{1-R} \times \frac{1-\hat{R}}{\hat{R}} \sim F_{2m_2, 2m_1} \tag{2.12}$$

Then the $100(1-\alpha)\%$ exact confidence interval of R is:

$$P\left(\frac{1}{1+F_{2m_1,2m_2,\alpha/2}(\frac{1}{\hat{R}}-1)} < R < \frac{1}{1+F_{2m_1,2m_2,1-\alpha/2}(\frac{1}{\hat{R}}-1)}\right) = 1-\alpha$$

Where α is the level of significance and $2m_1, 2m_2$ are the degree of freedom of F.

3 UMVUE of R

In this section the uniformly minimum variance unbiased estimator (UMVUE) is obtained for stress-strength parameter R. Let $X_{1:m_1:n_1}, \ldots, X_{m_1:m_1:n_1}$ be a progressive censored sample from $Ray(\sigma_1)$ with progressive censoring scheme $(n_1, m_1, r_1, \ldots, r_{m_1})$, and the progressive censored sample $Y_{1:m_2:n_2}, \ldots, Y_{m_2:m_2:n_2}$ from $Ray(\sigma_2)$ with progressive censoring scheme $(n_2, m_2, s_1, \ldots, s_{m_2})$, then from the jointly log-likelihood function of X,Y that given in equation (2.4), we obtained that $T = \sum_{i=1}^{m_1}(1+r_i)\, x_i^2$ is a sufficient statistics for σ_1 and $E = \sum_{j=1}^{m_2}(1+s_j)y_j^2$ is a sufficient statistics for σ_2. From equation (2.11)

$$f_T(t) = \frac{1}{\sigma_1^{m_1}\Gamma(m_1)} t^{m_1-1} exp(\frac{-t}{\sigma_1}), \quad 0 < t < \infty \tag{3.1}$$

Lemma 3.1 *The conditional pdf. of* $T_1 = X_1^2$ *given* T *is:*

$$f_{T_1|T}(x) = \frac{f_{T_1,T}(x)}{f_T(t)} = n_1(m_1-1)\frac{(T-n_1T_1)^{m_1-2}}{T^{m_1-1}}, \quad 0 < T_1 < T/n_1 \tag{3.2}$$

Proof. Let $W = \sum_{i=2}^{m_1} Z_i$ then clearly W & Z_1 are independent. Then the joint pdf. of $T_1 \& T$, $f_{T_1,T}(x)$ can be easily obtained from the jointly distribution of $W \& Z_1$ using the transformations $Z_1 = n_1 T_1$ & $W = T - Z_1 = T - T_1$ then

$$f_{W,Z_1} = f_W . f_{Z_1} = \frac{1}{\sigma_1^{m_1}\Gamma(m_1-1)} W^{m_1-2} exp(-\frac{W+Z_1}{\sigma_1}) \tag{3.3}$$

And

$$f_{T_1,T} = \frac{n_1}{\sigma_1^{m_1}\Gamma(m_1-1)}(T-n_1T_1)^{m_1-2} exp(-\frac{T}{\sigma_1}) \tag{3.4}$$

From equations (3.4), (3.1), we get the result.

Similarly, if $E = \sum_{j=1}^{m_2}(1+s_j)\,E_j$ where $E_j = Y_j^2$ Then

$$f_{E_1|E}(y) = n_2(m_2-1)\frac{(E-n_2E_1)^{m_2-2}}{E^{m_2-1}}, \quad 0 < E_1 < E/n_2 \tag{3.5}$$

Lemma 3.2 *The unbiased estimator of R is:*

$$\varphi(T_1,E_1) = \begin{cases} 1 & if \quad n_2E_1 < n_1T_1 \\ 0 & if \quad n_2E_1 \geq n_1T_1 \end{cases} \tag{3.6}$$

Where $E_1 = Y_1^2$ and $T_1 = X_1^2$.

