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Maximum Entropy for Image Segmentation based on an Adaptive Particle Swarm Optimization

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Abstract: Image segmentation is applied widely to image processing and object recognition. Threshold segmentation is a simple and important method in grayscale image segmentation. Information entropy can characterize the grayscale in formation of image and distinguish between the objectives and background. In this paper, we use exponential entropy instead of logarithmic entropy and propose a new multilevel thresholds image segmentation method based on maximum entropy and adaptive Particle Swarm Optimization (APSO). This proposed algorithm takes full account of the spatial information and the gray information to decrease the computing quantity. The APSO takes advantage of the characteristics of particle swarm optimization, through adaptively adjust particles flying speed to improve evolutional process of basic PSO. Standard test images and remote sensing image are segmented in experiment and compared with other related segmentation methods. Experimental results show that the APSO method can quickly converge with high computational efficiency.

Keywords: 2-D threshold, image segmentation, particle swarm optimization, maximum entropy

1 Introduction

Image analysis is based on the extraction of meaningful information. Pre-processing, segmentation and characterization of the identified objects are its attentions [1]. Image segmentation can be understood as the process of assigning a label to every pixel in an image, such that the same object is represented with the same label of pixels .

Image segmentation relates to computer vision technology. Image segmentation methods have been used to find ROI (regions of interest) in images. The importance of image segmentation can be illustrated in medical diagnosis [2] and face recognition [3]. Different algorithms have been proposed for image segmentation, such as histograms of gray levels [4],multi-scale segmentation [5]. Methods of physics concepts have also been applied for image segmentation, such as Markov random fields [6] and entropy [7]. The segmentation approach proposed in the current work is based on the concept of entropy.

In information theory, entropy is used to quantify the amount of information [8,22,23,24]. The entropy reflects the information content of symbols independent of any particular probability model [9,10]. Image analysis takes the concept of entropy in the sense of information theory (Shannon entropy), where entropy is used to quantify the minimum descriptive complexity of a random variable [8]. Because the entropy can provide a good level of information to describe a given image, we can compute the entropy of the distribution of gray levels and obtain an appropriate partition for target image.

The Particle Swarm Optimization (PSO) algorithm was originally introduced in [11] as a new, simple evolutionary algorithm. The idea is originated from the exchange and sharing of information between bird individuals in the process of searching food. In the simulation, the behavior of each individual is affected by either the best local or the best global individual. This makes particles constantly move to the optimal solution, and ultimately move to the global optimal solution.

In the past several years, PSO have been successfully applied in many research and application areas. Now, the PSO technique has been used to solve the problem of threshold segmentation. Zahara et al. in [12] applied the PSO technique to image threshold with multi-modal histograms. In [13], the PSO technique was used to find near-optimal thresholds by minimizing the cross entropy between the original image. Sathya et al. [14] presented a

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histogram-based method for multilevel threshold segmentation based on PSO. Hongmei et al. in [15] proposed a multilevel threshold method segmenting images based on the maximum entropy and an improved PSO. AndreL et al. [16] proposed an entropy-based image segmentation approach to color images.

In this paper, we propose a new multilevel threshold method segmenting images based on adaptive PSO (APSO). The proposed method treats the threshold segmentation problem as an optimization problem. APSO is used to find the best values of thresholds that can give us an appropriate partition for a target image.

2 Exponential Maximum Entropy threshold segmentation

Segmentation is an essential task for many applications in image processing. Maximum entropy threshold segmentation arithmetic based on gray-level change.

Shannon defined the entropy of an n-state system as

$$H = -\sum_{i} p_i ln p_i \; i = 1, 2, \cdots, n. \tag{1}$$

where p_i is the probability of occurrence of the event i and $\sum_i p_i = 1, 0 \leq p_i \leq 1$.

Pun [17] first used Shannon's concept to define the entropy of an image assuming that an image is entirely represented by its grey level histogram only. The entropy of the entire image which grey level is $\{0, 1, \cdots, L-1\}$:

$$H = -\sum_{i=0}^{L-1} p_i ln p_i \; i = 1, 2, \cdots, n. \tag{2}$$

Recently, two probability distributions of the entire image are considered: one for the object and the other for the background. Because One-dimension (1-D) entropy method doesn't take into account the spatial distribution of grey levels, two images with identical histogram but different spatial distributions will give rise to same threshold value. The two-dimension (2-D) histogram entropies are obtained from the 2-D histogram that is determined by using the gray value of the pixel and the local average gray value of the pixel.

