Journal of Nanotechnology & Advanced Materials
An International Journal

http://dx.doi.org/10.18576/jnam/070101

Two Temperature Effect on a Rotational Thermoelastic Medium with Diffusion due to Three-Phase-Lag Model

Sarhan Y. Atwa [1,*] *and Eman Ibrahim* [2]

[1] Higher Institue of Engineering, Dep. Of Eng. Math. And Physics, Shrouk Academy, Egypt.
[2] Faculty of Science, Mathematics Department, Helwan University, Egypt

Received: 21 Feb. 2019, Revised: 22 Mar. 2019, Accepted: 24 Mar. 2019.
Published online: 1 May 2019.

Abstract: In the present paper, we considered a rotational thermoelastic isotropic homogeneous half space medium with diffusion and two temperature effect in the context of three- phase- lag theory. Normal mode analysis is used to obtain the physical quantities. Comparisons are made graphically to show the effect of rotation, diffusion, and two- temperature under three- phase- lag (TPL) and Green- Nagdi type III (G-N) theories.

Keywords: Three-phase-lag model; diffusion; two temperature; rotation; generalized thermoelasticity.

1 Introduction

Generalized thermoelasticity theories have been developed in order to remove the paradox inherent in the convential coupled dynamical theory of thermoelasticity. There are two important generalized theories of thermoelasticity. The first generalization is due to Lord and Shulman [1] who developed the theory with one relaxation time. The second is due to Müller [2] which known as the theory of thermoelasticity with two relaxation times or the theory of temperature- rate- dependent thermoelasticity. A more explicit version was then introduced by Green and Laws [3] , and Green and Lindsay [4] , this theory contains two consatnts that acts as relaxtion timed and modify all the equations of the coupled theory, not only the heat equation.

Green and Naghdi [5-7] proposed three new thermoelastic theories which are called themoelasticity of type I, type II and type III. Type I is linearization of the classical system of thermoelasticity. Type II theory doesn't admit energy dissipation. Othman et al. [8] investigated the effect of diffusion on two-dimensional problem of generalized thermoelasticity with Green-Naghdi theory. Othman and Atwa [9] studied the thermoelastic plane waves for an elastic solid half- space under hydroelastic initial stress of type III. Othman et al. [10] solved a generalized thermoelasticity problem with temperature dependent properties without energy dissipation in two and three dimensions. Othman et al. [11] discussed the thermal loading effect due to laser pulse on 3-D problem of micropolar thermoelastic solid with energy dissipation.

Chen and Gurtin [12] and Chen et al. [13-14] formulated a theory of heat conduction in deformable bodies, which depends upon two distinct temperature, the conductive temperature ϕ and the thermodynamic temperature T . For time-dependent situations, the difference between these two temperature is proportional to the heat supply, and in the absence of any heat supply, the two temperature are identical. Singh and Bala [15] discussed thee propagation of waves in a two temperature rotating thermoelastic solid half-space without energy dissipation. Atwa [16] has discussed the generalized magneto-themoelasticity with two temperature and initial stress under (G-N) theory. Atwa and Jahangir [17] studied the two temperature effects on plane waves in generalized thermo microstretch elastic solid.

Recently, Roychoudhary [18] developed a new model of the linearized theory of coupled thermoelasticity called three-phase-

*Corresponding author e-mail: srhan_1@yahoo.com

lag by considering the heat conduction law that includes the temperature gradient and thermal displacement gradient among the constitutive variables. Stability were discussed by Quintanilla and Racke [19]. Kumar et al. [20] studied the effect of viscosity on wave propagation in anisotropic thermoelastic medium with three phase-lag model. Othman et al. [21] employed three-phase- lag model to study the deformation of thermoelastic solid half- space under hydrostatic initial stress and rotation with two- temperature. Othman et al. [22] studied the effect of magnetic field on generalized thermoelastic medium with two temperature in the context of TPL model.

Othman et al. [23] studied the effect of rotation on a magneto thermoviscoelastic plane waves in the context of (G-N) theory. Ailawila et al. [24] showed the effect of rotation in a generalized thermoelastic medium with two-temperature under the influence of gravity. Othman et al. [25] investigated the rotation of magneto- thermoelastic solid subjected to thermal loading due to laser pulse. Othman et al. [26] showed the effect of rotation on plane waves in generalized thermo-microstretch elastic solid for a mode- I crack under (G-N). Othman et al. [27] studied the effect of gravity on the general model of the equations of generalized thermo- microstretch for a homogeneous isotropic elastic half- space rotating solid whose surface is subjected to a mode- I crack. Othman and Atwa [28] solved a 2- D problem of anisotropic rotating thermoelastic half- space under (G-N) theory.

Diffusion is the process which atoms move in a material from regions of high concentration to a region of lower concentration. It can be defined as the mas flow process in which atoms change their positions relative to neighbors in agiven phase under the influence of thermal and agradient. Nowaki [29-31] used the coupled thermoelastic model to develop the theory of thermoelastic diffusion. Atwa [32] showed the effect of fractional parameter on plane waves of generalized thermoelastic diffusion with temperature-dependent elastic medium. Tripathi et al. [33] discussed the generalized thermoelastic diffusion problem in a thick circular plate with axisymmetric heat supply. Kumar [34] studied the wave propagation in microstretch thermoelastic diffusion solid.

Othman and Song [35] showed the effect of rotation on plane waves of the generalized electro magneto- thermo- viscoelasticity with two relaxation times. Othman, and Said [36] studied the effect of rotation on two- dimensional problem of a fiber-reinforced thermoelastic with one relaxation time. Zakaria [37] discussd the effect of hall current and rotation on magneto-micropolar generalized thermoelasticity due to ramp- type heating. Elmakizi and Othman [38] studied the effect of rotation on a thermoelastic diffusion with temperature dependent elastic moduli comparison of different theories. Othman et. al. [39] discused the effect of rotation on micro-polar generalized thermoelasticity with two temperature using dual- phase- lag model.

In this paper, the effect of diffusion and two- temperature on a rotational themoelastic material is discussed. Normal mode analysis was used in order to obtain the exact solution. Comparisons are made graphically in the presence and absence of rotation, diffusion, and two-temperature.

Nomenclature

σ_{ij} Components of stress tensor

e_{ij} Components of strain tensor

$e = e_{kk}$ Cubic dilatation

δ_{ij} Kronecker's delta

u, v Displacement vectors

T thermodynamic Temperature

T_0 reference Temperature $\left|(T - T_0)/T_0\right| < 1$

φ conductive temperature

P Chemical potential

C Concentration distribution

λ, μ Lame's constants

$\gamma = (3\lambda + 2\mu)\alpha_t$, α_t −Coefficient of linear thermal expansion

$\beta_1 = (3\lambda + 2\mu)\alpha_c$, α_c −Coefficient of diffusion thermal expansion

ρ Density

C_E Specific heat at constant strain

K Coefficient of thermal conductivity

K^* material characteristic of the theory

τ_v phase lag of thermal displacement gradient

τ_q phase lag of Heat flux

τ_T phase lag of temperature gradient

a Coefficient describing the measure of thermoelastic diffusion effects

b_1 Coefficient describing the measure of diffusive effects

d Thermoelastic diffusion constant

τ Diffusion relaxation time

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

2 Basic equations

We consider a homogenous thermoelastic half-space with two temperature rotating uniformly with angular velocity $\underline{\Omega} = \Omega\,\underline{n}$, where $\underline{n}$ is a unit vector representing the direction of the axis of rotation. Thus, all quantities are considered are functions of x, y and t.

