Mathematical Sciences Letters
An International Journal

http://dx.doi.org/10.12785/msl/040303

Soft Sets, Soft Semimodules and Soft Substructures of Semimodules

Akın Osman Atagün[1] *and Aslıhan Sezgin Sezer*[2,*]

[1] Department of Mathematics, Bozok University, Yozgat, Turkey
[2] Department of Mathematics, Amasya University, 05100 Amasya, Turkey

Received: 20 Jul. 2014, Revised: 22 Mar. 2015, Accepted: 25 Mar. 2015
Published online: 1 Sep. 2015

Abstract: Soft set theory, proposed by Molodtsov, has been regarded as an effective mathematical tool to deal with uncertainties. In this paper, we introduce and study soft semimodule and construct some basic properties by using semimodules and Molodtsov's definition of soft sets. We introduce the notions of Soft seminodule, Soft subsemimodule, Soft semimodule homomorphism. Furthermore, we introduce subsemimodule of a semimodule and some related properties about soft substructures of semimodules are investigated and illustrated by many examples.

Keywords: Soft sets, Soft semimodule, Soft subsemimodule, Soft semimodule homomorphism, Soft subsemimodule of a semimodule

1 Introduction

Molodtsov [18] introduced soft set theory in 1999 by for dealing with uncertainties and it has not continued to experience tremendous growth and diversification in the mean of algebraic structures as in [1, 2, 4, 8, 9, 10, 11, 12, 13, 14, 20, 23, 24, 25, 22, 26] but also operations of soft sets as in [3, 15, 21]. Furthermore, soft set relations and functions [5] and soft mappings [17] with many related concepts were discussed. The theory of soft set has also a wide-ranging applications especially in soft decision making as in the following studies: [6, **?**, **?**, **?**].

In this paper, we introduce a basic version of soft semimodules, which extends the notion of semimodules by including some algebraic structures in soft set theory. A soft semimodule defined in this paper is actually a parametrized family of subsemimodules and has some properties similar to those of semimodules.

2 Preliminaries

A *semiring* R is a structure consisting of a nonempty set R together with two binary operation on R called *addition* and

i)R together with addition is a semigroup,
ii)R together with multiplication is a semigroup,
iii)$(a+b)c = ac+bc$ and $a(b+c) = ab+ac$ for all $a,b,c \in R$.

A semiring R is said to be *additively commutative* if $a+b = b+a$ for all $a,b \in R$. Throughout this paper, R will always denote an additively commutative semiring. A zero element of a semiring R is an element 0 such that $0.x = x.0 = 0$ and $0+x = x+0 = x$ for all $x \in R$. A nonempty subset I of a semiring R is called a *left (resp. right) ideal of* R if I is closed under addition and $RI \subseteq I$ (resp. $IR \subseteq I$). We say that I is an ideal of R, denoted by $I \lhd R$, if it is both a left and right ideal of R. Given a semiring R, a *left R-semimodule* M is a nonempty set on which we have operations of addition and multiplication by elements of R (on the left side) such that

i)Addition is associative and commutative and has a neutral element, usually denoted by 0_M,
ii)$r(x+y) = rx+ry$,
iii)$(r+s)x = rx+sx$,
iv)$(rs)x = r(sx)$,
v)$0x = 0_M = r0_M$ and $1m = m$.

for all $r,s \in R$, $x,y \in M$. For example it is easy to see that if R is a semiring and A is a nonempty set, then the set R^A of all functions from A to R is a left R-semimodule, with scalar multiplication and addition being defined elementwise. Similarly R itself is a (left) R-semimodule by natural operations. Suppose M is a left R-module and

* Corresponding author e-mail: aslihan.sezgin@amasya.edu.tr

N is a subset of M. Then N is called a *subsemimodule* (or *R-subsemimodule*, to be more explicit) if, for any $n,n' \in N$ and any $r \in R$, $n+n' \in N$ and the product rn is in N.

Molodtsov [18] defined the soft set in the following manner: Let U be an initial universe set, E be a set of parameters, $P(U)$ be the power set of U and $A \subseteq E$.

Definition 1.[18] *A pair (F,A) is called a soft set over U, where F is a mapping given by*

$$F : A \to P(U).$$

In other words, a soft set over U is a parameterized family of subsets of the universe U.

Definition 2.[3] *Let (F,A) and (G,B) be two soft sets over a common universe U such that $A \cap B \neq \emptyset$. The restricted intersection of (F,A) and (G,B) is denoted by $(F,A) \Cap (G,B)$, and is defined as $(F,A) \Cap (G,B) = (H,C)$, where $C = A \cap B$ and for all $c \in C$, $H(c) = F(c) \cap G(c)$.*

Definition 3.[3] *Let (F,A) and (G,B) be two soft sets over a common universe U. The extended intersection of (F,A) and (G,B) is defined to be the soft set (H,C), where $C = A \cup B$ and for all $e \in C$,*

$$H(e) = \begin{cases} F(e) & \text{if } e \in A \setminus B, \\ G(e) & \text{if } e \in B \setminus A, \\ F(e) \cap G(e) & \text{if } e \in A \cap B. \end{cases}$$

This relation is denoted by $(F,A) \sqcap_{\varepsilon} (G,B) = (H,C)$.

Definition 4.[15] *Let (F,A) and (G,B) be two soft sets over a common universe U. The union of (F,A) and (G,B) is defined to be the soft set (H,C) satisfying the following conditions: (i) $C = A \cup B$; (ii) for all $e \in C$,*

$$H(e) = \begin{cases} F(e) & \text{if } e \in A \setminus B, \\ G(e) & \text{if } e \in B \setminus A, \\ F(e) \cup G(e) & \text{if } e \in A \cap B. \end{cases}$$

This relation is denoted by $(F,A)\widetilde{\cup}(G,B) = (H,C)$.

