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Generalized Magneto-Thermo-Viscoelastic Problem in an Infinite Circular Cylinder In Two Models Subjected to Rotation and Initial Stress

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Abstract: This paper is devoted to study the effect of rotating magnetic field and initial stress on the propagation of Rayleigh waves in a homogeneous isotropic, generalized thermo-viscoelastic body. The Hankel transform and Lame's potential have been applied in the basic equations of generalized thermoelasticity and analytical solution is presented. The frequency equation for thermoelastic body is obtained when the boundaries are stress free and is examined numerically. Numerical results for the frequency equation are given and illustrated graphically. Comparison is made with the results obtained in the presence and absence of the rotation, magnetic field and initial stress. The results indicate that the effect of rotation, magnetic field and initial stress are very pronounced.

Keywords: Rotation, initial stress, magneto-thermo-viscoelastic, infinite circular cylinder, isotropic

1 Introduction

Since decades, the study of generalized thermoelasticity has been addressed in different cases. Lord and Shulman [1] have found solution for the dynamical theory of generalized thermoelasticity taking into account the time needed for acceleration of the heat flow. The generalized thermoelastic waves are studied by Sharma and Singh [2] and Singh and Sharma [3]. Ponnusamy [4] investigated the wave propagation in a generalized thermoelastic solid cylinder of arbitrary cross-section. Sharma and Grover [5] have discussed body wave propagation in rotating thermoelastic media. Abd-Alla et al. [6] have discussed influences of rotation, magnetic field, initial stress, and gravity on Rayleigh waves in a homogeneous orthotropic elastic half-space. Time-harmonic sources in a generalized magneto-thermo-viscoelastic continuum with and without energy dissipation were investigated by Abd-Alla and Abo-Dahab [7]. The generalized magneto-thermoelasticity in a perfectly conducting medium was studied by Ezzat and Youssef [9]. Othman [10] used the normal mode analysis to show the effect of rotation on plane waves in generalized thermo- viscoelasticity with one relaxation time. Othman and Song [11] discussed the influence of rotation on plane waves of generalized electro- magneto-thermo-viscoelasticity with two relaxation times. Othman and Song [12] studied the reflection of magneto-thermoelastic waves from a rotating elastic half- space in generalized thermoelasticity under three theories. Abo-Dahab and Singh [13] investigated the influence of magnetic field on wave propagation in generalized thermoelastic solid with diffusion. Hussien et al [14] discussed the effect of initial stress, rotation and magnetic stress on isotropic elastic hollow cylinder. Hussein et al [15] studied the influence of the rotation on a non-homogeneous infinite cylinder of orthotropic material with external magnetic field. Bayones and Hussien [17] studied an analytical solution for the effect of rotation and magnetic field on the composite infinite cylinder in non-homogeneity viscoelastic media. Bayones and Hussien [18] investigated a fiber-reinforced generalized thermoelastic medium subjected to gravity field. Bayones [19] discussed the influence of rotation and initial magnetic field in fiber-reinforced anisotropic elastic media. Bayones [20] studied an effect of rotation on the composite an infinite cylinder in non-homogeneity viscoelastic media.

The aim of this paper is to investigate the propagation of Rayleigh wave velocity in in a homogeneous isotropic, generalized thermo-viscoelastic body in the presence of

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rotation, initial stress and magnetic field. The general Rayleigh wave velocity and attenuation coefficient is derived to study the effect of rotation, magnetic field and initial stress on Rayleigh wave velocity and attenuation coefficient are discussed. The results obtained in this investigation are more general in the sense that some earlier published results are obtained from our result as special cases.

2 Formulation of the problem and boundary conditions

In view of the electromagnetic field governed by Maxwell equations where the medium is characterized as a perfect electric conductor at the absence of the displacement current (SI) is considered in the form as in Roychoudhuri and Mukhopadhyay [21]

$$\begin{aligned}\vec{J} &= curl\,\vec{h},\\ -\mu_e\frac{\partial \vec{h}}{\partial t} &= curl\,\vec{E},\\ div\,\vec{h} &= 0,\\ div\,\vec{E} &= 0,\\ \vec{E} &= -\mu_e(\frac{\partial \vec{U}}{\partial t}\times\vec{H}).\end{aligned} \quad (1)$$

where

$$\vec{h} = curl(\vec{U}\times\vec{H}), \quad \vec{H} = \vec{H_o} + \vec{h}. \quad (2)$$

where $\vec{h}$ is the disturbed magnetic field over the primary magnetic field, $\vec{E}$ is the electric intensity, $\vec{J}$ is the electric current density, μ_e is the magnetic permeability, H_o is the constant primary magnetic field and $\vec{U}$ is the displacement vector.

Let us consider a homogeneous isotropic elastic solid with infinite circular cylinder under the initial stress P, the rotation Ω and initial temperature T_o and upon a primary magnetic field $\vec{H}$ acting on z-direction. When the temperature of infinite cylinder is changed, we find that the incremental thermal stress S_{ij} together with incremental strain e_{ij} are produced in it. The elastic medium is rotating uniformly with an angular velocity $\vec{\Omega} = \Omega\vec{n}$, where $\vec{n}$ is a unit vector which representing the direction of the axis of rotation. All quantities considered are functions of the time variable t and of the coordinates r and z. The displacement equation of motion in the rotating frame has additional term Schoenberg and Censor [22], centripetal acceleration, $\vec{\Omega}\times(\vec{\Omega}\times\vec{u})$ due to time varying motion only. In the absence of body forces the dynamic equation of motion under initial compression is given by abed-Alla [23] as follows:

$$\frac{\partial s_{rr}}{\partial r} + \frac{\partial s_{rz}}{\partial z} + \frac{1}{r}(S_{rr} - S_{\theta\theta})$$

$$+P\frac{\partial \omega_\theta}{\partial z} + f_r = \rho[\frac{\partial^2 u_r}{\partial t^2} - \Omega^2 u_r], \quad (3)$$

$$\frac{\partial s_{rz}}{\partial r} + \frac{\partial s_{zz}}{\partial z} + \frac{1}{r}S_{rz} + \frac{P}{r}\frac{\partial}{\partial r}(r\omega_\theta) = \rho[\frac{\partial^2 u_z}{\partial t^2} - \Omega^2 u_z]. \quad (4)$$

The generalized equation of heat conduction is given by Tanaka et al [24]:

$$K\nabla^2 T = \rho C_v(\dot{T} + \tau_1\ddot{T}) + \alpha\tau_m(3\lambda + 2\mu)T_0\underline{\nabla}\cdot(\dot{\vec{u}} + \delta\tau_1\ddot{\vec{u}}). \quad (5)$$

where ρ is the density of the material, K is the thermal conductivity, C_v is the specific heat of the material per unit mass, α is the coefficient of linear thermal expansion, λ, μ are Lame elastic constants, S_{rr}, $S_{\theta\theta}$, S_{zz} and S_{rz} are the incremental stresses, u_r and u_z are the displacement components and ω_θ is the rotating components,

$$\vec{u} = (u_r, 0, u_z), \vec{\Omega} = (0, \Omega, 0), \vec{H} = (0, 0, H_o), \omega_\theta = \frac{1}{2}(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r})$$