The constitutive equations

$$\sigma_{ij} = 2\mu e_{ij} + \delta_{ij}[\lambda e - \gamma(T - T_0) - \beta_1 C], \tag{1}$$

$$P = -\beta_1 e + b_1 C - a(T - T_0), \tag{2}$$

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}). \tag{3}$$

The equation of motion

$$\rho[\ddot{u}_i + \{\underline{\Omega} \times (\underline{\Omega} \times \underline{u})\}_i + 2(\underline{\Omega} \times \underline{\dot{u}})_i] = \sigma_{ij,j}. \tag{4}$$

The equation of mass diffusion

$$d\,\beta_1 e_{kk,ii} + daT_{,ii} + \dot{C} + \tau\ddot{C} - db_1 C_{,ii} = 0. \tag{5}$$

The equation of heat conduction

$$K^*\nabla^2\varphi + \tau_v^*\nabla^2\dot{\varphi} + K\tau_t\nabla^2\ddot{\varphi} = (1 + \tau_q\frac{\partial}{\partial t} + \frac{\tau_q^2}{2!}\frac{\partial^2}{\partial t^2})[\rho c_E\ddot{T} + \gamma T_0\ddot{e} + aT_0\ddot{C}], \tag{6}$$

Where, $\tau_v^* = (K + K^*\tau_v)$.

The equation of two temperature

$$T = (1 - b\nabla^2)\varphi. \tag{7}$$

Where, the list of symbols is given in the nomenclature.

3 Formulation of the problem

Eq.(1) can be written as

$$\sigma_{xx} = (\lambda + 2\mu)\frac{\partial u}{\partial x} + \lambda\frac{\partial v}{\partial y} - \gamma T - \beta_1 C, \tag{8}$$

$$\sigma_{yy} = (\lambda + 2\mu)\frac{\partial v}{\partial y} + \lambda\frac{\partial u}{\partial x} - \gamma T - \beta_1 C, \tag{9}$$

$$\sigma_{xy} = \mu(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}). \tag{10}$$

For a two dimensional problem in xy-plane, Eqs. (4) - (6) can be written as

$$\rho(\ddot{u} - \Omega^2 u - 2\Omega\dot{v}) = (\lambda + \mu)e_{,x} + \mu\nabla^2 u - \gamma(1 - b\nabla^2)\varphi_{,x} - \beta_1 C_{,x}, \tag{11}$$

$$\rho(\ddot{v} - \Omega^2 v + 2\Omega\dot{u}) = (\lambda + \mu)e_{,y} + \mu\nabla^2 v - \gamma(1 - b\nabla^2)\varphi_{,y} - \beta_1 C_{,y}, \tag{12}$$

$$d\,\beta_1\nabla^2 e + da(1-b\nabla^2)\nabla^2\varphi + \dot{C} + \tau\ddot{C} - db_1\nabla^2 C = 0, \tag{13}$$

$$K^*\nabla^2\varphi + \tau_v^*\nabla^2\dot{\varphi} + K\tau_t\nabla^2\ddot{\varphi} = (1+\tau_q\frac{\partial}{\partial t} + \frac{\tau_q^2}{2!}\frac{\partial^2}{\partial t^2})[\rho c_E(1-b\nabla^2)\ddot{\varphi} + \gamma T_0\ddot{e} + aT_0\ddot{C}]. \tag{14}$$

To facilitate the solution, the following dimensionless quantities are introduced

$$(x',y') = \frac{\omega^*}{c_o}(x,y),\quad (u',v') = \frac{\rho c_o\omega^*}{\gamma T_o}(u,v),\quad \{t',\tau',\tau_T',\tau_q',\tau_v'\} = \omega^*\{t,\tau,\tau_T,\tau_q,\tau_v\},\quad P' = \frac{P}{\beta_1},\ C' = \frac{C}{\rho},$$

$$\Omega' = \frac{\Omega}{\omega^*},\quad b' = \frac{c_o^2}{\omega^{*2}}b\quad \sigma'_{ij} = \frac{\sigma_{ij}}{\lambda+2\mu},\quad (T',\varphi') = \frac{\gamma}{\lambda+2\mu}(T,\varphi),$$

Where $\omega^* = \dfrac{\rho C_E c_o^2}{K}$ and $c_o^2 = \dfrac{\lambda+2\mu}{\rho}$.

Using the above non-dimensional quantities, Eqs. (11) - (14) will be written as

$$\ddot{u} - \Omega^2 u - 2\Omega\dot{v} = a_1 e_{,x} + a_2\nabla^2 u - a_3(1-b\nabla^2)\varphi_{,x} - a_4 C_{,x}, \tag{15}$$

$$\ddot{v} - \Omega^2 v + 2\Omega\dot{u} = a_1 e_{,y} + a_2\nabla^2 v - a_3(1-b\nabla^2)\varphi_{,y} - a_4 C_{,y}, \tag{16}$$

$$a_5\nabla^2 e + a_6(1-b\nabla^2)\nabla^2\varphi + \dot{C} + \tau\ddot{C} - a_7\nabla^2 C = 0, \tag{17}$$

$$\nabla^2\varphi + a_8\nabla^2\dot{\varphi} + a_9\nabla^2\ddot{\varphi} = (1+\tau_q\frac{\partial}{\partial t} + \frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2})[a_{10}(1-b\nabla^2)\ddot{\varphi} + a_{11}\ddot{e} + a_{12}\ddot{C}]. \tag{18}$$

Where

$$a_1 = \frac{\lambda+\mu}{\rho c_o^2},\ a_2 = \frac{\mu}{\rho c_o^2},\ a_3 = \frac{\lambda+2\mu}{\gamma T_o}, a_4 = \frac{\beta_1\rho}{\gamma T_o}, a_5 = \frac{d\beta_1\gamma T_o\omega^*}{\rho^2 c_o^4},\ a_6 = \frac{da\omega^*}{\gamma},\ a_7 = \frac{db_1\omega^*}{c_o^2},\ a_8 = \frac{K\omega^*}{K^*} + \tau_v,$$

$$a_9 = \frac{K\tau_T\omega^*}{K^*},\ a_{10} = \frac{\rho c_o^2 c_E}{K^*},\ a_{11} = \frac{\gamma^3 T_o^2}{\rho K^*(\lambda+2\mu)}, a_{12} = \frac{aT_o\gamma}{K^*}.$$

The displacement components $u(x,y,t)$ and $v(x,y,t)$ may be written in terms of potential functions $\varphi(x,y,t)$ and $\psi(x,y,t)$ as

$$u = q_{,x} - \psi_{,y}, v = q_{,y} + \psi_{,x}. \tag{19}$$

4 Normal mode analysis

The solution of the considered physical variables can be decomposed in terms of normal modes in the following form

$$[u,v,e,\varphi,\psi,q,T,C](x,y,t) = [u^*,v^*,e^*,\varphi^*,\psi^*,q^*,T^*,C^*](y)\exp(\omega t + imy), \tag{20}$$

Where ω is the complex time constant (frequency), i is the imaginary unit, m is the wave number in the $y-$direction and φ^*,ψ^*,T^* and C^* are the amplitudes of the functions.

