Mathematical Sciences Letters
An International Journal

http://dx.doi.org/10.18576/msl/100203

Application of COVID-19 Pandemic by Using Odd Lomax-*G* Inverse Weibull Distribution

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Received: 2 Feb. 2021, Revised: 2 Apr. 2021, Accepted: 21 Apr. 2021
Published online: 1 May 2021

Abstract: The aim of this paper is to introduce a new superior distribution for modeling of mortality rate for the COVID-19 pandemic of France from 1 January to 20 February 2021. A new distribution is a combination of the inverse Weibull distribution and the odd Lomax-*G* family to formulate the odd Lomax-*G* inverse Weibull (OLIW) distribution with four parameters. A simple linear representation, hazard function, hazard rate function, r^{th}-moment, moment generating function, and Rényi entropy have been obtained of OLIW distribution. To estimate the unknown parameters of OLIW distribution, we use maximum Liklihhod, maximum product spacing, and Bayesian estimation methods. For the Bayesian approximation, the Metropolis-Hasting algorithm and square error loss function are used. To evaluate the use of estimation methods, a numerical result from the Monte Carlo simulation is obtained.

Keywords: Odd Lomax-*G* family; Inverse Weibull; Bayesian; COVID-19; maximum product spacing

1 Introduction

One of the most important tasks of statistics is to model real-life events using probability distributions. Modeling and analyzing lifetime data is critical in many applied sciences, including medicine, engineering, and finance, among others. To model various types of survival data, several lifetime models have been used. The quality of statistical analysis procedures is heavily reliant on the generated family of distributions, and much effort has gone into developing new statistical models. However, there are still a number of significant issues concerning actual data that do not fit into any of the commonly used mathematical models. As a result, the statistical literature acknowledges the technique of extending a family of distributions by introducing new parameters.

Inverse (or inverted) distributions are significant in many fields, including biological sciences, life test problems, medical sciences, and so on, because of their applicability. Inverted conformation distributions have a different structure than non-inverted conformation distributions in terms of density and hazard ratio. Several researchers have addressed the applications of inverted distributions, and the reader can refer to them Folks and Chhikara [1], Rosaiah and Kantam [2], De Gusmao et al. [3], Joshi and Kumar [5], Almetwally [6], Ibrahim and Almetwally [7], Almongy et al. [4], Ramos et al. [8], Almetwally [9], and Hassan et al. [10], and among others. Let X be a random variable with the shape and scale parameters $\lambda,\theta > 0$, and the distribution is inverse Weibull (IW). The following are the functions for the cumulative distribution (CDF) and probability density (PDF):

$$G(x;\Theta) = e^{-\left(\frac{\theta}{x}\right)^{\lambda}}; \quad x > 0, \lambda, \theta > 0 \tag{1}$$

and,

$$g(x;\Theta) = \frac{\lambda\theta}{x^2}\left(\frac{\theta}{x}\right)^{\lambda-1} e^{-\left(\frac{\theta}{x}\right)^{\lambda}}; \; x, \lambda, \theta > 0, \tag{2}$$

where $\Theta = (\lambda,\theta)$ is a vector of parameters of IW distribution. In different statistical writings by various authors, a generalization of a different distribution of IW was discussed, mainly applied in reliability estimation. For example, the beta IW distribution was implemented by Hanook et al. [11]. Elbatal and Muhammed [12] proposed the exponentiated generalized IW distribution. Ibrahim and Almetwally [7] introduced a new extension of IW distribution by using X-Gamma family with applications of medical data. Okasha et al. [13] introduced extended IW distribution with reliability

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application. Basheer [14] discussed alpha power IW distribution with application. Muhammed and Almetwally [21] introduced bayesian and classical estimation for the Bivariate IW distribution under progressive Type-II censored sample.
We are introducing a new model with four parameters, called the distribution of odd Lomax inverse Weibull (OLIW). Centered on the odd Lomax-G (OL-G) family introduced by Cordeiro et al. [16]. Let $g(x;\Theta) = \frac{dG(x;\theta)}{dx}$ denote the survival function (S) and probability density function (PDF) of a baseline model with parameter vector θ respectively, so the CDF of the OL-G family is given by:

$$F(x;\Omega) = 1-\beta^{\alpha}\left[\beta+\frac{G(x;\Theta)}{1-G(x;\Theta)}\right]^{-\alpha}, x>0, \alpha, \beta>0, \quad (3)$$

where $\Omega = (\alpha, \beta, \Theta)$ is a vector of parameters of OL-G family. The corresponding PDF of (3) is defined by

$$f(x;\Omega) = \frac{\alpha\beta^{\alpha}g(x;\Theta)}{(1-G(x;\Theta))^2}\left[\beta+\frac{G(x;\Theta)}{1-G(x;\Theta)}\right]^{-\alpha-1} \quad (4)$$

where α and β are positive shape parameters. The random variable with PDF (4) is denoted by $X \sim$OL-G(Ω). A new extended four-parameter Weibull, Lomax, log-logistic, and log-Lindley distributions, called the OL-Weibull, OL-Lomax, OL-log-logistic, and OL-log-Lindley distributions respectively, were introduced by Cordeiro et al. [16]. Ogunsanya et al. [17] introduced odd Lomax-exponential distribution. Yakura et al. [18] introduced odd Lomax-Kumaraswamy distribution. Marzouk et al. [19] obtained a generalized odd Lomax generated family of distributions with applications. Abubakari et al. [20] discussed extended odd Lomax family of distribution.
The aim of this research is to look into the point estimation of the unknown four-parameter of the OLIW distribution using three estimation methods: maximum likelihood, maximum product spacing, and bayesian. Statistical analysis is carried out between these methods through simulation to test their efficiency and to investigate how these estimators work for various sample sizes and parameter values. It is addressed how COVID-19 data can be used. The remainder of this article is structured as follows. We define the OLIW distribution in Section 2. In Section 3, along with some of its statistical properties for the OLIW distribution are obtained. Section 4 studies three methods of point estimation. To compare the performance of these estimation methods, a simulation study is performed in Section 5. The Application of COVID-19 data is discussed in Section 6 to show the efficiency of the distribution of OLIW with respect to other distributions. Finally, in Section 7, conclusions are provided.