Definition 5.[15] *If (F,A) and (G,B) are two soft sets over a common universe U, then "(F,A) AND (G,B)" denoted by $(F,A)\widetilde{\wedge}(G,B)$ is defined by $(F,A)\widetilde{\wedge}(G,B) = (H, A \times B)$, where $H(x,y) = F(x) \cap G(y)$ for all $(x,y) \in A \times B$.*

Definition 6.[8] *Let $(F_i,A_i)_{i\in I}$ be a nonempty family of soft sets over a common universe U. The union of these soft sets is defined to be the soft set (G,B) such that $B = \bigcup_{i\in I} A_i$ and for all $x \in B$, $G(x) = \bigcup_{i\in I(x)} F_i(x)$ where $I(x) = \{i \in I \mid x \in A_i\}$. In this case we write $\widetilde{\bigcup}_{i\in I}(F_i,A_i) = (G,B)$.*

Definition 7.[8] *Let $(F_i,A_i)_{i\in I}$ be a nonempty family of soft sets over a common universe set U. The AND-soft set $\widetilde{\bigwedge}_{i\in I}(F_i,A_i)$ of these soft sets is defined to be the soft set (H,B) such that $B = \prod_{i\in I} A_i$ and $H(x) = \bigcap_{i\in I(x)} F_i(x)$ for all $x = (x_i)_{i\in I} \in B$.*

Note that if $A_i = A$ and $F_i = F$ for all $i \in I$, then $\widetilde{\bigwedge}_{i\in I}(F_i,A_i)$ is denoted by $\widetilde{\bigwedge}_{i\in I}(F,A)$. In this case, $\prod_{i\in I} A_i = \prod_{i\in I} A$ means the direct power A^I.

Definition 8.*Let $(F_i,A_i)_{i\in I}$ be a nonempty family of soft sets over a common universe set U. The restricted intersection of these soft sets is defined to be the soft set (G,B) such that $B = \bigcap_{i\in I} A_i \neq \emptyset$ and for all $x \in B$, $G(x) = \bigcap_{i\in I} F_i(x)$. In this case we write $\Cap_{i\in I}(F_i,A_i) = (G,B)$.*

3 Soft semimodules

From now on, let R be a semiring, M be a left R-semimodule and A be a nonempty set. For a soft set (F,A), the set $Supp(F,A) = \{x \in A \mid F(x) \neq \emptyset\}$ is called the *support* of the soft set (F,A). The null soft set is a soft set with an empty support, and a soft set (F,A) is non-null if $Supp(F,A) \neq \emptyset$ [12]. Note that, if N is a subsemimodule of M, then we write $N \leq M$. Now we are ready to give the definition of soft semimodule.

Definition 9.*Let (F,A) be a non-null soft set over a semimodule M. Then (F,A) is called a* soft semimodule *over M if $F(x)$ is a subsemimodule of M for all $x \in Supp(F,A)$.*

Example 31*Let $R = \{0,a,b,c\}$ be a semiring with the operation tables given by the following tables.*

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	a	0	a
b	0	0	b	b
c	0	a	b	c

Let $M = R$ and the soft set (F,A) over M, where $A = \{0,a,b\}$ and $F : A \to P(M)$ is a set-valued function defined by

$$F(x) = \{y \in M \mid y = x^n \ for\ some\ n \in \mathbb{N}\}$$

for all $x \in A$. Here, $x^n = xx...x$ means the n -fold product of x and $x^0 = 0$. Then $F(0) = \{0\}$, $F(a) = \{0,a\}$ and $F(b) = \{0,b\}$. Since $F(x)$ are all subsemimodules of M for all $x \in Supp(F,A)$, (F,A) is a soft semimodule over M. Similarly, if we define the soft set (G,B) over M, where $B = \{b,c\}$ and $G : B \to P(M)$ is a set-valued function defined by

$$G(x) = \{y \in M \mid xy \in \{0,b\}\}$$

for all $x \in B$, then $G(b) = \{0,a,b,c\}$ and $G(c) = \{0,b\}$. Since $G(x)$ are both subsemimodules of M for all $x \in Supp(G,B)$, (G,B) is a soft semimodule over M.

Let $R = \{0,a,b,c\}$ be a semiring with the operation tables given by the following tables.

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	a	0	b
b	0	0	0	0
c	0	b	0	a

Let $M = R$ and the soft set (H,C) over M, where $C = \{0,a,b,c\}$ and $H : C \to P(M)$ is a set-valued function defined by

$$H(x) = \{0\} \cup \{y \in M \mid x+y = 0\}$$

for all $x \in C$. Then $H(0) = \{0\}$, $H(a) = \{0,a\}$, $H(b) = \{0,b\}$ and $H(c) = \{0,c\}$. Since $H(a)$ and $H(c)$ are not subsemimodules of M, (H,C) is not a soft semimodule over M.

Theorem 32*Let (F,A) and (G,B) be soft semimodules over M. Then,*

a)If it is non-null, then the soft set $(F,A)\widetilde{\wedge}(G,B)$ is a soft semimodule over M.
b)If it is non-null, then the restricted intersection $(F,A) \Cap (G,B)$ is a soft semimodule over M.
c)If it is non-null, then the soft set $(F,A) \sqcap_{\varepsilon} (G,B)$ is a soft semimodule over M.
d)If A and B are disjoint, then $(F,A)\widetilde{\cup}(G,B)$ is a soft semimodule over M.

*Proof.*Let $(F,A)\widetilde{\wedge}(G,B) = (Q,A \times B)$, where $Q(x,y) = F(x) \cap G(y)$ for all $(x,y) \in A \times B$. Then by hypothesis, $(Q,A\times B)$ is a non-null soft set over M. If $(x,y) \in Supp(Q,A\times B)$, then $Q(x,y) = F(x) \cap G(y) \neq \emptyset$. It follows that $\emptyset \neq F(x)$ and $\emptyset \neq G(y)$ are both subsemimodules of M. Hence $Q(x,y)$ is a subsemimodule of M for all $(x,y) \in Supp(Q,A\times B)$. Therefore $(Q,A\times B)$ is a soft semimodule over M.