Applying Eqs. (19) and (20) on Eqs. (15) - (18)

$$(D^2 - a_{13})q^* + a_3(-bD^2 + a_{14})\varphi^* - a_4 C^* + 2\Omega\omega\psi^* = 0, \tag{21}$$

$$-2\Omega\omega q^* + (a_2 D^2 - a_{15})\psi^* = 0, \tag{22}$$

$$a_5(D^4 - 2m^2D^2 + m^4)q^* - a_6(bD^4 - a_{16}D^2 + a_{17})\varphi^* + (a_{18} - a_7D^2)C^* = 0, \tag{23}$$

$$a_{20}(D^2 - m^2)q^* + (a_{21}D^2 - a_{22})\varphi^* + a_{23}C^* = 0, \tag{24}$$

where

$$a_{13} = m^2 + \omega^2 - \Omega^2,\ a_{14} = 1 + bm^2, a_{15} = a_2m^2 + \omega^2 - \Omega^2, a_{16} = 1 + 2bm^2,\ a_{17} = m^2(1+bm^2),$$

$$a_{18} = \omega + \tau\omega^2 + a_7m^2,\ a_{19} = 1 + \omega\tau_q + 0.5\omega^2\tau_q^2,\ a_{20} = a_{11}a_{19}\omega^2, a_{21} = 1 + a_8\omega + a_9\omega^2 + b\omega^2 a_{10}a_{19},$$

$$a_{22} = m^2 + a_8\omega m^2 + a_9\omega^2m^2 + a_{10}a_{19}(1+bm^2),\ a_{23} = a_{12}a_{19}\omega^2.$$

Elimimating $C^*, \varphi^*, \psi^*, q^*$ between Eqs. (21)- (24), we get

$$(D^8 - L_1 D^6 + L_2 D^4 - L_3 D^2 + L_4)\{\psi^*, q^*, \varphi^*, C^*\} = 0, \tag{25}$$

where

$A_1 = a_{23} + a_4 a_{20}$, $A_2 = a_{23}a_{13} + m^2 a_4 a_{20}$, $A_3 = a_4 a_{21} - b a_3 a_{23}$, $A_4 = a_4 a_{22} - a_3 a_{14} a_{23}$, $A_5 = a_4 a_5 - a_7$,
$A_6 = a_7 a_{13} + a_{18} - 2m^2 a_4 a_5$, $A_7 = m^4 a_4 a_5 - a_{13} a_{18}$, $A_8 = b a_7 a_8 - b a_4 a_6$, $A_9 = b a_8 a_{18} + a_7 a_8 a_{14} - a_4 a_6 a_{16}$,
$A_{10} = a_8 a_{14} a_{18} - a_4 a_6 a_{17}$, $A_{11} = A_1 A_8 - A_3 A_5$, $A_{12} = A_1 A_9 + A_2 A_8 - A_4 A_5 + A_3 A_6$,
$A_{13} = A_2 A_9 + A_1 A_{10} + A_4 A_6 - A_3 A_7$, $A_{14} = A_4 A_7 - A_2 A_{10}$, $A_{15} = a_{23} A_8 + a_7 A_3$, $A_{16} = a_{23} A_9 + a_7 A_4 + a_{18} A_3$,
$A_{17} = a_{23} A_{10} + a_{18} A_4$, $A_{18} = a_2 A_{11}$, $A_{19} = a_{15} A_{11} + a_2 A_{12}$, $A_{20} = 4\Omega^2 \omega^2 A_{15} + a_{15} A_{12} + a_2 A_{13}$,
$A_{21} = 4\Omega^2 \omega^2 A_{16} + a_{15} A_{13} - a_2 A_{14}$, $A_{22} = 4\Omega^2 \omega^2 A_{17} - a_{15} A_{14}$, $L_1 = \dfrac{A_{19}}{A_{18}}, L_2 = \dfrac{A_{20}}{A_{18}}, L_3 = \dfrac{A_{21}}{A_{18}}, L_4 = \dfrac{A_{22}}{A_{18}}$.

The solution of Eq.(25) have the form

$$q^*(x) = \sum_{n=1}^{4} M_n e^{-k_n x}, \tag{26}$$

$$\psi^*(x) = \sum_{n=1}^{4} H_{1n} M_n e^{-k_n x}, \tag{27}$$

$$\varphi^*(x) = \sum_{n=1}^{4} H_{2n} M_n e^{-k_n x}, \tag{28}$$

$$C^*(x) = \sum_{n=1}^{4} H_{3n} M_n e^{-k_n x}, \tag{29}$$

where M_n (n=1, 2, 3, 4) are some constants, k_n^2 are the roots of the characteristic equation of Eq. (25).
The displacement components take the form

$$u^*(x) = \sum_{n=1}^{4} H_{4n} M_n e^{-k_n x}, \tag{30}$$

$$v^*(x) = \sum_{n=1}^{4} H_{5n} M_n e^{-k_n x}. \tag{31}$$

The stress components yield the following:

$$\sigma_{xx}^*(x) = \sum_{n=1}^{4} H_{6n} M_n e^{-k_n x}, \tag{32}$$

$$\sigma_{yy}^*(x) = \sum_{n=1}^{4} H_{7n} M_n e^{-k_n x}, \tag{33}$$

$$\sigma_{xy}^*(x) = \sum_{n=1}^{4} H_{8n} M_n e^{-k_n x}, \tag{34}$$

$$P^*(x) = \sum_{n=1}^{4} H_{9n} M_n e^{-k_n x}. \tag{35}$$

Where

$$H_{1n} = \frac{2\Omega\omega}{a_2 k_n^2 - a_{15}}, H_{2n} = \frac{a_{23}(a_{13} - k_n^2 - 2\Omega\omega H_{1n}) + a_4 a_{20}(m^2 - k_n^2)}{a_{23} a_3 (a_{14} - b k_n^2) + a_4 (a_{21} k_n^2 - a_{22})},$$