Let the average gray level values of each pixel neighborhood be from *0* to *L-1*, as well as the gray level values of each pixel. Then every pixel in original image corresponds to the pair of the pixel gray level and the average gray level of the neighborhood. Let n_{ij} n be the total number of grey level i of the pixel and its neighborhood grey level j. p_{ij} is the probability of the pair of grey level i of the pixel and its neighborhood grey level j, $p_{ij} = \frac{n_{ij}}{N \times N}$,where $N \times N$ is the size of the image.

Fig.1 is the XOY plane of 2-D histogram, A and B regions along the diagonal represent the object and background. C and D regions represent the boundary and noise. Supposing A and B have different probability

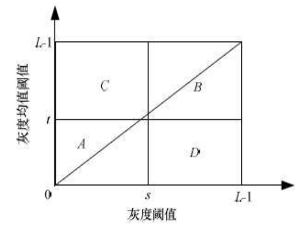

Fig. 1: XOY plane of 2D histogram

distribution, and thus the probability p_{ij} is normalized by prior-probability so that the entropy of every region has additivity. Let *(s, t)* be the threshold vector,

$$P_A = \sum_{i=0}^{s-1} \sum_{j=0}^{t-1} p_{ij}$$

$$P_B = \sum_{i=0}^{L-1} \sum_{j=0}^{L-1} p_{ij}$$

The 2-D entropy of A and B region is

$$H(A) = -\sum_{i=0}^{s-1} \sum_{j=0}^{t-1} \frac{p_{ij}}{P_A} ln \frac{p_{ij}}{P_A} = ln(P_A) + \frac{H_A}{P_A} \tag{3}$$

Similarly

$$H(B) = ln(P_B) + \frac{H_B}{P_B} \tag{4}$$

where

$$H_A = \sum_{i=0}^{s-1} \sum_{j=0}^{t-1} p_{ij} ln p_{ij}$$

$$P_B = \sum_{i=0}^{L-1} \sum_{j=0}^{L-1} p_{ij} ln p_{ij}$$

Because the probability of the information of boundary and noise contained in C and D is small, so it ignores. In the other word is the probability p_{ij} of C and D is *0*. The discriminant function of entropy is

$$H(s,t) = H(A) + H(B) = ln[P_A(1-P_A)] + \frac{H_A}{P_A} + \frac{H_L - H_A}{1-P_A} \tag{5}$$

When H(s,t) is maximized, (s, t) is the best threshold.

In information theory, information quantity is defined as:

$$\triangle I(p_i) = ln \frac{1}{p_i} = -ln p_i \tag{6}$$

where $\sum_{i=0}^{L-1} p_i = 1, (0 \leq p_i \leq 1)$

Nakib et. al. [18] proposed two-dimensional exponential entropy with PSO. The PSO is used for optimization of two-dimensional exponential entropy to solve the segmentation problem. Result show that the exponential entropy method can produce better results than two-dimensional Shannon entropy [19].

exponential entropy overcame the deficiency of the logarithmic information entropy through improving undefined value and zero problems of logarithmic entropy. In addition, because logarithmic computational speed is slow, exponentiation computation can be greatly reduce computational time.

In this paper, we defined information quantity of an event i with probability p_i as:

$$\triangle I(p_i) = e^{(1-p_i)} \tag{7}$$

exponential entropy is:

$$E_H = E(\triangle I) = \sum_{i=0}^{L-1} p_i e^{(1-p_i)} \tag{8}$$

Therefore, exponential entropy of the segmented image is:

$$E_H = \sum_{i=0}^{t} \frac{p_i}{P(t)} e^{(1-\frac{p_i}{P(t)})} + \sum_{i=t+1}^{L-1} \frac{p_i}{1-P(t)} e^{(1-\frac{p_i}{1-P(t)})} \tag{9}$$

optimal threshold of the segmented image is:

$$Th = arg \max_{t \in C_l}(E_H) \tag{10}$$

3 APSO for Image Segmentation

In this section, we introduce the basic PSO (BPSO) and propose a new adaptive PSO for image segmentation.

3.1 Fundamental Principle of Particle Swarm Optimization

PSO is first initialized into a swarm of random particles. By following current optimal particle, all particles search in the solution space until the optimal solution is found.

