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Bayesian Estimation for Linear Exponential Distribution Under Progressive Type -II Censoring in Constant-Stress Partially-Accelerated Life Tests

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Abstract: In this paper, the estimations of linear exponential distribution parameters and the acceleration factor in constant-stress partially-accelerated life tests based on progressive type -II censoring are considered. Maximum likelihood estimations for the considered parameters are obtained. Observed Fisher information matrix is used to construct asymptotic confidence interval to estimate the model parameters through normal approximation. By using Lindley's approximation and Markov chain Monte Carlo (MCMC) method, approximate Bayes estimates under loss functions are obtained. Finally, the accuracy of the maximum likelihood estimations and Bayesian estimations for the parameters is investigated through simulation studies.

Keywords: Constant-stress partially-accelerated life test, Progressive type -II censored, Linear exponential distribution, Maximum likelihood method, Bayesian estimation, Markov chain Monte Carlo.

1 Introduction

In reliability analysis, life tests are performed to observe the life of experimental items during the test. In such a life test, some surviving items are censored because of time and cost constraints or because of immediate needs of experimental items for other purposes. The goal of accelerated life analysis is to utilize the test data to extrapolate a product's life distribution and its associated parameters at a normal stress level. In a constant stress accelerated test, each unit is run at a prespecified constant stress level which does not vary with time and most products such as capacitors, microelectronics, semiconductors, lamps . . . etc, run at a constant stress.

One way to accelerate failure is the constant stress where each item runs at either use or accelerated conditions only, see Bai and Chung [8]. According to Nelson [27], the stress can be applied in various ways. Another way is step stress, where one increases the stress applied to test product in a specified discrete sequence. In other words, as indicated by Xiong and Ji [33], a test unit starts at a specified low stress. If the unit does not fail at a specified time, stress on it is raised and held in a specified time. Stress is repeatedly increased and held, until the test unit fails or a censoring time is reached.

Recently, Bayes and maximum likelihood methods of estimation are applied on Constant-Stress Partially-Acclelerated Life Tests (CSPALT) to finite mixtures of distributions by AL-Hussaini and Abdel-Hamid [5,6]. Mohie El-Din et al. [26] developed Bayes estimation for CSPALT to extension of the exponential distribution under progressive censoring. The estimation of parameters from different lifetime distributions based on progressive type -I and type -II censored samples are studied by several authors including Childs and Balakrishnan [14], Balakrishnan and Kannan [11], Mousa and Jaheen [7], Balakrishnan, et al. [12] and Soliman [32]. Jaheen et al. [23] considered the CSPALT under progressive censoring for generalized exponential distribution. Abdel-Hamid [2] considered CSPALT when the lifetime of units under use condition follows Burr XII distribution. Also, Ismail [22] studied inference in the generalized exponential distribution

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under PALT with progressive type -II censoring. Guan et al. [18] obtained the optimal CSPALT with complete sample for the generalized exponential distribution. Finally, EL-Sagheer [15] provided inferences in CSPALT based on progressive type -II Censoring. Abdel Ghaly and Aly [4] provided different estimation methods for CSPALT under the family of the exponentiated distributions. Based on progressive type -II censored samples. Ahmad and fawzy [3] studied non-Bayesian estimation in CSPALT for linear exponential (LE) distribution with progressive type -II censoring.
In the Step Stress PALT (SSPALT), a test unit is first run at use condition and if it does not fail in a specified time, then it is run at accelerated condition until failures occurs or the observation is censored. More details about the PALT are considered by different author, Abd-Elfattah et al. [1]. Ismail [20] obtained the Bayesian estimates of the Pareto distribution parameters under SSPALT with censored data. Ismail [21] estimated the parameters of Weibull distribution and the acceleration factor from hybrid partially accelerated life test.
The main aim in this paper is to obtain the Bayesian Estimations (BEs) of the acceleration factor and the distribution parameters and to compare them with the Maximum Likelihood Estimations (MLEs) counterparts by Monte Carlo simulations. The squared-error loss function is considered to make the comparison more meaningful.
The integration of modern statistical approaches into oceanographic data assimilation is in its infancy. The integration draws samples from the required distribution and then forms sample averages to approximate expectations; see Geman and Geman [16] and Hastings [19].
The linear exponential distribution with the parameters $\theta > 0$ and $\beta > 0$, will be denoted by LE(θ, β) distribution. Two special cases of LE distribution are exponential distribution (when $\beta = 0$) and Rayleigh distributions (when $\theta = 0$). The corresponding cumulative distribution function (CDF), probability density function (PDF), survival function (SF) and the hazard rate function (HRF) are given for $t > 0,\ \theta,\ \beta > 0$, respectively, by

$$\left.\begin{aligned} F_1(t) &= 1-\exp\left\{-(\theta t+\tfrac{\beta}{2}t^2)\right\}, \\ f_1(t) &= (\theta+\beta t)\exp\left\{-(\theta t+\tfrac{\beta}{2}t^2)\right\}, \\ \overline{F}_1(t) &= \exp\left\{-(\theta t+\tfrac{\beta}{2}t^2)\right\}, \\ h_1(t) &= \theta+\beta t. \end{aligned}\right\} \tag{1.1}$$

The LE distribution has many applications in applied statistics and reliability analysis. The LE distribution was also used by Carbone et al. [13] to study the survival pattern of patients with plasmacytic myeloma.