Proof.

$$E(\varphi) = 1.\,P(n_2E_1 < n_1T_1) = P(Y_1 < [\frac{n_1x_1^2}{n_2}]^{1/2}) = P(Y_1 < a)$$

$$= \int_0^\infty \int_0^a f_{X_1}(x)\, f_{Y_1}(y)dydx \tag{3.7}$$

Where the distributions of order statistics X_1 and Y_1 are

$$f_{X_1}(x) = n_1\frac{x}{\sigma_1^2}e^{-n_1x^2/2\sigma_1^2}$$

$$f_{Y_1}(y) = n_2\frac{y}{\sigma_2^2}e^{-n_2y^2/2\sigma_2^2} \tag{3.8}$$

Then by using equations (3.8) we get

$$E(\varphi) = \frac{\sigma_1^2}{\sigma_1^2+\sigma_2^2} = R \tag{3.9}$$

Theorem 3.3 *Based on the sufficient statistics T and E, as defined before for σ_1 and σ_2 respectively and the unbiased statistics φ, the UMVUE of R, say $\tilde{R}$, for $m_1 \geq 2$ and $m_2 \geq 2$ can be expressed as follows:*

$$\tilde{R} = \begin{cases} 1 - \sum_{k=0}^{m_2-1}(-1)^k\left(\frac{T}{E}\right)^k \frac{\binom{m_2-1}{k}}{\binom{m_1+k-1}{k}} & if \quad T < E \\ \sum_{k=0}^{m_1-1}(-1)^k\left(\frac{E}{T}\right)^k \frac{\binom{m_1-1}{k}}{\binom{m_2+k-1}{k}} & if \quad T \geq E \end{cases} \quad 3.10)$$

Proof. For $T < E$ using the Rao-Blackwell theorem

$$\tilde{R} = E(\varphi(T_1,E_1)|T,E) = \int\int_A f_{(T_1|T)}\, f_{(E_1|E)}dE_1dT_1, \tag{3.11}$$

Where $A = \{(E_1,T_1): 0 < T_1 < \frac{T}{n_1}, 0 < E_1 < \frac{E}{n_2}$ and $n_2E_1 < n_1T_1\}$ and $f_{(T_1|T)} \& f_{(E_1|E)}$ are defined in equations (3.2), (3.5) respectively. Then $\tilde{R}$ becomes:

$$\tilde{R} = \int_0^{T/n_1}\int_0^{n_1T_1/n_2} n_1(m_1-1)\frac{(T-n_1T_1)^{m_1-2}}{T^{m_1-1}}.\,n_2(m_2-1)\frac{(E-n_2E_1)^{m_2-2}}{E^{m_2-1}}dE_1dT_1$$

$$= 1 - \int_0^{T/n_1} n_1(m_1-1)\frac{(T-n_1T_1)^{m_1-2}}{T^{m_1-1}}\frac{(E-n_1T_1)^{m_2-1}}{E^{m_2-1}}dT_1 \tag{3.12}$$

let $c = \frac{n_1T_1}{T}$, then $\tilde{R}$ becomes:

$$\tilde{R} = 1 - \int_0^1(m_1-1)\,(1-c)^{m_1-2}\left(1-c\frac{T}{E}\right)^{m_2-1}dc, \tag{3.13}$$

since the binomial expansion of $(1-c\frac{T}{E})^{m_2-1} = \sum_{k=0}^{m_2-1}(-1)^k\binom{m_2-1}{k}(\frac{cT}{E})^k$, then $\tilde{R}$ is obtained as following:

$$\tilde{R} = 1 - \sum_{k=0}^{m_2-1}(-1)^k\binom{m_2-1}{k}\left(\frac{T}{E}\right)^k\int_0^1 c^k\,(1-c)^{m_1-2}dc$$

$$= 1 - \sum_{k=0}^{m_2-1}(-1)^k\left(\frac{T}{E}\right)^k\frac{\binom{m_2-1}{k}}{\binom{m_1+k-1}{k}} \tag{3.14}$$

Similarly, if $T \geq E$ then $\tilde{R}$ becomes:

$$\tilde{R} = \sum_{k=0}^{m_1-1}(-1)^k \left(\frac{E}{T}\right)^k \frac{\binom{m_1-1}{k}}{\binom{m_2+k-1}{k}} \tag{3.15}$$