Also, f_r and f_z are Lorentz's force, Kraus [25], are defined by

$$f_r = \mu_e H_0^2[\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r}\frac{\partial u_r}{\partial r} - \frac{u_r}{r^2} + \frac{\partial^2 u_r}{\partial r\partial z}]$$

$$f_z = 0$$

The stress- strain relations with incremental isotropy under initial stress are given by Abd-Alla [22]:

$$S_{rr} = (\tau_m(\delta_* + \mu) + p)e_{rr}$$

$$+(\tau_m(\delta_* - \mu) + p)e_{zz} + (\tau_m(\delta_* - \mu) + p)e_{\theta\theta} - \frac{\gamma}{\chi_\theta}(T + \tau_o\dot{T}), \quad (6)$$

$$S_{\theta\theta} = (\tau_m(\delta_* + \mu) + p)e_{\theta\theta}$$

$$+(\tau_m(\delta_* - \mu) + p)e_{rr} + (\tau_m(\delta_* - \mu) + p)e_{zz} - \frac{\gamma}{\chi_\theta}(T + \tau_o\dot{T}), \quad (7)$$

$$S_{zz} = \tau_m(\delta_* + \mu)e_{zz}$$

$$+\tau_m(\delta_* - \mu)e_{rr} + \tau_m(\delta_* - \mu)e_{\theta\theta} - \frac{\gamma}{\chi_\theta}(T + \tau_o\dot{T}), \quad (8)$$

$$S_{rz} = 2\mu\tau_m e_{rz}, \qquad S_{r\theta} = S_{\theta z} = 0. \quad (9)$$

The magnetic stress is

$$\tau_{rr} = \mu_e H_0^2(\frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_r}{\partial z}) \quad (10)$$

where $\delta_* = (\lambda + \mu), \gamma = \alpha\tau_m(3\lambda + 2\mu), \tau_m = (1 + \tau_0\frac{\partial}{\partial t})$ and χ_θ is the isothermal compressibility, τ_0 and τ_1 are thermal relaxation parameters.

The incremental strain components and the rotation are given by Bagri and Eslami [26] as follows:

$$e_{rr} = \frac{\partial u_r}{\partial r}, e_{\theta\theta} = \frac{u_r}{r}, e_{zz} = \frac{\partial u_z}{\partial z},$$
$$e_{rz} = \frac{1}{2}(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}), \omega_\theta = \frac{1}{2}(\frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r}). \quad (11)$$

By using Eqs. (6) - (10), Eqs. (3) and (4) can be written as:

$$(\tau_m(\lambda+2\mu)+P+\mu_e H_0^2)\frac{\partial}{\partial r}(\frac{\partial u_r}{\partial r}+\frac{1}{r}u_r)$$
$$+(\tau_m(\lambda+\mu)+P+\mu_e H_0^2)\frac{\partial^2 u_z}{\partial r\partial z}+(\tau_m\mu+P)\frac{\partial^2 u_r}{\partial z^2}$$
$$-\frac{\gamma}{\chi_\theta}\frac{\partial}{\partial r}(T+\tau_0\dot{T}) = \rho(\frac{\partial^2 u_r}{\partial t^2}-\Omega^2 u_r) \quad (12)$$

$$(\tau_m(\lambda+\mu)+\frac{P}{2})\frac{\partial^2 u_r}{\partial r\partial z}+(\tau_m\mu-\frac{P}{2})\frac{\partial^2 u_z}{\partial r^2}$$
$$+\tau_m(\lambda+2\mu)\frac{\partial^2 u_z}{\partial z^2}+\frac{1}{r}(\tau_m(\lambda+\mu)+\frac{P}{2})\frac{\partial u_r}{\partial z}$$
$$+\frac{1}{r}(\tau_m\mu-\frac{P}{2})\frac{\partial u_z}{\partial r}-\frac{\gamma}{\chi_\theta}(\frac{\partial T}{\partial z}+\tau_0\frac{\partial \dot{T}}{\partial z}) = \rho(\frac{\partial^2 u_z}{\partial t^2}-\Omega^2 u_z) \quad (13)$$

By Helmholtz's theorem [27] , the displacement vector $\overrightarrow{u}$ can be written in the form:

$$\overrightarrow{u} = grad\phi + curl\overrightarrow{\psi}, \quad (14)$$

where, ϕ is the scalar and $\overrightarrow{\psi}$ is the vector represent irrotational and rotational parts of the displacement $\overrightarrow{u}$. The cylinder being bounded by the curved surface, therefore the stress distribution includes the effect of both ϕ and $\overrightarrow{\psi}$. It is possible to take only one component of the vector $\overrightarrow{\psi}$ to be non-zero, as

$$\overrightarrow{\psi} = (0,-\psi,0). \quad (15)$$

From qs.(14) and (15) we obtain

$$u_r = \frac{\partial\phi}{\partial r}+\frac{\partial\psi}{\partial z}, \qquad u_z = \frac{\partial\phi}{\partial z}-\frac{\partial\psi}{\partial r}-\frac{\psi}{r}. \quad (16)$$

Substituting from (14)-(16) into (11) and (12), we get two independent equations for ϕ and $\overrightarrow{\psi}$ as follows:

$$K\nabla^2 T = \rho C_v\frac{\partial}{\partial r}\left(1+\tau_1\frac{\partial}{\partial t}\right)T$$
$$+\gamma T_0\frac{\partial}{\partial t}\left(1+\delta\tau_1\frac{\partial}{\partial t}\right)\nabla^2\phi, \quad (17)$$

$$\nabla^2\phi - \frac{1}{(\tau_m(\lambda+2\mu)+P+\mu_e H_0^2)}\left[\rho\left(\frac{\partial^2}{\partial r^2}-\Omega^2\right)\right]\phi +$$
$$+\frac{\gamma}{\chi_\theta(\tau_m(\lambda+2\mu)+P+\mu_e H_0^2)}(T+\tau_0\dot{T}), \quad (18)$$

$$\nabla^2\phi = \frac{1}{\tau_m(\lambda+2\mu)}\left[\frac{\partial^2}{\partial r^2}-\Omega^2\right]\phi$$
$$+\frac{\gamma}{\chi_\theta(\tau_m(\lambda+2\mu))}(T+\tau_0\dot{T}), \quad (19)$$

$$[\nabla^2-\frac{1}{r^2}]\psi = \frac{1}{(\tau_m\mu+\frac{P}{2}+\mu_e H_0^2))}$$
$$\times\left[\rho\left(\frac{\partial^2\psi}{\partial t^2}-\Omega^2\psi\right)\right], \quad (20)$$

$$\nabla^2\psi = \frac{\rho}{(\mu\tau_m-\frac{P}{2})}\left(\frac{\partial^2\psi}{\partial t^2}-\Omega^2\psi\right), \quad (21)$$

where

$$\nabla^2 = \frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}. \quad (22)$$