$$H_{3n}=\frac{a_{20}(m^2-k_n^2)+(a_{22}-a_{21}k_n^2)H_{2n}}{a_{23}},\ H_{4n}=-imH_{1n}-k_n,\ H_{5n}=im-k_nH_{1n},$$
$$H_{6n}=-k_nH_{4n}+imB_2H_{5n}-(1-b(k_n^2-m^2))H_{2n}-B_3H_{3n},$$
$$H_{7n}=imB_1H_{5n}-B_2k_nH_{4n}-(1-b(k_n^2-m^2))H_{2n}-B_3H_{3n},H_{8n}=a_2B_1(imH_{4n}-k_nH_{5n}),$$
$$H_{9n}=B_1(k_nH_{4n}-imH_{5n})+B_4H_{3n}-B_5(1-b(k_n^2-m^2))H_{2n},$$
$$B_1=\frac{\gamma T_o}{\lambda+2\mu},\ B_2=\frac{\lambda\gamma T_o}{\rho c_o^2(\lambda+2\mu)},\ B_3=\frac{\beta_1\rho}{\lambda+2\mu},B_4=\frac{b_1\rho}{\beta_1},B_5=\frac{a(\lambda+2\mu)}{\gamma\beta_1}.$$

5 Boundary conditions

The boundary conditions at $x=o$ are considered as

$$\frac{\partial\varphi}{\partial x}=0,\ \frac{\partial C}{\partial x}=0,\ \sigma_{xx}=fe^{i(\omega t+my)},\ \text{and}\ \sigma_{xy}=0. \tag{36}$$

Where f is a constant.

Substituting from Eq.(36) in Eqs. 28,29,32,34, we can obtain the following equations satisfied by the parameters

$$\sum_{n=1}^{4}k_nH_{2n}M_n=0, \tag{37}$$

$$\sum_{n=1}^{4}k_nH_{3n}M_n=0, \tag{38}$$

$$\sum_{n=1}^{4}H_{6n}M_n=f, \tag{39}$$

$$\sum_{n=1}^{4}H_{8n}M_n=0. \tag{40}$$

6 Numerical results

Copper is used to study the effect of rotation, diffusion, and two temperature. We take the following values of the different physical constants.

$$T_o=293K,\ C_E=383.1J/(kg.K),\alpha_t=1.78\times10^{-5}K^{-1},\rho=8954kg/m^3,\ \lambda=7.76\times10^{10}kg/(m\,s^2)$$
$$,\mu=3.86\times10^{10}kg/(m.s^2),\alpha_c=1.98\times10^{-4}m^3/kg,\ K=300W,d=0.85\times10^{-8}(kg.s)/m^3$$
$$,a=1.2\times10^4m^2/(s^2.K),b_1=0.9\times10^6m^5/(kg.s^2),\ w_o=-0.3,\xi=2,w=w_o+i\xi,y=2$$
$$K^*=2.97\times10^{13},m=0.5,f=2,\ b=0.2,\ \tau=0.02,\ \tau_v=0.05,\tau_T=0.2,\tau_q=0.8.$$

The numerical values written above, was used to show the distribution of the displacement v, the stress components σ_{xx}, σ_{xy}, the temperature T, the concentration distribution C, and the chemical potential p.

Fig. 1-6 show the distribution of the physical quantities in the presence and absence of rotation using two theories (TPL) and (G-N) III.

Fig. 1 indicates the distribution of the displacement v, TPL curves decrease with the increase of rotation, while G-N curves increase with the increasing of rotation. Fig. 2 shows the distribution of the stress function σ_{xx}, all curves start from positive value. The curves are decreasing in the interval $0<x<1$, and then increase in the interval $1<x<2.2$, then acting as a decreasing wave until vanishing. Fig. 3 shows the distribution of the stress function σ_{xy}, all curves satisfy the boundary condition and start from zero. Curves are inversely proportional with rotation. Fig. 4 shows the distribution of the temperature T, curves are decreasing with the increasing of rotation. Fig. 5 shows the distribution of the concentration C, all curves start from positive values. Curves are smooth decreasing until reaching zero. Fig. 6 shows the distribution of the potential P, the curves in case of presence of rotation is lower than the other curves.

Fig. 7-12 show the distribution of the physical quantities in the presence and absence of two- temperature using two theories (TPL) and (G-N III).
Fig. 7 depicts the distribution of the displacement v, all curves start from positive values. Curves are smooth decreasing until vanishing. Fig. 8 shows the distribution of the stress function σ_{xx}, all curves start from positive values. G- N Curves have a higher value than TPL curves. Fig. 9 shows the distribution of the stress function σ_{xy}, all curves satisfy the boundary condition and start from zero. Rotation is more influenced in TPL curves. Fig. 10 shows the distribution of the temperature T, all curves start from negative value. Curves are increasing in $0 < x < 1$, and then decreasing in $1 < x < 6$. Fig. 11 shows the distribution of the concentration C, TPL curves inversely proportional with two- temperature, while G-N curves directly proportional with two- temperature. Fig. 12 shows the distribution of the potential chemical P, all curves start from positive values and then smooth decreasing until vanishing.
Fig. 13-18 show the distribution of the physical quantities in the presence and absence of diffusion (D -WD) respectively using two theories (TPL) and (G-N III).
Fig. 13 depicts the distribution of the displacement v. TPL curves are more effective by diffusion than G-N curves. Fig. 14 indicates the distribution of the stress function σ_{xx}. In the presence of diffusion, TPL curves are more influenced by diffusion. In the absence of diffusion curves are decreasing until vanishing. Fig. 15 indicates the distribution of the stress function σ_{xy}. All curves satisfy the boundary conditions and start from zero. In the absence of diffusion, curves at $0 < x < 0.6$, then increase at $0.6 < x < 1.6$, and then decreasing until vanishing. In the presence of diffusion curves are decreasing until disappearing. Fig. 16 depicts the distribution of the temperature T. All curves start from negative value. In presence of diffusion, curves are increasing until reaching to zero. Fig. 17 and 18 depicts the distribution of the concentration C and the chemical potential P. TPL curve is lower than G-N curve.

Conclusion

1. Analytical solutions based upon normal mode analysis of the thermoelastic problem have been developed.
2. The method used in the present article is applicable to a wide range of problems in thermoelasticity.
3. The presence of rotation plays a siginifiacnt role in all physical quantities.
4. The value of all the physical quantities converges to zero, and all the functions are continuous.