2 OLIW Distribution

A special model of the OL-G family with IW distribution as a baseline function is the four-parameters OLIW distribution. By substituting the IW model CDF and PDF files (1) and (2) of the OL-G family (3) and (4), the OLIW distribution CDF and PDF are obtained as;

$$F(x;\Omega) = 1-\beta^{\alpha}\left[\beta+\frac{e^{-\left(\frac{\theta}{x}\right)^{\lambda}}}{1-e^{-\left(\frac{\theta}{x}\right)^{\lambda}}}\right]^{-\alpha}, \quad (5)$$

where $x>0, \alpha, \beta, \lambda, \theta>0$.

$$f(x;\Omega) = \alpha\lambda\beta^{\alpha}\frac{\frac{1}{x}\left(\frac{\theta}{x}\right)^{\lambda}e^{-\left(\frac{\theta}{x}\right)^{\lambda}}}{\left(1-e^{-\left(\frac{\theta}{x}\right)^{\lambda}}\right)^2}\left[\beta+\frac{e^{-\left(\frac{\theta}{x}\right)^{\lambda}}}{1-e^{-\left(\frac{\theta}{x}\right)^{\lambda}}}\right]^{-\alpha-1}, \quad (6)$$

Therefore, a random variable with PDF (6) is denoted by $X \sim$OLIW$(\alpha, \beta, \lambda, \theta)$, see Figure 1. The hazard rate function (HR) of the OLIW distribution are given by

$$hr(x;\Omega) = \alpha\lambda\frac{\frac{1}{x}\left(\frac{\theta}{x}\right)^{\lambda}e^{-\left(\frac{\theta}{x}\right)^{\lambda}}}{\left(1-e^{-\left(\frac{\theta}{x}\right)^{\lambda}}\right)^2}\left[\beta+\frac{e^{-\left(\frac{\theta}{x}\right)^{\lambda}}}{1-e^{-\left(\frac{\theta}{x}\right)^{\lambda}}}\right]^{-1}.$$

The hazard function of the OLIW distribution are given by

$$h(x;\Omega) = \alpha\left\{\ln(\beta)+\ln\left[\beta+\frac{e^{-\left(\frac{\theta}{x}\right)^{\lambda}}}{1-e^{-\left(\frac{\theta}{x}\right)^{\lambda}}}\right]\right\}.$$

The odds ratio of failure (ORF) of the OLIW distribution are given by

$$ORF(x;\Omega) = \beta^{-\alpha}\left[\beta+\frac{e^{-\left(\frac{\theta}{x}\right)^{\lambda}}}{1-e^{-\left(\frac{\theta}{x}\right)^{\lambda}}}\right]^{\alpha}-1.$$

Figures 1 2, and 3 are separate shapes of the OLIW distribution's PDF, HR, and hazard. The PDF of the OLIW distribution can be right-skewed, symmetric, or decreasing curves, as seen in these figures. The HR of the OLIW distribution has some interesting forms, such as constant, decreasing, and upside down curves, all of which are appealing features for any lifetime model. The OLIW distribution, as seen in the application section, has a lot of versatility and can be used to model distorted data, so it's commonly used in fields like biomedical studies, biology, reliability, physical engineering, and survival analysis.

3 Statistical Properties of OLIW Distribution

In this section, we observe some statistical properties of the OLIW distribution, namely, the linear representation, which is useful in finding the moments, moment generating function (MGF), and Rényi entropy. Also, we obtain the mean residual life and mean inactivity time.

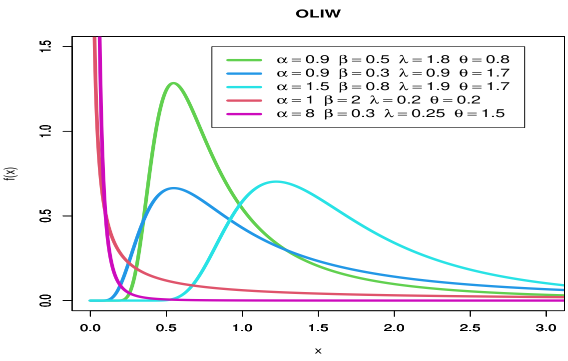

Fig. 1: pdf of OLIW distribution

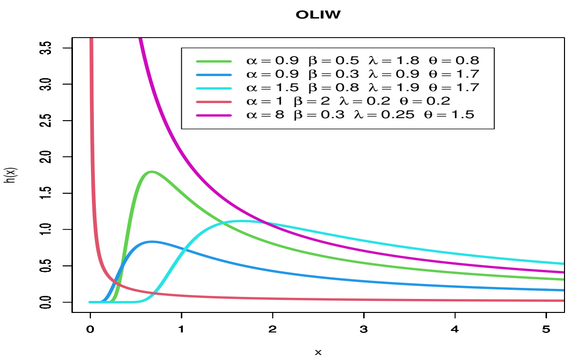

Fig. 2: HR of OLIW distribution

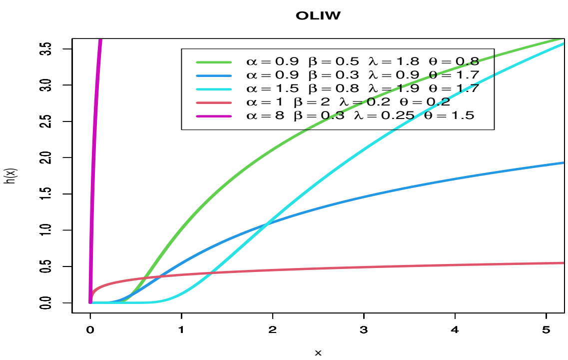

Fig. 3: Hazard of OLIW distribution

3.1 Linear Representation

According to Cordeiro et al. [16], which they discussed the density linear representation for the OL-G family as follows

$$f(x,\Omega)=\sum_{k,j=0}^{\infty}\Delta_{k,j}(k+j+1)g(x,\Theta)G(x,\Theta)^{k+j}, \quad (7)$$

where $\Delta_{k,j}=\frac{(-1)^{j}\alpha}{(k+j+1)\beta^{k+1}}\binom{-\alpha-1}{k}\binom{-k-2}{j}$. The cumulative linear representation for the OL-G family is as follows

$$F(x,\Omega)=\sum_{k,j=0}^{\infty}\Delta_{k,j}G(x,\Theta)^{k+j+1}, \quad (8)$$

According to Equations (7) and (8), the Linear representation of pdf for the OLIW density can be written as

$$f(x,\Omega)=\sum_{k,j=0}^{\infty}\Delta_{k,j}(k+j+1)\frac{\lambda\theta}{x^{2}}\left(\frac{\theta}{x}\right)^{\lambda-1}e^{-(k+j+1)\left(\frac{\theta}{x}\right)^{\lambda}}, \quad (9)$$

Equation (9) denotes the IW density with parameters λ and $(k+j+1)\theta^{\lambda}$. By integrating Equation (9), we obtain the Linear representation of cdf for the OLIW density can be written as

$$F(x,\Omega)=\sum_{k,j=0}^{\infty}\Delta_{k,j}(k+j+1)\,e^{-(k+j+1)\left(\frac{\theta}{x}\right)^{\lambda}}, \quad (10)$$

3.2 Quantile for The OLIW Distribution

The quantile function of the OLIW distribution, say $x=Q(x)=F(x,\Omega)^{-1}(Q)$ is derived by inverting Equation (5) as follows:

$$x_{q}=\theta\left(\ln\left[1+\frac{1}{\beta\left((1-q)^{\frac{-1}{\alpha}}-1\right)}\right]\right)^{\frac{-1}{\lambda}}\ ;\ 0<q<1 \quad (11)$$

In particular, the first quartile, say Q1, the second quartile, say Q2, and the third quartile, say Q3 are obtained by setting $Q=0.25, 0.5, 0.75$, respectively, in (11).