3 Boundary conditions

Consider a homogeneous and isotropic elastic solid with an infinite circular cylinder of radius R. The axis of the cylinder is taken along the z-axis. It is subjected to the boundary conditions which are given as traction free (at r=R)

$$s_{rr}(r,z,t)+\tau_{zz}(r,z,t) = 0, s_{rz}(r,z,t) = 0 \quad (23)$$

The thermal boundary condition is:

$$\frac{\partial T(r,z,t)}{\partial r} = 0, \qquad \text{at } r = \text{R}. \quad (24)$$

4 Solution of the problem

Assuming a simple harmonic time dependent factor $e^{i\omega t}$ for all the quantities and omitting the factor $e^{i\omega t}$ throughout the equations (17), (18) and (21) which yields a set of differential equations for $T^*e^{i\omega t}, \phi^*e^{i\omega t}$ and $\psi^*e^{i\omega t}$ i.e.,

$$[\nabla^2-\ell_1]T^* = \ell_2\nabla^2\phi^*, \quad (25)$$

$$[\nabla^2+\ell_3]\phi^* = \ell_4 T^*, \quad (26)$$

$$[\nabla^2+\ell_5]\psi^* = 0. \quad (27)$$

where ∇^2 is Laplace operator, T can be eliminated from Eq. (26) by substituting it in Eq.(25), we have

$$\left[\nabla^4+A\nabla^2-B\right]\phi^* = 0, \quad (28)$$

where

$$\nabla^4 = \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}\right)^2, \ell_1$$

$$= \frac{\rho C_v (i\omega - \omega^2 \tau_1)}{K}, \ell_2 = \gamma T_0 (i\omega - \delta \tau_1 \omega^2),$$

$$\ell_3 = \frac{\rho(\omega^2 + \Omega^2)}{(\tau_m(\lambda + 2\mu) + P + \mu_e H_0^2)},$$

$$\ell_4 = \frac{\gamma(1 + i\omega\tau_0)}{(\tau_m(\lambda + 2\mu) + P + \mu_e H_0^2)},$$

$$\ell_5 = \frac{\rho(\omega^2+\Omega^2)}{(\tau_m \mu - \frac{P}{2})}, \quad A = (\ell_3 - \ell_1 - \ell_2 \ell_4), \quad B = \ell_1 \ell_3. \tag{29}$$

General solution of equations (27) and (28) can be found. If we introduce the inversion of the Hankel transform is defined by:

$$\phi^*(r,z,\omega) = \int_0^\infty \widehat{\phi}(\eta, z, \omega) J_0(\eta r) \eta \, d\eta \tag{30}$$

Substituting equation (29) into equation (28), we obtain:

$$(\eta^2 - \frac{d^2}{dz^2})^2 \widehat{\phi} + A(\eta^2 - \frac{d^2}{dz^2})\widehat{\phi} - B\widehat{\phi} = 0 \tag{31}$$

By putting $f^2 = (\eta^2 - \frac{d^2}{dz^2})$, the indicial equation governing (31) is:

$$f^4 + Af^2 - B = 0 \tag{32}$$

It can be seen that, the factor $e^{i\omega t}$ has been omitted in the expression for ϕ, ψ and T, f_1^2, f_2^2 are the roots of the equation (32). The roots of the equation (32) take the form

$$f_1 = \pm \frac{\sqrt{-A - \sqrt{A^2 + 4B}}}{\sqrt{2}},$$

$$f_2 = \pm \frac{\sqrt{-A + \sqrt{A^2 + 4B}}}{\sqrt{2}} \tag{33}$$

Putting $\zeta_j^2 = \eta^2 - f_1^2$, $R_e(\zeta_j) \geq 0$ and $j = 1,2$ The the solution of of the equation (31) is

$$\widehat{\phi}(\eta, z, \omega) = A_1(\eta) e^{-\zeta_1 z} + B_1(\eta) e^{-\zeta_2 z}$$

which leads to

$$\phi(r,z,t) = \int_0^\infty [A_1(\eta) e^{-\zeta_1 z + i\omega t}$$

$$+ B_1(\eta) e^{-\zeta_2 z + i\omega t}] J_0(\eta r) \eta \, d\eta \tag{34}$$

Similarly one can obtain the solution of equation (27) which leads to

$$\psi(r,z,t) = \int_0^\infty C(\eta) e^{-\zeta_3 z + i\omega t} J_0(\eta r) \eta \, d\eta \tag{35}$$

where $\zeta_3^2 = \eta^2 - l_5$ and $R_e \geq 0$. To determine the temperature deviation T, by substituting equation (34) into (26), we have

$$T = \frac{1}{\ell_4} \int_0^\infty \Big(\big(\eta + (\zeta_1^2 + \ell_3)\big) A_1(\eta) e^{-\zeta_1 z + i\omega t} + \big(\eta + (\zeta_2^2 + \ell_3)\big) B_1(\eta) e^{-\zeta_2 z + i\omega t} \Big) J_0(\eta r) \eta \, d\eta \tag{36}$$

Substituting equations (34), (35) and (36) into equations (6), (8) and (16), we get

$$u_r = -\int_0^\infty \Big([A(\eta) e^{-\zeta_1 z + i\omega t} + B(\eta) e^{-\zeta_2 z + i\omega t}] \eta^2 J_1(\eta r) + C(\eta) \zeta_3 e^{-\zeta_3 z + i\omega t} \eta J_0(\eta r) \Big) d\eta, \tag{37}$$

$$u_Z = -\int_0^\infty \{ [A(\eta)\zeta_1 e^{-\zeta_1 z + i\omega t} - B(\eta)\zeta_2 e^{-\zeta_2 z + i\omega t} + \frac{1}{r} C(\eta) e^{-\zeta_3 z + i\omega t}] \eta J_0(\eta r) - C(\eta) e^{-\zeta_3 z + i\omega t} \eta^2 J_1(\eta r) \} d\eta, \tag{38}$$

$$\begin{aligned} S_{rr} = A_1(\eta) \int_0^\infty \{ & -(\tau_m^*(\delta_* + \mu) + P)(\eta^2 J_0(\eta r) + \frac{\eta}{r} J_1(\eta r)) \\ & + (\tau_m^*(\delta_* - \mu) + P)(\zeta_1^2 \eta J_0(\eta r) - \frac{1}{r}\eta^2 J_1(\eta r)) \\ & - \frac{\gamma}{\ell_4 \chi_\theta}(1 + i\omega\tau_0)(\eta + (\zeta_1^2 + \ell_3))\eta J_0(\eta r) \} e^{-\zeta_1 z + i\omega t} d\eta \\ & + B_1(\eta) \int_0^\infty \{ -(\tau_m^*(\delta_* + \mu) + P)(\eta^2 J_0(\eta r) + \frac{\eta}{r} J_1(\eta r)) \\ & + (\tau_m^*(\delta_* - \mu) + P)(\zeta_2^2 \eta J_0(\eta r) - \frac{1}{r}\eta^2 J_1(\eta r)) \\ & - \frac{\gamma}{\ell_4 \chi_\theta}(1 + i\omega\tau_0)(\eta + (\zeta_2^2 + \ell_3))\eta J_0(\eta r) \} e^{-\zeta_2 z + i\omega t} d\eta \\ & + C(\eta) \int_0^\infty \{ (\tau_m^*(\delta_* + \mu) + P)\zeta_3 \eta^2 J_1(\eta r) \\ & - (\tau_m^*(\delta_* - \mu) + P)(\zeta_3 \eta^2 J_1(\eta r)) \} e^{-\zeta_3 z + i\omega t} d\eta, \end{aligned} \tag{39}$$

