Information Sciences Letters
An International Journal

http://dx.doi.org/10.18576/isl/100117

Optimal Multiplicative Generalized Linear Search Plan for a Discrete Randomly Located Target

W. A. Afifi[1,2,*] *and Abd Al-Aziz Hosni El-Bagoury*[2]

[1]Mathematics and Statistics Department, College of Science, Taibah University, Yanbu, Kingdom of Saudi Arabia.
[2]Mathematics Department, Faculty of Science, Tanta University,Egypt.

Received: 11 Jun. 2020, Revised: 25 Sep. 2020, Accepted: 29 Sep. 2020
Published online: 1 Jan. 2021

Abstract: In this paper we study a novel model of determining the existence of the multi generalized linear search problem to detect a lost target in one of several real lines. Every line has one searcher or robot. We have n searchers starting from some points on n lines. The existence of the optimal search plan which minimizes the expected value of the first meeting time between one of the searchers and the lost target is proved. The effectiveness of this strategy is illustrated by introducing a real life application.

Keywords: Located target , Linear search technique, Expected value, Optimal search plan.

1 Introduction

Concepts in search theory have been introduced since 1975 with the World War II. Many of important applications of search strategies have emerged since then, Koopman [1–3] and Stone [4, 5] offered advanced studies in this area from an operational research point of view, as a result, in the linear search case, more extensions and variants of this problem have been introduced in wide variety directions in both the operations research and statistical literature. Reyniers [6, 7] discussed the problem in which the searchers starting r from the origin point of the line where the speed equals to one, rather calculating the expected time for detecting a lost target, In an earlier work, Beck et al. [8–12] illustrated the target in both cases located and moved, the time plays critical issue if the target is important, in case of located target on the real line with a known probability and velocity, the searcher wishes to detect the target in minimal expected time. It is supposed the searcher can change its direction without any loss of time. The target can be detected only, if the searcher reaches the target. W. Afifi et al. [14–16] presented both symmetric and asymmetric cooperative search for a randomly located target on two intersected lines. Recently, W. Afifi et al. [17, 18] illustrated a random walker target on one of two and n disjoint lines. Regarding the two-Dimensional search problem which know as searching in the plane, was presented by Edelsbrunner and Maurer [19], they determined the optimal solutions for one of famous problem which called post office problem in the plane. F. Bourgault et al. [20, 21] concerned with the same problem when the target moves on the plane, like submarines and missing system by applying Bayesian Search and Tracking (SAT). About the 3-Dimensional search, A. H. El-Bagoury et al. [22–25] proposed a modern search model in the three dimensional space by one searcher, two searchers and four searchers. S. N. Al-Aziz et al. [26] illustrated new mechanism of discovering missing target using two searchers or robots is starting from the origin point of the circle. The competition between searchers begins to win the discovery of the target in the search area designated for each of them rather than the calculating the expected time to detect the target.

In this paper we generalize the technique which presented in Afifi. W. A. [13]. We use n searchers or sensors to detect a located target with monotonic decreasing or increasing discrete distribution in one of the n disjoint cylinders (real lines). This problem contributed to solving complicated life problems, such as discovering places where the cable of internet cut under water. Each line has only single searcher starting from some point on the line, where at the same time the other searchers start searching from the same point on their lines. We aim to determine the existence of an optimal search plan that

* Corresponding author e-mail: walaa_affy@yahoo.com

reduce the first meeting time between the lost target and the searcher.

2 The Searching Framework

The space of search: n disjoint axis of n cylinders (n real lines L_i, $i = 1,2,...,n$).

The target: The target with out aim and located on one of n disjoint straight lines and it has a discrete distribution and the probability distribution of it is known for the searchers.

The means of search: Looking for the lost located target performed by one searcher on each line. The searchers start searching for the target from some point $a_{0i} \neq 0$, $i = 1,2,...,n$ on the lines with continuous paths and with equal speeds, go to the right as far as a_{1i}. If the target is not found there, turn back and search in the left part of a_{0i} as far as a_{2i}. If the target is still not found, turn back and search in the right part of a_{1i} as far as a_{3i}, and so on until the searcher meet the target.