b) Let $(F,A) \Cap (G,B) = (H,C)$, where $H(x) = F(x) \cap G(x)$ for all $x \in C = A \cap B \neq \emptyset$. Suppose that (H,C) is a non-null soft set over M. If $x \in Supp(H,C)$, then $H(x) = F(x) \cap G(x) \neq \emptyset$. It follows that $\emptyset \neq F(x)$ and $\emptyset \neq G(x)$ are both subsemimodules of M. Hence $H(x)$ is a subsemimodule of M for all $x \in Supp(H,C)$. Thus, (H,C) is a soft semimodule over M.

c) Let $(F,A) \sqcap_{\varepsilon} (G,B) = (K,A\cup B)$, where

$$K(x) = \begin{cases} F(x) & \text{if } x \in A \setminus B, \\ G(x) & \text{if } x \in B \setminus A, \\ F(x) \cap G(x) & \text{if } x \in A \cap B \end{cases}$$

for all $x \in A\cup B$. Suppose that $(K,A\cup B)$ is a non-null soft set over M. Let $x \in Supp(K,A\cup B)$. If $x \in A \setminus B$, then $\emptyset \neq K(x) = F(x) \leq M$. If $x \in B\setminus A$, then $\emptyset \neq K(x) = G(x) \leq M$ and if $x \in A\cap B$, then $K(x) = F(x) \cap G(x) \neq \emptyset$. Since $\emptyset \neq F(x) \leq M$ and $\emptyset \neq G(x) \leq M$, it follows that $K(x) \leq M$ for all $x \in Supp(K,A\cup B)$. Therefore $(F,A) \sqcap_{\varepsilon} (G,B) = (K,A\cup B)$ is a soft semimodule over M.

d) We can write $(F,A)\widetilde{\cup}(G,B) = (T,A\cup B)$, where

$$T(x) = \begin{cases} F(x) & \text{if } x \in A \setminus B, \\ G(x) & \text{if } x \in B \setminus A, \\ F(x) \cup G(x) & \text{if } x \in A \cap B \end{cases}$$

for all $x \in A\cup B$. Since $A\cap B = \emptyset$, it follows that either $x \in A\setminus B$ or $x \in B\setminus A$ for all $x \in A\cup B$. If $x \in A\setminus B$, then $T(x) = F(x)$ is a subsemimodule of M and if $x \in B\setminus A$, then $T(x) = G(x)$ is a subsemimodule of M. Thus, $(T,A\cup B)$ is a soft semimodule over M.

Definition 10.*Let (F,A) and (G,B) be two soft semimodules over M_1 and M_2, respectively. The product of soft semimodules (F,A) and (G,B) is defined as $(F,A) \times (G,B) = (U,A \times B)$, where $U(x,y) = F(x) \times G(y)$ for all $(x,y) \in A\times B$.*

Proposition 33*Let (F,A) and (G,B) be two soft semimodules over M_1 and M_2, respectively. Then if it is non-null, the product $(F,A)\times(G,B)$ is a soft semimodule over $M_1 \times M_2$.*

*Proof.*Let $(F,A) \times (G,B) = (U,A \times B)$, where $U(x,y) = F(x) \times G(y)$ for all $(x,y) \in A\times B$. Then by hypothesis, $(U,A\times B)$ is a non-null soft set over $M_1 \times M_2$. If $(x,y) \in Supp(U,A \times B)$, then $U(x,y) = F(x) \times G(y) \neq \emptyset$. Since $\emptyset \neq F(x)$ is a subsemimodule of M_1 and $\emptyset \neq G(y)$ is a subsemimodule of M_2, it follows that $U(x,y)$ is a subsemimodule of $M_1 \times M_2$ for all $(x,y) \in Supp(U,A\times B)$. Therefore $(U,A\times B)$ is a soft semimodule over $M_1\times M_2$.

It is worth nothing that if N_1 and N_2 are two subsemimodules of M, then the sum of these two subsemimodules is defined as the following: $N_1+N_2 = \{n_1+n_2 \mid n_1 \in N_1 \wedge n_2 \in N_2\}$.

Definition 11.*Let (F,N_1) and (G,N_2) be two semimodules over M. If $N_1 \cap N_2 = \{0_M\}$, then the sum of soft semimodules (F,N_1) and (G,N_2) is defined as $(F,N_1) + (G,N_2) = (H,N_1 + N_2)$, where $H(x+y) = F(x)+G(y)$ for all $x+y \in N_1+N_2$.*

Proposition 34*Let (F,N_1) and (G,N_2) be two soft semimodules over M where $N_1\cap N_2 = \{0_M\}$. Then if it is non-null, the sum $(F,N_1)+(G,N_2)$ is a soft semimodule over M.*

*Proof.*Let $(F,N_1) + (G,N_2) = (H,N_1 + N_2)$, where $H(x+y) = F(x)+G(y)$ for all $x+y \in N_1+N_2$. Then by hypothesis, (H,N_1+N_2) is a non-null soft set over M. If $x + y \in Supp(H,N_1 + N_2)$, then $H(x+y) = F(x)+G(y) \neq \emptyset$. It is seen that H is well defined because $N_1\cap N_2 = \{0_M\}$. Since $\emptyset \neq F(x)$ is a subsemimodule of M and $\emptyset \neq G(y)$ is a subsemimodule of M, it follows that $H(x+y)$ is a subsemimodule of M for all $x+y \in Supp(H,N_1+N_2)$. Therefore (H,N_1+N_2) is a soft semimodule over M.

Example 35*Let consider the soft semimodules (F,A) and (G,B) in Example 31. Let $(F,A)\widetilde{\wedge}(G,B) = (Q,A\times B)$, where $Q(x,y) = F(x) \cap G(y)$ for all $(x,y) \in A\times B = \{(0,b),(0,c),(a,b),(a,c),(b,b),(b,c)\}$. Then $Q(0,b) = Q(0,c) = Q(a,c) = \{0\}$, $Q(a,b) = \{0,a\}$ and $Q(b,b) = Q(b,c) = \{0,b\}$. Since $Q(x,y)$ is a subsemimodule of $M = R$ for all $(x,y) \in Supp(Q,A\times B)$, $(Q,A\times B)$ is a soft semimodule over M.*

Let $(F,A) \Cap (G,B) = (T,C)$, *where* $H(x) = F(x) \cap G(x)$ *for all* $x \in C = A \cap B = \{b\}$. *Since* $T(b) = F(b) \cap G(b) = \{0,b\}$ *is a subsemimodule of* $M = R$, (T,C) *is a soft semimodule over* M.