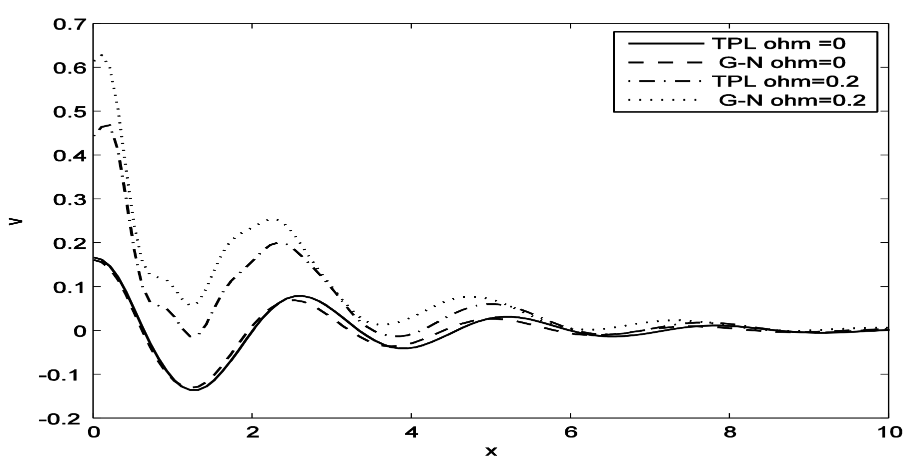

Fig. 1 Distribution of the displacement v in the absence and presence of rotation.

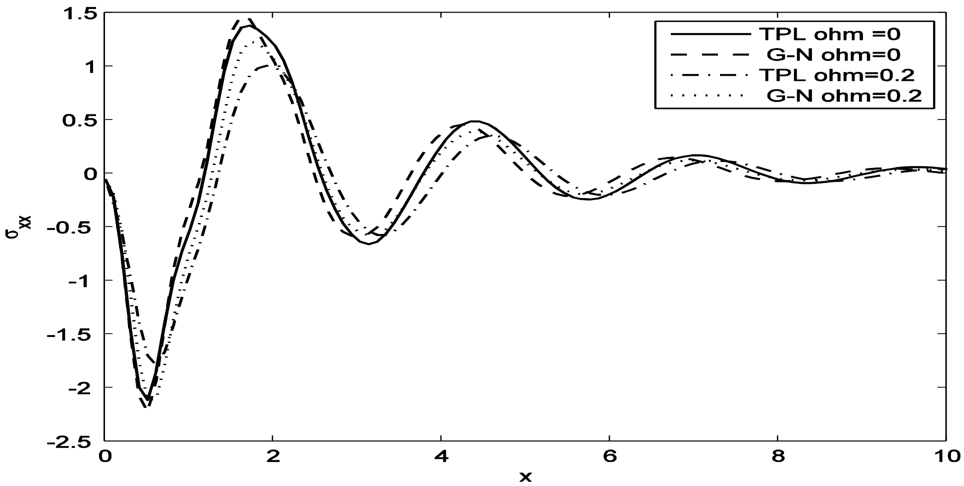

Fig. 2 Distribution of the stress component σ_{xx} in the absence and presence of rotation.

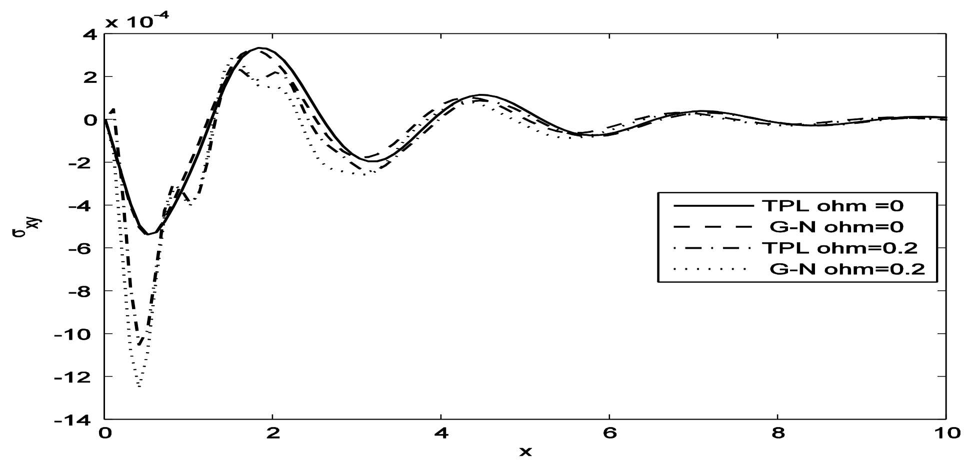

Fig. 3 Distribution of the stress component σ_{xy} in the absence and presence of rotation.

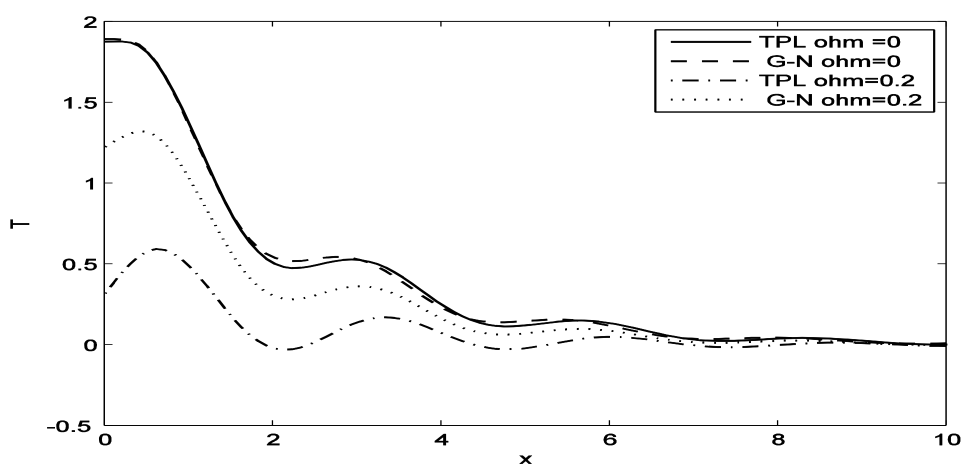

Fig. 4 Distribution of the temperature T in the absence and presence of rotation.

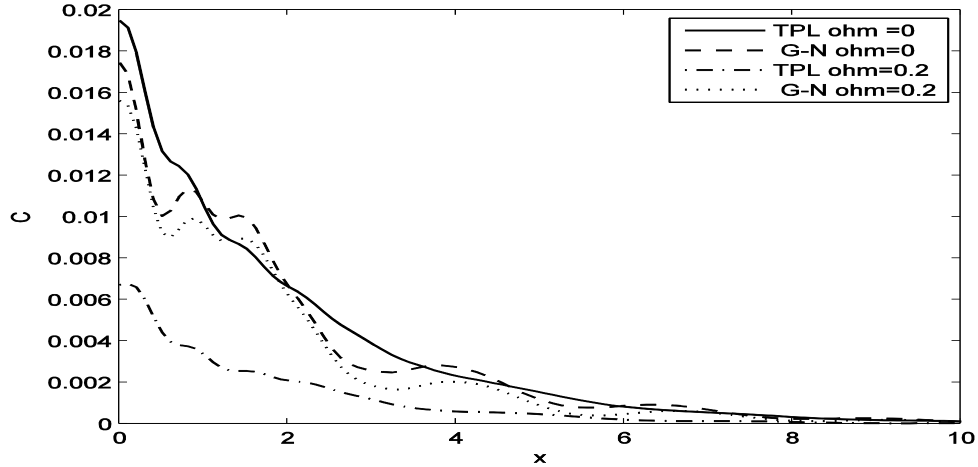

Fig. 5 Distribution of the concentration C in the absence and presence of rotation.