3 Problem Formulation

On 27 March 2013, Internet service in Egypt and the UAE has been affected as a result of a major submarine cable cut in the Mediterranean. Whereas, the "Smw4" cable was cut off near the coastal city of Alexandria, which weakened the country's Internet service for a short period. What if the cable cut (lost target) problem is found in one of n disjoint cables (lines) which need to n sensors (searchers) to detect it, see Fig. (1). Of course the problem will be more complicated. However, the probability distribution function of the cable cut location is known to the sensor, which searches for it and aims to discover it in the shortest possible time. Our goal is to calculate the optimal search plan which minimize the lost target detection.

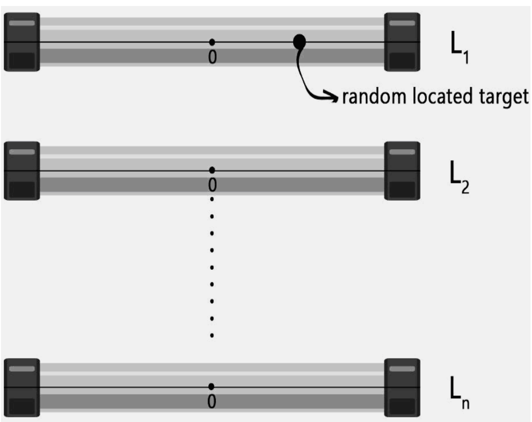

Fig. 1: Linear search for a random located target inside disjoint system of cables.

4 The Searching Technique

Let us have a n searcher $S_1,S_2,...,S_n$ start searching for the located lost target on one of n disjoint lines, where the searcher S_1 starts looking for the lost target from some point a_{01} on L_1, and the second searcher S_2 looking for the lost target from any point a_{02} on the second line L_2, and so on until the searchers S_n looking for the lost target from any point a_{0n} on the n^{th} line L_n. We assume that the random variable X_0 be the position of the lost target with a probability of the position of the target at each point in $[d_i, c_i]$ can be calculated from a given distribution with a density function $p_i(x)$ and a distribution function $F_i(x)$ on the line L_i, $i = 1,2,...,n$. We assume the searchers $S_1,S_2,...,S_n$ begin their search path from any points $a_{01},a_{02},...,a_{0n}$ on $L_1,L_2,...,L_n$ respectively, with speeds $V_1,V_2,...,V_n$. The search plans of the n be represented by $\hat{\phi} = (\phi_1,\phi_2,\phi_3,...,\phi_n) \in \hat{\Phi}$ where $\hat{\Phi}$ is the set of all search plans, and ϕ_i be the search path of the first searcher S_i which defined by the sequence $a_i = \{a_{hi}$, where $h = 0,1,2,...$ and $i = 1,2,...,n\}$, with c_i as the maximum value of a_{2h+G1i} and d_i is the minimum value of a_{2hi}, where h is nonnegative integer such that:

$$d_i = \inf\{x : F_i(x) > 0\} \quad \text{and} \quad c_i = \sup\{x : F_i(x) < 1\}.$$

There is a known probability measure $\nu_1 + \nu_2 + ... + G\nu_n = 1$ on $L_1 \cup L_2 \cup ... \cup L_n$, which describes the location of the target, where ν_i, $i = 1,2,...,n$ is probability measure induced by the position of the target on L_i, where $\nu_i(a_{hi},a_{h+1i}) = F_i(a_{h+1i}) - F_i(a_{hi})$.

The searchers S_i, $i = 1,2,...,n$ follow the following search path which is functions $\phi_i : R^+ \to R$ such that $|\phi_i(t_1) - \phi_i(t_2) \leq v_i|t_2 - t_1|$, $\forall i = 1,2,...,n$ and $t_1,t_2 \in R^+$.