$$S_{rz} = \tau_m^* \mu \int_0^\infty \{ A(\eta)\zeta_1 \eta^2 J_1(\eta r)) e^{-\zeta_1 z + i\omega t} + B(\eta)\zeta_2 \eta^2 J_1(\eta r)) e^{-\zeta_2 z + i\omega t} + C(\eta)\zeta_3^2 \eta J_0(\eta r)) e^{-\zeta_3 z + i\omega t} \} d\eta, \tag{40}$$

$$\tau_{rr} = \mu_e H_0^2 \int_0^\infty \{ A_1(\eta)(\frac{1}{r} J_1(\eta r) - \eta J_0(\eta r)) \eta^2 e^{-\zeta_1 z + i\omega t} + B_1(\eta)(\frac{1}{r} J_1(\eta r) - \eta J_0(\eta r)) \eta^2 e^{-\zeta_2 z + i\omega t} - C(\eta)\zeta_3 \eta^2 J_1(\eta r)) e^{-\zeta_3 z + i\omega t} \} d\eta, \tag{41}$$

5 Frequency equation

In this section, the frequency equation obtained by applying the boundary conditions (23) and (24) on equations (36) and (39-41), then we get three homogeneous linear equations in $A_1(\eta)$, $B_1(\eta)$ and $C(\eta)$:

$$A_1(\eta)\{-(\tau_m^*(\delta_*+\mu)+P)(\eta^2J_0(\eta R)+\frac{\eta}{R}J_1(\eta R))$$
$$+(\tau_m^*(\delta_*-\mu)+P)(\zeta_1^2\eta J_0(\eta R)-\frac{1}{R}\eta^2J_1(\eta R))$$
$$-\frac{\gamma}{\ell_4\chi_\theta}(1+i\omega\tau_0)(\eta+(\zeta_1^2+\ell_3))\eta J_0(\eta R)$$
$$+\mu_eH_0^2(\frac{1}{R}J_1(\eta R)-\eta J_0(\eta R))\eta^2\}e^{-\zeta_1 z}$$
$$+B_1(\eta)\{-(\tau_m^*(\delta_*+\mu)+P)(\eta^2J_0(\eta R)+\frac{\eta}{R}J_1(\eta R))$$
$$+(\tau_m^*(\delta_*-\mu)+P)(\zeta_2^2\eta J_0(\eta R)-\frac{1}{R}\eta^2J_1(\eta R))$$
$$-\frac{\gamma}{\ell_4\chi_\theta}(1+i\omega\tau_0)(\eta+(\zeta_2^2+\ell_3))\eta J_0(\eta R)$$
$$+\mu_eH_0^2(\frac{1}{R}J_1(\eta R)-\eta J_0(\eta R))\eta^2\}e^{-\zeta_2 z}$$
$$+C(\eta)\{(\tau_m^*(\delta_*+\mu)+P)\zeta_3\eta^2J_1(\eta R)$$

$$-(\tau_m^*(\delta_*-\mu)+P)(\zeta_3\eta^2J_1(\eta R)+\zeta_3\eta^2J_1(\eta R)\}e^{-\zeta_3 z}=0, \tag{42}$$

$$A_1(\eta)(\zeta_1\eta^2J_1(\eta R))e^{-\zeta_1 z}+B_1(\eta)(\zeta_2\eta^2J_1(\eta R))e^{-\zeta_2 z}+C(\eta)\zeta_3^2\eta J_0(\eta R)e^{-\zeta_3 z}=0 \tag{43}$$

$$A_1(\eta)(\eta+(\zeta_1^2+\ell_3))e^{-\zeta_1 z}+B_1(\eta)(\eta+(\zeta_2^2+\ell_3))e^{-\zeta_2 z}=0 \tag{44}$$

Eliminating the constants $A_1(\eta)$, $B_1(\eta)$ and $C(\eta)$ we obtain the frequency equation in the form of a third order determinant as:

$$\begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & 0 \end{bmatrix}=0 \tag{45}$$

where

$$D_{11}=\{-(\tau_m^*(\delta_*+\mu)+P)(\eta^2J_0(\eta R)+\frac{\eta}{R}J_1(\eta R))$$
$$+(\tau_m^*(\delta_*-\mu)+P)(\zeta_1^2\eta J_0(\eta R)-\frac{1}{R}\eta^2J_1(\eta R))$$
$$-\frac{\gamma}{\ell_4\chi_\theta}(1+i\omega\tau_0)(\eta+(\zeta_1^2+\ell_3))\eta J_0(\eta R)$$
$$+\mu_eH_0^2(\frac{1}{R}J_1(\eta R)-\eta J_0(\eta R))\eta^2\}e^{-\zeta_1 z}$$

$$D_{12}=\{-(\tau_m^*(\delta_*+\mu)+P)(\eta^2J_0(\eta R)+\frac{\eta}{R}J_1(\eta R))$$
$$+(\tau_m^*(\delta_*-\mu)+P)(\zeta_2^2\eta J_0(\eta R)-\frac{1}{R}\eta^2J_1(\eta R))$$
$$-\frac{\gamma}{\ell_4\chi_\theta}(1+i\omega\tau_0)(\eta+(\zeta_2^2+\ell_3))\eta J_0(\eta R)$$
$$+\mu_eH_0^2(\frac{1}{R}J_1(\eta R)-\eta J_0(\eta R))\eta^2\}e^{-\zeta_2 z}$$

$$D_{13}=\{(\tau_m^*(\delta_*+\mu)+P)\zeta_3\eta^2J_1(\eta R)$$
$$-(\tau_m^*(\delta_*-\mu)+P)(\zeta_3\eta^2J_1(\eta R)+\zeta_3\eta^2J_1(\eta R)\}e^{-\zeta_3 z}$$

$$D_{21}=\zeta_1\eta^2J_1(\eta R)e^{-\zeta_1 z},\quad D_{22}=\zeta_2\eta^2J_1(\eta R)e^{-\zeta_2 z},\quad D_{23}=\zeta_3^2\eta J_0(\eta R)e^{-\zeta_3 z},$$
$$D_{31}=(\eta+(\zeta_1^2+\ell_3))e^{-\zeta_1 z},\quad D_{32}=(\eta+(\zeta_2^2+\ell_3))e^{-\zeta_2 z},\quad D_{33}=0.$$

The transcendental Eq. (45), in the determinant form, has complex roots. The real part (Re) gives the velocity of Rayleigh waves and the imaginary part (Im) gives the attenuation coefficient due to the desirable nature of the medium. We discuss this case and special cases in two models as the following:
(I) LS-model ($\tau_0=0,\tau_1>0,\delta=1$) and (II) GL-model ($\tau_0\geq\tau_1>0,\delta=0$).