3.3 Moments for The OLIW Distribution

Let X be a random variable having OLIW distribution. Then the r^{th} moment of X follows simply from Equation (9) as

$$\begin{aligned}\acute{\mu}_{r}&=E(X^{r})\\&=\sum_{k,j=0}^{\infty}\Delta_{k,j}\,\theta^{r}(k+j+1)^{\frac{r}{\lambda}}\Gamma\left(1-\frac{r}{\lambda}\right),\ r<\lambda\end{aligned} \quad (12)$$

The moment generating function of OLIW distribution is given by

$$\begin{aligned}M_{X}^{'}(t)&=E(e^{xt})\\&=\sum_{k,j=0}^{\infty}\Delta_{k,j}\sum_{q=0}^{\infty}\frac{t^{q}}{q!}\,\theta^{q}(k+j+1)^{\frac{q}{\lambda}}\Gamma\left(1-\frac{q}{\lambda}\right),\ q<\lambda\end{aligned} \quad (13)$$

3.4 Rényi Entropy

Rényi entropy of order δ is given by

$$I_{\delta}(x)=\frac{1}{\delta-1}\ln\left(\sum_{k,q=0}^{\infty}\Psi_{k,q}\int_{0}^{\infty}g(x,\Theta)^{\delta}G(x,\Theta)^{k,q}dx\right), \quad (14)$$

where $\Psi_{k,q}=(-1)^{k}\beta^{-q}\left(\frac{\alpha}{\beta}\right)^{\delta}\binom{-2\delta-k}{q}\binom{-\theta(\alpha+1)}{k}$. If X has OLIW distribution with vector parameteres Ω, then the Rényi entropy of a random variable X, is given by

$$I_{\delta}(x)=\frac{\left(\frac{\lambda}{\theta}\right)^{\delta-1}}{\delta-1}\ln\left(\sum_{k,q=0}^{\infty}\Psi_{k,q}\Gamma\left[\frac{\delta(\lambda+1)-1}{\lambda}\right]\right)\Xi_{k,q}, \quad (15)$$

where $\Xi_{k,q}=(k+q+1)^{\frac{1-\delta(\lambda+1)}{\lambda}}$.

4 Estimation Methods

The estimation problem of the OLIW distribution parameters is studied in this Section using three different estimation methods called: maximum likelihood estimators (MLE), maximum product spacing estimator (MPSE), and Bayesian estimation based on the function of square error loss.

4.1 Maximum Likelihood Estimators

Let $x_1,...,x_n$ be a random sample with the parameters α,β,λ and θ from the OLIW distribution. The log-likelihood feature for the distribution of OLIW is provided by

$$\begin{aligned}l(\Omega)=&n\left[\ln(\alpha)+\ln(\lambda)+\alpha\ln(\beta)+\lambda\ln(\theta)\right]-\\&(\lambda+1)\sum_{i=1}^{n}ln(x_i)-2\sum_{i=1}^{n}\ln\left[1-e^{-\left(\frac{\theta}{x_i}\right)^{\lambda}}\right]-\\&\sum_{i=1}^{n}\left(\frac{\theta}{x_i}\right)^{\lambda}-(\alpha+1)\sum_{i=1}^{n}\ln\left[\beta+\frac{e^{-\left(\frac{\theta}{x_i}\right)^{\lambda}}}{1-e^{-\left(\frac{\theta}{x_i}\right)^{\lambda}}}\right]\end{aligned} \quad (16)$$

The partial derivatives of $l(\Omega)$ with respect to the model parameters α,β,λ and θ are

$$\frac{\partial l(\Omega)}{\partial\alpha}=\frac{n}{\alpha}+n\ln(\beta)-\sum_{i=1}^{n}\ln\left[\beta+\frac{e^{-\left(\frac{\theta}{x_i}\right)^{\lambda}}}{1-e^{-\left(\frac{\theta}{x_i}\right)^{\lambda}}}\right] \quad (17)$$

$$\frac{\partial l(\Omega)}{\partial\beta}=\frac{n\alpha}{\beta}-(\alpha+1)\sum_{i=1}^{n}\frac{1}{\beta+\frac{e^{-\left(\frac{\theta}{x_i}\right)^{\lambda}}}{1-e^{-\left(\frac{\theta}{x_i}\right)^{\lambda}}}} \quad (18)$$

$$\begin{aligned}\frac{\partial l(\Omega)}{\partial\lambda}=&\frac{n}{\lambda}+n\ln(\theta)-2\sum_{i=1}^{n}\frac{\left(\frac{\theta}{x_i}\right)^{\lambda}\ln\left(\frac{\theta}{x_i}\right)e^{-\left(\frac{\theta}{x_i}\right)^{\lambda}}}{1-e^{-\left(\frac{\theta}{x_i}\right)^{\lambda}}}-\\&\sum_{i=1}^{n}\left(\frac{\theta}{x_i}\right)^{\lambda}\ln\left(\frac{\theta}{x_i}\right)-\sum_{i=1}^{n}ln(x_i)+\\&(\alpha+1)\sum_{i=1}^{n}\frac{\frac{\left(\frac{\theta}{x}\right)^{\lambda}\ln\left(\frac{\theta}{x}\right)e^{-\left(\frac{\theta}{x}\right)^{\lambda}}}{\left[1-e^{-\left(\frac{\theta}{x}\right)^{\lambda}}\right]^{2}}}{\beta+\frac{e^{-\left(\frac{\theta}{x}\right)^{\lambda}}}{1-e^{-\left(\frac{\theta}{x}\right)^{\lambda}}}}\end{aligned} \quad (19)$$