4 Bayes Estimator of R

In this section, the Bayes estimator of $R = P(Y < X)$ is obtained based on the progressive type-II censoring. Let $X_{1:m_1:n_1}, \dots, X_{m_1:m_1:n_1}$ be a progressive censored sample from $Ray(\sigma_1)$ with progressive censoring scheme $(n_1, m_1, r_1, \dots, r_{m_1})$, and the progressive censored sample $Y_{1:m_2:n_2}, \dots, Y_{m_2:m_2:n_2}$ from $Ray(\sigma_2)$ with progressive censoring scheme $(n_2, m_2, s_1, \dots, s_{m_2})$. Assuming that the parameters σ_1, σ_2 are random variables having a gamma prior distribution, for simplicity we assume that the parameters $\lambda_1 = \sigma_1^2$ and $\lambda_2 = \sigma_2^2$. The conjugate priors for λ_1, λ_2 are:

$$\pi(\lambda_1) = \frac{\beta_1^{\alpha_1}}{\Gamma(\alpha_1)} \lambda_1^{-\alpha_1+1} e^{-\beta_1/\lambda_1} \tag{4.1}$$

and

$$\pi(\lambda_2) = \frac{\beta_2^{\alpha_2}}{\Gamma(\alpha_2)} \lambda_2^{-\alpha_2+1} e^{-\beta_2/\lambda_2} \tag{4.2}$$

The likelihood function $L(\lambda_1) = f(\underline{X}|\lambda_1)$ for random variable X is:

$$f(\underline{X}|\lambda_1)) = \frac{k_1}{\lambda_1^{m_1}} \left(\prod_{i=1}^{m_1} x_i\right) exp[-\frac{1}{2\lambda_1} \sum_{i=1}^{m_1}(1 + r_i)\, x_i^2] \tag{4.3}$$

and the marginal distribution for $X \geq 0$ is given by:

$$f(\underline{X}) = \int_0^\infty f(\underline{X}|\lambda_1)\, \pi(\lambda_1) d\lambda_1$$

$$= \frac{k_1 \beta_1^{\alpha_1} \Gamma(\alpha_1 + m_1 - 2)}{A^{\alpha_1+m_1-2}\Gamma(\alpha_1)} \left(\prod_{i=1}^{m_1} x_i\right) \tag{4.4}$$

where $A = \beta_1 + 0.5 \sum_{i=1}^{m_1}(1 + r_i)\, x_i^2$, then the posterior distribution of λ_1 is given by:

$$\pi(\lambda_1|\underline{X}) = \frac{A^{\alpha_1+m_1-2} e^{-A/\lambda_1}}{\lambda_1^{\alpha_1+m_1-1} \Gamma(\alpha_1 + m_1 - 2)} \tag{4.5}$$

Similarly, for $\underline{Y}$ the posterior distribution of λ_2 is given by:

$$\pi(\lambda_2|\underline{Y}) = \frac{B^{\alpha_2+m_2-2} e^{-B/\lambda_2}}{\lambda_2^{\alpha_2+m_2-1} \Gamma(\alpha_2 + m_2 - 2)} \tag{4.6}$$

where $B = \beta_2 + 0.5 \sum_{j=1}^{m_2}(1 + s_j) y_j^2$. Now the joint posterior distribution for λ_1, λ_2 is given by:

$$\pi(\lambda_1, \lambda_2|\underline{X}, \underline{Y}) = H \frac{e^{-A/\lambda_1} e^{-B/\lambda_2}}{\lambda_1^{\alpha_1+m_1-1} \lambda_2^{\alpha_2+m_2-1}} \tag{4.7}$$

where $H = \frac{A^{\alpha_1+m_1-2} B^{\alpha_2+m_2-2}}{\Gamma(\alpha_1+m_1-2)\Gamma(\alpha_2+m_2-2)}$. Now by taking the following transformations $R = \frac{\lambda_1}{\lambda_1+\lambda_2}$ and $\xi = \lambda_1 + \lambda_2$ where $0 \leq R \leq 1, \xi > 0$, then the joint posterior distribution for R and ξ is derived as following:

$$\pi(R, \xi|\underline{X}, \underline{Y}) = H \xi^{3-m_1-m_2-\alpha_1-\alpha_2} \frac{exp[-\frac{1}{\xi}(\frac{A}{R} + \frac{B}{(1-R)})]}{R^{\alpha_1+m_1-1}(1-R)^{\alpha_2+m_2-1}} \tag{4.8}$$

