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An Improvement in Variance Estimation Using Information on Median and Coefficient of Kurtosis of an Auxiliary Variable

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Abstract: The objective of the present work is to develop an efficient estimation procedure to enhance the precision of estimate of a finite population variance under simple random sampling without replacement (SRSWOR) scheme. Utilizing information on median and coefficient of kurtosis of an auxiliary variable, an improved ratio-type estimator of population variance has been suggested. The properties of the proposed estimation procedure have been examined and empirical studies are performed to show the dominance over some contemporary estimators. Suitable recommendations are made to the survey practitioners.

Keywords: Study variable, auxiliary variable, variance estimation, median, coefficient of kurtosis, bias, mean square error.

1 Introduction

The estimation of finite population variance attracts the attention of survey practioners for many practical applications. For example, a physician requires sufficient knowledge about different pathological parameters of human body such as variations in degree of blood pressure, pulse rate, blood sugar level to provide adequate prescriptions to the patients. Similarly farmers need adequate information regarding the patterns of variations in various weather parameters for cultivation of different crops. The use of auxiliary information has been a popular technique for enhancing the precision of estimates for long time and its application in estimation of finite population variance goes back to [3] and followed by [15], [4], [13], [1], and [2] among others.

Several authors have introduced the modified versions of ratio, product and linear regression methods of estimation for finite population variance by utilizing the information on auxiliary variable. Singh et al. [12] utilized known coefficient of kurtosis of auxiliary variable to estimate the population variance of the study variable. In follow up Upadhyaya and Singh [19,20] , Kadilar and Cingi [5,6], Khan and Shabbir [7] and Subramani and Kumarapandeyan [16,17,18] proposed the modified estimation procedures of population variance which made the use of various known parameters of auxiliary variable. Motivated with the above works, the aim of the present work is to propose an improved estimation procedure for estimation of finite population variance by using information on known population median and coefficient of kurtosis of an auxiliary variable under SRSWOR scheme. Properties of the suggested estimation procedure are deeply examined and supplemented with empirical studies.

2 Description of notations and some existing estimators of population variance

Let y_i and x_i be the values of study variable y and auxiliary variable x respectively for the i^{th} unit of a finite population of size N. To estimate the population variance ${S_y}^2$ of study variable y, a random sample of size n under without replacement scheme is drawn from the population and surveyed for the study variable y under the assumption that the information on auxiliary variable x readily available for all the units of the population. The following notations have been adopted for the further use:

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$\bar{X}(\bar{Y})$: Population means of auxiliary (study) variable $x(y)$.
$\bar{x}(\bar{y})$: Sample means of auxiliary (study) variable $x(y)$.
$s_x^2(s_y^2)$: Sample variance of the variable $x(y)$.
C_y, C_x : Population coefficients of variation for the variables shown in subscripts.
ρ_{yx} : Population correlation coefficient between the variables y and x.
B (.) : Bias of the estimator.
M (.) : Mean square error (MSE) of the estimator.
$f = \frac{n}{N}$: Sampling Fraction.
$\mu_{st} = \frac{1}{(N-1)}\sum\limits_{i=1}^{N}(y_i-\bar{Y})^s(x_i-\bar{X})^t$,$\delta_{st} = \frac{\mu_{st}}{\mu_{20}^{s/2}\mu_{02}^{t/2}}$ and $\zeta = \left(\frac{1}{n}-\frac{1}{N}\right)$
$\kappa_{(x)} = \frac{\mu_{04}}{\mu_{02}^2}$: Population coefficient of kurtosis of the auxiliary variable.
$\kappa_{(y)} = \frac{\mu_{40}}{\mu_{20}^2}$: Coefficient of kurtosis of the study variable.

M_d : Population median of the auxiliary variable.

$Q_i\,(i = 1,2,3)$ denote the first, second and third quartiles of the auxiliary variable. Q_r, Q_d and Q_a are the functions of quartile defined as

$Q_r = (Q_3 - Q_1),\ Q_d = \frac{(Q_3-Q_1)}{2}$

$Q_r = (Q_3 - Q_1),\ Q_d = \frac{(Q_3-Q_1)}{2}$ and $Q_a = \frac{(Q_3+Q_1)}{2}$

For sample observations

$$\bar{y} = (n)^{-1}\sum_{i=1}^{n} y_i,\ \bar{x} = (n)^{-1}\sum_{i=1}^{n} x_i,\ s_x^2 = (n-1)^{-1}\sum_{i=1}^{n}(x_i-\bar{x})^2,$$

$$s_y^2 = (n-1)^{-1}\sum_{i=1}^{n}(y_i-\bar{y})^2,\quad s_{yx} = (n-1)^{-1}\sum_{i=1}^{n}(y_i-\bar{y})(x_i-\bar{x})$$