and

$$\begin{aligned}\frac{\partial l(\Omega)}{\partial\theta}=&\frac{n\lambda}{\theta}-2\lambda\theta^{\lambda-1}\sum_{i=1}^{n}\frac{\left(\frac{1}{x_i}\right)^{\lambda}e^{-\left(\frac{\theta}{x_i}\right)^{\lambda}}}{1-e^{-\left(\frac{\theta}{x_i}\right)^{\lambda}}}-\\&\lambda\theta^{\lambda-1}\sum_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\lambda}+\\&\lambda\theta^{\lambda-1}(\alpha+1)\sum_{i=1}^{n}\frac{\frac{\left(\frac{1}{x_i}\right)^{\lambda}e^{-\left(\frac{\theta}{x_i}\right)^{\lambda}}}{\left[1-e^{-\left(\frac{\theta}{x_i}\right)^{\lambda}}\right]^{2}}}{\beta+\frac{e^{-\left(\frac{\theta}{x_i}\right)^{\lambda}}}{1-e^{-\left(\frac{\theta}{x_i}\right)^{\lambda}}}}\end{aligned} \quad (20)$$

It is possible to obtain the MLE of α,β,λ and θ by maximizing the last equation with respect to α,β,λ and θ, equal to zero. Using the Newton-Rapshon method, R packages can be used to maximize the log-likelihood function to obtain MLE parameters.

4.2 Maximum Product of Spacings Method

The MPS method is used as an alternative to the MLE method adopted by Cheng and Amin [23] to estimate the parameters of continuous univariate models. Many authors used MPS to estimate model parameters based on a complete and different censored sample by Almetwally and Almongy [24], Basu et al. [25], Almetwally et al. [24], El-Sherpieny et al. [26] and Alshenawy et al. [22]. Let $x_1<x_2<...<x_n$ then x_i is order of data.

$$\begin{aligned}D_i(\Omega)&=F(x_{(i)},\Omega)-F(x_{(i-1)},\Omega)\\&=\beta^{\alpha}\left[\beta+\frac{e^{-\left(\frac{\theta}{x_{i-1}}\right)^{\lambda}}}{1-e^{-\left(\frac{\theta}{x_i}\right)^{\lambda}}}\right]^{-\alpha}-\beta^{\alpha}\left[\beta+\frac{e^{-\left(\frac{\theta}{x_{i-1}}\right)^{\lambda}}}{1-e^{-\left(\frac{\theta}{x_i}\right)^{\lambda}}}\right]^{-\alpha}\\&;\ i=1,...,n+1\end{aligned} \quad (21)$$

where $D_i(\Delta)$ denotes to the uniform spacings, $F(x_{(0)}, \Omega) = 0$, $F(x_{(n+1)}, \Omega) = 1$ and $\sum_{i=1}^{n+1} D_i(\Omega) = 1$. with respect to α, β, λ and θ. Further, the MPSE of the OLIW parameters can also be obtained by first derivatives with parameter and equalize it to zero.

4.3 Bayesian Estimation

As random and parameter uncertainties are represented by a previous joint distribution that is established prior to the data collected on the failure, the Bayesian approach deals with the parameters. The ability to incorporate prior knowledge into research makes the Bayesian method very useful in the analysis of reliability, as one of the main problems associated with reliability analysis is the limited availability of data. The α, β, λ and θ parameters have prior gamma distributions. The α, β, λ and θ independent joint prior density function can be written as follows:

$$\Pi(\Omega) \propto \alpha^{a_1-1}\beta^{a_2-1}\lambda^{a_3-1}\theta^{a_4-1}e^{-(b_1\alpha+b_2\beta+b_3\lambda+b_4\theta)} \tag{22}$$

From the likelihood function and joint prior function, the joint posterior density function of Ω is obtained. The joint posterior of the distribution of OLIW distribution can then be written as

$$\Pi(\Omega|x) \propto \alpha^{n+a_1-1}\beta^{n\alpha+a_2-1}\lambda^{n+a_3-1}e^{-(b_1\alpha+b_2\beta+b_3\lambda+b_4\theta)} e^{-\sum_{i=1}^{n}\left(\frac{\theta}{x_i}\right)^{\lambda}} \prod_{i=1}^{n} \frac{1}{x_i^{\lambda+1}\left[1-e^{-\left(\frac{\theta}{x_i}\right)^{\lambda}}\right]^2} \theta^{n\lambda+a_4-1}\left[\beta+\frac{e^{-\left(\frac{\theta}{x_i}\right)^{\lambda}}}{1-e^{-\left(\frac{\theta}{x_i}\right)^{\lambda}}}\right]^{-\alpha-1}, \tag{23}$$

Using the most common function for symmetric loss, which is a function for squared error loss. Bayes estimators of $\hat{\Omega}$ based on the squared error loss function are defined by the squared error loss function.

$$S(\tilde{\Omega}) = E\left(\tilde{\Omega}-\Omega\right)^2 \int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\left(\tilde{\Omega}-\Omega\right)^2 \Pi(\Omega|x)d\Omega_1 d\Omega_2 d\Omega_3 d\Omega_4 \tag{24}$$

It should be noted that the integrals given by (24) cannot be obtained directly. As a consequence, we employ the Markov chain Monte Carlo (MCMC) to estimate the value of integrals. Gibbs sampling and more general Metropolis-Hesting within Gibbs samplers are important sub-classes of MCMC techniques. The Metropolis Hasting (MH) algorithm and Gibbs sampling are the two most popular applications of the MCMC method. The MH algorithm, like acceptance-rejection sampling, assumes that a candidate value from a proposal distribution can be generated for each iteration of the algorithm. The MH algorithm, like acceptance-rejection sampling, claims that a candidate value from a proposal distribution can be generated for each iteration of the algorithm. To generate random samples of conditional posterior densities from the OLIW distribution family, we use the MH within the Gibbs sampling. We need the conditional distribution of posterior as following:

$$\Pi(\alpha|\beta,\lambda,\theta,x) \propto \alpha^{n+a_1-1}e^{-(b_1\alpha)}\prod_{i=1}^{n}\left[\beta+\frac{e^{-\left(\frac{\theta}{x_i}\right)^{\lambda}}}{1-e^{-\left(\frac{\theta}{x_i}\right)^{\lambda}}}\right]^{-\alpha-1}, \tag{25}$$

$$\Pi(\beta|\alpha,\lambda,\theta,x) \propto \beta^{n\alpha+a_2-1}e^{-b_2\beta} \prod_{i=1}^{n}\left[\beta+\frac{e^{-\left(\frac{\theta}{x_i}\right)^{\lambda}}}{1-e^{-\left(\frac{\theta}{x_i}\right)^{\lambda}}}\right]^{-\alpha-1}, \tag{26}$$