1.1 Basic assumptions

1. The lifetime of an item tested at use condition follows LE distribution.
2. The HRF of an item tested at accelerated condition is given by $h_2(t) = \rho h_1(t)$, where ρ is an acceleration factor satisfying $\rho > 1$. So, the HRF, CDF, PDF and SF under accelerated condition are given, for $t > 0, \theta, \beta > 0$ and $\rho > 1$,respectively, by

$$\left.\begin{aligned} h_2(t) &= \rho(\theta+\beta t), \\ F_2(t) &= 1-\exp\left\{-\rho(\theta t+\tfrac{\beta}{2}t^2)\right\}, \\ f_2(t) &= \rho(\theta+\beta t)\exp\left\{-\rho(\theta t+\tfrac{\beta}{2}t^2)\right\}, \\ \overline{F}_2(t) &= \exp\left\{-\rho(\theta t+\tfrac{\beta}{2}t^2)\right\}. \end{aligned}\right\} \tag{1.2}$$

3. The identically-distributed lifetimes $T_{ji},\ j = 1,2,\ i = 1,...,n_j$ of items allocated to use condition $(j = 1)$ and accelerated condition $(j = 2)$ are mutually independent.
According to CSPALT, group 1 consists of n_1 items randomly chosen among n test items is allocated to use condition and group 2 consists of $n_2 = n - n_1$ remaining items are subjected to an accelerated condition. Progressive type -II censoring is applied as follows: In group $j,\ j = 1,2$, at the time of the first failure, R_{j1} items are randomly withdrawn from the remaining n_{j1} surviving items. At the second failure, R_{j2} items from the remaining $n_j - 2 - R_{j1}$ items are randomly

withdrawn. The test continues until the m_j^{th} failure at which time, all remaining $R_{jm_j} = n_j - m_j - R_{j1} - R_{j2} - R_{j(m_j-1)}$ items are withdrawn. The R_{ji} are fixed prior to the study, $m_j \leq n_j$ and

$$m_j = np_j - \sum_{i=j}^{n-1} R_{ji} \tag{1.3}$$

There are many situations in life-testing and reliability studies in which the experimenter may be unable to obtain complete information on failure times for all experimental items. There are also situations wherein the removal of items prior to failure is pre-planned in order to reduce the cost and time associated with testing. The conventional type -I, and type -II censoring schemes do not have the flexibility of allowing removal of items at points other than the terminal point of the experiment. We consider here a more general censoring scheme, known as progressively type -II censoring, details of the scheme are discussed in Balakrishnan and Aggarwala [10].

The rest of the paper is arranged as follows: In Section 2, the MLEs of the involved parameters are derived. in Section 3, approximate BEs for the parameters under consideration are derived using Lindley's approximation and Markov chain Monte Carlo (MCMC) methods. A simulation study is presented in Section 4. Finally, some concluding remarks are introduced in Section 5.

2 Maximum likelihood estimation

Based on progressive type -II censored data under CSPALT model, the MLEs of the parameters θ, β and ρ are obtained in this section. Both point and interval estimations of the parameters are derived.

The ML method is used to obtain the estimates of the parameter of the population distribution. Let $T_{j1:m_j:n_j}^{(R_{j1},\dots,R_{jmj})} < T_{j2:m_j:n_j}^{(R_{j1},\dots,R_{jmj})} < \dots < T_{jm_j:m_j:n_j}^{(R_{j1},\dots,R_{jmj})}$, $j = 1,2$, be the progressive type -II censored data from two populations whose SFs and PDFs are given by (1.1) and (1.2), with $R_{j1},\dots,R_{jm_j}$ are the two progressive censoring schemes. In the sequel, we use $T_{jm_j:m_j:n_j}$ instead of $T_{jm_j:m_j:n_j}^{(R_{j1},\dots,R_{jm_j})}$. Also, let $t_{j1:m_j:n_j} < t_{j2:m_j:n_j} < \dots < t_{jm_j:m_j:n_j}$ be the corresponding observed values. The likelihood function, $L(\theta,\beta,\rho)$, based on the two progressively type -II censored samples when the lifetimes of the tested units follow the LE distribution that is given by

$$\begin{aligned} L(\theta,\beta,\rho) &= \prod_{j=1}^{2}\left[C_j \prod_{i=1}^{m_j} (f_j(t_{ji:m_j:n_j}))[\bar{F}(t_{ji:m_j:n_j})]^{R_{ji}}\right] \\ &= \prod_{j=1}^{2}\left[C_j \prod_{i=1}^{m_j} \rho^{j-1}(\theta+\beta t_{ji:m_j:n_j})\left[\exp\left\{-\rho^{j-1}\left(\theta t_{ji:m_j:n_j} + \frac{\beta}{2} t_{ji:m_j:n_j}^2\right)\right\}\right]^{1+R_{ji}}\right], \end{aligned} \tag{2.4}$$

where $j = 1,2$ and $C_j = n_j(n_j-1-R_{j1})(n_j-2-R_{j1}-R_{j2})(n_j-m_j+1-R_{j1}-R_{j2}-\dots-R_{j(m_j-1)})$.

Hence, the log-likelihood function, $\ell(\theta,\beta,\rho)$ is

$$\ell(\theta,\beta,\rho) = C + m_2 \ln\rho + \sum_{j=1}^{2}\sum_{i=1}^{m_j}\left\{\ln(\theta+\beta t_{ji:m_j:n_j}) - \rho^{j-1}(R_{ji}+1)\left(\theta t_{ji:m_j:n_j} + \frac{\beta}{2} t_{ji:m_j:n_j}^2\right)\right\} \tag{2.5}$$

where $C = \ln C_1 + \ln C_2$.