The first meeting time between one of the searchers and the lost target is a random vairable

$$D(\hat{\phi}) = \inf\{t : \text{either}\, D(\phi_1) = X_0 \quad \text{or}\, D(\phi_2) = X_0 \quad \text{or ... or}\, D(\phi_n) = X_0\}$$

without loss of generality we put $V_i = 1$, i. e., the path length of the searcher S_i from starting point a_{0i} until reaching the target equal to the time (or the cost) of the search. The searcher S_i starts looking for the target from some points $a_{0i} \neq 0$ on the line L_i.

The probable search path ϕ_i of the searcher S_i follow one of the following cases:

Case (0): in this case we have either

$$... \leq a_{3i} \leq a_{1i} \leq 0 = a_{0i} \leq a_{2i} \leq a_{4i} \leq ...,$$

with $a_{2hi} \to c_i$ and $a_{2h+1i} \to d_i$ or

$$... \leq a_{4i} \leq a_{2i} \leq 0 = a_{0i} \leq a_{1i} \leq a_{3i} \leq ...,$$

with $a_{2h+1i} \to c_i$ and $a_{2hi} \to d_i$

Case (1): in this case we have either

$$... \leq a_{3i} \leq a_{1i} \leq 0 \leq a_{0i} \leq a_{2i} \leq a_{4i} \leq ...,$$

with $a_{2hi} \to c_i$ and $a_{2h+1i} \to d_i$ or

$$... \leq a_{4i} \leq a_{2i} \leq a_{0i} \leq 0 \leq a_{1i} \leq a_{3i} \leq ...,$$

with $a_{2h+1i} \to c_i$ and $a_{2hi} \to d_i$

Case (2): in this case we have either

$$... \leq a_{4i} \leq a_{2i} \leq 0 \leq a_{0i} \leq a_{1i} \leq a_{3i} \leq ...,$$

with $a_{2h+1i} \to c_i$ and $a_{2hi} \to d_i$ or

$$... \leq a_{3i} \leq a_{1i} \leq 0 \leq a_{0i} \leq a_{2i} \leq a_{4i} \leq ...,$$

with $a_{2hi} \to c_i$ and $a_{2h+1i} \to d_i$

Case (3): let H be a finite and non empty set of positive integer numbers. For $h \in H$, $h = 0, 1, 2, ...$ we have either

$$... \leq a_{h+3i} \leq 0 \leq a_{2h+1i} \leq a_{2h-1i} \leq ... \leq a_{1i} \leq a_{0i} \leq a_{2i} \leq ... \leq a_{2h-2i} \leq a_{2hi} \leq ...$$

with $a_{2hi} \to c_i$ and $a_{2h+1i} \to d_i$ or

$$... \leq a_{2h-2i} \leq ... \leq a_{4i} \leq a_{2i} \leq a_{0i} \leq a_{1i} \leq ... \leq a_{2h-1i} \leq a_{2h+1i} \leq 0 \leq a_{2h+3i} \leq ...$$

with $a_{2h+1i} \to c_i$ and $a_{2hi} \to d_i$

Case (4): let H be a finite and non empty set of positive integer numbers. For $h \in H$, $h = 0, 1, 2, ...$ we have either

$$... \leq a_{2h+2i} \leq 0 \leq a_{2hi} \leq a_{2h-2i} \leq ... \leq a_{2i} \leq a_{0i} \leq a_{1i} \leq a_{3i} \leq ... \leq a_{2h-1i} \leq a_{2h+1i} \leq ...$$

with $a_{2h+1i} \to c_i$ and $a_{2hi} \to d_i$ or

$$... \leq a_{2h-1i} \leq ... \leq a_{3i} \leq a_{1i} \leq a_{0i} \leq a_{2i} \leq ... \leq a_{2h-2i} \leq a_{2hi} \leq 0 \leq a_{2h+2i} \leq ...$$

with $a_{2hi} \to c_i$ and $a_{2h+1i} \to d_i$.

Now we need finding the optimal search plane to find the target when this target has a discrete distribution.

We will take case (1) as an example to our problem. The following Fig. (2) accurately shows the search path for the searchers in case (2) who are looking for a lost located target on one of n disjoint lines.