$$\Pi(\lambda|\alpha,\beta,\theta,x) \propto \lambda^{n+a_3-1}e^{-b_3\lambda}e^{-\sum_{i=1}^{n}\left(\frac{\theta}{x_i}\right)^{\lambda}} \prod_{i=1}^{n}\frac{1}{x_i^{\lambda+1}\left[1-e^{-\left(\frac{\theta}{x_i}\right)^{\lambda}}\right]^2}\left[\beta+\frac{e^{-\left(\frac{\theta}{x_i}\right)^{\lambda}}}{1-e^{-\left(\frac{\theta}{x_i}\right)^{\lambda}}}\right]^{-\alpha-1}, \tag{27}$$

and

$$\Pi(\theta|\alpha,\beta,\lambda,x) \propto \theta^{n\lambda+a_4-1}e^{-b_4\theta} e^{-\sum_{i=1}^{n}\left(\frac{\theta}{x_i}\right)^{\lambda}}\prod_{i=1}^{n}\frac{1}{x_i^{\lambda+1}\left[1-e^{-\left(\frac{\theta}{x_i}\right)^{\lambda}}\right]^2} \left[\beta+\frac{e^{-\left(\frac{\theta}{x_i}\right)^{\lambda}}}{1-e^{-\left(\frac{\theta}{x_i}\right)^{\lambda}}}\right]^{-\alpha-1}. \tag{28}$$

5 Simulation

In this part, the Monte-Carlo simulation procedure is used to compare the classical estimation methods: MLE, MPS, and Bayesian estimation method based on the function of square error loss for MCMC, for estimation of OLIW lifetime distribution parameters by R language. Monte-Carlo experiments are carried out using 10000 randomly generated OLIW distribution samples, where x represents the OLIW lifetime for various parameter actual values and sample sizes n: (35, 75, and 150). Different actual parameters of the OLIW distribution have been obtained as follows:

Case I=$\alpha = 0.5; \beta = 0.5; \lambda = 0.5; \theta = 0.5$.
Case II=$\alpha = 0.5; \beta = 1.5; \lambda = 0.5; \theta = 1.5$.
Case III=$\alpha = 1.5; \beta = 1.5; \lambda = 2.5; \theta = 1.5$.
Case IV=$\alpha = 1.5; \beta = 1.5; \lambda = 2.5; \theta = 3$.
Case V=$\alpha = 3; \beta = 0.5; \lambda = 0.5; \theta = 0.5$.
Case VI=$\alpha = 3; \beta = 3; \lambda = 2.5; \theta = 3$.

Asymptotic confidence intervals for MLE and MPS have been done. The Bayesian credible intervals have been obtained.
The best estimator methods could be defined as minimizing estimator relative bias (RB), mean squared error (MSE), and confidence interval duration (L.CI). Tables 1, 2 and 3 summarise the simulation results of the methods discussed in this paper for point estimation. In order to perform the necessary comparison between various point estimation methods, we consider the RB, MSE, and L.CI values. From these tables, the following observations can be made:

1. The RB, MSE, and L.CI decrease as n increases for actual parameters of the OLIW distribution.

2. Bayesian credible intervals is better than asymptotic confidence intervals for MLE and MPS.

3. Bayesian estimation is the best estimation method.

4. MPS estimation is a better alternative method of MLE.

6 Analysis of COVID-19 Data of France

This section includes COVID-19 data from France to assess the OLIW distribution's accuracy. Other related models such as generalized inverse Weibull (GIW) [De Gusmao et al. [3]], generalized inverse generalized Weibull (GIGW) [Jain et al. [27]], Exponentiated generalized inverse Weibull (EGIW) [Elbatal and Muhammed [12]], Kumaraswamy inverse Topp-Leone (KITL) [Hassan et al. [28]] and Marshall-Olkin Alpha Power IW (MOAPIW) [Basheer et al. [29]] are compared with the OLIW model. Table 4 provide the Cramer-von Mises (W*), Anderson-Darling (A*) and the Kolmogorov Smirnov (KS) statistics, along with the P-value for all models fitted on the basis of COVID-19 data of France. We plot the total time on test (TTT) plot in Figure 5 to classify the possible shapes behind these unknown hrf results. As a result, the OLIW distribution, where "probTV" is the cumulative probability distribution for the time value and sorted time value is the vector "*sorted_time_value*", is sufficient to suit the data.
The considered COVID-19 data belong to France of 51 days that is recorded from 1 January to 20 February 2021. This data formed of mortality rate. The data are as follows: 0.0995, 0.0525, 0.0615, 0.0455, 0.1474, 0.3373, 0.1087, 0.1055, 0.2235, 0.0633, 0.0565, 0.2577, 0.1345, 0.0843, 0.1023, 0.2296, 0.0691, 0.0505, 0.1434, 0.2326, 0.1089, 0.1206, 0.2242, 0.0786, 0.0587, 0.1516, 0.2070, 0.1170, 0.1141, 0.2705, 0.0793, 0.0635, 0.1474, 0.2345, 0.1131, 0.1129, 0.2054, 0.0600, 0.0534, 0.1422, 0.2235, 0.0908, 0.1092, 0.1958, 0.0580, 0.0502, 0.1229, 0.1738, 0.0917, 0.0787, and 0.1654.
Table 4 shows that the OLIW distribution has minimum values for all information parameters as compared to other distributions. As a result, we assume that OLIW best suits the two real data sets. Figure 4 shows the fitted OLIW CDF, PP, and QQ-plots of the two data sets. The Q-Q and P-P plots in Figure 4 indicate that our distribution is a good fit for modeling the actual data above. The Bayesian estimation method of the OLIW distribution is the best estimation method, according to Table 5. Figure 7 depicts history plots, estimated marginal posterior density, and MCMC convergence of α, β, λ and θ.

7 Conclusion

This paper proposes the OLIW distribution, a modern generalization of the inverse Weibull and odd Lomax distribution. We looked into its statistical properties and came up with the following: linear representation, quantile function of moments, moment generation functions, and Rényi entropy. Point estimation of the OLIW unknown parameters α, β, λ, and θ were considered by MLE, MPS, and Bayesian estimation methods. Interval estimation of the OLIW parameters α, β, λ, and θ were considered by MLE, MPS, and Bayesian estimation methods. To distinguish the performance of different estimation methods, a comparison was carried out through Monte-Carlo simulation analysis using the R package. For that reason, the COVID-19 data sets were also considered, and OLIW was shown to match these data better compared to other competitive distributions. Bayesian estimation is the best estimation method for estimate parameters of OLIW distribution.