Then by applying integration on equation 4.8 with respect to ξ, we get the posterior distribution for R

$$\pi(R|\underline{X}, \underline{Y}) = H \times \Gamma[m_1 + m_2 + \alpha_1 + \alpha_2 - 4] \frac{R^{\alpha_2+m_2-3}(1-R)^{\alpha_1+m_1-3}}{[A(1-R)+B.R]^{m_1+m_2+\alpha_1+\alpha_2-4}} \tag{4.9}$$

Then under the squared error loss function the Bayes estimator of R is given by:

$$\hat{R}_3 = \int_0^1 R.\pi(R|\underline{X}, \underline{Y}) dR$$

$$= H.\Gamma[m_1 + m_2 + \alpha_1 + \alpha_2 - 4] \int_0^1 \frac{R^{\alpha_2+m_2-2}(1-R)^{\alpha_1+m_1-3}}{[A(1-R)+B.R]^{m_1+m_2+\alpha_1+\alpha_2-4}} dR \tag{4.10}$$

5 Simulation Study

Within this section, the Monte Carlo simulation is performed to check the performance of the different estimators of R under several types of progressive censoring schemes. Samples are generated under progressive type-II censoring with many different schemes for the (n-m) removed items. These schemes are described as follows: Scheme I: complete sample (n=m) i.e there is no removed items. Scheme II: $(r_1 = 0, \dots, r_{m-1} = 0, r_m = n - m)$. Scheme III: $(r_1 = n - m, \dots, r_{m-1} = 0, r_m = 0)$. Scheme IV: The remaining items (n-m) are removed equally at each failure time. For example, if n=10 and m=5 then scheme IV become $(r_1 = 1, r_2 = 1, \dots, r_5 = 1)$. Simulation is performed 1000 times with different sample sizes $n_1, n_2 = 10,20,30$ and the number of failures $m_1, m_2 = 5,10,15,20,30$ for X and Y. Different values of parameters $(\sigma_1, \sigma_2) = (1,1), (1,2), (2,1)$ for the distributions of X and Y are used. The average estimates of MLE for R and the average of MSE's are reported is Table 1,2,3 for each set of parameters. With each table the 95% exact confidence interval of R. Also, simulation is constructed 1000 times for UMVUE of R suggested in Section (3) at each set of values in table 4. Finally in table 5 the Bayes estimator of R is obtained using different values for the parameters $\alpha_1, \alpha_2, \beta_1$ and β_2 when $n_1, n_2 = 20,30$ and $m_1, m_2 = 10,15$. All tables are founded in the appendix. We note that in such cases as the effective sample size increases the estimates of R become better. When $n = m$ i.e in case of complete samples the biased is decreased. Also, when $(n_1, m_1) = (n_2, n_2)$ the estimates are good. We note that MLE of R give results better than the UMVUE of R.

Table 1: MLE, MSE and Exact 95% C.I for R=0.5 when $\sigma_1 = \sigma_2 = 1$ with different scheme types r, s.

(n_1, m_1)	(n_2, m_2)	r	s	MLE		95%C.I.	
				$\hat{R}_1$	MSE	Lower	Upper
(10,10)	(10,10)	Comp	Comp	0.499078	$1.1 .10^{-2}$	0.28789	0.71060
(10,5)	(10,5)	II	II	0.501915	$2.205 .10^{-2}$	0.213291	0.789268
		III	III	0.498651	$1.30 .10^{-2}$	0.211109	0.787089
		IV	IV	0.506229	$1.34 .10^{-2}$	0.216201	0.792125
(20,20)	(10,10)	Comp	Comp	0.50448	$7.95 .10^{-3}$	0.26163	0.677949
(10,10)	(20,20)	Comp	Comp	0.49126	$8.92 .10^{-3}$	0.315959	0.732939
(20,10)	(10,5)	II	II	0.516308	$1.79 .10^{-2}$	0.237949	0.74752
		III	III	0.512684	$9.64 .10^{-3}$	0.235328	0.744772
		IV	IV	0.527223	$1.10 .10^{-2}$	0.245972	0.755686
(10,5)	(20,10)	II	II	0.483445	$1.81 .10^{-2}$	0.252294	0.761872
		III	III	0.486932	$1.03 .10^{-2}$	0.254936	0.764395
		IV	IV	0.467997	$1.09 .10^{-2}$	0.240789	0.750452
(20,10)	(20,10)	II	II	0.502365	$1.18 .10^{-2}$	0.290589	0.713296
		III	III	0.500906	$6.55 .10^{-3}$	0.289388	0.712101
		IV	IV	0.499608	$6.44 .10^{-3}$	0.288321	0.711035
(20,20)	(20,20)	Comp	Comp	0.49969	$6.08 .10^{-3}$	0.347527	0.651923
(30,15)	(30,15)	II	II	0.49626	$8.27 .10^{-3}$	0.322042	0.671396
		III	III	0.50307	$4.16 .10^{-3}$	0.328014	0.677373
		IV	IV	0.498426	$4.58 .10^{-3}$	0.323934	0.673302
(30,30)	(30,30)	Comp	Comp	0.50059	$3.94 .10^{-3}$	0.37554	0.62557