Assume that $(F,A) \sqcap_{\varepsilon} (G,B) = (K, A \cup B)$, *where*

$$K(x) = \begin{cases} F(x) & \text{if } x \in A \setminus B = \{0,a\}, \\ G(x) & \text{if } x \in B \setminus A = \{c\}, \\ F(x) \cap G(x) & \text{if } x \in A \cap B = \{b\} \end{cases}$$

for all $x \in A \cup B$. *Then,* $K(0) = \{0\}$, $K(a) = \{0,a\}$, $K(c) = \{0,b\}$ *and* $K(b) = \{0,b\}$. *Then, it is obvious that* $(K, A \cup B)$ *is a semimodule over* M.

Let $(F,A) \times (G,B) = (Z, A \times B)$, *where* $Z(x,y) = F(x) \times G(y)$ *for all* $(x,y) \in A \times B = \{(0,b),(0,c),(a,b),(a,c),(b,b),(b,c)\}$. *Then* $Z(0,b) = \{(0,0),(0,a),(0,b),(0,c)\}$, $Z(0,c) = \{(0,0),(0,b)\}$, $Z(a,b) = \{(0,0),(0,a),$ $(0,b),(0,c),(a,0),(a,a),(a,b),(a,c)\}$, $Z(a,c) = \{(0,0),(0,b),(a,0),(a,b)\}$, $Z(b,b) = \{(0,0),(0,a),(0,b),(0,c),(b,0),(b,a),(b,b),(b,c)\}$ *and* $Z(b,c) = \{(0,0),(0,b),(b,0),(b,b)\}$. *Since* $Z(x,y)$ *are all subsemimodules of* $M \times M$ *for all* $(x,y) \in Supp(Z, A \times B)$, $(Z, A \times B)$ *is a soft semimodule over* $M \times M$.

Definition 12. *Let* (F,A) *and* (G,B) *be two soft semimodules over* M. *Then* (F,A) *is called a* soft subsemimodule *of* (G,B) *if it satisfies:*

i) $A \subseteq B$
ii) $F(x)$ *is a subsemimodule of* $G(x)$ *for all* $x \in Supp(F,A)$.

Example 36 *Let* $R = \mathbb{Z}^* = \mathbb{Z}^+ \cup \{0\}$ *be the semiring under ordinary addition and multiplication and* $M = \mathbb{Z}^* \times \mathbb{Z}^*$ *be the left R-semimodule of R with the usual scalar multiplication. Let* (F,A) *be a soft set over* M, *where* $A = \mathbb{Z}^*$ *and* $F : A \to P(M)$ *is a set-valued function defined by* $F(x) = \{0\} \times 2x\mathbb{Z}^*$ *for all* $x \in A$. *It is obvious that* (F,A) *is a soft semimodule over* M. *Let* (G,B) *be a soft set over* M , *where* $B = \{0,1,\ldots 40\} \subseteq A$ *and* $G : B \to P(M)$ *is a set-valued function defined by* $G(x) = \{0\} \times 4x\mathbb{Z}^*$ *for all* $x \in B$. *It is obvious that* $G(x)$ *is a subsemimodule of* $F(x)$ *for all* $x \in Supp(G,B)$. *Therefore,* (G,B) *is a soft subsemimodule of* (F,A).

Theorem 37 *Let* (F,A), (G,A) *and* (H,B) *be soft semimodules over* M. *Then we have the following:*

a) *If* $F(x) \subseteq G(x)$ *for all* $x \in A$, *then* (F,A) *is a soft subsemimodule* (G,A).
b) $(F,A) \Cap (H,B)$ *is a soft subsemimodule both* (F,A) *and* (H,B) *if it is non-null.*
c) $(F,A) \sqcap_{\varepsilon} (G,A)$ *is a soft subsemimodule of both* (F,A) *and* (G,A) *if it is non-null.*

Proof. **a)** If $F(x) \subseteq G(x)$ for all $x \in A$, it is clear that $F(x)$ is a subsemimodule $G(x)$. Therefore the result is obvious.

b) It is obvious that $A \cap B \subseteq A$ (and $A \cap B \subseteq B$). Let $(F,A) \Cap (H,B) = (K,C)$, where $C = A \cap B$ and $K(x) = F(x) \cap H(x)$ for all $x \in C$. Since $K(x) = F(x) \cap H(x) \subseteq F(x)$ and $K(x) = F(x) \cap H(x) \subseteq H(x)$ for all $x \in C$, the proof is completed.

c) Let $(F,A) \sqcap_{\varepsilon} (G,A) = (Q,A)$ where $Q(x) = F(x) \cap G(x)$ for all $x \in A$. Since $Q(x) = F(x) \cap G(x) \subseteq F(x)$ and $Q(x) = F(x) \cap G(x) \subseteq G(x)$ for all $x \in A$, the proof is completed.

Theorem 38 *Let* (F,A) *be a soft semimodule over* M *and* $(F_i,A_i)_{i\in I}$ *be a nonempty family of soft subsemimodules of* (F,A). *Then we have the following:*

a) $\Cap_{i\in I}(F_i,A_i)$ *is a soft subsemimodule of* (F,A), *if it is non-null.*
b) $\widetilde{\bigwedge}_{i\in I}(F_i,A_i)$ *is a soft subsemimodule of* $\widetilde{\bigwedge}_{i\in I}(F,A)$, *if it is non-null.*
c) *If* $\{A_i \mid i \in I\}$ *are pairwise disjoint, i.e.,* $i \neq j$ *implies* $A_i \cap A_j = \emptyset$, *then* $\widetilde{\bigcup}_{i\in I}(F_i,A_i)$ *is a soft subsemimodule of* (F,A).