Acknowledgement

The authors are grateful to the anonymous referee for a careful checking of the details and for helpful comments that improved this paper.

Conflict of interest

The author declares no conflicts of interest.

Funding

The author received no financial support for the research.

Table 1: RB, MSE, and lenght of CI of OLIW distribution for MLE, MPS and Bayesian: Case I and II

			MLE			MPS			Bayesian		
case	n		RB	MSE	L.CI	RB	MSE	L.CI	RB	MSE	L.CCI
I	35	α	-0.0425	0.3150	1.2331	0.0827	0.2862	1.1113	-0.0058	0.1027	0.3957
		β	0.4008	0.5103	1.8413	0.0945	0.4005	1.5607	0.0094	0.1271	0.4947
		λ	0.4327	0.3977	1.3096	0.0578	0.2381	0.9274	0.0338	0.0891	0.3459
		θ	-0.0245	0.4194	1.6452	0.4483	0.4967	1.7392	-0.0059	0.1450	0.5592
	75	α	-0.0148	0.2116	0.8300	0.0776	0.2046	0.7884	-0.0016	0.0610	0.2365
		β	0.2284	0.3371	1.2445	0.0311	0.3057	1.1979	-0.0152	0.0760	0.2943
		λ	0.2171	0.2511	0.8884	0.0122	0.1855	0.7273	0.0079	0.0517	0.2024
		θ	0.0342	0.3398	1.3316	0.4868	0.4734	1.5932	-0.0058	0.0793	0.3099
	150	α	-0.0705	0.1369	0.5191	0.0224	0.1249	0.4882	0.0002	0.0180	0.0703
		β	0.1937	0.2732	1.0023	-0.0154	0.2099	0.8229	0.0020	0.0188	0.0737
		λ	0.1619	0.1820	0.6397	-0.0035	0.1238	0.4855	-0.0038	0.0169	0.0636
		θ	-0.0010	0.3308	1.2981	0.3695	0.3959	1.3739	-0.0016	0.0173	0.0646
II	35	α	0.0263	0.2875	1.1269	0.0710	0.2551	0.9912	0.0211	0.1062	0.4040
		β	0.1215	0.9201	3.5386	0.0146	0.7469	2.9295	-0.0050	0.1680	0.6712
		λ	0.2500	0.3028	1.0820	0.0393	0.2071	0.8088	0.0134	0.0870	0.3297
		θ	0.0392	0.5134	2.0013	0.0096	0.3286	1.2883	0.0056	0.1849	0.7166
	75	α	-0.0090	0.1782	0.6992	0.0251	0.1643	0.6428	0.0072	0.0644	0.2509
		β	0.0392	0.5662	2.2095	-0.0155	0.5060	1.9835	-0.0060	0.0943	0.3582
		λ	0.1232	0.1682	0.6140	0.0157	0.1243	0.4869	0.0006	0.0534	0.2022
		θ	0.0006	0.3220	1.2634	-0.0048	0.2318	0.9092	-0.0044	0.0896	0.3528
	150	α	-0.0019	0.1188	0.4660	0.0284	0.1181	0.4602	-0.0008	0.0171	0.0687
		β	0.0332	0.3916	1.5243	0.0073	0.3761	1.4752	-0.0011	0.0185	0.0710
		λ	0.0517	0.0830	0.3094	-0.0071	0.0701	0.2746	-0.0019	0.0178	0.0672
		θ	0.0014	0.2201	0.8634	0.0063	0.1716	0.6723	-0.0005	0.0187	0.0694

Table 2: RB, MSE, and lenght of CI of OLIW distribution for MLE, MPS and Bayesian: Case III and IV

			MLE			MPS			Bayesian		
	n		RB	MSE	L.CI	RB	MSE	L.CI	RB	MSE	L.CCI
III	35	α	0.1842	1.1147	4.2376	0.0335	0.7507	2.9391	-0.0014	0.1719	0.6512
		β	0.2037	1.3443	5.1367	-0.1089	0.8856	3.4154	-0.0011	0.1703	0.6925
		λ	0.0831	0.6468	2.4033	-0.0504	0.4951	1.8789	-0.0093	0.1670	0.6333
		θ	0.0458	0.3361	1.2909	0.0913	0.3588	1.3014	0.0045	0.1166	0.4515
	75	α	0.1602	0.8387	3.1528	0.0876	0.6267	2.4046	-0.0013	0.0895	0.3484
		β	0.1131	0.8871	3.4165	-0.0850	0.6730	2.5929	0.0010	0.0885	0.3429
		λ	0.0250	0.5347	2.0836	-0.0576	0.4321	1.5986	-0.0026	0.0881	0.3528
		θ	0.0582	0.3493	1.3270	0.0853	0.3312	1.1988	-0.0036	0.0587	0.2295
	150	α	0.0990	0.6091	2.3178	0.0611	0.4803	1.8499	-0.0006	0.0182	0.0700
		β	0.0590	0.6485	2.5210	-0.0618	0.5230	2.0195	0.0000	0.0182	0.0689
		λ	0.0051	0.3860	1.5137	-0.0426	0.3227	1.1952	-0.0003	0.0183	0.0749
		θ	0.0410	0.2504	0.9525	0.0535	0.2122	0.7709	0.0003	0.0174	0.0664
IV	35	α	0.4288	1.7544	6.4050	0.0911	0.7577	2.9245	-0.0130	0.1663	0.6223
		β	0.7394	2.6397	9.3993	-0.0100	0.8559	3.3579	-0.0014	0.1807	0.7173
		λ	0.1091	0.8387	3.1123	-0.0533	0.4684	1.7618	0.0002	0.1753	0.6617
		θ	0.0201	0.6591	2.5756	0.0466	0.4056	1.4941	-0.0031	0.1459	0.5934
	75	α	0.1667	1.1090	4.2395	0.0476	0.6055	2.3593	-0.0043	0.0832	0.3240
		β	0.3297	1.3732	5.0268	-0.0448	0.6248	2.4374	-0.0011	0.0877	0.3439
		λ	0.0901	0.6831	2.5306	-0.0274	0.3760	1.4508	-0.0004	0.0893	0.3557
		θ	0.0115	0.6223	2.4379	0.0336	0.3497	1.3141	-0.0027	0.0868	0.3413
	150	α	0.0995	0.7250	2.7838	0.0489	0.4456	1.7246	-0.0002	0.0176	0.0660
		β	0.1574	0.9561	3.6355	-0.0534	0.5048	1.9557	-0.0005	0.0177	0.0684
		λ	0.0319	0.5426	2.1059	-0.0336	0.2724	1.0171	0.0001	0.0187	0.0721
		θ	0.0449	0.6460	2.4791	0.0370	0.2810	1.0131	-0.0001	0.0176	0.0678