Then we have the likelihood equations for θ, β and ρ, respectively, by solving the following equations

$$\left.\begin{aligned} \frac{\partial \ell(\theta,\beta,\rho)}{\partial\theta} &= \sum_{j=1}^{2}\sum_{i=1}^{m_j}\left\{\frac{1}{\theta+\beta t_{ji:m_j:n_j}} - \rho^{j-1}(R_{ji}+1)t_{ji:m_j:n_j}\right\} = 0, \\ \frac{\partial \ell(\theta,\beta,\rho)}{\partial\beta} &= \sum_{j=1}^{2}\sum_{i=1}^{m_j}\left\{\frac{t_{ji:m_j:n_j}}{\theta+\beta t_{ji:m_j:n_j}} - \frac{1}{2}\rho^{j-1}(R_{ji}+1)t_{ji:m_j:n_j}^2\right\} = 0 \\ \frac{\partial \ell(\theta,\beta,\rho)}{\partial\rho} &= \frac{m_2}{\rho} - \sum_{i=1}^{m_2}(R_{2i}+1)\left(\theta t_{2i:m_2:n_2} + \frac{\beta}{2} t_{2i:m_2:n_2}^2\right) = 0. \end{aligned}\right\} \tag{2.6}$$

Now, we have a system of three non-linear likelihood equations in three unknowns θ, β and ρ. It cannot be solved analytically. An iterative method such as the Newton–Raphson is used to obtain the MLEs of the unknown parameters.

3 Bayes Estimations

In this section, the squared error loss function is considered to obtain BE of the parameters and acceleration factor. Unfortunately, in many cases, the BEs cannot be expressed in explicit forms. So, approximate BEs are obtained under Informative Priors (INP) using Lindley's approximation and MCMC methods.

3.1 Priors distributions

We shall consider two cases of informative priors of the parameters θ,β and ρ

Case 1: θ and β are independent and ρ is unknown.

The parameters θ and β behave as independent random variables and ρ is unknown with the following prior PDF:

$$\nu(\theta)=\alpha\exp\{-\alpha\theta\},\ \ \alpha>0,\ \ \theta>0, \tag{3.7}$$

$$\nu(\beta)=\lambda\exp\{-\lambda\beta\},\ \ \lambda>0,\ \ \lambda>0 \tag{3.8}$$

and

$$\nu(\rho)=\rho^{-1},\ \ \rho>1. \tag{3.9}$$

where α and β are assumed to be known. The joint prior PDF of θ, β and ρ, is

$$\nu_2(\theta,\beta,\rho)=\nu(\theta)\nu(\beta)\nu(\rho)=\rho^{-1}\alpha\lambda\exp\{-\alpha\theta-\lambda\beta\},\ \ \theta,\ \ \beta>0. \tag{3.10}$$

Case 2. θ and β are dependent and ρ is unknown.

Here, we assume the parameter θ to have the gamma prior

$$\nu(\theta)=\frac{1}{\gamma^{\varsigma}\Gamma(\varsigma)}\theta^{\varsigma-1}\exp(\frac{-\theta}{\gamma})\qquad \theta>0,\ \gamma>0,\ \varsigma>0, \tag{3.11}$$

while the density function of $\beta|\theta$ has the gamma prior

$$\nu(\beta|\theta)=\frac{\theta^{\varepsilon}}{\Gamma(\varepsilon)}\beta^{\varepsilon-1}\exp(-\theta\beta),\qquad \beta>0,\ \varepsilon>0 \tag{3.12}$$

and the density function of ρ is

$$\nu(\rho)=\rho^{-1},\ \ \rho>1. \tag{3.13}$$

Then, the joint prior density of θ, β and ρ can be written as

$$\nu_3(\theta,\beta,\rho)=\nu(\rho)\nu(\theta)\nu(\beta|\theta)\ =\rho^{-1}\theta^{\varsigma+\varepsilon-1}\beta^{\varepsilon-1}\exp[-\theta(\beta+\gamma)],\qquad \theta>0,\ \beta>0,\rho>1. \tag{3.14}$$

3.2 Bayes estimation using Lindley's approximation

Under a squared-error loss function, the Bayes estimator of a parameter is its posterior expectation. To obtain the posterior mean and posterior variance of the acceleration factor ρ and parameters θ, β, an approximation due to Lindley [24] is used. The approximation is evaluated at the MLEs of the parameters. Now, let Θ be a set of parameters Θ_1, $\Theta_2,...,\Theta_m$, where m is the number of parameters, then the posterior expectation of an arbitrary function $u(\Theta)$ can be asymptotically estimated by:

$$E(u(\Theta)|data)=\frac{\int_{\Theta}u(\Theta)\nu(\Theta)e^{lnL(data|\Theta)}d\Theta}{\int_{\Theta}\nu(\Theta)e^{lnL(data|\Theta)}d\Theta}$$

$$\approx[\hat{u}_{ML}+\frac{1}{2}\sum_{i,j}(u_{ij}^{(2)}+2u_i^{(1)}\rho_j^{(1)})\sigma_{ij}+\frac{1}{2}\sum_{i,j,k,s}L_{ijk}^{(3)}\sigma_{ij}\sigma_{k_s}u_s^{(1)}]\downarrow\hat{\Theta}_{ML} \tag{3.15}$$

which is the Bayes estimator of $u(\Theta)$ under a squared-erorr loss function, where $i,j,k,s=1,2,\cdots,m,\nu(\Theta)$ is the prior distribution of Θ, $\hat{u}=u(\hat{\Theta}_{ML})$, $L=L(\Theta)$ is the likelihood function, $\rho\equiv\rho(\Theta)=log\nu(\Theta)$, σ_{ij} are the elements of the inverse of the asymptotic Fisher-information matrix of θ, β and ρ

$$u_i^{(1)}=\frac{\partial u}{\partial\Theta_i},\ u_{i,j}^{(2)}=\frac{\partial^2 u}{\partial\Theta_i\partial\Theta_j},\ \rho_j^{(1)}=\frac{\partial log\nu(\Theta)}{\partial\Theta_j}\ \text{and}\ L_{ijk}^{(3)}=\frac{\partial^3 lnL(x|\Theta)}{\partial\Theta_i\partial\Theta_j\partial\Theta_k}.$$