For population observations

$$\bar{Y} = (N)^{-1}\sum_{i=1}^{N} Y_i,\ \bar{X} = (N)^{-1}\sum_{i=1}^{N} X_i,\ S_x^2 = (N-1)^{-1}\sum_{i=1}^{N}\left(x_i-\bar{X}\right)^2, C_x = (S_x/\bar{X})$$

$$S_y^2 = (N-1)^{-1}\sum_{i=1}^{N}\left(y_i-\bar{Y}\right)^2,\ S_{xy} = (N-1)^{-1}\sum_{i=1}^{N}\left(y_i-\bar{Y}\right)\left(x_i-\bar{X}\right), C_y = (S_y/\bar{Y})$$

2.1 Some existing estimators of population variance

In this section, we have revisited to some existing estimators of the population variance ${S_y}^2$, which will provide the strong basis for proposition of an estimator. The estimators and their expression of the bias and mean square errors are shown below in Table 1. The usual unbiased estimator of population variance and the expression of their variance is presented as

$$d_0 = {s_y}^2 \tag{1}$$

and

$$V(d_0) = \zeta S_y^4(\kappa_{(y)} - 1) \tag{2}$$

Table 1: Existing estimators and their bias and mean square errors

Estimator and Sources	Bias(d_j)	MSE(d_j)
$d_1 = s_y^2(S_x^2/s_x^2)$ [4]	$\zeta S_y^2\left[L_1(\kappa_{(x)}-1)(L_1-\psi)\right]$	$\zeta S_y^4\left[\begin{matrix}(\kappa_{(y)}-1)\\+L_1(\kappa_{(x)}-1)(L_1-2\psi)\end{matrix}\right]$
$d_2 = s_y^2\left[(S_x^2+\kappa_{(x)})\big/(s_x^2+\kappa_{(x)})\right]$ [20]	$\zeta S_y^2\left[L_2(\kappa_{(x)}-1)(L_2-\psi)\right]$	$\zeta S_y^4\left[\begin{matrix}(\kappa_{(y)}-1)\\+L_2(\kappa_{(x)}-1)(L_2-2\psi)\end{matrix}\right]$
$d_3 = s_y^2\left[(S_x^2-C_x)/(s_x^2-C_x)\right]$ [5]	$\zeta S_y^2\left[L_3(\kappa_{(x)}-1)(L_3-\psi)\right]$	$\zeta S_y^4\left[\begin{matrix}(\kappa_{(y)}-1)\\+L_3(\kappa_{(x)}-1)(L_3-2\psi)\end{matrix}\right]$
$d_4 = s_y^2\left[(S_x^2-\kappa_{(x)})\big/(s_x^2-\kappa_{(x)})\right]$ [5]	$\zeta S_y^2\left[L_4(\kappa_{(x)}-1)(L_4-\psi)\right]$	$\zeta S_y^4\left[\begin{matrix}(\kappa_{(y)}-1)\\+L_4(\kappa_{(x)}-1)(L_4-2\psi)\end{matrix}\right]$
$d_5 = s_y^2\left[(S_x^2\kappa_{(x)}-C_x)\big/(s_x^2\kappa_{(x)}-C_x)\right]$ [5]	$\zeta S_y^2\left[L_5(\kappa_{(x)}-1)(L_5-\psi)\right]$	$\zeta S_y^4\left[\begin{matrix}(\kappa_{(y)}-1)\\+L_5(\kappa_{(x)}-1)(L_5-2\psi)\end{matrix}\right]$
$d_6 = s_y^2\left[(S_x^2C_x-\kappa_{(x)})\big/(s_x^2C_x-\kappa_{(x)})\right]$ [5]	$\zeta S_y^2\left[L_6(\kappa_{(x)}-1)(L_6-\psi)\right]$	$\zeta S_y^4\left[\begin{matrix}(\kappa_{(y)}-1)\\+L_6(\kappa_{(x)}-1)(L_6-2\psi)\end{matrix}\right]$