The discussion is clear from Figs. (1-10), In general case:
(I) LS-model ($\tau_0=0,\tau_1>0,\delta=1$). (II) GL-model ($\tau_0\geq\tau_1>0,\delta=0$), $\tau_0=2\tau_1$.

6 Special cases

i) When, neglecting of rotation, i.e. $\Omega=0$
(I) LS-model ($\tau_0=0,\tau_1>0,\delta=1$)
(II) GL-model ($\tau_0\geq\tau_1>0,\delta=0$), $\tau_0=2\tau_1$.
ii) When, neglecting of magnetic field, i.e. $H=0$
(I) LS-model ($\tau_0=0,\tau_1>0,\delta=1$)
(II) GL-model ($\tau_0\geq\tau_1>0,\delta=0$), $\tau_0=2\tau_1$.
iii) When, neglecting of initial stress , i.e. $P=0$
(I) LS-model ($\tau_0=0,\tau_1>0,\delta=1$)
(II) GL-model ($\tau_0\geq\tau_1>0,\delta=0$), $\tau_0=2\tau_1$
iv) When, neglecting of rotation and magnetic field , i.e. $\Omega=H=0$
(I) LS-model ($\tau_0=0,\tau_1>0,\delta=1$)
(II) GL-model ($\tau_0\geq\tau_1>0,\delta=0$), $\tau_0=2\tau_1$.

7 Numerical results and discussion

The numerical results for the frequency equation are computed for the thermoelastic body. Since the frequency equation is transcendental in nature, there are an infinite number of roots for the frequency equation. In order to clarify theoretical results that obtained in the previous section, we now illustrate some numerical results. The material chosen for this purpose of Carbon steel, the physical data for which is given below [28].

$$\begin{aligned}\rho=&\ 2\ \mathrm{kgm}^{-3}, \lambda=9.3\times10^{10}N\mathrm{m}^{-1},\ \mu=8.4\times10^{10}N\mathrm{m}^{-1},\ \varepsilon=0.34,\\ T_0=&\ 2931k,\ \tau_0=(0,2\tau_1)\ \mathrm{sec}, C_v=6.4\times10^{10}JKg^{-1}\deg^{-1},\\ K=&\ 50Wm^{-1}k^{-1},\quad \chi_\theta=0.321JKg^{-1}\deg^{-1},\end{aligned} \tag{46}$$

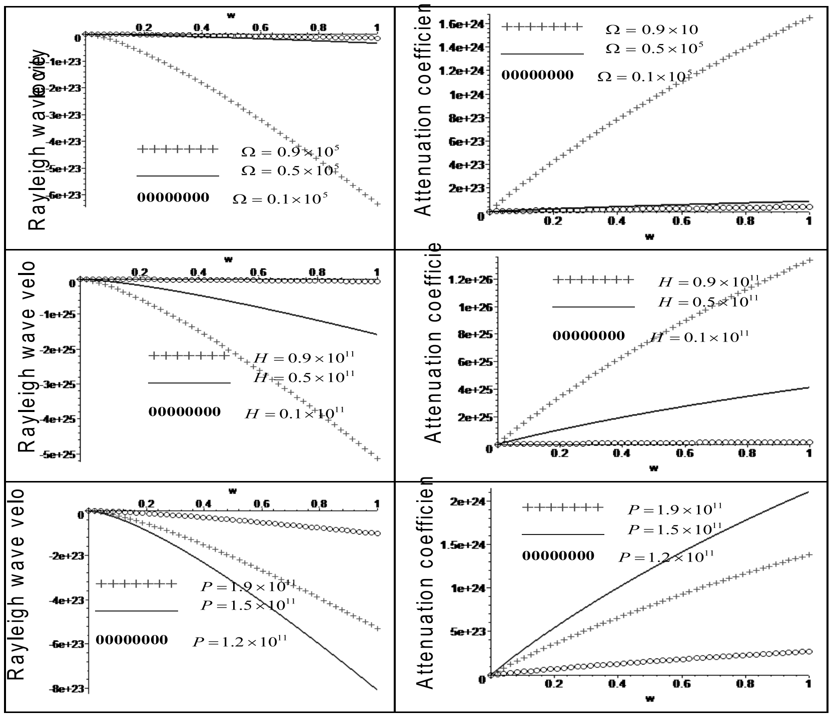

Fig. 1: Variation of Rayleigh wave velocity (Re) and attenuation coefficient (Im) for different values of magnetic field, rotation and initial stress with respect wave number ω.

Using these values, it is found that:

$$\mu_e = 0.2, \quad \tau_1 = 0.01, \quad R = 2$$

8 General case:

Fig. 1 shows the variation of the Rayleigh wave velocity (Re) and attenuation coefficient (Im) with respect to the value of the wave number ω for different values of Ω rotation, magnetic field H and initial stress P for LS-model. It is clear that the Rayleigh wave velocity decreases with increasing rotation, magnetic field and initial stress, while attenuation coefficient increases with increasing rotation, magnetic field and initial stress.

Fig. 2 shows the variation of the Rayleigh wave velocity (Re) and attenuation coefficient (Im) with respect to the value of the wave number ω for different values of Ω rotation, magnetic field H and initial stress P for GL-model. It is clear that the Rayleigh wave velocity decreases with increasing rotation, magnetic field and initial stress, while attenuation coefficient increases with increasing rotation, magnetic field and initial stress.

9 Special cases:

Case (i): If the rotation is assumed to be zero, in this case discussed the effect of magnetic field and initial stress with absence of Ω.

Fig. 3 shows the variation of the Rayleigh wave velocity (Re) and attenuation coefficient (Im) with respect to the value of the wave number ω for different values of magnetic field H and initial stress P for LS-model. It is clear that the Rayleigh wave velocity decreases with increasing magnetic field, while it increases with increasing initial stress, as well the attenuation coefficient increases with increasing magnetic field, while it decreases with increasing initial stress.

Fig. 4 shows the variation of the Rayleigh wave velocity (Re) and attenuation coefficient (Im) with respect to the value of the wave number ω for different values of magnetic field H and initial stress P for GL-model. It is clear that the Rayleigh wave velocity decreases with increasing of magnetic field and initial stress, while the attenuation coefficient increases with increasing magnetic field, while it decreases with increasing of initial stress.

Case (ii): If the magnetic field is assumed to be zero, in this case discussed the effect of rotation and initial stress with absence of H.