Proposition 39 *Let* (F,A) *be a soft semimodule over* M *and* $(F_i,A_i)_{i\in I}$ *be a nonempty family of soft subsemimodules of* (F,A). *Then* $\Cap_{i\in I}(F_i,A_i)$ *is a soft subsemimodule of* (F_i,A_i) *for each* $i \in I$, *if it is non-null.*

Proof. Let $\Cap_{i\in I}(F_i,A_i) = (H,C)$, where $C = \bigcap_{i\in I} A_i \neq \emptyset$ and $H(x) = \bigcap_{i\in I} F_i(x)$ for all $x \in C$. The parameter set of the soft set $\Cap_{i\in I}(F_i,A_i)$, that is, $\bigcap_{i\in I} A_i$ is a subset of the parameter set of the soft set $(F_i,A_i)_{i\in I}$ for all $i \in I$. Suppose that (H,C) is a non-null soft set over M. If $x \in Supp(H,C)$, then $H(x) = \bigcap_{i\in I} F_i(x) \neq \emptyset$. Thus $\emptyset \neq F_i(x)$ are subsemimodules over M for all $i \in I$. Therefore $H(x) = \bigcap_{i\in I} F_i(x)$ is a subsemimodule over M. Moreover, since $\bigcap_{i\in I} F_i(x) \subseteq F_i(x)$, for all $i \in I$ and for all $x \in \bigcap_{i\in I} A_i$, the rest of the proof is obvious.

Proposition 310 *If* (F,A) *be a soft semimodule over* M *and* $B \subset A$, *then so is* (F,B), *whenever* (F,B) *is non-null.*

Definition 13. *Let* (F,A) *be a soft semimodule over* M. *Then,*

a) *If* M *is a left R-semimodule with zero and if* $F(x) = \{0_M\}$ *for all* $x \in Supp(F,A)$, *then* (F,A) *is called trivial.*
b) (F,A) *is said to be whole if* $F(x) = M$ *for all* $x \in Supp(F,A)$.

Example 311 *Let R be the semiring in Example 31 with the second operation tables. Let* $M = R$ *and the soft set* (Q,A) *over* M, *where* $A = \{0,a,b,c\}$ *and* $Q : A \to P(M)$ *is a set-valued function defined by*

$$Q(x) = \{y \in M \mid x0 = y\}$$

for all $x \in A$. *Then* $Q(0) = Q(a) = Q(b) = Q(c) = \{0\}$. *Since* $Q(x) = \{0_M\}$ *for all* $x \in Supp(Q,A)$, (Q,A) *is a trivial soft semimodule over* M.

Let the soft set (T,B) *over* M, *where* $B = \{0,b\}$ *and* $T : B \to P(M)$ *is a set-valued function defined by*

$$T(x) = \{y \in M \mid xy = 0\}$$

for all $x \in B$. Then $T(0) = T(b) = M$. It follows that (T,B) is a whole soft semimodule over M.

Proposition 312_Let (F,A) and (G,B) be soft semimodules over M. Then,_

i)If (F,A) and (G,B) are trivial (resp., whole) soft semimodules over M, then $(F,A) \Cap (G,B)$ is a trivial (resp., whole) soft semimodule over M.
ii)If (F,A) is a trivial soft semimodule over M and (G,A) is a whole soft semimodule over M, then $(F,A) \Cap (G,B)$ is a trivial soft semimodule over M.
iii)If (F,A) and (G,B) are trivial (resp., whole) soft semimodules over M where $A \cap B = \{0_M\}$, then $(F,A) + (G,B)$ is a trivial (resp., whole) soft semimodule over M.
iv)If (F,A) is a trivial soft semimodule over M and (G,B) is a whole soft semimodule over M where $A \cap B = \{0_M\}$, then $(F,A) + (G,B)$ is a whole soft semimodule over M.

Proposition 313_Let (F,N_1) and (G,N_2) be two soft semimodules over M_1 and M_2, respectively. Then,_

i)If (F,N_1) and (G,N_2) are trivial soft semimodules over N_1 and N_2, respectively, then $(F,N_1) \times (G,N_2)$ is a trivial soft semimodule over $M_1 \times M_2$.
ii)If (F,N_1) and (G,N_2) are whole soft semimodules over M_1 and M_2, respectively, then $(F,N_1) \times (G,N_2)$ is a whole soft semimodule over $M_1 \times M_2$.

Let (F,A) be a soft semimodule over M and let $f : M_1 \to M_2$ be a mapping of semimodules. Then the soft set $(f(F), Supp(F,A))$ over M_2 can be defined, where

$$f(F) : Supp(F,A) \to P(M_2)$$

is given by $f(F)(x) = f(F(x))$ for all $x \in Supp(F,A)$. It is also worth nothing that $Supp(F,A) = Supp(f(F), Supp(F,A))$.

Proposition 314_Let $f : M_1 \to M_2$ be a semimodule epimorphism. If (F,A) is a non-null soft semimodule over M_1 , then $(f(F), Supp(F,A))$ is a non-null soft semimodule over M_2._

*Proof.*Note first that since (F,A) is is a non-null soft semimodule over M_1, then so is $(f(F), Supp(F,A))$ over M_2. We have $f(F)(x) = f(F(x)) \neq \emptyset$ for all $x \in Supp(f(F), Supp(F,A))$. Because of the fact that (F,A) is a soft semimodule over M_1, the nonempty set $F(x)$ is a subsemimodule of M_1. Thus, we can conclude that its onto homomorphic image $f(F(x))$ is a subsemimodule over M_2. So, $f(F(x))$ is a subsemimodule over M_2 for all $x \in Supp(f(F), Supp(F,A))$. It means that $(f(F), Supp(F,A))$ is a soft semimodule over M_2.

Theorem 315_Let (F,A) be a soft semimodule over M_1 and let $f : M_1 \to M_2$ be a surjective homomorphism of semimodules. Then_

a)If $F(x) = Ker f$ for all $x \in Supp(F,A)$, then $(f(F), Supp(F,A))$ is a trivial soft semimodule over M_2.
b)If (F,A) is whole, then $(f(F), Supp(F,A))$ is a whole soft semimodule over M_2.