Such an approximation is easy to use and does not require innovative programming and extensive computer time. The linear Bayes estimator in (3.15) is a practical approximation of multi-dimensional integrals, according to Green [17]. It has led to many useful applications Sinha [30]. However, if the domain of the parameters is a function of the parameters, Bayes estimators using Lindley expansion are not obtainable unless the MLEs exist Sinha and Sloan [31].
Let the subscripts 1, 2, 3 refer to θ, β and ρ, respectively. Therefore, the posterior means (Bayes estimators) of θ, β and ρ, can be obtained as follows:

$$\theta^* = E(\theta|data) = \left[\hat{a}_{ML} - \left(\frac{\sigma_{11}}{\theta} + \frac{\sigma_{12}}{\beta} + \frac{\sigma_{13}}{\rho}\right) + \frac{1}{2}(\sigma_{11}D_1 + \sigma_{12}D_2 + \sigma_{13}D_3)\right]_{\downarrow\Theta=\hat{\Theta}_{ML}}, \tag{3.16}$$

$$\beta^* = E(\beta|data) = \left[\hat{\beta}_{ML} - \left(\frac{\sigma_{21}}{\theta} + \frac{\sigma_{22}}{\beta} + \frac{\sigma_{23}}{\rho}\right) + \frac{1}{2}(\sigma_{21}D_1 + \sigma_{22}D_2 + \sigma_{23}D_3)\right]_{\downarrow\Theta=\hat{\Theta}_{ML}} \tag{3.17}$$

and

$$\rho^* = E(\rho|data) = \left[\hat{\rho}_{ML} - \left(\frac{\sigma_{31}}{\theta} + \frac{\sigma_{32}}{\beta} + \frac{\sigma_{33}}{\rho}\right) + \frac{1}{2}(\sigma_{31}D_1 + \sigma_{32}D_2 + \sigma_{33}D_3)\right]_{\downarrow\Theta=\hat{\Theta}_{ML}}. \tag{3.18}$$

where

$$D_1 = \sum_{i,j}\sigma_{ij}L^{(3)}_{ij1} \equiv \sigma_{11}L^{(3)}_{111} + \sigma_{12}L^{(3)}_{121} + \sigma_{13}L^{(3)}_{131} + \sigma_{22}L^{(3)}_{221} + \sigma_{23}L^{(3)}_{231} + \sigma_{33}L^{(3)}_{331},$$

$$D_2 = \sum_{i,j}\sigma_{ij}L^{(3)}_{ij2} \equiv \sigma_{11}L^{(3)}_{112} + 2\sigma_{12}L^{(3)}_{122} + 2\sigma_{13}L^{(3)}_{132} + \sigma_{22}L^{(3)}_{222} + 2\sigma_{23}L^{(3)}_{232} + \sigma_{33}L^{(3)}_{332}$$

and

$$D_3 = \sum_{i,j}\sigma_{ij}L^{(3)}_{ij3} \equiv \sigma_{11}L^{(3)}_{113} + 2\sigma_{12}L^{(3)}_{123} + 2\sigma_{13}L^{(3)}_{133} + \sigma_{22}L^{(3)}_{223} + 2\sigma_{23}L^{(3)}_{233} + \sigma_{33}L^{(3)}_{333}, \quad i,j = 1,2,3.$$

To compute the posterior means of the three parameters θ, β and ρ, both second and third derivatives of the natural logarithm of the likelihood function must be obtained. The log-likelihood function is shown in (2.5). As for the third derivations of the natural logarithm of the likelihood function are given by

$$L^{(3)}_{111} = \frac{\partial^3\ell}{\partial\theta^3} = \sum_{i=1}^{m_1}\frac{2}{(\theta+\beta t_{1i:m_1:n_1})^3} + \sum_{i=1}^{m_2}\frac{2}{(\theta+\beta t_{2i:m_2:n_2})^3}, \tag{3.19}$$

$$L^{(3)}_{222} = \frac{\partial^3\ell}{\partial\beta^3} = \sum_{i=1}^{m_1}\frac{2t^3_{2i:m_1:n_1}}{(\theta+\beta t_{1i:m_1:n_1})^3} + \sum_{i=1}^{m_2}\frac{2t^3_{2i:m_2:n_2}}{(\theta+\beta t_{2i:m_2:n_2})^3}, \tag{3.20}$$

$$L^{(3)}_{333} = \frac{\partial^3\ell}{\partial\rho^3} = \frac{2m_2}{\rho^3}, \tag{3.21}$$

$$L^{(3)}_{112} = L^{(3)}_{121} = L^{(3)}_{211} = \frac{\partial^3\ell}{\partial\theta^2\partial\beta} = \sum_{i=1}^{m_1}\frac{2t_{1i:m_1:n_1}}{(\theta+\beta t_{1i:m_1:n_1})^3} + \sum_{i=1}^{m_2}\frac{2t_{i:m_2:n_2}}{(\theta+\beta t_{2i:m_2:n_2})^3}, \tag{3.22}$$

$$L^{(3)}_{221} = L^{(3)}_{212} = L^{(3)}_{122} = \frac{\partial^3\ell}{\partial\beta^2\partial\theta} = \sum_{i=1}^{m_1}\frac{2t^2_{1i:m_1:n_1}}{(\theta+\beta t_{1i:m_1:n_1})^3} + \sum_{i=1}^{m_2}\frac{2t^2_{i:m_2:n_2}}{(\theta+\beta t_{2i:m_2:n_2})^3}, \tag{3.23}$$

and

$$L^{(3)}_{113} = L^{(3)}_{131} = L^{(3)}_{311} = L^{(3)}_{223} = L^{(3)}_{232} = L^{(3)}_{322} = L^{(3)}_{331} = L^{(3)}_{313} = L^{(3)}_{133} = L^{(3)}_{332}$$

$$= L^{(3)}_{323} = L^{(3)}_{233} = L^{(3)}_{123} = L^{(3)}_{132} = L^{(3)}_{213} = L^{(3)}_{231} = L^{(3)}_{312} = L^{(3)}_{321} = 0. \tag{3.24}$$