$d_7 = s_y^2\left[(S_x^2+M_d)/(s_x^2+M_d)\right]$ [16]	$\zeta S_y^2\left[L_7(\kappa_{(x)}-1)(L_7-\psi)\right]$	$\zeta S_y^4\left[\begin{matrix}(\kappa_{(y)}-1)\\+L_7(\kappa_{(x)}-1)(L_7-2\psi)\end{matrix}\right]$
$d_8 = s_y^2\left[(S_x^2+Q_1)/(s_x^2+Q_1)\right]$ [17]	$\zeta S_y^2\left[L_8(\kappa_{(x)}-1)(L_8-\psi)\right]$	$\zeta S_y^4\left[\begin{matrix}(\kappa_{(y)}-1)\\+L_8(\kappa_{(x)}-1)(L_8-2\psi)\end{matrix}\right]$
$d_9 = s_y^2\left[(S_x^2+Q_3)/(s_x^2+Q_3)\right]$ [17]	$\zeta S_y^2\left[L_9(\kappa_{(x)}-1)(L_9-\psi)\right]$	$\zeta S_y^4\left[\begin{matrix}(\kappa_{(y)}-1)\\+L_9(\kappa_{(x)}-1)(L_9-2\psi)\end{matrix}\right]$
$d_{10} = s_y^2\left[(S_x^2+Q_r)/(s_x^2+Q_r)\right]$ [17]	$\zeta S_y^2\left[L_{10}(\kappa_{(x)}-1)(L_{10}-\psi)\right]$	$\zeta S_y^4\left[\begin{matrix}(\kappa_{(y)}-1)\\+L_{10}(\kappa_{(x)}-1)(L_{10}-2\psi)\end{matrix}\right]$
$d_{11} = s_y^2\left[(S_x^2+Q_d)/(s_x^2+Q_d)\right]$ [17]	$\zeta S_y^2\left[L_{11}(\kappa_{(x)}-1)(L_{11}-\psi)\right]$	$\zeta S_y^4\left[\begin{matrix}(\kappa_{(y)}-1)\\+L_{11}(\kappa_{(x)}-1)(L_{11}-2\psi)\end{matrix}\right]$
$d_{12} = s_y^2\left[(S_x^2+Q_a)/(s_x^2+Q_a)\right]$ [17]	$\zeta S_y^2\left[L_{12}(\kappa_{(x)}-1)(L_{12}-\psi)\right]$	$\zeta S_y^4\left[\begin{matrix}(\kappa_{(y)}-1)\\+L_{12}(\kappa_{(x)}-1)(L_{12}-2\psi)\end{matrix}\right]$
$d_{13} = s_y^2\left[(S_x^2\rho+Q_3)/(s_x^2\rho+Q_3)\right]$ [7]	$\zeta S_y^2\left[L_{13}(\kappa_{(x)}-1)(L_{13}-\psi)\right]$	$\zeta S_y^4\left[\begin{matrix}(\kappa_{(y)}-1)\\+L_{13}(\kappa_{(x)}-1)(L_{13}-2\psi)\end{matrix}\right]$
$d_{14} = s_y^2\left[(S_x^2C_x+M_d)/(s_x^2C_x+M_d)\right]$ [18]	$\zeta S_y^2\left[L_{14}(\kappa_{(x)}-1)(L_{14}-\psi)\right]$	$\zeta S_y^4\left[\begin{matrix}(\kappa_{(y)}-1)\\+L_{14}(\kappa_{(x)}-1)(L_{14}-2\psi)\end{matrix}\right]$

where

$L_1 = 1, L_2 = \frac{S_x^2}{s_x^2+\kappa_{(x)}}, L_3 = \frac{S_x^2}{s_x^2-C_x}, L_4 = \frac{S_x^2}{s_x^2-\kappa_{(x)}}, L_5 = \frac{S_x^2\kappa_{(x)}}{s_x^2\kappa_{(x)}-C_x}, L_6 = \frac{S_x^2C_x}{s_x^2C_x-\kappa_{(x)}}, L_7 = \frac{S_x^2}{s_x^2+M_d}, L_8 = \frac{S_x^2}{s_x^2+Q_1},$
$L_9 = \frac{S_x^2}{s_x^2+Q_3}, L_{10} = \frac{S_x^2}{s_x^2+Q_r}, L_{11} = \frac{S_x^2}{s_x^2+Q_d}, L_{12} = \frac{S_x^2\rho}{s_x^2\rho+Q_a}, L_{13} = \frac{S_x^2\rho}{s_x^2\rho+Q_3}, L_{14} = \frac{S_x^2C_x}{s_x^2C_x+M_d}$ and $\psi = \frac{\delta_{22}-1}{\kappa_{(x)}-1}$