*Proof.***a)** Assume that $F(x) = Ker f$ for all $x \in Supp(F,A)$. Then $f(F)(x) = f(F(x)) = 0_M$ for all $x \in Supp(F,A)$. That is to say $(f(F), Supp(F,A))$ is a trivial soft semimodule over M_2.

b) Suppose that (F,A) is whole. Then, $F(x) = V$ for all $x \in Supp(F,A)$. It follows that $f(F)(x) = f(F(x)) = F(V) = W$ for all $x \in Supp(F,A)$, which means that $(f(F), Supp(F,A))$ is a whole soft semimodule over M_2.

Definition 14._Let (F,A) and (G,B) be soft semimodule over M_1 and M_2, respectively. Let $f : M_1 \to M_2$ and $g : A \to B$ be two mappings. Then the pair (f,g) is called a_ soft semimodule homomorphism _if it satisfies the conditions below:_

i)f is an epimorphism.
ii)g is a surjective mapping.
iii)$f(F(x)) = G(g(x))$ for all $x \in A$.

If there exists a soft homomorphism between (F,A) and (G,B), we mention that (F,A) is soft homomorphic *to (G,B), which is denoted by $(F,A) \sim (G,B)$. Furthermore, if f is an isomorphism of semimodules and g is a bijective mapping, then (f,g) is said to be a* soft semimodule isomorphism. *In this case, we say that (F,A) is* soft isomorphic *to (G,B), which is denoted by $(F,A) \simeq_M (G,B)$.*

Example 316_Let the semiring $R = \mathbb{Z}^* = \{0\} \cup \mathbb{Z}^+$ and $M = \mathbb{Z}^* \times \mathbb{Z}^*$ be the left R-semimodule of R with the usual scalar multiplication. Let (F,A) be a soft set over M, where $A = \mathbb{Z}^*$ and $F : A \to P(M)$ is a set-valued function defined by $F(x) = \{0\} \times 2x\mathbb{Z}^*$ for all $x \in A$. It is obvious that (F,A) is a soft semimodule over M. Let the semiring $R' = \mathbb{Z}^*$ and $M' = \mathbb{Z}^*$ be the left R'-semimodule of R'. Let (G,B) be a soft set over M', where $B = \mathbb{Z}^*$ and $G : B \to P(M')$ is a set-valued function defined by $G(x) = 2xk$ $(k \in \mathbb{Z})$ for all $x \in B$. It is obvious that (G,B) is a soft semimodule over M'. Let $f : \mathbb{Z}^* \times \mathbb{Z}^* \to \mathbb{Z}^*$ be the mapping defined by $f(x,y) = y$. One can easily say that f is an epimorphism of semimodules. Let $g : \mathbb{Z}^* \to \mathbb{Z}^*$ be the mapping defined by $g(x) = x$ for all $x \in \mathbb{Z}^*$. Then one can easily say that g is surjective. Since $f(F(x)) = f(\{0\} \times 2x\mathbb{Z}^*) = 2x\mathbb{Z}^*$ and $(G(g(x)) = G(x) = 2xk = 2x\mathbb{Z}^*$ is satisfied for all $x \in \mathbb{Z}$, it follows that (f,g) is a soft semimodule homomorphism and $(F,A) \sim (G,B)$._

4 Soft substructures of semimodules

Definition 15._Let N be a subsemimodule of M and let (F,N) be a soft set over M. If for all $x,y \in N$ and for all $r \in R$,_

s1) $F(x+y) \supseteq F(x) \cap F(y)$ *and*
s2) $F(rx) \supseteq F(x)$,

then the soft set (F,N) *is called a soft subsemimodule of* M *and denoted by* $(F,N)\tilde{<}M$ *or simply* $F_N\tilde{<}M$.

Example 41*Let* R *be the semiring in Example 31 with the first operation tables. Let* $M = R$ *be a left* R*-semimodule and* $N_1 = \{0,a\}$ *be a subsemimodule of* M*. Let the soft set* (F,N_1) *over* M*, where* $F : N_1 \to P(M)$ *is a set valued function by* $F(0) = \{0,a,b\}$ *and* $F(a) = \{0,b\}$*. Then it can be easily seen that* $(F,N_1)\tilde{<}M$.

Let $N_2 = \{0,b\} < M$ *and the soft set* (G,N_2) *over* M*, where* $G : N_2 \to P(M)$ *is a set valued function by* $G(0) = \{0,b,c\}$ *and* $G(b) = \{b\}$*. Then* $(G,N_2)\tilde{<}M$*, too. However if we define the soft set* (H,N_2) *over* M *such that* $H(0) = \{a,c\}$ *and* $H(b) = \{0,b,c\}$*, then* $H(a.b) = H(0) = \{a,c\} \not\supseteq H(b) = \{0,b,c\}$*. Therefore,* (H,N_2) *is not a soft subsemimodule over* M.

Theorem 42*If* $F_{N_1}\tilde{<}M$ *and* $G_{N_2}\tilde{<}M$*, then* $F_{N_1} \Cap G_{N_2}\tilde{<}M$.

*Proof.*Since N_1 and N_2 are subsemimodules of M,then it follows that $N_1 \cap N_2 \neq \emptyset$ and $N_1 \cap N_2$ is a subsemimodule of M. Let $F_{N_1} \Cap G_{N_2} = (F,N_1) \Cap (G,N_2) = (H,N_1 \cap N_2)$, where $H(x) = F(x) \cap G(x)$ for all $x \in N_1 \cap N_2 \neq \emptyset$. Then for all $x,y \in N_1 \cap N_2$ and $r \in R$,

s1) $H(x+y) = F(x+y) \cap G(x+y) \supseteq (F(x) \cap F(y)) \cap (G(x) \cap G(y)) = (F(x) \cap G(x)) \cap (F(y) \cap G(y)) = H(x) \cap H(y)$,
s2) $H(rx) = F(rx) \cap G(rx) \supseteq F(x) \cap G(x) = H(x)$.

Therefore $F_{N_1} \Cap G_{N_2} = H_{N_1\cap N_2}\tilde{<}M$.