3.3 Bayesian estimation using MCMC

In this section, we consider the MCMC method to generate samples from the posterior distributions and then compute the Bayes estimates of θ, β and ρ under constant PALT for LE distribution using progressive type -II censored. A wide variety of MCMC schemes are available, and it can be difficult to choose among them. An important sub-class of MCMC methods are Gibbs sampling and more general Metropolis-Hastings (MH)-within-Gibbs samplers, see, for example, Robert and Casella [29] and Recently, Rezaei et al. [28].

• **Posterior distribution in the informative priors**

case I: θ and β are independent and ρ is unknown.

By using the prior distribution in (3.10), the joint posterior distribution of θ , β and ρ is given by

$$g_2^*(\theta,\beta,\rho|\underline{t}) \propto \rho^{-1}\alpha\lambda\exp\{-\alpha\theta-\lambda\beta\}\exp\left\{\sum_{j=1}^{2}\sum_{i=1}^{m_j}\ln(\theta+\beta t_{ji:m_j:n_j}) -\rho^{j-1}(1+R_{ji})\left(\theta t_{ji:m_j:n_j}+\frac{\beta}{2}t^2_{ji:m_j:n_j}\right)\right\}. \tag{3.25}$$

From (3.25), the conditional posterior distributions of θ, β and ρ are given, respectively, by

$$g_{21}^*(\theta|\beta,\rho,\underline{t}) \propto \exp\left\{\left[\sum_{j=1}^{2}\sum_{i=1}^{m_j}[\ln(\theta+\beta t_{ji:m_j:n_j})-\rho^{j-1}(1+R_{ji})t_{ji:m_j:n_j}]-\alpha\right]\theta\right\}, \tag{3.26}$$

$$g_{22}^*(\beta|\theta,\rho,\underline{t}) \propto \exp\left\{\left[\sum_{j=1}^{2}\sum_{i=1}^{m_j}[\ln(\theta+\beta t_{ji:m_j:n_j})-0.5\rho^{j-1}(1+R_{ji})t^2_{ji:m_j:n_j}]-\lambda\right]\beta\right\}, \tag{3.27}$$

$$g_{23}^*(\rho|\theta,\beta,\underline{t}) \propto \rho^{m_2-1}\exp\left\{-\rho\sum_{i=1}^{m_2}(1+R_{2i})\left(\theta t_{2i:m_2:n_2}-0.5\beta t^2_{2i:m_2:n_2}\right)\right\}. \tag{3.28}$$

case II: θ and β are dependent and ρ is unknown.

By using (3.14), the joint posterior distributions of θ , β and ρ are given, respectively, by

$$g_3^*(\theta,\beta,\rho|\underline{t}) \propto \rho^{-1}\pi\theta^{\varsigma+\varepsilon-1}\beta^{\varepsilon-1}\exp[-\theta(\beta+\gamma)]exp\left\{\sum_{j=1}^{2}\sum_{i=1}^{m_j}\ln(\theta+\beta t_{ji:m_j:n_j}) -\rho^{j-1}(1+R_{ji})\left(\theta t_{ji:m_j:n_j}+\frac{\beta}{2}t^2_{ji:m_j:n_j}\right)\right\}. \tag{3.29}$$

where $\pi^{-1}=\Gamma(\varsigma)\Gamma(\varepsilon+1)\gamma^{\varsigma}$.

Then the conditional posterior distributions of θ , β and ρ by using (3.29), are given respectively, by

$$g_{31}^*(\theta|\beta,\rho,\underline{t}) \propto \theta^{\varsigma+\varepsilon-1}\exp\left\{\left[\sum_{j=1}^{2}\sum_{i=1}^{m_j}[\ln(\theta+\beta t_{ji:m_j:n_j})-\rho^{j-1}(1+R_{ji})t_{ji:m_j:n_j}]+(\beta+\gamma)\right]\theta\right\}, \tag{3.30}$$

$$g_{32}^*(\beta|\theta,\rho,\underline{t}) \propto \beta^{\varepsilon-1}\exp\left\{\left[\sum_{j=1}^{2}\sum_{i=1}^{m_j}[\ln(\theta+\beta t_{ji:m_j:n_j})-0.5\rho^{j-1}(1+R_{ji})t^2_{ji:m_j:n_j}]-\theta\right]\beta\right\}, \tag{3.31}$$

$$g_{33}^*(\rho|\theta,\beta,\underline{t}) \propto \rho^{m_2-1}\exp\left\{-\rho\sum_{i=1}^{m_2}(1+R_{2i})\left(\theta t_{2i:m_2:n_2}-0.5\beta t^2_{2i:m_2:n_2}\right)\right\}. \tag{3.32}$$

Remark

It can be seen that Equations (3.28) and (3.32) have a gamma density with shape parameter m_2 and scale parameter $\sum_{i=1}^{m_2}(1+R_{2i})\left(\theta t_{2i:m_2:n_2}-0.5\beta t^2_{2i:m_2:n_2}\right)$ and, therefore, samples of ρ can be easily generated using any gamma-generating routine. However, in our case, the conditional posterior distribution of θ in Equations (3.26) and (3.30) and conditional

posterior distribution of β in Equations (3.27) and (3.31) cannot be reduced analytically to well known distributions and therefore it is not possible to sample directly by standard methods, but it plot shows that it is similar to normal distribution. So, to generate random numbers from this distribution, we use the MH method with normal proposal distribution.
Now, we propose the following scheme to generate (θ,β,ρ) from the posterior density function and in turn obtain the Bayes estimates and the corresponding credible intervals.