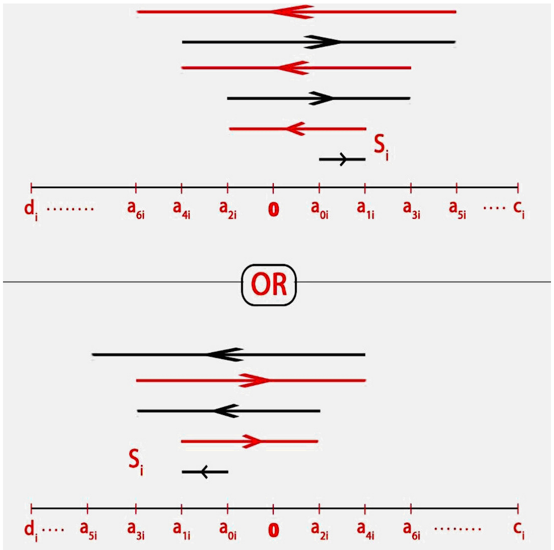

Fig. 2: The search path of n searchers on n lines in case 2.

5 Existence of an Optimal Search Plan

Lemma 5.1. Let x_i be the position of S_i on the line L_i, where $x_i \geq a_{0i}$, and X_0 be a discrete random variable representing the position of a target located on one of n disjoint straight line, and $p_i(x_{wi})$ be its probability distribution function decreasing on the closed $[d_i, c_i]$, where

$$-\infty < d_i < 0, \quad 0 < c_i < \infty, \quad p_i(x_{wi}) > 0,$$

$$\sum_{w=1}^{Q} p_i(x_{wi}) = v_i(d_i, c_i), \quad d_i \leq w_{wi} \leq c_i, \quad i = 1, 2, ..., n$$

$$\text{and} w = 1, 2, ..., Q.$$

Such that

$$v_1(d_1, c_1) + v_2(d_2, c_2) + ... + v_n(d_n, c_n) = 1,$$

then

$$v_1(a_{0i}, x_{wi}) \leq \frac{(x_{wi} - a_{0i}) v_i(d_i, c_i)}{(x_{wi} - d_i)}, \quad x_{wi} \geq a_{0i}, \tag{1}$$

and

$$v_i(a_{0i}, x_{wi}) \leq \frac{(|x_{wi}| + a_{0i}) v_i(d_i, c_i)}{|x_{w_i}| + c_i}, \quad x_{wi} \leq a_{0i}. \tag{2}$$

Proof. Let the number of the points x_{wi} in an interval $[a_{0i}, x_{wi}] = c(|x_{wi}| + a_{0i}) = c(x_{wi} - a_{0i})$, where c is the constant dependent on the natural of the distribution and a_{0i}, x_{wi} are real numbers. We divided the interval $[d_i, c_i]$ into the following sub intervals $[d_i, a_{0i}]$, (a_{0i}, x_{wi}), $(x_{wi}, c_i]$ on the line L_i

$$v_i(d_i, c_i) = v_i(d_i, a_{0i}) + v_i(a_{0i}, x_{wi}) + v_i(x_{wi}, c_i)$$

$$= \sum_{w=1}^{n} p_i(x_{wi}) + \sum_{w=n+1}^{n+m} p_i(x_{wi}) + \sum_{w=n+m+1}^{Q} p_i(x_{wi}).$$

Let $x_{wi} \geq a_{0i}$

$$\begin{aligned} v_i(a_{0i},x_{wi}) &= v_i(d_i,c_i) - \sum_{w=1}^{n} p_i(x_{wi}) - \sum_{w=n+m+1}^{Q} p_i(x_{w_i}) \\ &= \sum_{w=n+1}^{n+m} p_i(x_{wi}) \\ &\leq v_i(d_i,c_i) - nf(a_{0i}) \\ &\leq v_i(d_i,c_i) - c(|d_i| - |a_{0i}|)f(a_{0i}) \\ &\leq v_i(d_i,c_i) - c(-d_i + a_{0i})f(a_{0i}). \end{aligned}$$