Definition 16.*Let* M_1 *and* M_2 *be left* R*-subsemimodules and let* (F,N_1) *and* (G,N_2) *be two soft subsemimodules of* M_1 *and* M_2*, respectively. The product of soft subsemimodules* (F,N_1) *and* (G,N_2) *is defined as* $(F,N_1) \times (G,N_2) = (Q,N_1 \times N_2)$*, where* $Q(x,y) = F(x) \times G(y)$ *for all* $(x,y) \in N_1 \times N_2$.

Theorem 43*If* $F_{N_1}\tilde{<}M_1$ *and* $G_{N_2}\tilde{<}M_2$*, then* $F_{N_1} \times G_{N_2}\tilde{<}M_1 \times M_2$.

*Proof.*Since N_1 and N_2 are subsemimodules of M_1 and M_2, respectively, then $N_1 \times N_2$ is a subsemimodule of $M_1 \times M_2$. Let $F_{N_1} \times G_{N_2} = (F,N_1) \times (G,N_2) = (Q,N_1 \times N_2)$, where $Q(x,y) = F(x) \times G(y)$ for all $(x,y) \in M_1 \times M_2$. Then for all $(x_1,y_1),(x_2,y_2) \in M_1 \times M_2$ and $r \in R$,

s1) $Q((x_1,y_1)+(x_2,y_2)) = Q(x_1+x_2,y_1+y_2) = F(x_1+x_2) \times G(y_1+y_2) \supseteq (F(x_1) \cap F(x_2)) \times (G(y_1) \cap G(y_2)) = (F(x_1) \times G(y_1)) \cap (F(x_2) \times G(y_2)) = Q(x_1,y_1) \cap Q(x_2,y_2)$,
s2) $Q(r(x_1,y_1)) = Q(rx_1,ry_1) = F(rx_1) \times G(ry_1) \supseteq F(x_1) \times G(y_1) = Q(x_1,y_1)$.

Hence $F_{N_1} \times G_{N_2} = Q_{N_1\times N_2}\tilde{<}M_1 \times M_2$.

Example 44*Let* $(F,N_1)\tilde{<}M$ *and* $(G,N_2)\tilde{<}M$ *in Example 41.* $(F,N_1) \Cap (G,N_2) = (T,N_1 \cap N_2)$*, where* $T(x) = F(x) \cap G(x)$ *for all* $x \in N_1 \cap N_2 = \{0\}$*. Then* $T(0) = F(0) \cap G(0) = \{0,b\}$*. It is obvious that* $(T,N_1 \cap N_2)\tilde{<}M$.

Let $F_{N_1} \times G_{N_2} = (F,N_1) \times (G,N_2) = (Q,N_1 \times N_2)$*, where* $Q(x,y) = F(x) \times G(y)$ *for all* $(x,y) \in N_1 \times N_2 = \{(0,0),(0,b),(a,0),(a,b)\}$*. Then it can be easily seen that* $Q_{N_1\times N_2}\tilde{<}\mathbb{Z}_{10} \times \mathbb{Z}_{10}$*. We show the operations for some elements of* $N_1 \times N_2$*:*

$$\begin{aligned} Q((a,0)+(a,b)) = Q(a+a,0+b) &= Q(0,b) \\ &= F(0) \times G(b) = \{0,a,b\} \times \{b\} \\ &= \{(0,b),(a,b),(b,b)\} \\ Q(a,0) \cap Q(a,b) &= (F(a) \times G(0)) \cap (F(a) \times G(b)) \\ &= (\{0,b\} \times \{0,b,c\}) \cap (\{0,b\} \times \{b\}) \\ &= \{(0,b),(b,b)\} \\ Q(a(a,b)) = Q(aa,ab) &= Q(a,0) \\ &= F(a) \times G(0) = (\{0,b\} \times \{0,b,c\}) \\ &= \{(0,0),(0,b),(0,c),(b,0),(b,b),(b,c)\} \end{aligned}$$

It is seen that $Q((a,0)+(a,b)) \supseteq Q(a,0) \cap Q(a,b)$ *and* $Q(a(a,b)) \supseteq Q(a,b) = F(a) \times G(b) = \{(0,b),(b,b)\}$.

Definition 17.*Let* (F,N) *and* (G,K) *be two soft subsemimodules of* M*. If* $N \cap K = \{0_M\}$*, then the sum of soft subsemimodules* (F,N) *and* (G,K) *is defined as* $(F,N) + (G,K) = (T,N+K)$*, where* $T(x+y) = F(x) + G(y)$ *for all* $x+y \in N+K$.

Theorem 45*If* $F_N\tilde{<}M$ *and* $G_K\tilde{<}M$*, where* $N \cap K = \{0_M\}$*, then* $F_N + G_M\tilde{<}M$.

*Proof.*Since N and K are subsemimodules of M, then $N+K$ is a subsemimodule of M. Let $F_N + G_K = (F,N) + (G,K) = (T,N+K)$, where $T(x) = F(x) + G(x)$ for all $x \in N+K$. Then for all $x_1+y_1, x_2+y_2 \in N+K$ and $r \in R$,

$$\begin{aligned} T((x_1+y_1)+(x_2+y_2)) &= T((x_1+x_2)+(y_1+y_2)) \\ &= F(x_1+x_2)+G(y_1+y_2) \\ &\supseteq (F(x_1) \cap F(x_2)) + (G(y_1) \cap G(y_2)) \\ &= (F(x_1)+G(y_1)) \cap (F(x_2)+G(y_2)) \\ &= T(x_1+y_1) \cap T(x_2+y_2), \\ T(r(x_1+y_1)) &= T(rx_1+ry_1) \\ &= F(rx_1)+G(ry_1) \\ &\supseteq F(x_1)+G(y_1) \\ &= T(x_1+y_1). \end{aligned}$$

Therefore $F_N + G_K = T_{N+K}\tilde{<}M$.

Proposition 46*Let* M *be an* R*-semimodule such that* $(M,+)$ *is a group. If* $F_N\tilde{<}M$*, then* $F(0_M) \supseteq F(x)$ *for all* $x \in N$.

*Proof.*Since (F,N) is a soft subsemimodule of M, then for all $x,y \in N$, $F(x+y) \supseteq F(x) \cap F(y)$. Since $(M,+)$ is a group, if we take $y = -x$ then $F(x-x) = F(0_M) \supseteq F(x) \cap F(x) = F(x)$ for all $x \in N$.