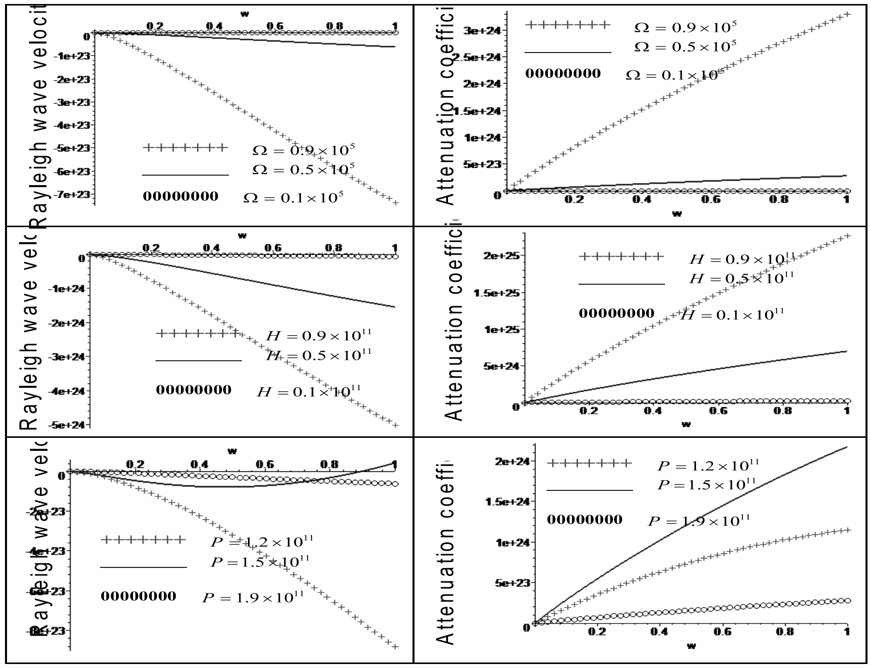

Fig. 2: Variation of Rayleigh wave velocity (Re) and attenuation coefficient (Im) for different values of magnetic field, rotation and initial stress with respect wave number ω.

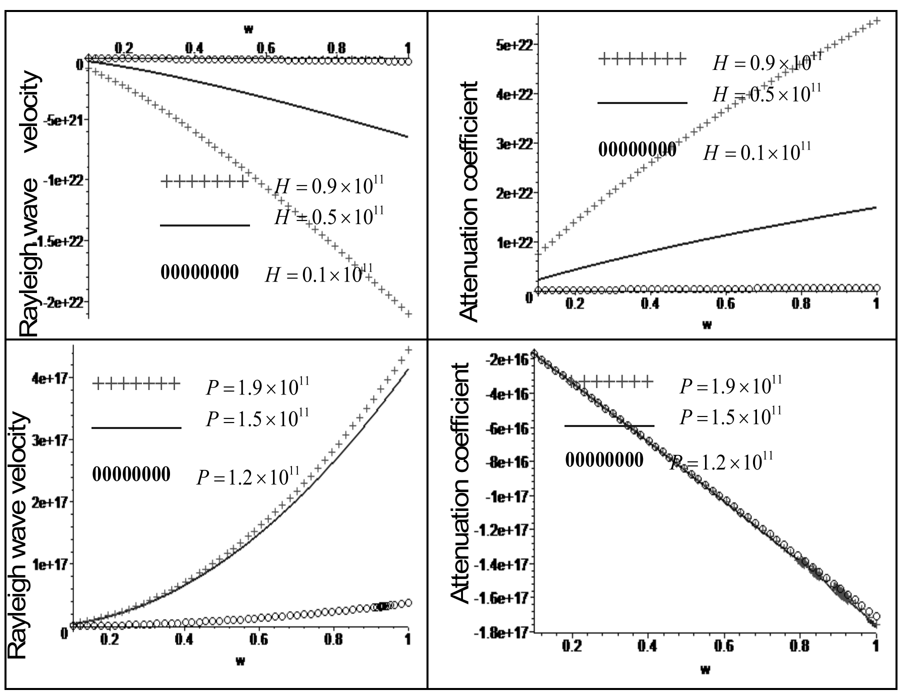

Fig. 3: Variation of Rayleigh wave velocity (Re) and attenuation coefficient (Im) for different values of magnetic field and initial stress with respect wave number ω.

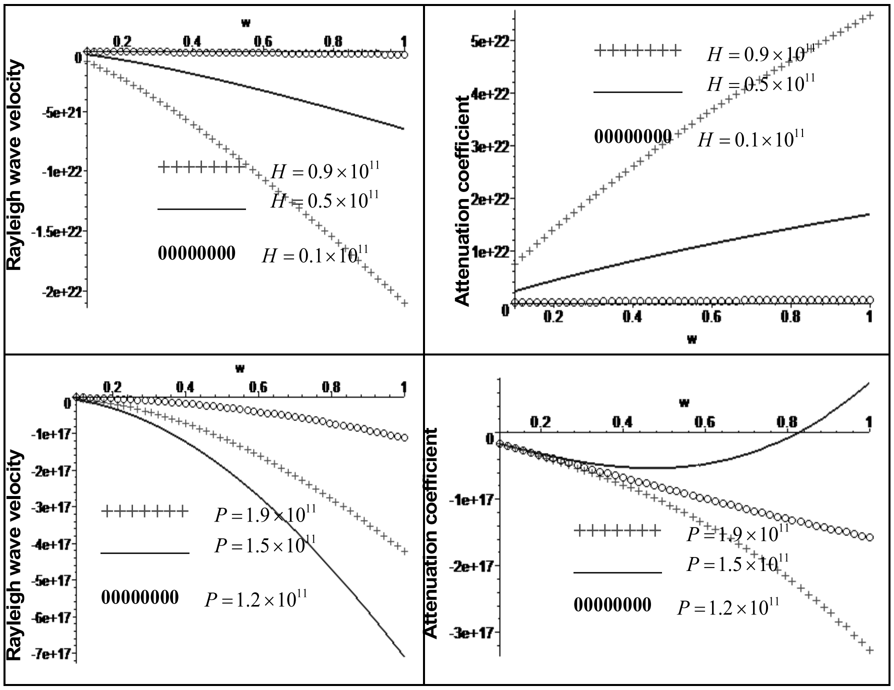

Fig. 4: Variation of Rayleigh wave velocity (Re) and attenuation coefficient (Im) for different values of magnetic field and initial stress with respect wave number ω.