Hence

$$v_i(a_{0i},x_{wi}) \leq v_i(d_i,c_i) + c(d_i - a_{0i})f(a_{0i}). \quad (3)$$

Also

$$\begin{aligned} v_i(a_{0i},x_{wi}) &= \sum_{w=n+1}^{n+m} p_i(x_{wi}) \\ &\leq mf(a_{0i}) \\ &\leq c(|x_{wi}| + |a_{0i}|)f(a_{0i}) \\ &\leq c(x_{wi} - a_{0i})f(a_{0i}). \end{aligned}$$

Hence

$$f(a_{0i}) \geq \frac{v_i(a_{0i},x_{wi})}{c(x_{wi} - a_{0i})}. \quad (4)$$

From (3) and (4) we get

$$v_i(a_{0i},x_{wi}) \leq v_i(d_i,c_i) + c(d_i - a_{0i})\frac{v_i(a_{0i},x_{wi})}{c(x_{wi} - a_{0i})},$$

$$\begin{aligned} (x_{wi} - a_{0i})v_i(a_{0i},x_{wi}) \leq\ & (x_{wi} - a_{0i})v_i(d_i,c_i) \\ & + (d_i - a_{0i})v_i(a_{0i},x_{wi}). \end{aligned}$$

Then

$$(x_{wi} - a_{0i} + a_{0i} - d_i)v_i(a_{0i},x_{wi}) \leq (x_{wi} - a_{0i})v_i(d_i,c_i).$$

Hence

$$v_i(a_{0i},x_{wi}) \leq \frac{(x_{wi} - a_{0i})v_i(d_i,c_i)}{(x_{wi} - d_i)}, \quad x_{wi} \geq a0i.$$

Similarly we can prove that:

$$v_i(a_{0i},x_{wi}) \geq \frac{(|x_{wi}| + a_{0i})v_i(d_i,c_i)}{(|x_{wi}| + c_i)}, \quad x_{wi} \leq a0i.$$

W. Afifi [13] calculated expected cost of the multiple search by following relation,

$$\begin{aligned} E_{\tau\Psi_{xi}} = E|x_0| &- 2\sum_{i=1}^{n}\sum_{x_0}|x_0|p_i(x) + \sum|a_{0i}| \\ &+ 2\sum_{i=1}^{n}\sum_{h=1}^{\infty}|a_{hi}|(v_i(d_i,c_i) - v_i(a_{h-1i},a_{hi})). \quad (5) \end{aligned}$$

Theorem 3.1. Suppose that X_0 has a probability functions $p_i(x)$ which are monotonic on the intervals $[d_i,c_i]$, then the optimal search plan is from d_i to c_i, if $p_i(x)$ are monomtonic decreasing, but from c_i to d_i if $p_i(x)$ they are monotonic increasing.

Proof. According to L. D. Stone [5] optimality condition, $p_i(x)$ are decreasing on intervals $[d_i,c_i]$, the other case is similar, the optimal search plan $\Psi = (\Psi_{xi})$ containing at most $4i$ element, let Ψ_{xi} be search plans defined by elements.