We define the notation adopted in this paper: assuming that the search space is N-dimensional, the number of particle is n, the i-th particle of the swarm is represented by the N-dimensional vector $X_i = (x_{i1}, x_{i2}, \cdots, x_{iN})$ and the best particle of the swarm, i.e. the particle with the lowest function value, is denoted by index g. The best previous position (i.e. the position giving the best function value) of the i-th particle is recorded and represented by $X_i = (x_{i1}, x_{i2}, \cdots, x_{iN})$, and the position change (velocity) of the i-th particle is $V_i = (v_{i1}, v_{i2}, \cdots, v_{iN})$. The particles are manipulated according to the following equations (the superscripts denote the iteration number):

$$v_{il}^{k+1} = \omega v_{il}^{k} + c_1 \times rand_1() \times (p_{il}^{k} - x_{il}^{k})$$

$$+ c_2 \times rand_2() \times (p_{gl}^{k} - x_{il}^{k}) \tag{11}$$

$$x_{id} = x_{id} + v_{id} \tag{12}$$

where $1 \le i \le m, 1 \le d \le D$, w is the inertia weight, c_1 and c_2 are two positive constants, called the cognitive and social parameter respectively; $rand_1()$ and $rand_2()$ are two random numbers uniformly distributed within the range [0, 1]. Some variants of PSO impose a maximum allowed velocity V_{max} to prevent the swarm from explosion (i.e. if $v_{il}^{k+1} > V_{max}$, then $v_{il}^{k+1} = V_{max}$)[20].

Eq.(11) is used to calculate the i-th particle's new velocity, at each iteration. Three terms are taken into consideration. The first term, v_{il}^{k}, is the particle's previous velocity. The second term, $p_{il}^{k} - x_{il}^{k}$, is the distance between the particle's best previous position, and its current position. Finally, the third term, $p_{gl}^{k} - x_{il}^{k}$, is the distance between the swarm's best experience, and the i-th particle's current position. The parameters c_1, c_2, $rand_1()$ and $rand_2()$, provide randomness that makes the technique less predictable yet more flexible[21].

Eq.(12) provides the new position of the i-th particle, adding its new velocity, to its current position. The inertia weight is employed to control the impact of the previous history of velocities on the current velocity. In this way, the parameter w regulates the trade-off between the global and local exploration abilities of the swarm and influences PSO convergence behavior. A small inertia weight facilitates local exploration, while a large one tends to facilitate global exploration.

3.2 Image Segmentation Method Based On Adaptive Particle Swarm Optimization

This subsection develops a new multilevel threshold method segmenting images using adaptive PSO.

In BPSO algorithms, the particle swarm is initialized randomly in feasible solution space and each particle has initial speed and initial position. The track is updated through individual best position P_{best} and the global best location G_{best} found by the entire population. This makes particles constantly move to the optimal solution, and ultimately move to the global optimal solution.

In practical application, the BPSO algorithm is liable to trap in local optimization and cause premature convergence. In order to overcome the problem of premature convergence and balance the global and local search capability of the algorithm. we propose a new method, named APSO, which uses adaptive flying time. The only difference between a standard particle and an APSO particle is an additional random perturbation for every element of velocity vector v_i in time t as description in Eq.(13). Random perturbation is added according to the difference of particles in different position, which can balance local and globe search scope.

$$x_{id} = x_{id} + rand() \times (1 - k\frac{iter \times ln(t)}{I_{max} \times ln(t+b)}) \times v_{id} \quad (13)$$

where rand() is random number uniformly distributed within the range *[0,1]*. I_{max} is the total number of iterations. *iter* is current iteration size. Constant k balances I_{max} with *iter* and b is a constant. Usually, there is $I_{max} \geq iter$.

APSO tries to find near-optimal solution of thresholds that can give us a near-optimal segmentation. First, the APSO algorithm initializes a random swarm of m particles, where each one has its k thresholds. Second, particles fly to look after the target partition on a search space according to the fitness function Eqs.(5) and (10). The APSO algorithm uses the gray levels $\{g_{min}, \ldots, g_{max}\}$ as a search space, where g_{min} and g_{max} are the minimum and maximum gray levels in a given image respectively.

The steps of image segmentation method based on APSO are described as follow:

Step 1 For each particle, random initialize the population of n, v_0 and p_0.

Step 2 From initial position, produce initial individual optimum P_{best} ;

Step 2.1 Use Eqs.(5) and (10) to calculate fitness and store to the position of the particle;

Step 2.2 Take out the particle with minimal fitness as global optimal G_{best}.

Step 3 Update the velocity and position of next iteration by Eqs.(11) and (13);

Step 3.1 Compute $iter/I_{max}$ and $ln(t)/ln(t+b)$, and then compute v_{id} with given k according to Eq.(13).

Step 3.2 Calculate fitness of all particles, produce current individual optimum P_{best} and current global optimum G_{best}.

Step 4 Calculate the fitness E_H; if E_H is superior to the optimal value of G_{best}, modify G_{best} and corresponding optimal value replace corresponding information.