Algorithm (1)

1. Start with an $(\theta^{(0)},\beta^{(0)})$.

2. Set $y=1$.

3. Generate $\rho^{(y)}$ from Gamma $\left(m_2,\sum_{i=1}^{m_2}(1+R_{2i})\left(\theta t_{2i:m_2:n_2}-0.5\beta t^2_{2i:m_2:n_2}\right)\right)$.

4. Using Metropolis-Hastings (MH)-within-Gibbs samplers (see, Metropolis et al. [25]), generate $\theta^{(y)}$ and $\beta^{(y)}$ from $g^*_{dk}(\theta|\beta,\rho,\underline{t})$ and $g^*_{dk}(\beta|\theta,\rho,\underline{t})$ with the $N(\theta^{(y-1)},\sigma_1^2)$ and $N(\beta^{(y-1)},\sigma_2^2)$ proposal distributions.
Where σ_1^2 and σ_2^2 are the variance obtained using variance-covariance matrix with $d=1,2,3$ and $k=1,2$.

5. Compute $\theta^{(y)}$, $\beta^{(y)}$ and $\rho^{(y)}$.
6. Set $y=y+1$.

7. Repeat Steps 3 - 6 N times.

8. Obtain the Bayes estimates of θ, β and ρ with respect to SEL function as

$$\hat{\theta}_{MCMC}=E(\theta|t)=\frac{1}{N-M}\sum_{i=M+1}^{N}\theta_i\ ,$$

$$\hat{\beta}_{MCMC}=E(\beta|t)=\frac{1}{N-M}\sum_{i=M+1}^{N}\beta_i$$

and

$$\hat{\rho}_{MCMC}=E(\rho|t)=\frac{1}{N-M}\sum_{i=M+1}^{N}\rho_i\ .$$

where M is burn-in.

4 Simulation Study

In this section, we discuss results of a simulation study testing the performance of MLE for different sample sizes, and censoring schemes. Using the algorithm presented in Balakrishnan and Sandhu [9], we have generated progressively type -II censored samples of different sizes from LE distribution as follows.

Algorithm (2)

1. Specify the values of $\theta,\ \beta,\ \rho,\ n_1,\ n_2,\ m_1,\ m_2,\alpha,\lambda,\ \varepsilon,\ \mu,\ \gamma,\ \varsigma$.

2. Generate two independent random samples of sizes m_1 and m_2 from Uniform $(0,1)$ distribution.

3. Specify the values of the censored scheme $R_{ji},\ j=1,2; i=1,...,m_j$.

4. Set $E_{ji}=1/(i+\sum_{d=m_j-i+1}^{m_j}R_{jd})$ for $j=1,2;\ \ i=1,...,m_j$.

5. Set $U_{ji}=V_{ji}^{E_{ji}}$ for $j=1,2$ and $i=1,...,m_j$.

6. For $j=1,2$, obtain the two progressive type -II right censored samples,

$(W_{j1},W_{j2},...,W_{jmj})$ from the Uniform $(0,1)$ distribution, where $W_{1i}=1-\prod_{d=m_1-i+1}^{m_1}U_{1d}$ $,i=1,...,m_1$ and $W_{2i}=1-\prod_{d=m_2-i+1}^{m_2}U_{2d}$ $,i=1,...,m_2$

7. Using step 5, generate two random samples $(t_{j1},t_{j2},...,t_{jmj})$, from $CDFs$ $F_1(t)$ and $F_2(t)$ given in (1.1) and (1.2) respectively, as follows $t_{1i}=(-\theta+\sqrt{\theta^2-2\beta\log(1-W_{1i})})/\beta$ and $t_{2i}=(-\theta\rho+\sqrt{(\theta\rho)^2-2\beta\log(1-W_{2i})})/\rho\beta$, $i=1,...,m_j$ and $j=1,2$.
and hence obtain the two ordered samples $(t_{1:m_1:n_1},...,t_{m_1:m_1:n_1})$, and $(t_{2:m_2:n_2},...,t_{m_2:m_2:n_2})$ which represent two progressive type -II right-censored samples from linear failure rate distribution under CSPALT.

8. For each sample, the ML estimates of acceleration factor and the parameters of LE distribution (θ,β,ρ) are obtained by solving the nonlinear equation (2.6).

9. The Bayes estimates of (θ,β,ρ) are computed based on squared error loss function.

10. Steps 2-9 were repeated 1000 times for different sample sizes.

11. The average of ML, Bayes estimates and mean square error (MSE) are computed.

Now, we compare several progressive type -II censoring schemes on these data. We consider the following three sampling plans

scheme $I: R_{ji}=n-m;\ i\neq 1,\ j=2$

scheme $II: \begin{cases} R_{ji}=n-m; \quad i\neq \frac{m+1}{2}, & j\neq \frac{m+1}{2} \quad if\ m\ odd.\\ R_{ji}=n-m; i\neq \frac{m}{2}+1, & j\neq \frac{m}{2}+1 \quad if\ m\ even.\end{cases}$

scheme $III: R_{ji}=n-m; i\neq m, j=2$

4.1 Simulation procedure

In this subsection the MLEs of the unknown parameters θ, β and ρ are computed. The Newton–Raphson technique is connected for solving the nonlinear system and find the CIs. When the NIP distributionis used, we compute the Bayes estimates of the unknown parameters by the MCMC method. The CIs are computed in view of 10000 MCMC tests. Table 1 lists the different censoring SCs ($n_1=n_2=n$ and $m_1=m_2=m$) and Table 2 lists the different censoring SCs ($n_1\neq n_2$ and $m_1\neq m_2$), used in the simulation study, for different choices of sample sizes, n. Table 3 and Table 4 show the MSE for θ,β and ρ using the ML and MCMC methods under progressive censoring SCs listed in Tables 1, 2.