Table 2: MLE, MSE and Exact 95% C.I for R=0.2 when $(\sigma_1, \sigma_2) = (1,2)$ with different scheme types r, s.

(n_1, m_1)	(n_2, m_2)	r	s	MLE $\hat{R}_1$ MSE	95%C.I. Lower Upper
(10,10)	(10,10)	Comp	Comp	0.21291 5.89 .10^{-3}	0.0989044 0.399994
(10,5)	(10,5)	II III IV	II III IV	0.21925 1.29 .10^{-2} 0.21216 5.74 .10^{-3} 0.208308 6.16 .10^{-3}	0.0702452 0.510697 0.0675582 0.500226 0.0661114 0.494427
(20,20)	(10,10)	Comp	Comp	0.21169 4.33 .10^{-3}	0.08547 0.35701
(10,10)	(20,20)	Comp	Comp	0.19994 3.72 .10^{-3}	0.10783 0.41797
(20,10)	(10,5)	II III IV	II III IV	0.22161 9.23 .10^{-3} 0.21439 4.98 .10^{-3} 0.22960 6.47 .10^{-3}	0.07687 0.44124 0.07392 0.430831 0.08018 0.452546
(10,5)	(20,10)	II III IV	II III IV	0.201719 7.44 .10^{-3} 0.19692 4.26 .10^{-3} 0.19005 4.27 .10^{-3}	0.08349 0.463472 0.08123 0.45601 0.07799 0.445103
(20,10)	(20,10)	II III IV	II III IV	0.20661 5.19 .10^{-3} 0.20345 3.04 .10^{-3} 0.20333 2.77 .10^{-3}	0.09557 0.39091 0.09390 0.38630 0.093843 0.38613
(20,20)	(20,20)	Comp	Comp	0.20485 2.85 .10^{-3}	0.12079 0.32573
(30,15)	(30,15)	II III IV	II III IV	0.20828 3.79 .10^{-3} 0.20735 2.01 .10^{-3} 0.20148 1.91 .10^{-3}	0.11257 0.35301 0.1120 0.35171 0.10846 0.34353
(30,30)	(30,30)	Comp	Comp	0.20296 1.79 .10^{-3}	0.13253 0.29797

Table 3: MLE, MSE and Exact 95% C.I for R=0.8 when $(\sigma_1, \sigma_2) = (2,1)$ with different scheme types r, s.

(n_1, m_1)	(n_2, m_2)	r	s	MLE $\hat{R}_1$ MSE	95%C.I. Lower Upper
(10,10)	(10,10)	Comp	Comp	0.78995 5.84.10^{-3}	0.60411 0.90261
(10,5)	(10,5)	II III IV	II III IV	0.779165 1.2.10^{-2} 0.78699 6.18.10^{-3} 0.78757 6.36.10^{-3}	0.48699 0.92915 0.4985 0.93212 0.49937 0.93234
(20,20)	(10,10)	Comp	Comp	0.79852 3.86.10^{-3}	0.57973 0.89124
(10,10)	(20,20)	Comp	Comp	0.78893 4.45.10^{-3}	0.64383 0.91482
(20,10)	(10,5)	II III IV	II III IV	0.79655 7.60.10^{-3} 0.79974 4.12.10^{-3} 0.81033 4.22.10^{-3}	0.53386 0.91568 0.53878 0.917195 0.55551 0.92218
(10,5)	(20,10)	II III IV	II III IV	0.780636 8.89.10^{-3} 0.78366 4.66.10^{-3} 0.773707 6.23.10^{-3}	0.56198 0.924043 0.56634 0.9246 0.55211 0.92119
(20,10)	(20,10)	II III IV	II III IV	0.78873 5.14.10^{-3} 0.79276 2.81.10^{-3} 0.79387 3.31.10^{-3}	0.60236 0.90197 0.60818 0.9041 0.60979 0.90469