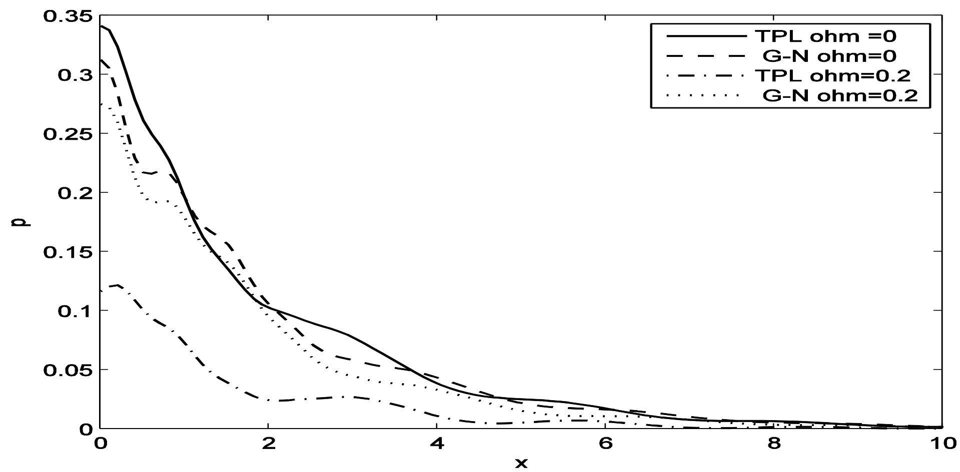

Fig. 6 Distribution of the potential P in the absence and presence of rotation.

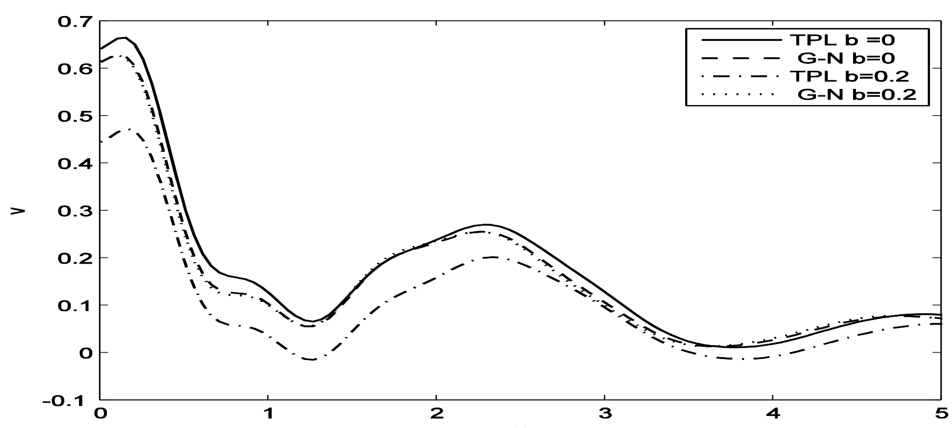

Fig. 7 Distribution of the displacement v in the absence and presence of two- temperature.

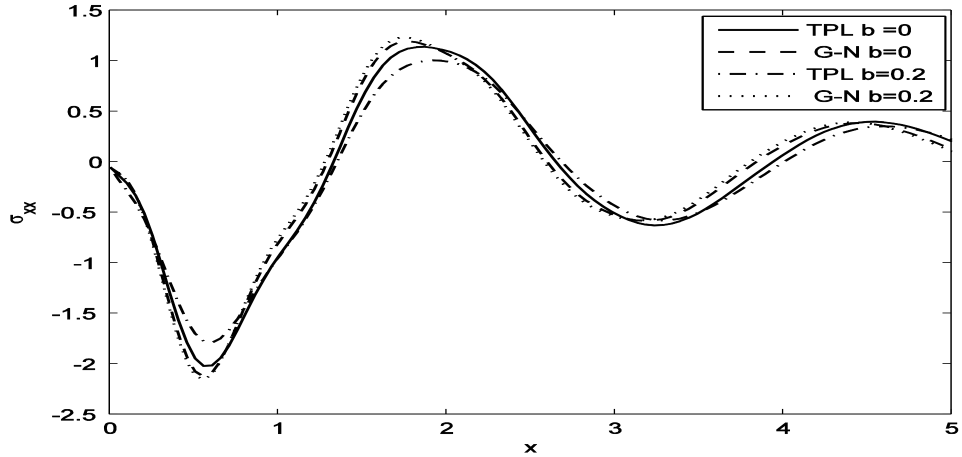

Fig. 8 Distribution of the stress component σ_{xx} in the absence and presence of two- temperature.

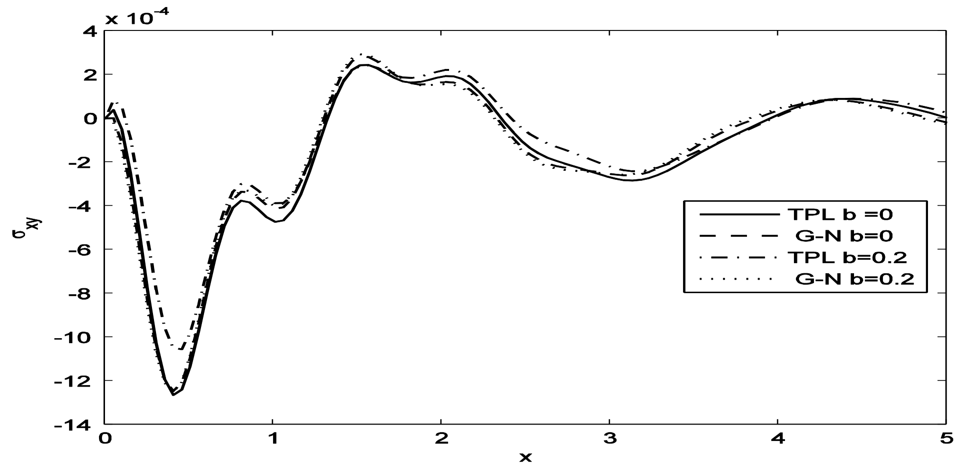

Fig. 9 Distribution of the stress component σ_{xy} in the absence and presence of two- temperature.