Table 1 : Progressive censoring schemes used in the Monte Carlo simulation study at $n_1=n_2=n$ and $m_1=m_2=m$.

n m	Sc	$(R_1,...,R_m)$	Sc	$(R_1,...,R_m)$	Sc	$(R_1,...,R_m)$
15 9	$[I]$	$R_1=6$ $R_i=0,\ i\neq 1$	$[II]$	$R_5=6$ $R_i=0,\ i\neq 5$	$[III]$	$R_9=6$ $R_i=0,\ i\neq 9$
50 30	$[I]$	$R_1=20$ $R_i=0,\ i\neq 1$	$[II]$	$R_{16}=20$ $R_i=0,\ i\neq 16$	$[III]$	$R_{30}=20$ $R_i=0,\ i\neq 30$
100 60	$[I]$	$R_1=40$ $R_i=0,\ i\neq 1$	$[II]$	$R_{31}=40$ $R_i=0,\ i\neq 31$	$[III]$	$R_{60}=40$ $R_i=0,\ i\neq 60$

Table 2 : Progressive censoring schemes used in the Monte Carlo simulation study at $n_1 \neq n_2$ and $m_1 \neq m_2$.

n_1 / n_2	m_1 / m_2	Sc	$(R_1,...,R_{m_1})$ / $(R_1,...,R_{m_2})$	Sc	$(R_1,...,R_{m_1})$ / $(R_1,...,R_{m_2})$	Sc	$(R_1,...,R_{m_1})$ / $(R_1,...,R_{m_2})$
20	12	$[I]$	$R_1 = 8$	$[II]$	$R_7 = 8$	$[III]$	$R_{12} = 8$
			$R_i = 0,\ i \neq 1$		$R_i = 0,\ i \neq 7$		$R_i = 0,\ i \neq 12$
25	15		$R_1 = 10$		$R_8 = 10$		$R_{15} = 10$
			$R_j = 0,\ j \neq 1$		$R_j = 0,\ j \neq 8$		$R_j = 0,\ j \neq 15$
50	30	$[I]$	$R_1 = 20$	$[II]$	$R_{16} = 20$	$[III]$	$R_{30} = 20$
			$R_i = 0,\ i \neq 1$		$R_i = 0,\ i \neq 16$		$R_i = 0,\ i \neq 30$
60	35		$R_1 = 25$		$R_{18} = 25$		$R_{36} = 25$
			$R_j = 0,\ j \neq 1$		$R_j = 0,\ j \neq 18$		$R_j = 0,\ j \neq 35$
100	60	$[I]$	$R_1 = 40$	$[II]$	$R_{31} = 40$	$[III]$	$R_{60} = 40$
			$R_i = 0,\ i \neq 1$		$R_i = 0,\ i \neq 31$		$R_i = 0,\ i \neq 60$
120	72		$R_1 = 48$		$R_{37} = 48$		$R_{72} = 48$
			$R_j = 0,\ j \neq 1$		$R_j = 0,\ j \neq 37$		$R_j = 0,\ j \neq 72$

Table 3 : $MSEs$ for the estimates of the parameters θ, β and ρ with $(\theta = 0.3,\ \beta = 1.3$ and $\rho = 1.1)$ and values of the prior parameters $(\alpha = 1.2,\ \lambda = 0.9,\ \gamma = 1.1,\ \varepsilon = 0.95$ and $\varsigma = 0.65)$ with $n_1 = n_2 = n$ and $m_1 = m_2 = m$.

(n,m)	SCs	MLE	$Bayes-Lindley$		$Bayes-MCMC$	
		$\hat{\theta}$	Prior 1 $(\hat{\theta})$	Prior 2$(\hat{\theta})$	Prior 1 $(\hat{\theta})$	Prior 2 $(\hat{\theta})$
		β	Prior 1 (β)	Prior 2(β)	Prior 1 (β)	Prior 2 (β)
		$\hat{\rho}$	Prior 1$(\hat{\rho})$	Prior 2 $(\hat{\rho})$	Prior 1 $(\hat{\rho})$	Prior 2 $(\hat{\rho})$
(15,9)	[I]	0.3503	1.8820	0.0645	0.0366	0.0581
		1.0119	1.0719	1.3699	0.7216	0.3056
		0.6145	0.8615	2.296	0.3327	0.5514
	[II]	0.2725	0.4788	0.1769	0.0383	0.1299
		1.4050	2.1199	2.6421	0.9041	0.8286
		0.8370	1.3384	2.7432	0.4443	0.8038
	[III]	0.2960	0.3598	0.0834	0.0576	0.0612
		1.6753	2.4065	1.7393	0.8811	0.3067
		0.6805	0.3502	0.667	0.3468	0.3681
(15,15)		0.2304	0.1136	0.0549	0.0286	0.0474
		0.6569	0.4304	0.2822	0.1462	0.166
		0.4435	0.2509	0.1364	0.2021	0.1629
(50,30)	[I]	0.2328	0.3478	0.0636	0.0751	0.0476
		0.6815	2.0567	0.3189	0.4893	0.2621
		0.4510	0.3628	0.6031	0.2675	0.2087
	[II]	0.2695	0.1484	0.0506	0.0436	0.0490
		0.9141	1.5982	0.3849	0.7347	0.2964
		0.5279	0.3754	0.6311	0.2304	0.2207
	[II]	0.2483	2.9312	2.4969	0.0200	0.0438
		1.2680	1.2977	1.8432	0.7848	0.2818
		0.5789	0.4474	0.7902	0.2264	0.2566
(50,50)		0.1737	0.0703	0.0278	0.0171	0.0277
		0.5097	0.2871	0.1708	0.1174	0.1522
		0.2598	0.1005	0.1277	0.0987	0.0824
(100,60)	[I]	0.1968	0.0952	0.0474	0.0298	0.0295
		0.5603	0.4778	0.2097	0.6591	0.239
		0.2907	0.1769	0.2511	0.1654	0.1205
	[II]	0.2178	0.0708	0.0455	0.0703	0.0432
		0.6756	1.3560	1.356	0.6631	0.2529
		0.3644	0.2349	0.2594	0.1718	0.1344
	[III]	0.1971	0.0993	0.0664	0.0405	0.0354
		0.8645	1.0662	1.0137	0.8692	0.2762
		0.3775	0.1809	0.2617	0.1495	0.1328
(100,100)		0.1474	0.058	0.0244	0.0191	0.0209
		0.3790	0.3166	0.1094	0.0831	0.0728
		0.2117	0.0823	0.0578	0.0482	0.0418