Proposition 47*Let M be an R-semimodule such that $(M,+)$ is a group. If $F_N \widetilde{<} M$, then $N_F = \{x \in N \mid F(x) = F(0_M)\}$ is a subsemimodule of N.*

*Proof.*We need to show that $x+y \in N_F$ and $nx \in N_F$ for all $x,y \in N_F$ and $n \in N$, which means that $F(x+y) = F(0_M)$ and $F(nx) = F(0_M)$ have to be satisfied. Since $x,y \in N_F$, then $F(x) = F(y) = F(0_M)$. Since (F,N) is a soft subsemimodule of M, then $F(x+y) \supseteq F(x) \cap F(y) = F(0_M)$ and $F(nx) \supseteq F(x) = F(0_M)$ for all $x,y \in N_F$ and and $n \in N$. Moreover, $F(0_M) \supseteq F(x+y)$ and $F(0_M) \supseteq F(nx)$. Therefore N_F is a subsemimodule of N.

Definition 18.*Let (F,N) be a soft subsemimodule of M. Then,*

i)If M is a left R-semimodule with zero 0_M and if $F(x) = \{0_M\}$ for all $x \in N$, then (F,A) is called trivial.
ii)(F,N) is said to be whole if $F(x) = M$ for all $x \in N$.

Proposition 48*Let (F,N_1) and (G,N_2) be soft subsemimodules of M. Then,*

i)If (F,N_1) and (G,N_2) are trivial (resp., whole) soft subsemimodules of M, then $(F,N_1) \Cap (G,N_2)$ is a trivial (resp., whole) soft subsemimodule of M.
ii)If (F,N_1) is a trivial soft subsemimodule of M and (G,N_2) is a whole soft subsemimodule of M, then $(F,N_1) \Cap (G,N_2)$ is a trivial soft subsemimodule of M.
iv)If (F,N_1) and (G,N_2) are trivial (resp., whole) soft subsemimodules of M where $N_1 \cap N_2 = \{0_M\}$, then $(F,N_1) + (G,N_2)$ is a trivial (resp., whole) soft subsemimodule of M.
v)If (F,N_1) is a trivial soft subsemimodule of M and (G,N_2) is a whole soft subsemimodule of M where $N_1 \cap N_2 = \{0_M\}$, then $(F,N_1) + (G,N_2)$ is a whole soft subsemimodule of M.

Proposition 49*Let (F,N_1) and (G,N_2) be two soft subsemimodules of M_1 and M_2, respectively. Then,*

i)If (F,N_1) and (G,N_2) are trivial soft subsemimodules of M_1 and M_2, respectively, then $(F,N_1) \times (G,N_2)$ is a trivial soft subsemimodule of $M_1 \times M_2$.
ii)If (F,N_1) and (G,N_2) are whole soft subsemimodules of M_1 and M_2, respectively, then $(F,N_1) \times (G,N_2)$ is a whole soft subsemimodule of $M_1 \times M_2$.

Theorem 410*Let M_1 be a R-semimodules with zero 0_{M_1} and M_2 be a R-semimodules with zero 0_{M_2}, $(F_1,N_1) \widetilde{<} M_1$, $(F_2,N_2) \widetilde{<} M_2$. If $f : N_1 \to N_2$ is a semimodule homomorphism, then*

a)If f is an epimorphism, then $(F_1, f^{-1}(N_2)) \widetilde{<} M_1$,
b)$(F_2, f(N_1)) \widetilde{<} M_2$,
c)$(F_1, Ker f) \widetilde{<} M_1$.

*Proof.***a)** Since $N_1 < M_1$, $N_2 < M_2$ and $f : N_1 \to N_2$ is a semimodule epimorphism, then it is clear that $f^{-1}(N_2) < M_1$. Since $(F_1,N_1) \widetilde{<} M_1$ and $f^{-1}(N_2) \subseteq N_1$, $F_1(x+y) \supseteq F_1(x) \cap F_1(y)$ and $F_1(rx) \supseteq F_1(x)$ for all $x,y \in f^{-1}(N_2)$ and $r \in R$. Hence $(F_1, f^{-1}(N_2)) \widetilde{<} M_1$.

b) Since $N_1 < M_1$, $N_2 < M_2$ and $f : N_1 \to N_2$ is a semimodule homomorphism, then $f(N_1) < M_2$. Since $f(N_1) \subseteq N_2$, the result is obvious.

c) Since $Ker f < M_1$ and $Ker f \subseteq N_1$, the rest of the proof is clear.

Corollary 411*Let $(F_1,N_1) \widetilde{<} M_1$, $(F_2,N_2) \widetilde{<} M_2$ and $f : N_1 \to N_2$ is a semimodule homomorphism, then $(F_2, \{0_{N_2}\}) \widetilde{<} M_2$.*

*Proof.*Since $(F_1, Ker f) \widetilde{<} M_1$. Then $(F_2, f(Ker f)) = (F_2, \{0_{N_2}\}) \widetilde{<} M_2$.

5 Conclusion

Throughout this paper, in a semimodule structure, we have studied the algebraic properties of soft sets which were introduced by Molodtsov as a new mathematical tool for dealing with uncertainty. This work bears soft semimodule, soft subsemimodule and soft semimodule homomorphism. Moreover, we deal with the algebraic soft substructures of a semimodule. We have introduced soft subsemimodule of a semimodule and study its related properties with some examples. To extend this work, one could study the soft substructures of other algebraic structures.

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Akın Osman Atagün is Associate Professor of Mathematics at Bozok University. He received the PhD degree in Mathematics at Erciyes University (Turkey). His main research interests are: Near-ring, Prime ideals of near-rings and near-ring modules, Soft set and its algebraic applications.

Aslıhan Sezgin Sezer is Associate Professor Doctor of Mathematics at Amasya University. She received the PhD degree in Mathematics at Gaziosmanpaşa University (Turkey). Her research interests are in the areas of soft set and its algebraic applications and general near-ring theory.