In general the MSEs of the estimators shown in Table 1, may be written as

$$\mathrm{MSE}(d_j) = \zeta S_y^4\left[(\kappa_{(y)}-1)+L_j(\kappa_{(x)}-1)(L_j-2\psi)\right]; (j = 1,2,....,14) \tag{3}$$

3 Proposition of the estimator and its bias and mean square error

Following the previously discussed estimation procedures, we propose a general class of estimators of population variance $S_y{}^2$which utilizes the readily available information on median and coefficient of kurtosis of an auxiliary variable x and defined as

$$d_{PS} = s_y^2\left\{\lambda+(1-\lambda)\frac{(S_x^2\kappa_{(x)}+M_d^2)}{(s_x^2\kappa_{(x)}+M_d^2)}\right\} \tag{4}$$

where λ is a scalar quantity to be determined under certain criterions so as to minimise the MSE of the estimator d_{PS}. To obtain the bias and mean square error of the proposed estimator d_{PS} up to the first order of approximations, we use the following transformations:

$s_y^2 = S_y^2(1+e_1), s_x^2 = S_x^2(1+e_2)$ and $E(e_i) = 0$ for $(i = 1,2)$.

We use the following expected values for further derivations:

$E(e_1^2) = \zeta(\kappa_{(y)}-1), E(e_2^2) = \zeta(\kappa_{(x)}-1)$ and $E(e_1e_2) = \zeta(\delta_{22}-1)$

To derive the expression of bias and mean square error of the proposed class of estimators d_{PS}, the structure of estimator is expressed in terms of e_i's as

$$d_{PS} = S_y^2(1+e_1)\left\{\lambda+(1-\lambda)\frac{\kappa_{(x)}S_x^2+M_d^2}{\kappa_{(x)}S_x^2(1+e_2)+M_d^2}\right\}$$

$$= S_y^2(1+e_1)\left[\lambda+(1-\lambda)(1+L^*e_2)^{-1}\right] \tag{5}$$

where $L^* = \frac{S_x^2\kappa_{(x)}}{s_x^2\kappa_{(x)}+M_d^2}$ and $|L^*e_1| < 1$ so that the term $(1+L^*e_2)^{-1}$ is convergent.

Now expanding the expression in equation (5) binomially and simplifying we have

$$d_{PS} = S_y^2\left(1+e_1-L^*(1-\lambda)e_2-L^*(1-\lambda)e_1e_2+L^{*2}(1-\lambda)e_2^2+L^*(1-\lambda)e_1e_2^2-.....\right) \tag{6}$$

Taking expectation and retaining the terms up to the order n^{-1}, we have the expression of bias and mean square error of the estimator d_{PS} as

$$\begin{aligned} B(d_{PS}) &= E(d_{PS}-S_y^2) \\ &= (1-\lambda)\zeta S_y^2(\kappa_{(x)}-1)L^*[L^*-\psi] \end{aligned} \tag{7}$$

$$\begin{aligned} M(d_{PS}) &= E[d_{PS}-S_y^2]^2 \\ &= S_y^4\zeta\left[(\kappa_{(y)}-1)+(1-\lambda)^2L^{*2}(\kappa_{(x)}-1)-2(1-\lambda)L^*(\delta_{22}-1)\right] \end{aligned} \tag{8}$$

The expression of mean square error of the estimator d_{PS} in equation (8) is consist the unknown scalar λ, hence to obtain the optimum value of λ, the expression of MSE (d_{PS}) is minimized with respect to λ and subsequently the optimum value of λ say λ_{opt} is obtained as

$$\lambda_{opt} = 1-\frac{(\delta_{22}-1)}{L^*(\kappa_{(x)}-1)} \tag{9}$$

Further substituting the value of λ_{opt} in equation (8), we get the optimum MSE (d_{PS}) as

$$MSE_{\min.}(d_{PS}) = S_y^4\zeta\left[(\kappa_{(y)}-1)-\frac{(\delta_{22}-1)^2}{(\kappa_{(x)}-1)}\right] \tag{10}$$

3.1 Practicability of the proposed estimator d_{PS}

In order to make the proposed class of estimators d_{PS} practicable, the unknown scalar λ will be replaced by λ_{opt} and λ_{opt} consist of several unknown parameters such as δ_{22} and S_y^4 . Hence for practical application, these unknown parameters may be replaced by their guess values available from the past surveys or pilot surveys. If such guess values are not readily available then they may be estimated by their corresponding sample estimates, see [8],[9,10],[14] , [11].