Table 3: RB, MSE, and lenght of CI of OLIW distribution for MLE, MPS and Bayesian: Case V and VI

			MLE			MPS			Bayesian		
	n		RB	MSE	L.CI	RB	MSE	L.CI	RB	MSE	L.CCI
V	35	α	-0.0195	0.5722	2.2335	-0.0548	0.6080	2.2969	-0.0021	0.1809	0.7001
		β	0.2621	0.3284	1.1814	-0.0537	0.2113	0.8224	0.0517	0.1297	0.4883
		λ	0.1062	0.1060	0.3600	-0.0452	0.0726	0.2706	0.0216	0.0605	0.2283
		θ	-0.0987	0.2370	0.9097	0.2629	0.2715	0.9320	0.0133	0.1296	0.4853
	75	α	-0.0134	0.4980	1.9477	-0.0392	0.4442	1.6808	-0.0004	0.0893	0.3407
		β	0.1657	0.2580	0.9587	-0.0836	0.1631	0.6185	0.0147	0.0727	0.2665
		λ	0.0621	0.0805	0.2914	-0.0379	0.0607	0.2261	0.0071	0.0374	0.1394
		θ	-0.0278	0.2226	0.8716	0.2434	0.2383	0.8039	-0.0010	0.0762	0.2921
	150	α	-0.0026	0.3541	1.3892	-0.0225	0.3181	1.2196	-0.0001	0.0177	0.0702
		β	0.1324	0.2037	0.7559	-0.0588	0.1350	0.5172	-0.0005	0.0179	0.0720
		λ	0.0412	0.0515	0.1851	-0.0255	0.0435	0.1632	-0.0002	0.0151	0.0575
		θ	-0.0521	0.1646	0.6378	0.1652	0.1876	0.6609	0.0001	0.0175	0.0677
VI	35	α	0.0462	1.7628	6.8957	-0.1040	0.9547	3.5409	0.0037	0.1793	0.7235
		β	0.2582	2.6650	10.0060	-0.1021	1.0249	3.8379	-0.0022	0.1786	0.7012
		λ	0.1604	0.8232	2.8212	0.0014	0.3917	1.5368	-0.0022	0.1610	0.6168
		θ	-0.0123	0.5512	2.1580	-0.0089	0.2073	0.8068	0.0018	0.1501	0.5596
	75	α	0.0730	1.5260	5.9259	-0.0520	0.7115	2.7241	0.0005	0.0914	0.3395
		β	0.2034	2.0340	7.6137	-0.0651	0.7433	2.8143	-0.0003	0.0883	0.3386
		λ	0.0834	0.5591	2.0359	-0.0046	0.2737	1.0730	-0.0028	0.0865	0.3290
		θ	0.0022	0.5178	2.0315	-0.0039	0.1551	0.6067	-0.0008	0.0790	0.3103
	150	α	0.0548	1.3748	5.3560	-0.0446	0.5374	2.0421	0.0002	0.0190	0.0729
		β	0.1000	1.5390	5.9231	-0.0516	0.5279	1.9802	0.0004	0.0183	0.0677
		λ	0.0417	0.4122	1.5647	-0.0035	0.1873	0.7340	-0.0006	0.0189	0.0730
		θ	0.0114	0.4606	1.8024	-0.0028	0.1143	0.4472	-0.0002	0.0185	0.0715

Table 4: MLE estimates, SE, KS with P-Value, W*, and A* for COVID-19 data of France

		α	β	λ	θ	KS	P-Value	W*	A*
OLIW	estimate	14.9557	19.3057	1.5303	0.0949	0.1014	0.6713	0.0626	0.5007
	SE	45.7564	58.1673	0.4803	0.0437				
GIW	estimate	0.2717	0.0906	2.1119		0.1028	0.6539	0.1271	0.8535
	SE	1.0731	0.7552	0.2291					
GIGW	estimate	0.2194	1.3436	0.8670	7.3580	0.1020	0.6685	0.0698	0.5715
	SE	2.5270	13.5052	0.5567	13.3671				
EGIW	estimate	37.4564	96.3594	0.2214	8.8510	0.1013	0.6709	0.0716	0.5809
	SE	408.6375	313.0758	0.3331	35.1811				
MOAPIW	estimate	30.0093	3.9397	2.8427	0.0425	0.1033	0.6484	0.0812	0.6459
	SE	5.9456	4.6320	0.4469	0.0185				
KITL	estimate	4.0006	0.1380	552.7278		0.1310	0.3456	0.0662	0.5251
	SE	1.2628	0.0200	0.0026					

Table 5: MLE, MPS, and Bayesian estimates, SE of OLIW distribution for COVID-19 data of France

	MLE		Bayesian		MPS	
	estimate	SE	estimate	SE	estimate	SE
α	14.9557	45.7564	16.5751	11.5479	8.4539	87.4296
β	19.3057	58.1673	29.3807	22.6591	8.7650	74.4226
λ	1.5303	0.4803	1.6240	0.3795	1.4108	5.3033
θ	0.0949	0.0437	0.0932	0.0368	0.1016	0.5586