Table 4 : $MSEs$ for the estimates of the parameters θ, β and ρ with $(\theta = 0.3,\ \beta = 1.3$and $\rho = 1.1)$ and values of the prior parameters $(\alpha = 1.2,\ \lambda = 1.3,\ \gamma = 1.1,\ \varepsilon = 0.95$ and $\varsigma = 0.65)$ when $n_1 \neq n_2$ and $m_1 \neq m_2$.

(n_1,n_2) (m_1,m_2)	SCs	MLE $\hat{\theta}$ β $\hat{\rho}$	$Bayes-Lindley$ Prior 1 $(\hat{\theta})$ Prior 1 (β) Prior 1 $(\hat{\rho})$	Prior 2$(\hat{\theta})$ Prior 2(β) Prior 2$(\hat{\rho})$	$Bayes-MCMC$ Prior 1 $(\hat{\theta})$ Prior 2 (β) Prior 1 $(\hat{\rho})$	Prior 2$(\hat{\theta})$ Prior 2(β) Prior 2$(\hat{\rho})$
(20,25)	[I]	0.3388	2.4238	0.8459	0.0205	0.2376
(12,15)		4.2571	8.456	8.674	0.853	0.1978
		1.6623	3.5058	6.2763	0.5819	0.6217
	[II]	0.4269	5.5308	3.6111	0.0126	0.1831
		1.18608	5.0631	8.5511	0.8156	0.1658
		1.7185	3.5849	6.5872	0.5208	0.6808
	[III]	0.4761	5.5895	0.9425	0.0084	0.1091
		1.8669	7.9311	3.408	0.8121	0.1212
		1.0224	2.8433	2.002	0.3369	0.5116
		0.3295	1.1789	0.1395	0.0333	0.0722
(20,25)		0.8518	4.5531	5.5982	0.6459	0.1821
		0.5425	0.3663	0.9138	0.2553	0.1784
(50,60)	[I]	0.3452	0.0671	0.0713	0.0707	0.0905
(30,35)		0.9857	8.7653	7.3394	0.8191	0.2673
		0.5771	0.5464	0.946	0.3337	0.3051
	[II]	0.3852	0.1607	0.056	0.0181	0.0542
		1.0432	4.7532	5.6799	0.7564	0.3001
		0.6648	0.7115	1.2506	0.3688	0.4013
	[III]	0.3570	1.3951	1.2646	0.0121	0.0411
		1.3657	5.2120	4.9311	0.8593	0.2837
		0.7434	0.6586	1.0144	0.3026	0.4003
		0.2050	0.044	0.0399	0.0768	0.0373
(50,60)		0.4538	0.1374	0.1485	0.4283	0.1443
		0.3196	0.1227	0.1637	0.1355	0.0941
(100,120)	[I]	0.2386	0.0551	0.0535	0.0597	0.0493
(60,72)		0.6045	0.1491	0.2242	0.5285	0.2116
		0.4009	0.2193	0.3185	0.1989	0.1508
	[II]	0.2197	0.0597	0.0455	0.0492	0.0414
		0.6349	0.2489	0.2222	0.6001	0.2197
		0.4001	0.2146	0.3071	0.1865	0.147
	[III]	0.2023	0.2195	0.2211	0.0271	0.0284
		0.9256	2.6236	2.6698	0.8394	0.3069
		0.4141	0.2067	0.2947	0.1457	0.1361
		0.1562	0.0211	0.0235	0.1031	0.0234
(100,120)		0.3196	0.0742	0.0866	0.3634	0.0883
		0.2214	0.0559	0.0671	0.0851	0.0479

5 Concluding Remarks

From the simulation study, we can conclude that:
1. Table 3 and Table 4 show that for small samples (< 30), MLE is better than Lindley's approximation while for the samples (≥ 30) Lindley's approximation is better than MLE. Also, MCMC is the best in both two types of samples, except for few cases. This may be due to fluctuations in data.
2. As the sample sizes n_1 and n_2 increase, the MSEs for estimating the parameters $(\theta,\ \beta, \rho)$ decrease in the cases of MLE and BE.
3. For the sample sizes n_1 and n_2 and with increasing the censoring sizes m_1 and m_2, the MSEs for estimating the parameters $(\theta,\ \beta, \rho)$ decrease.
Based on the above remarks, one can say that large sample sizes with large censored samples give better estimates, in the sense of having smaller MSE.

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