(20,20)	(20,20)	Comp	Comp	0.79693 $2.53.10^{-3}$	0.67666 0.88037
(30,15)	(30,15)	II III IV	II III IV	0.795464 $3.42.10^{-3}$ 0.798036 $1.69.10^{-3}$ 0.79720 $1.68.10^{-3}$	0.652201 0.889696 0.655795 0.891244 0.654632 0.89074
(30,30)	(30,30)	Comp	Comp	0.79717 $1.67.10^{-3}$	0.70220 0.86756

Table 4: UMVUE $\hat{R}_2$and MSE for R at different sets of values of $\sigma_1, \sigma_2$with different scheme types r, s.

(n_1, m_1)	(n_2, m_2)	r	s	$(\sigma_1, \sigma_2) = (1,1)$ R=0.5 $\hat{R}_2$ MSE	$(\sigma_1, \sigma_2) = (1,2)$ R=0.2 $\hat{R}_2$ MSE	$(\sigma_1, \sigma_2) = (2,1)$ R=0.8 $\hat{R}_2$ MSE
(10,10)	(10,10)	Comp	Comp	0.50808 $1.35.10^{-2}$	0.1979 $5.29.10^{-3}$	0.80086 $5.38.10^{-3}$
(10,5)	(10,5)	II III IV	II III IV	0.495515 $2.82.10^{-2}$ 0.5045 $1.52.10^{-2}$ 0.49617 $1.66.10^{-2}$	0.196772 $1.13.10^{-2}$ 0.18978 $6.19\ .10^{-3}$ 0.19043 $5.48\ .10^{-3}$	0.80204 $1.27.10^{-2}$ 0.81112 $6.29.10^{-3}$ 0.810735 $6.38.10^{-3}$
(20,20)	(10,10)	Comp	Comp	0.49997 $8.62\ .10^{-3}$	0.20129 $4.41\ .10^{-3}$	0.80175 $3.78.10^{-3}$
(10,10)	(20,20)	Comp	Comp	0.50002 $1.00\ .10^{-2}$	0.19802 $3.78\ .10^{-3}$	0.80355 $3.89.10^{-3}$
(20,10)	(10,5)	II III IV	II III IV	0.50059 $2.15\ .10^{-2}$ 0.5036 $1.15.10^{-2}$ 0.52110 $1.21\ .10^{-2}$	0.20457 $9.18\ .10^{-3}$ 0.19251 $4.62\ .10^{-3}$ 0.20198 $4.81\ .10^{-3}$	0.80413 $7.97.10^{-3}$ 0.80694 $4.18.10^{-3}$ 0.81298 $4.13.10^{-3}$
(10,5)	(20,10)	II III IV	II III IV	0.49943 $2.05\ .10^{-2}$ 0.49673 $1.15.10^{-2}$ 0.47983 $1.24\ .10^{-2}$	0.20083 $8.82\ .10^{-3}$ 0.19241 $4.18\ .10^{-3}$ 0.18361 $4.48\ .10^{-3}$	0.80030 $9.25.10^{-3}$ 0.80489 $4.46.10^{-3}$ 0.80132 $5.29.10^{-3}$
(20,10)	(20,10)	II III IV	II III IV	0.50182 $1.29\ .10^{-2}$ 0.49939 $7.09\ .10^{-3}$ 0.50416 $6.80\ .10^{-3}$	0.20285 $5.59\ .10^{-3}$ 0.19574 $2.93\ .10^{-3}$ 0.19113 $2.64\ .10^{-3}$	0.79938 $5.39.10^{-3}$ 0.80537 $3.09.10^{-3}$ 0.80283 $2.91.10^{-3}$
(20,20)	(20,20)	Comp	Comp	0.50026 $6.29\ .10^{-3}$	0.20091 $2.67\ .10^{-3}$	0.79727 $2.74.10^{-3}$
(30,15)	(30,15)	II III IV	II III IV	0.50353 $8.53\ .10^{-3}$ 0.49911 $4.64\ .10^{-3}$ 0.50236 $4.88\ .10^{-3}$	0.19523 $3.54\ .10^{-3}$ 0.19582 $1.72\ .10^{-3}$ 0.19517 $1.84\ .10^{-3}$	0.79645 $3.56.10^{-3}$ 0.80565 $1.88.10^{-3}$ 0.80332 $1.89.10^{-3}$
(30,30)	(30,30)	Comp	Comp	0.49885 $4.23\ .10^{-3}$	0.20195 $1.68\ .10^{-3}$	0.80127 $1.72.10^{-3}$