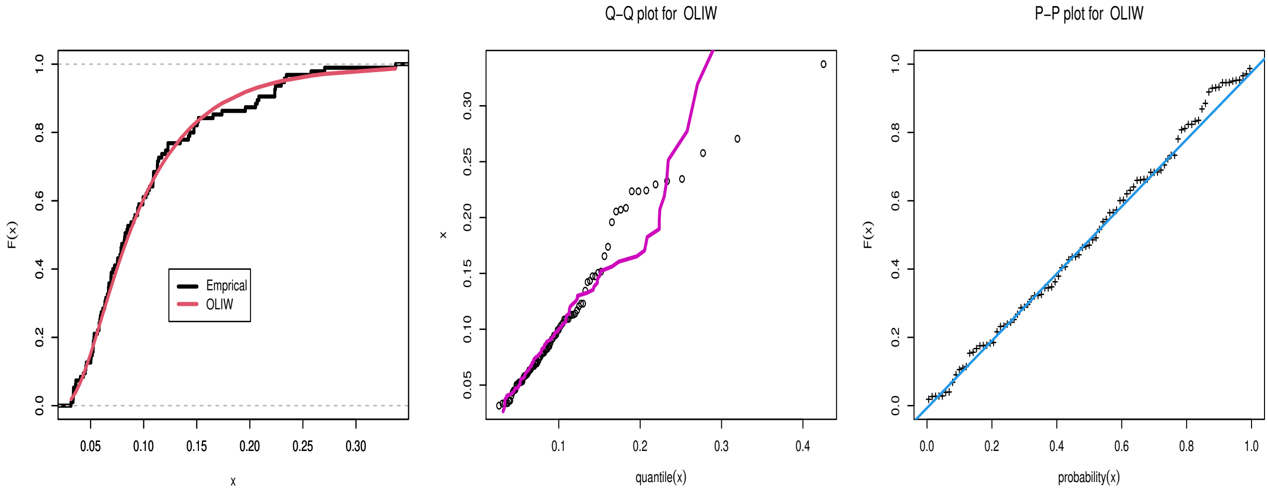

Fig. 4: Different measures of OLIW distribution for COVID-19 data of France

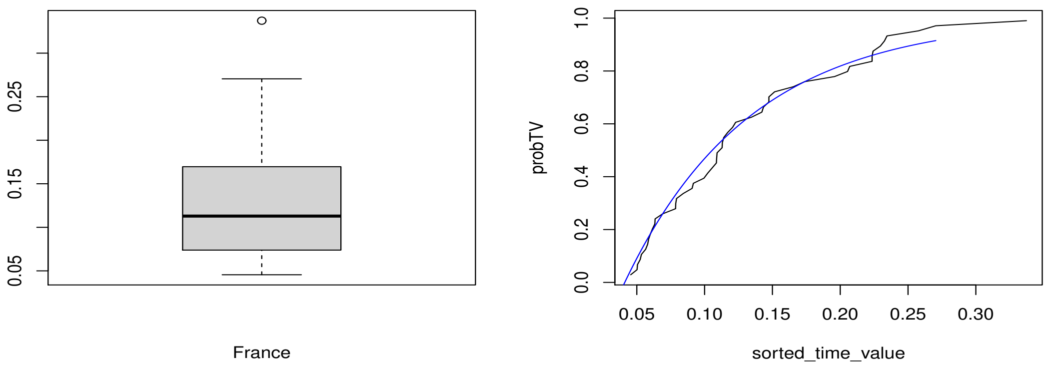

Fig. 5: Boxplot and TTT plot of COVID-19 data of France

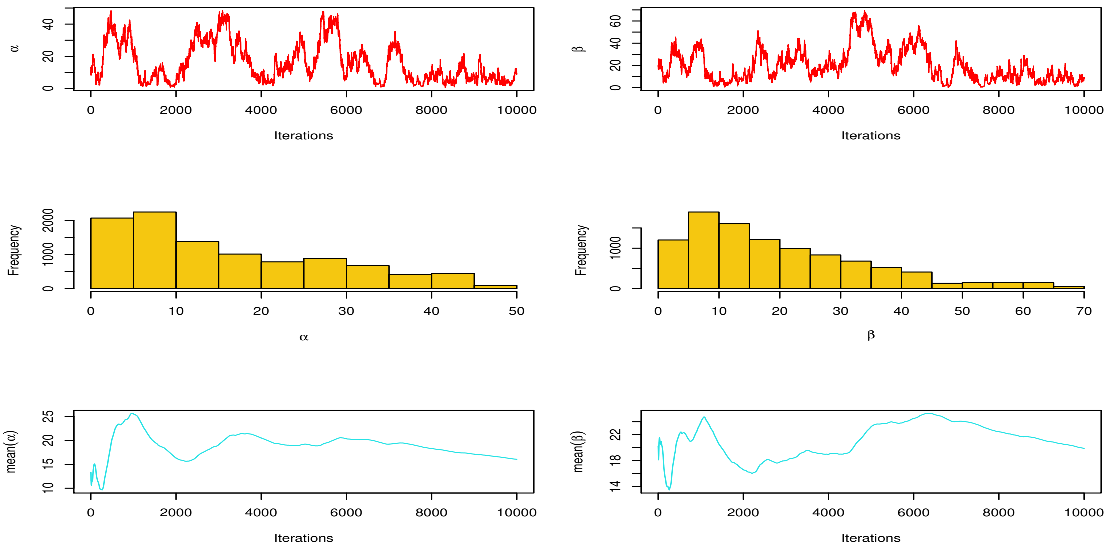

Fig. 6: Convergence of MCMC estimation of OLIW distribution for COVID-19 data of France: α, and β

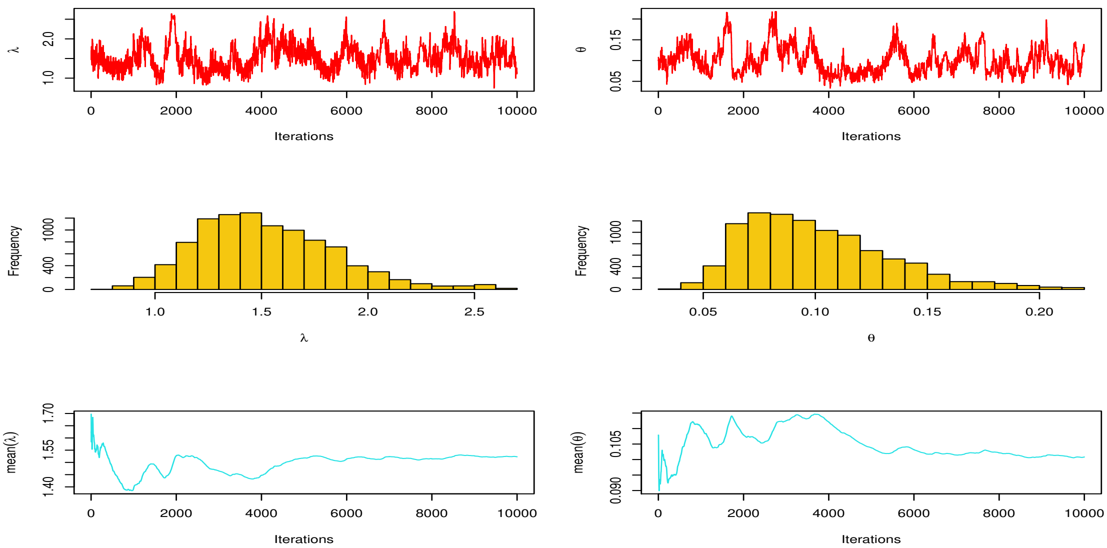

Fig. 7: Convergence of MCMC estimation of OLIW distribution for COVID-19 data of France: λ, and θ

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