4 Theoretical Comparisons

The estimator d_{PS} is more efficient than the existing estimators $d_j (j = 1, 2...., 14)$ under the following conditions which are obtained by comparing their respective variance/MSEs. Using equations (1) and (10), we have the following results

$MSE_{\min.}(d_{ps}) < V(d_0)$, if $(\delta_{22} - 1) > 0$

From equations (3) and (10), we have the following conditions:

$MSE_{\min.}(d_{ps}) < MSE_{\min.}(d_{ps}$, if $(\delta_{22} - 1) > (\kappa_{(x)} - 1)$

5 Empirical studies

To examine the validity of theoretical comparisons of proposed estimator and other available estimators discussed in this work, we borrowed the following numerical values of different population parameters for real population datasets available in [16].

Population I

$$N = 80,\ n = 40,\ \overline{Y} = 51.8264,\ \overline{X} = 11.2646,\ \rho_{xy} = 0.9413,\ S_y = 18.3569,\ C_y = 0.3542,$$

$$S_x = 8.4563,\ C_x = 0.7507,\ \kappa_{(x)} = 2.8664,\ \kappa_{(y)} = 2.2667,\ \delta_{22} = 2.2209,\ M_d = 7.5750,$$

$$Q_1 = 5.1500,\ Q_3 = 16.975,\ Q_r = 11.825,\ Q_d = 5.9125,\ Q_a = 11.0625.$$

Population II

$$N = 70,\ n = 20,\ \overline{Y} = 96.70,\ \overline{X} = 175.2671,\ \rho_{xy} = 0.7293,\ S_y = 160.7140,\ C_y = 0.6279,$$

$$S_x = 140.8572,\ C_x = 0.8037,\ \kappa_{(x)} = 7.0952,\ \kappa_{(y)} = 4.7596,\ \delta_{22} = 4.6038,\ M_d = 121.50,$$

$$Q_1 = 80.1500,\ Q_3 = 225.0250,\ Q_r = 144.8750,\ Q_d = 72.4375,\ Q_a = 152.5875.$$

To assess the performance of the proposed estimator, the bias and mean square errors of all the discussed estimators are calculated and presented in Table 2. Further the percent relative efficiencies (PREs) of proposed estimator and other existing estimators are calculated with respect to the natural sample variance estimator using the following formula and shown in Table 3.

$$PRE(t, s_y^2) = \frac{V(s_y^2)}{MSE(t)} \times 100 \tag{11}$$

where t is the estimator of our interest.

6 Interpretations of results and conclusions

From Table 2 it is observed that the bias and mean square errors of the proposed estimator is less than the bias and mean square error of other existing estimators and subsequently from Table 3 it is visible that the percent relative efficiencies of the proposed estimator are appreciably higher than other discussed estimators for both the population values. From the above interpretations we may conclude that the structure of the proposed estimator is justified and it may be considered for practical applications by the survey practitioners.

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Table 2: Bias B (.) and MSE M (.) of existing and proposed estimators