Table 5: Bayes estimator of R, $\hat{R}_3$ and MSE when $\sigma_1 = 1, \sigma_2 = 2$

(α_1,β_1)	(α_2,β_2)	r	s	(n_1,n_2,m_1,m_2)=(20,20,10,10) $\hat{R}_3$ MSE	(n_1,n_2,m_1,m_2)=(30,30,15,15) $\hat{R}_3$ MSE
(0,0)	(0,0)	II III IV	II III IV	0.22119 $5.90.10^{-3}$ 0.21705 $3.27.10^{-3}$ 0.21461 $2.86.10^{-3}$	0.21197 $3.57.10^{-3}$ 0.21186 $1.97.10^{-3}$ 0.21050 $1.95.10^{-3}$
(0,1)	(0,1)	II III IV	II III IV	0.23210 $6.41.10^{-3}$ 0.22379 $3.25.10^{-3}$ 0.22554 $3.56.10^{-3}$	0.22299 $4.10.10^{-3}$ 0.21600 $2.02.10^{-3}$ 0.21363 $2.02.10^{-3}$
(1,1)	(1,1)	II III IV	II III IV	0.22805 $5.71.10^{-3}$ 0.21738 $2.86.10^{-3}$ 0.22286 $3.26.10^{-3}$	0.220073 $4.31.10^{-3}$ 0.21280 $1.92.10^{-3}$ 0.21337 $1.95.10^{-3}$
(2,1)	(2,1)	II III IV	II III IV	0.23215 $6.42.10^{-3}$ 0.22133 $3.25.10^{-3}$ 0.22160 $3.29.10^{-3}$	0.22391 $3.91.10^{-3}$ 0.21549 $2.12.10^{-3}$ 0.213616 $2.01.10^{-3}$
(1,2)	(1,2)	II III IV	II III IV	0.24156 $6.17.10^{-3}$ 0.228882 $3.36.10^{-3}$ 0.22876 $3.49.10^{-3}$	0.22672 $4.29.10^{-3}$ 0.21822 $2.12.10^{-3}$ 0.2177 $2.06.10^{-3}$
				(n_1,n_2,m_1,m_2)=(20,20,20,20)	(n_1,n_2,m_1,m_2)=(30,30,30,30)
(α_1,β_1)	(α_2,β_2)	r	s	$\hat{R}_3$ MSE	$\hat{R}_3$ MSE
(0,0) (0,1) (1,1) (2,1) (1,2)	(0,0) (0,1) (1,1) (2,1) (1,2)	Comp Comp Comp Comp Comp	Comp Comp Comp Comp Comp	0.21128 $2.80.10^{-3}$ 0.21521 $2.78.10^{-3}$ 0.21656 $2.86.10^{-3}$ 0.21768 $2.84.10^{-3}$ 0.219668 $2.84.10^{-3}$	0.20507 $1.66.10^{-3}$ 0.21269 $2.03.10^{-3}$ 0.20979 $1.72.10^{-3}$ 0.20784 $1.73.10^{-3}$ 0.21345 $1.92.10^{-3}$

6 Conclusions

We have presented some efficient estimators of the stress-strength parameter R using MLE and UMVUE methods. The methods are very efficient. We have found that, our estimates of R using progressive censoring schemes are very close to estimates in case of complete samples so these estimates are better to accelerate the life testing. This work gives a general estimate since the case when sample sizes equal the number of failures is a special case. The exact confidence intervals based on MLE of Rare obtained. The choice of removing schemes effect on the estimates. Numerical results are presented which exhibit the performance of the proposed methods. From the mean square error (MSE) we found that UMVUE of R is more effective than MLE and Bayes estimator of R in many cases.

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this article

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