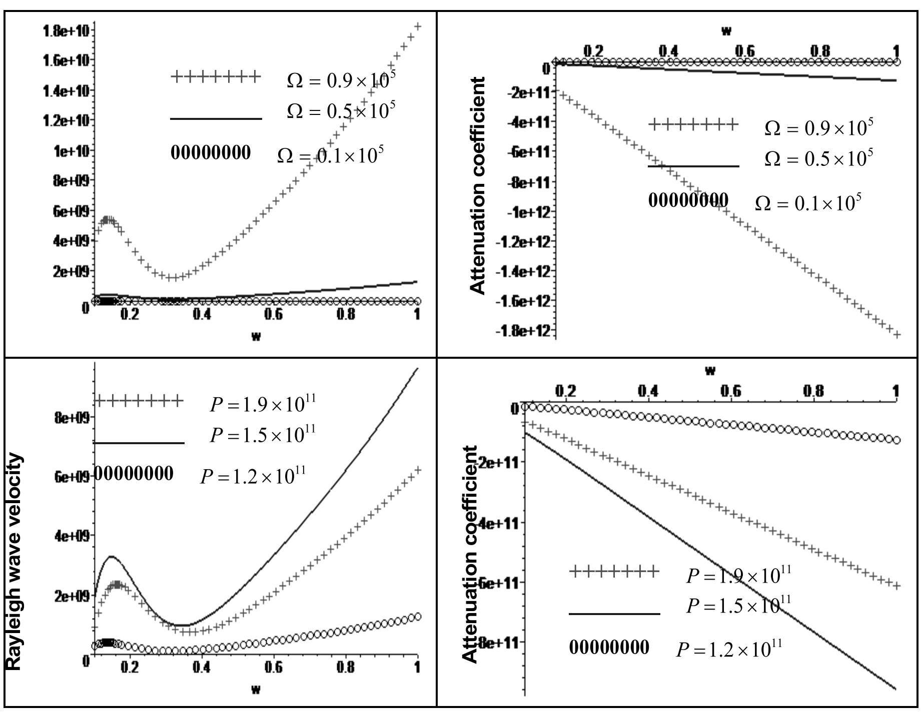

Fig. 5: Variation of Rayleigh wave velocity (Re) and attenuation coefficient (Im) for different values of rotation and initial stress with respect wave number ω.

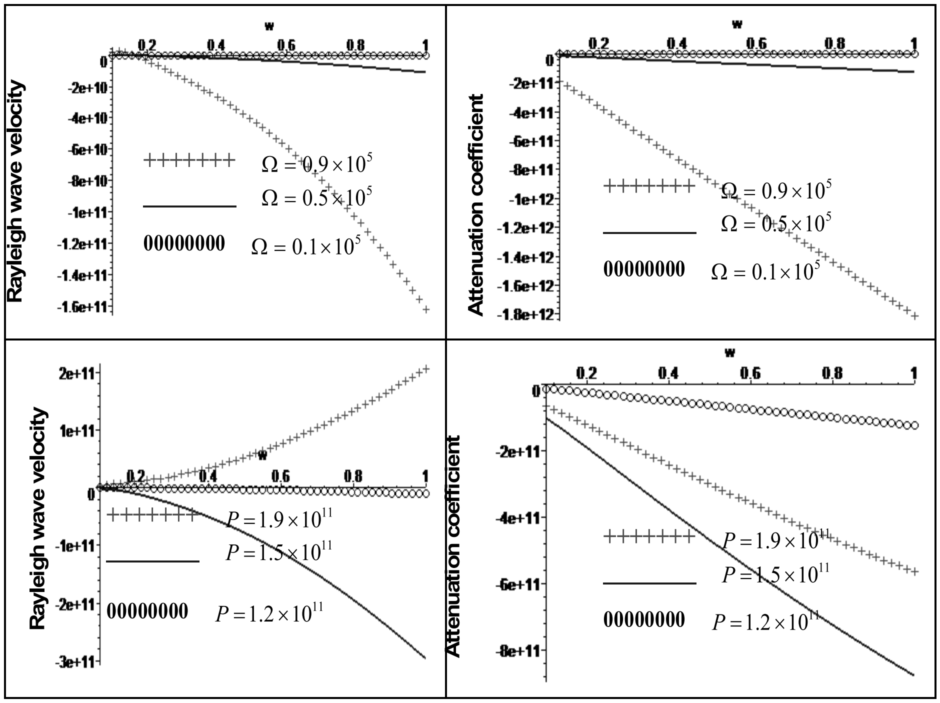

Fig. 6: Variation of Rayleigh wave velocity (Re) and attenuation coefficient (Im) for different values of rotation and initial stress with respect wave number ω.

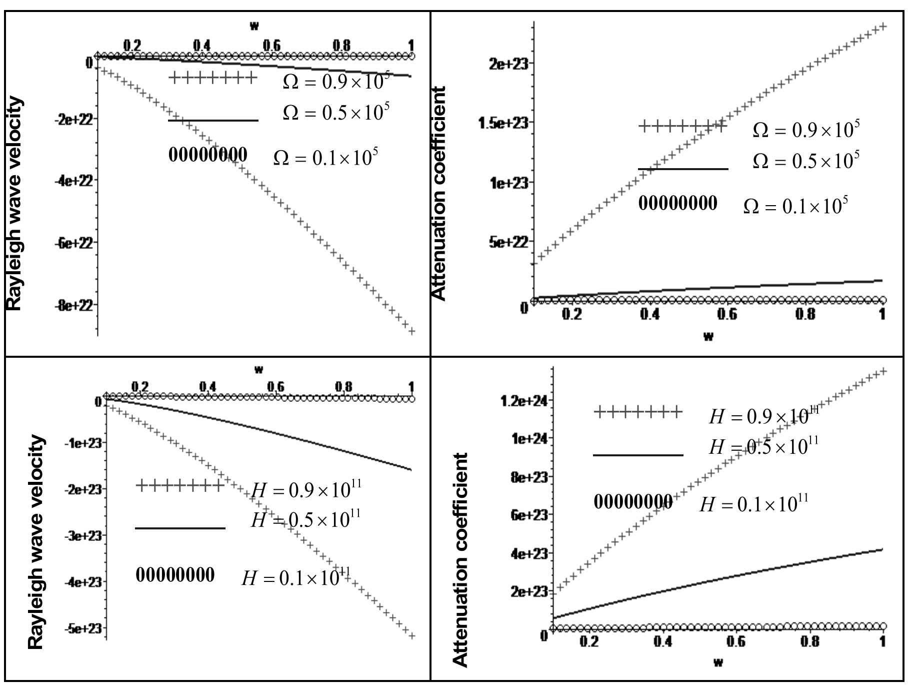

Fig. 7: Variation of Rayleigh wave velocity (Re) and attenuation coefficient (Im) for different values of rotation and magnetic field with respect wave number ω.