Step 5: If the terminal condition is fulfilled, output the optimal solution G_{best} and the optimal value, otherwise turn to Step 3.

Fig. 2: Original image of Lena and cameraman

Fig. 3: Original image of Peppers

Fig. 4: Segmented with various methods on Lena test image

4 Experimental Results

In this section, we compare the segmentation performance of the 1-D logarithmic entropy method with that of the 1-D exponential method and the 2-D maximum entropy method based APSO with that of the 2-D maximum entropy method based BPSO.

In order to validate the effectiveness of the proposed method, standard test images Lena, Cameraman and Peppers are selected as experimental data. The segmentation algorithms were developed in MATLAB 2010a on a Pentium IV, 4GMB of RAM, 2.6 GHz computer. In this part test, the parameters of the proposed method were: the size of the swarm was 20, the maximum iterative time was 50, c1 = c2 = 1.6, X_{max} = 0.9, X_{min} = 0.4.

Original Lena and Cameraman images contain 256x256's grayscale image data in Fig. 2. Original Peppers image contains 512x512'sgrayscale image data in Fig. 3. Their segmentation results by the proposed method are shown in Figs. 4 - 6, where (a) is the segmented result with 1-D logarithmic entropy method, (b) the 1-D exponential method, (c) the 2-D maximum entropy method based BPSO, and (d) the 2-D maximum entropy method based APSO, respectively.

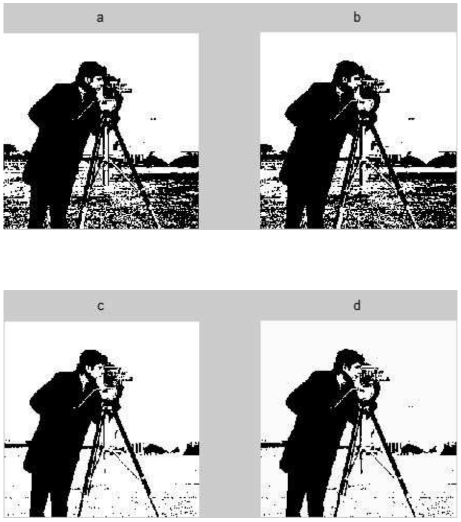

Fig. 5: Segmented with various methods on Cameraman test image

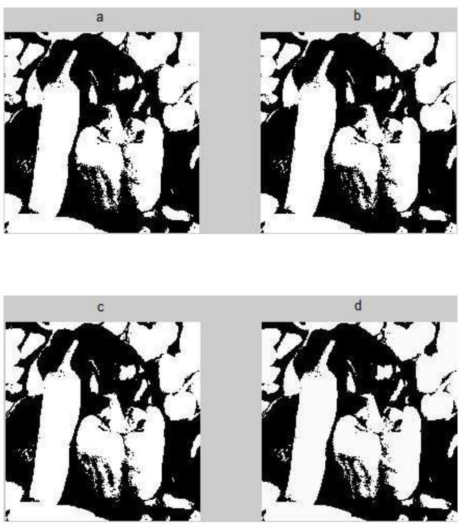

Fig. 6: Segmented with various methods on Peppers test image

The algorithm of APSO starts by segmenting the target image using the gray level ranges of each particle and computing the fitness of each one using the evaluation function Eq.(10).

Because APSO is a stochastic optimization algorithm, we repeated experiments to test the stability of the APSO method. Results and time of the APSO method in 50 repeated experiments are shown in Tables 1 - 3.

The first column of Tables is various algorithms. In the last four columns, the optimal threshold ($Threshold_{opt}$), the maximum entropy ($entropy_{max}$), the average number of the iterations ($Iteration_{avg}$), the average time of the 50 runs of the algorithm ($Time_{avg}$), respectively. The four rows is four algorithms, the 1-D logarithmic entropy method, the 1-D exponential method, the 2-D maximum entropy method based BPSO, the 2-D maximum entropy method based APSO, respectively.

Fig. 7: Original image of Remote Sensing

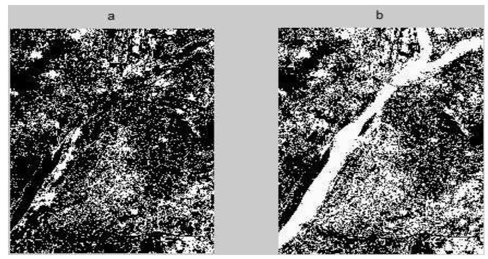

Fig. 8: Segmented with GA and PSO on Remote Sensing image

The speed of the 1-D entropy is quick, but the 1-D entropy method is one of those methods that do not take into account the spatial correlation. The 2-D entropy method makes a fuller use of the original image and yields better results than 1-D entropy method. Because it uses both the spatial gray level distribution and the gray level distribution, it is time-consuming.