$$\Psi_{xi} = (x_i,d_i,c_i,a_{0i}), \quad x_i \geq a_{0i},$$

then

$$\begin{aligned} E_{\tau\Psi_{xi}} &= E|x_0| - 2\sum_{i=1}^{n}\sum_{x_0}|x_0|p_i(x) + \sum_{i=1}^{n}|a_{0i}| \\ &\quad + 2\sum_{i=1}^{n}\sum_{h=1}^{\infty}|a_{hi}|(v_i(d_i,c_i) - v_i(a_{h-1i},a_{hi})) \\ &= E|x_0| - 2\sum_{i=1}^{n}\sum_{x_0}|x_0|p_i(x) + \sum_{i=1}^{n}|a_{0i}| \\ &\quad + 2\sum_{i=1}^{n}[x_i(v_i(d_i,c_i) - v_i(a_{0i},x_i)) + |d_i|(v_i(d_i,c_i) \\ &\quad - v_i(x_i,d_i))] \\ &= E|x_0| - 2\sum_{i=1}^{n}\sum_{x_0}|x_0|p_i(x) + \sum_{i=1}^{n}|a_{0i}| \\ &\quad + 2\sum -i = 1^n[x_i(v_i(d_i,c_i) - v_i(a_{0i},x_i)) \\ &\quad - d_i(v_i(x_i,c_i))] \\ &= E|x_0| - 2\sum_{i=1}^{n}\sum_{x_0}|x_0|p_i(x) + \sum_{i=1}^{n}|a_{0i}| \\ &\quad + 2\sum_{i=1}^{n}[x_i(v_i(d_i,c_i) - v_i(a_{0i},x_i)) - d_i(v_i(a_{0i},c_i) \\ &\quad - v_i(a_{0i},x_i))] \\ &= E|x_0| - 2\sum_{i=1}^{n}\sum_{x_0}|x_0|p_i(x) + \sum_{i=1}^{n}|a_{0i}| \\ &\quad + 2\sum_{i=1}^{n}[x_iv_i(d_i,c_i) - (x_i - d_i)v_i(a_{0i},x_i)) \\ &\quad - d_iv_i(a_{0i},c_i)] \\ &= E|x_0| - 2\sum_{i=1}^{n}\sum_{x_0}|x_0|p_i(x) + \sum_{i=1}^{n}|a_{0i}| \\ &\quad + 2\sum_{i=1}^{n}[(x_iv_i(d_i,c_i) - (x_i - d_i)\frac{(x_i - a_{0i})v_i(d_i,c_i)}{(x_i - d_i)} \\ &\quad - d_iv_i(a_{0i},c_i)] \\ &= E|x_0| - 2\sum_{i=1}^{n}\sum_{x_0}|x_0|p_i(x) + \sum_{i=1}^{n}|a_{0i}| \\ &\quad + 2\sum_{i=1}^{n}[a_{0i}v_i(d_i,c_i) - d_iv_i(a_{0i},c_i)]. \end{aligned}$$

Hence,

$$E_{\tau}\Psi_{x_i} = E_{\tau}\Psi_{a_{0i}}, \tag{6}$$

the optimal search plane at $x_i = a_{0i}$.

The plane of optimal defined by the elements $\{x_i, d_i, c_i, a_{0i}\}$, $x_i = a_{0i}$.

By the similar way we can find the optimal search plan when the searcher has any path following any other case.

6 Application

In one of two disjoint Internet cables, two sensors S_1 and S_2 are ready to discover the located cut inside one of the two cables. Let x_1, x_2 be a random variables, which represent to the place of cable cut on one of two disjoint cables L_1 and L_2 respectively, follow truncated geometric distribution, with probability density functions

$$p_1(x_1) = p(1-p)^{x_1-k},$$

where $0 \leq p \leq 1$, $x_1 \geq k$, $k < 1$, and

$$p_2(x_2) = p(1-p)^{x_2-r},$$

where $0 \leq p \leq 1$, $x_2 \geq r$, $r < 1$, which monotonic on the intervals $[d_1, c_1]$ on L_1 and $[d_2, c_2]$ on L_2, find the optimal search plan when $p_1(x_1)$ and $p_2(x_2)$ are monotonic decreasing on the intervals $[d_1, c_1]$ and $[d_2, c_2]$ respectively, and searchers begins searchers from any points $0 \geq a_{01} \geq x_1$ on L_1 , and $0 \leq a_{02} \leq x_2$ on L_2.

Solution. Let

$$f_t(x_1) = \frac{p(1-p)^{x_1}}{q^{k+1} - q^{c_1+1}}, \quad k < c_1,$$

$$f_t(x_2) = \frac{p(1-p)^{x_2}}{q^{r+1} - q^{c_2+1}}, \quad r < c_2,$$

Ψ_{x_1} be the plan defined by elements

$$\{x_1, d_1, c_1, a_{01}\}, \quad 0 \geq a_{01} \geq x_1,$$

let $d_1 = k = -200$, $c_1 = 1000$, $a_{01} = -100$. Ψ_{x_2} be the plan defined by elements

$$\{d_2, c_2, a_{02}, x_2\}, \quad 0 \leq a_{02} \leq x_2,$$

let $d_2 = r = -200$, $c_2 = 1000$, $a_{02} = 100$.