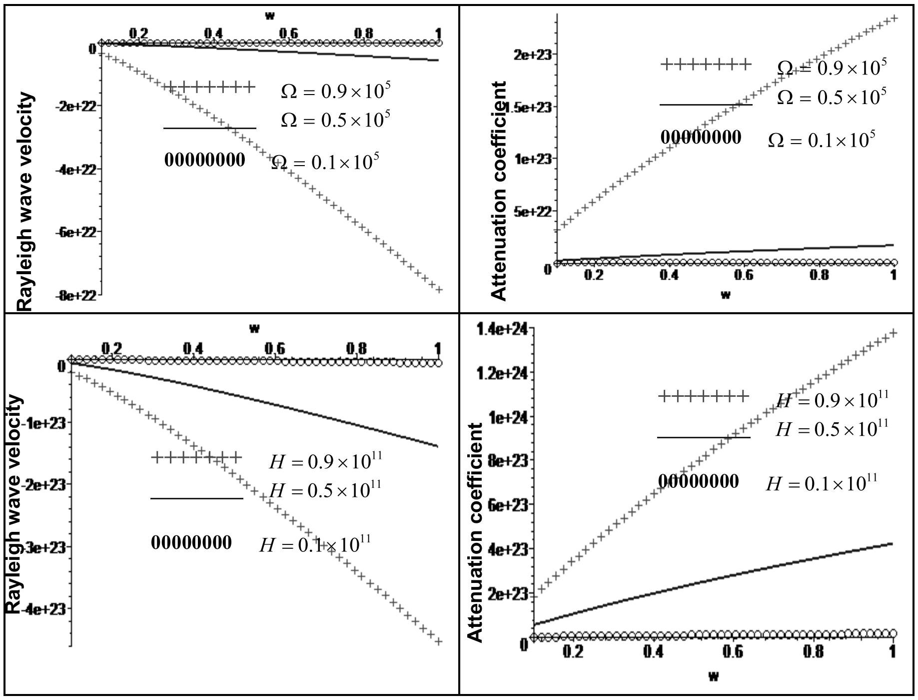

Fig. 8: Variation of Rayleigh wave velocity (Re) and attenuation coefficient (Im) for different values of rotation and magnetic field with respect wave number ω.

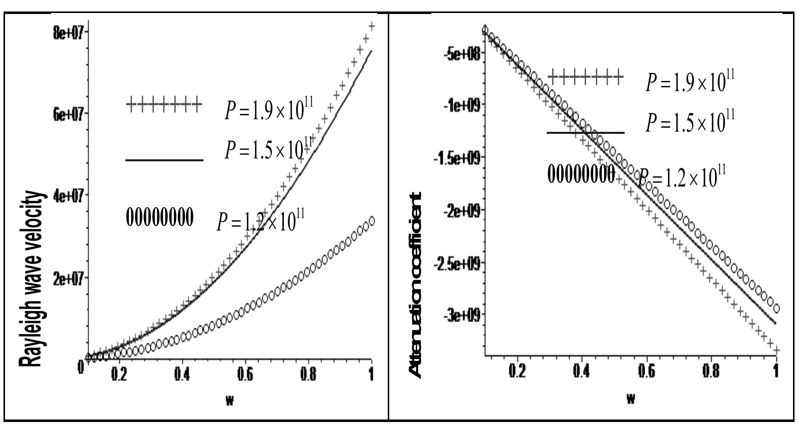

Fig. 9: Variation of Rayleigh wave velocity (Re) and attenuation coefficient (Im) for different values of initial stress with respect wave number ω.

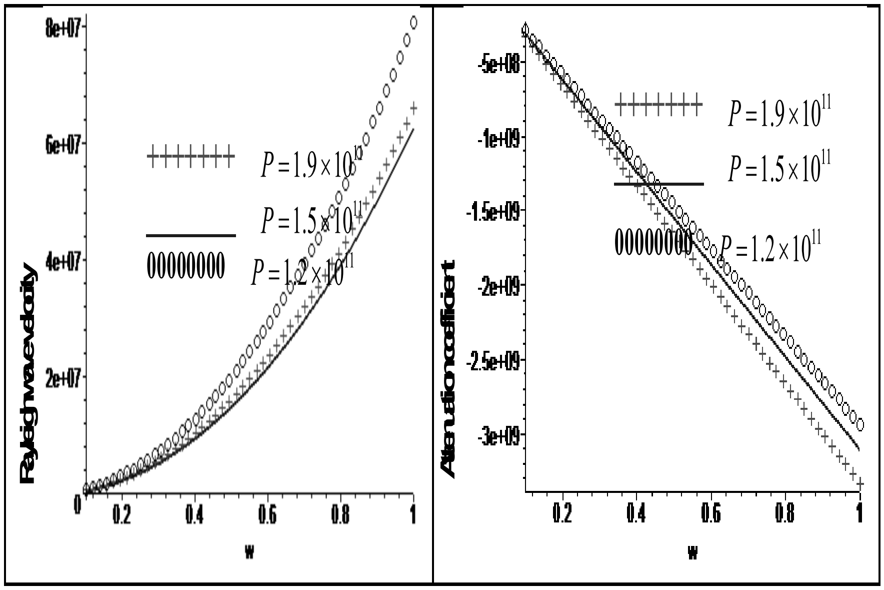

Fig. 10: Variation of Rayleigh wave velocity (Re) and attenuation coefficient (Im) for different values of initial stress with respect wave number ω.

Fig. 5 shows the variation of the Rayleigh wave velocity (Re) and attenuation coefficient (Im) with respect to the value of the wave number ω for different values of Ω rotation and initial stress P for LS-model. It is clear that the Rayleigh wave velocity increases with increasing rotation, while the attenuation coefficient decreases with increasing initial stress.

Fig. 6 shows the variation of the Rayleigh wave velocity (Re) and attenuation coefficient (Im) with respect to the value of the wave number ω for different values of Ω rotation and initial stress P for GL-model. It is clear that the Rayleigh wave velocity decreases with increasing rotation, while it increases and decreases with increasing of initial stress, as well the attenuation coefficient decreases with increasing rotation and initial stress.

Case (iii): If the initial stress is assumed to be zero, in this case discussed the effect of magnetic field and rotation with absence of P.

Fig. 7 shows the variation of the Rayleigh wave velocity (Re) and attenuation coefficient (Im) with respect to the value of the wave number ω for different values of Ω rotation and magnetic field H for LS-model. It is clear that the Rayleigh wave velocity decreases with increasing rotation and magnetic field, while the attenuation coefficient increases with increasing rotation and magnetic field.

Fig. 8 shows the variation of the Rayleigh wave velocity (Re) and attenuation coefficient (Im) with respect to the value of the wave number ω for different values of Ω rotation and magnetic field H for GL-model. It is clear that the Rayleigh wave velocity decreases with increasing rotation and magnetic field, while the attenuation coefficient increases with increasing rotation and magnetic field.

Case (iv): If the rotation and magnetic field both assumed to be zero, in this case discussed the effect of initial stress with absence of $\Omega = H = 0$.

Fig. 9 shows the variation of the Rayleigh wave velocity (Re) and attenuation coefficient (Im) with respect to the value of the wave number ω for different values of initial stress P for LS-model. It is clear that the Rayleigh wave velocity increases with increasing initial stress, while the attenuation coefficient decreases with increasing initial stress.

Fig. 10 shows the variation of the Rayleigh wave velocity (Re) and attenuation coefficient (Im) with respect to the value of the wave number ω for different values initial stress P for GL-model. It is clear that the Rayleigh wave velocity increases with increasing initial stress, while the attenuation coefficient decreases with increasing of initial stress.

10 Perspective

The method which is used in the present article is applicable to a wide range of problems in magneto-thermo elasticity with voids. The presence of magnetic field, rotation and initial stress play a significant role in all the physical quantities. Deformation of a body depends on the nature of forced applied as well as the type of boundary conditions. Study of the phenomenon of rotation, magnetic field and initial stress are also used to improve the conditions of oil extractions.Finally, if the magnetic field, rotation and initial stress are neglected, the relevant results obtained are deduced to the results obtained by [1].

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