APSO is paralleled optimum algorithm and it can shorten the time of looking for threshold, so the speed of multithreshold segmentation is distinctly improved. As is shown in Tables, the proposed method makes up the drawback of 2-D entropy method of being time consuming, and yields satisfactory segmentation results.

Furthermore, in order to verify the efficiency of the proposed method, we choose a remote sensing image in experiment. Image is shown in Fig.7, which contains 512x700's truecolor image data. Image contain river, bridge and plant. We care for the profile of river.

In order to compare the performance of the proposed method on remote sensing image, we carry out independent experiment on APSO and genetic algorithm (GA). The parameters of the proposed method were: the size of the swarm was 20, the maximum iterative time was 100, V_{max} = 3, c1 = c2 =1.49. The parameters of GA were: the size of the population elements was 20, the maximum iterative time was 100, crossover probability was 0.8, mutation probability 0.02. 2-D maximum entropy threshold segmenting result is shown in Fig.8, where (a) is the segmented result with 2-D maximum

Table 1: Comparison of the APSO method and BPSO on Lena test image

Algorithm	$Threshold_{opt}$	$Entropy_{max}$	$Iteration_{avg}$	$Time_{avg}(s)$
APSO-Log	137	14.6577	37	0.43
APSO-Exp	137	14.6577	28	0.23
BPSO-2D	127,127	14.6577	43	0.87
APSO-2D	127,127	14.6577	35	0.65

Table 2: Comparison of the APSO method and BPSO on Cameraman test image

Algorithm	$Threshold_{opt}$	$Entropy_{max}$	$Iteration_{avg}$	$Time_{avg}(s)$
APSO-Log	196	14.2210	43	0.35
APSO-Exp	196	14.2210	26	0.21
BPSO-2D	189,189	14.2210	43	0.88
APSO-2D	189,189	14.2210	32	0.67

Table 3: Comparison of the APSO method and BPSO on Peppers test image

Algorithm	$Threshold_{opt}$	$Entropy_{max}$	$Iteration_{avg}$	$Time_{avg}(s)$
APSO-Log	108	14.4268	45	0.71
APSO-Exp	108	14.4268	32	0.49
BPSO-2D	112,105	14.4268	43	2.63
APSO-2D	112,105	14.4268	33	2.12

entropy method based GA and (b) the 2-D maximum entropy method based APSO, respectively.

We can see in Fig.8(a), that the main features of the image (river), is also well preserved, while Fig.8(b) leads to the hopeless loss of some main features. With human visual perception, our Objective, say river, is clear. It can be seen that the results produced by APSO are better than the results produced by GA.

Experiment has been run 10 times independently. The result is shown in Table 4. The first column of Table 4 is various algorithms. Here *(s,t)* in GA(s),GA(t) and APSO(s), APSO(t) denote the best threshold when *H(s,t)* is maximized (Eq.5). The other is each optimal threshold, the average of threshold in 10 runs and the total time of the 10 runs of the algorithms ($Time_{total}$), respectively.

As can be seen from Table 4 that the APSO method has converged optimal threshold, while optimal threshold of GA is volatile. Moreover, time cost APSO method is less than GA for 10 runs. Therefore, it can be concluded that the APSO method is both fast and good search stability in selecting the optimal threshold adaptively when applied to remote sensing image.

5 Conclusion

Image segmentation is the central task of pattern recognition system. It is a significant step in image processing. Information theory is used widely as an efficient method of Image segmentation based on thresholds. In this work, APSO has been used to produce a new entropy-based image segmentation method. In the APSO method, the algorithm of APSO tries to find a near optimal segmentation for a test image using a fitness function. The segmented results of standard test image and remote sensing image shown the APSO method has good search stability and speed in the repeated experiments. Therefore, maximum entropy image segmentation based on APSO is an efficient segmentation method.

As a future work, the results of our research could be extended to the object with noise or real-time constraints. It would also be an interesting task to consider new simulation experiments with image sequences.

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Table 4: Comparison of the APSO method and GA on Remote Sensing image

Algorithms	1	2	3	4	5	6	7	8	9	10	average	$Time_{total}(s)$
GA(s)	116	126	123	119	114	186	122	190	128	107	133	35
GA(t)	111	113	185	197	147	104	163	96	129	124	137	35
APSO(s)	92	91	94	92	92	92	92	92	93	92	92	21
APSO(t)	121	121	122	121	121	120	121	120	121	121	121	21

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