According to Theorem 3.1 the optimal search plan from $d_1 = k = -200$ to $c_1 = 1000$ on L_1, also from $d_2 = r = -200$ to $c_2 = 1000$ on L_2.

Figures (3) and (4) show the first meeting time between the path of the sensors and the located cable cut where, the target may be located in the first cable or second cable.

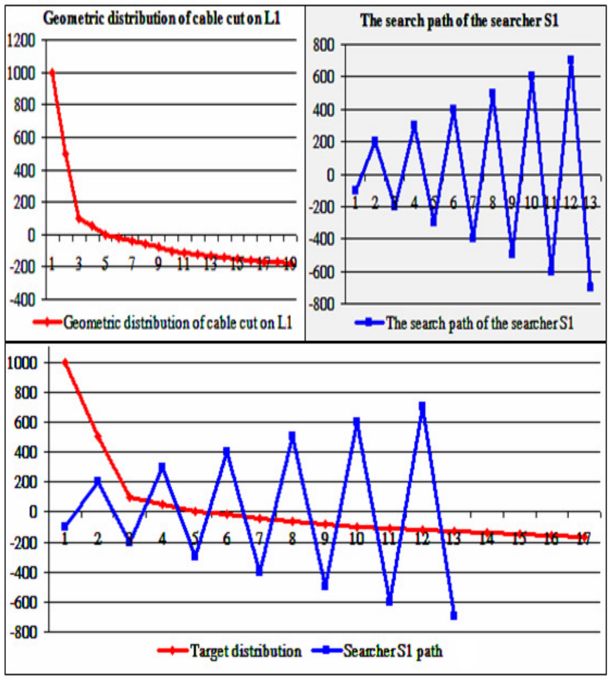

Fig. 3: The first meeting time between the sensor S_1 and the cable cut on L_1.

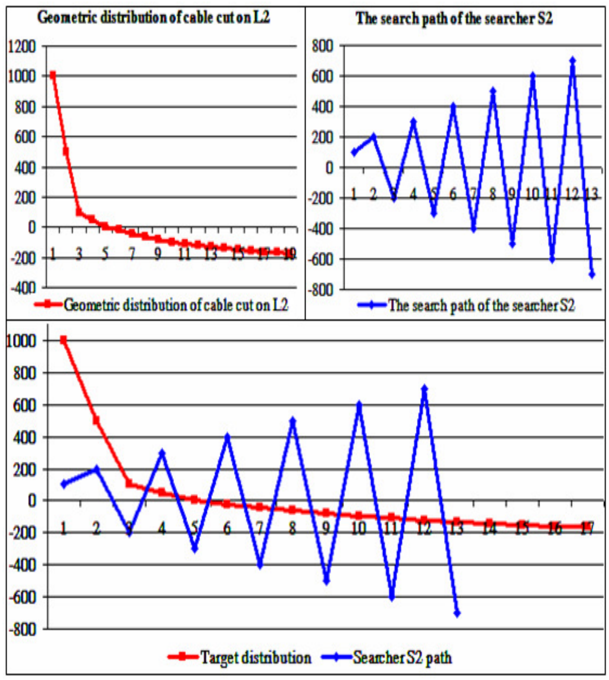

Fig. 4: The first meeting time between the sensor S_2 and the cable cut on L_2.

7 Conclusions

(1) We have designed and generalized a new search technique, we calculated the expected value of the time and obtained the optimal search plan to detect the lost location of the cable cut as soon as possible, the importance of this technique is illustrated using a numerical example.

(2) In this model, the motion of the searchers on n lines are independent; this helps us to find the lost target without wasting time and cost. The importance of this technique is illustrated using a real life numerical example.

(3) In future research, one can study generalized the multiplicative semi-coordinated linear search plan of the expected value of the first meeting time between one of n searchers and one moving target and calculate the optimal search plan.

Acknowledgment

The authors are grateful to the anonymous referee, the editorial manger and the editor on-chief for a careful checking of the details and for helpful comments that improved this paper.

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this article.

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