Estimator and Source	Bias B (.)		MSE M (.)	
	Population I	Population II	Population I	Population II
$d_1 = s_y^2(S_x^2/s_x^2)$ [4]	8.1569	236.1542	2943.7110	924946.50
$d_2 = s_y^2\left[(S_x^2+\kappa_{(x)})\big/(s_x^2+\kappa_{(x)})\right]$ [20]	6.9686	235.8633	2743.6490	924324.40
$d_3 = s_y^2\left[(S_x^2-C_x)/(s_x^2-C_x)\right]$ [5]	8.4963	236.1871	3002.9290	925017.00
$d_4 = s_y^2\left[(S_x^2-\kappa_{(x)})\big/(s_x^2-\kappa_{(x)})\right]$ [5]	9.5235	236.4454	3187.1310	925569.60
$d_5 = s_y^2\left[(S_x^2\kappa_{(x)}-C_x)\big/(s_x^2\kappa_{(x)}-C_x)\right]$ [5]	8.2739	236.1588	2964.0260	924956.40
$d_6 = s_y^2\left[(S_x^2C_x-\kappa_{(x)})\big/(s_x^2C_x-\kappa_{(x)})\right]$ [5]	10.0225	236.5166	3279.0970	925721.90
$d_7 = s_y^2\left[(S_x^2+M_d)/(s_x^2+M_d)\right]$ [16]	5.3329	233.2011	2490.0660	918641.40
$d_8 = s_y^2\left[(S_x^2+Q_1)/(s_x^2+Q_1)\right]$ [17]	6.1309	232.8888	2610.2640	917976.10
$d_9 = s_y^2\left[(S_x^2+Q_3)/(s_x^2+Q_3)\right]$ [17]	2.9355	227.0994	2181.580	905689.90
$d_{10} = s_y^2\left[(S_x^2+Q_r)/(s_x^2+Q_r)\right]$ [17]	4.1277	230.2846	2323.6670	912437.80
$d_{11} = s_y^2\left[(S_x^2+Q_d)/(s_x^2+Q_d)\right]$ [17]	5.8707	233.2011	2570.24	918641.40
$d_{12} = s_y^2\left[(S_x^2+Q_a)/(s_x^2+Q_a)\right]$ [17]	4.3276	229.9762	2349.8550	911783.20
$d_{13} = s_y^2\left[(S_x^2\rho+Q_3)/(s_x^2\rho+Q_3)\right]$ [7]	2.7208	223.8262	2158.9170	898785.40
$d_{14} = s_y^2\left[(S_x^2C_x+M_d)/(s_x^2C_x+M_d)\right]$ [18]	4.5924	232.4854	2385.4380	917116.90
$d_{ps} = s_y^2\left[\lambda+(1-\lambda)\frac{(S_x^2\kappa_{(x)}+M_d^2)}{(s_x^2\kappa_{(x)}+M_d^2)}\right]$	1.5164	107.2051	1993.0700	569127.70

Table 3: PREs of various estimators with respect to $d_0 = s_y^2$

Estimator and Source	PREs	
	Population I	Population II
$d_1 = s_y^2(S_x^2/s_x^2)$ [4]	183.2345	142.0218
$d_2 = s_y^2\left[(S_x^2+\kappa_{(x)})\big/(s_x^2+\kappa_{(x)})\right]$ [20]	196.5956	142.1173
$d_3 = s_y^2\left[(S_x^2-C_x)/(s_x^2-C_x)\right]$ [5]	179.6211	142.0109
$d_4 = s_y^2\left[(S_x^2-\kappa_{(x)})\big/(s_x^2-\kappa_{(x)})\right]$ [5]	169.2398	141.9261
$d_5 = s_y^2\left[(S_x^2\kappa_{(x)}-C_x)\big/(s_x^2\kappa_{(x)}-C_x)\right]$ [5]	181.9786	142.0202
$d_6 = s_y^2\left[(S_x^2C_x-\kappa_{(x)})\big/(s_x^2C_x-\kappa_{(x)})\right]$ [5]	164.4933	141.9028
$d_7 = s_y^2\left[(S_x^2+M_d)/(s_x^2+M_d)\right]$ [16]	216.6165	142.9965
$d_8 = s_y^2\left[(S_x^2+Q_1)/(s_x^2+Q_1)\right]$ [17]	206.6417	143.1002
$d_9 = s_y^2\left[(S_x^2+Q_3)/(s_x^2+Q_3)\right]$ [17]	247.2471	145.0414
$d_{10} = s_y^2\left[(S_x^2+Q_r)/(s_x^2+Q_r)\right]$ [17]	232.1285	143.9687
$d_{11} = s_y^2\left[(S_x^2+Q_d)/(s_x^2+Q_d)\right]$ [17]	209.8595	142.9965
$d_{12} = s_y^2\left[(S_x^2+Q_a)/(s_x^2+Q_a)\right]$ [17]	229.5416	144.0721
$d_{13} = s_y^2\left[(S_x^2\rho+Q_3)/(s_x^2\rho+Q_3)\right]$ [7]	249.8426	146.1556
$d_{14} = s_y^2\left[(S_x^2C_x+M_d)/(s_x^2C_x+M_d)\right]$ [18]	226.1175	143.2342
$d_{ps} = s_y^2\left[\lambda+(1-\lambda)\frac{(S_x^2\kappa_{(x)}+M_d^2)}{(s_x^2\kappa_{(x)}+M_d^2)}\right]$	270.6320	230.8138

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