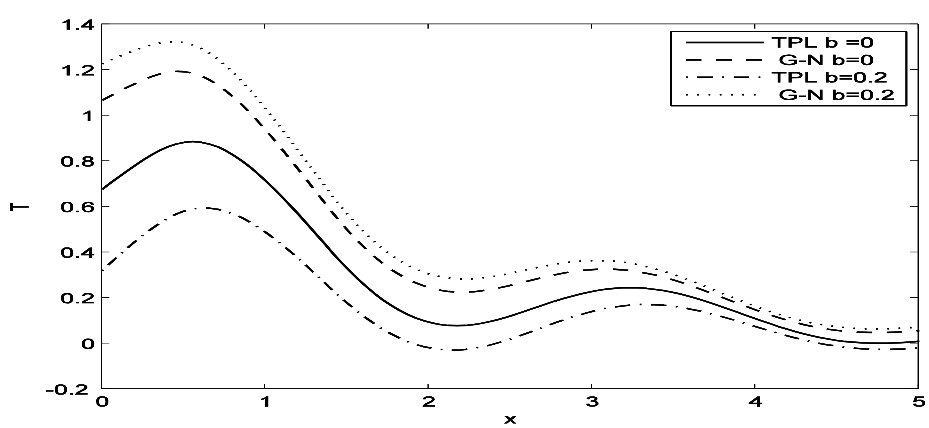

Fig. 10 Distribution of the temperature T in the absence and presence of two- temperature.

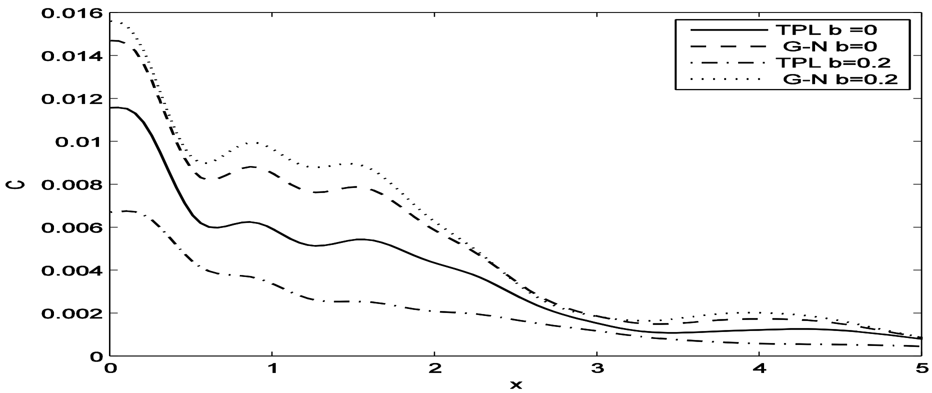

Fig. 11 Distribution of the concentration C in the absence and presence of two- temperature.

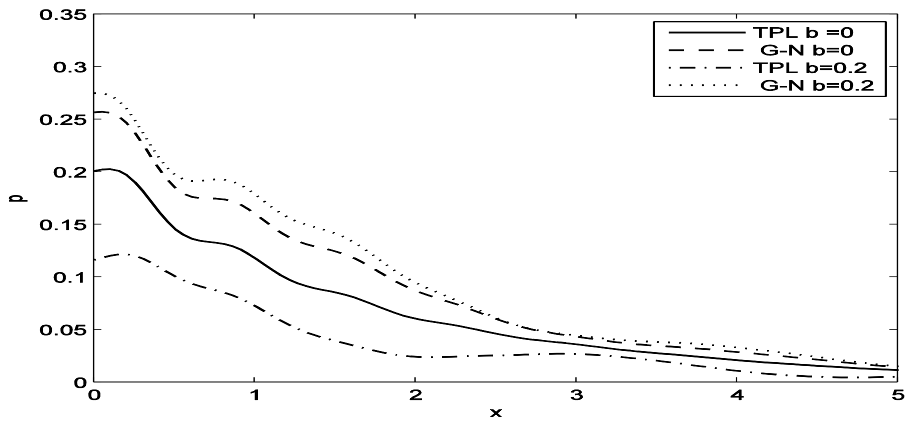

Fig. 12 Distribution of the potential function P in the absence and presence of two- temperature.

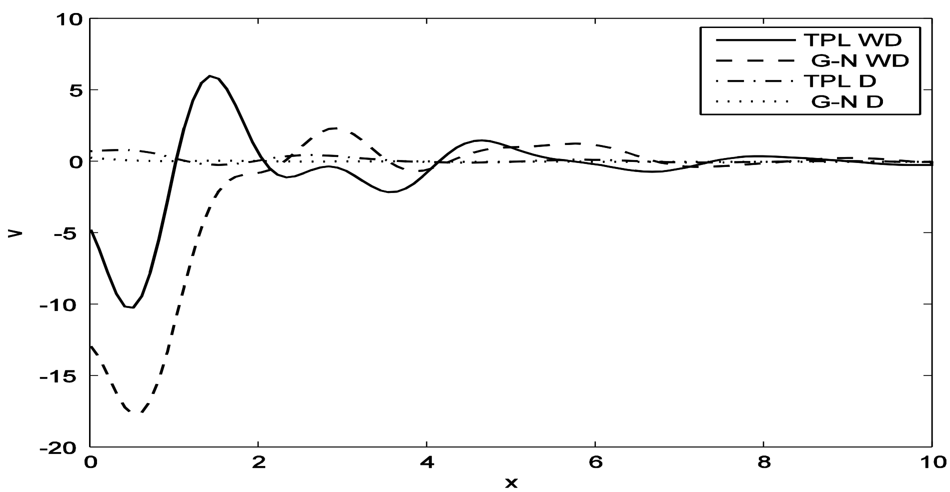

Fig. 13 Distribution of the displacement v in the absence and presence of diffusion.

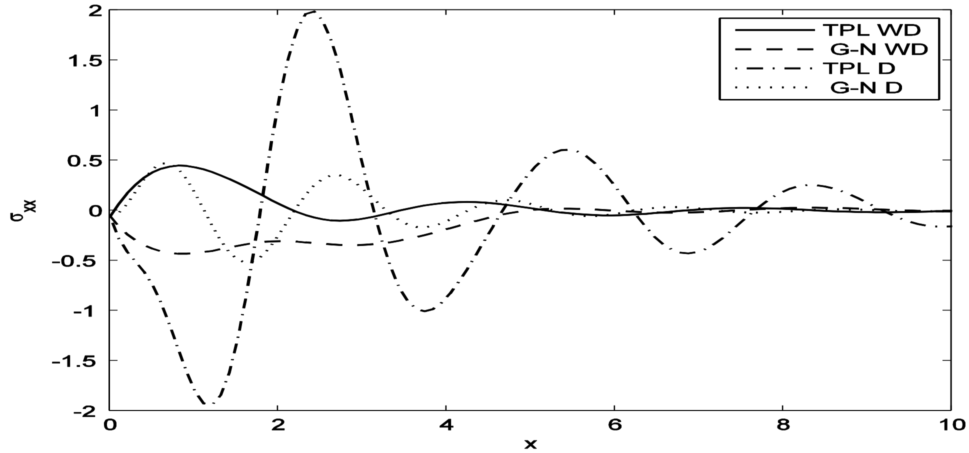

Fig. 14 Distribution of the stress function σ_{xx} in the absence and presence of diffusion.

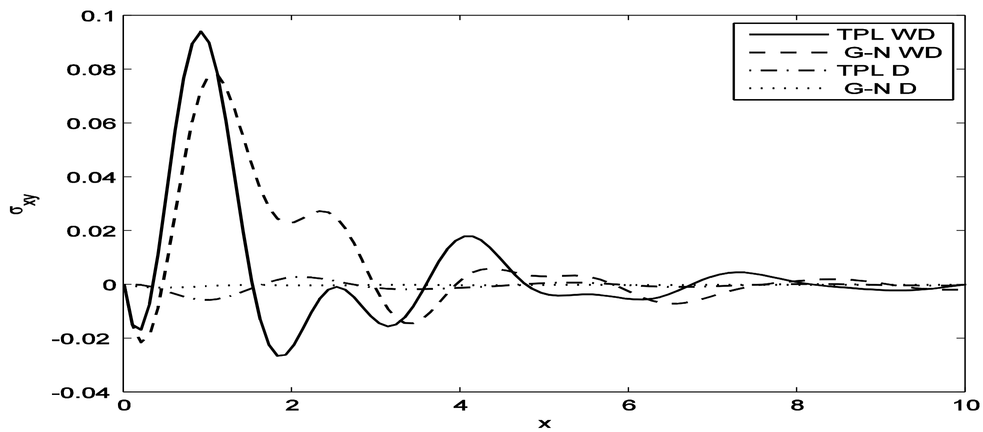

Fig. 15 Distribution of the stress function σ_{xy} in the absence and presence of diffusion.

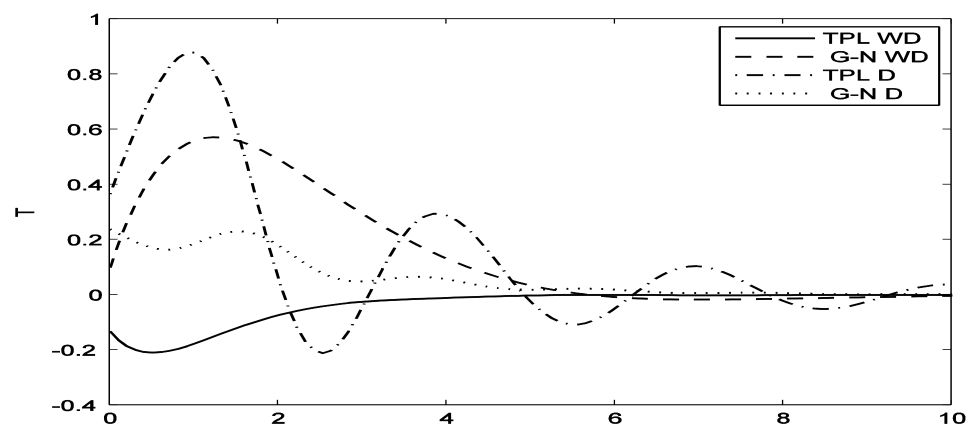

Fig. 16 Distribution of the temperature T in the absence and presence of diffusion.

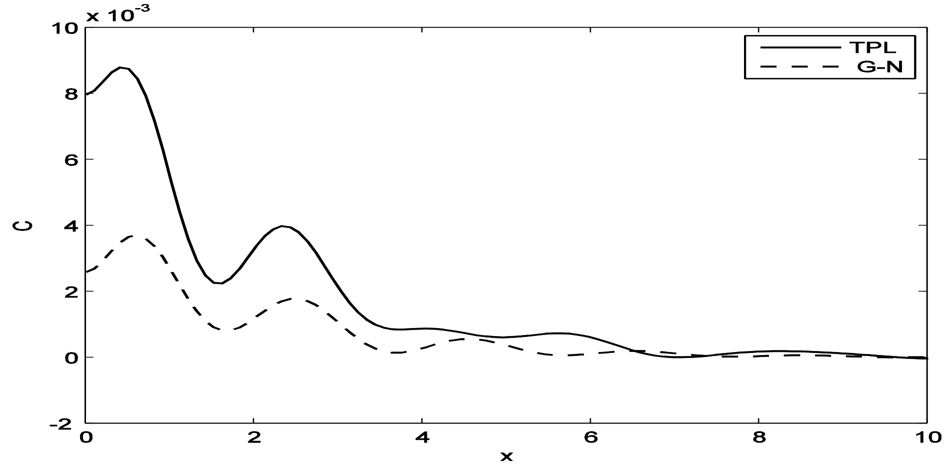

Fig. 17 Distribution of the concentration function C in the presence of diffusion.

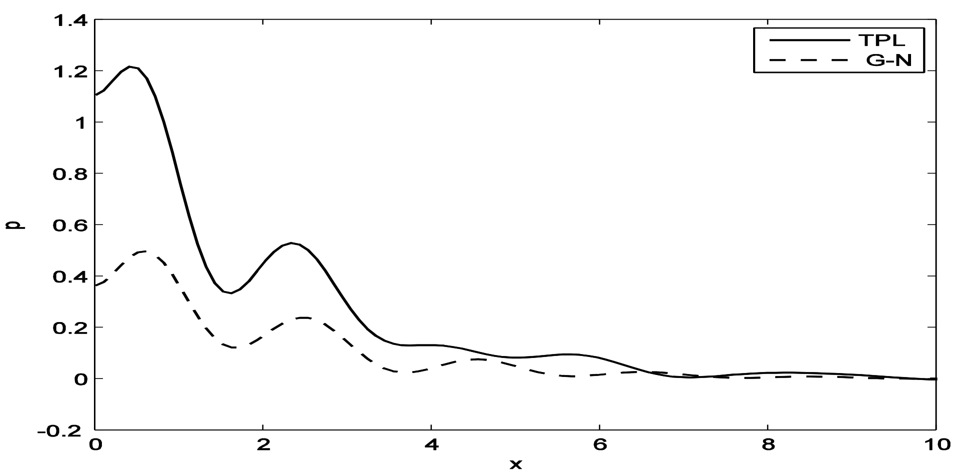

Fig. 18 Distribution of the potential function P in the presence of diffusion.

References

[1] H. W. Lord, Y. Shulman, A generalized dynamical theory of thermoelasticity, Journal of Mechanics and Physics of Solid, 15, 299-309,1967.
[2] I. Muller, The coldness, a universal function in thermoelastic bodies, Archive of Rational Mechanics and Analysis, 41,319-332,1971.
[3] A. E. Green, N. Laws, On the entropy production inequality, Archive for Rational Mechanics and Analysis, 45, 47-53,1972.
[4] A. E. Green, K. A. Lindsay, Thermoelasticity, Journal of Elasticity, 2,1 – 7,1972.
[5] A. E. Green, P. M. Nagdhi, A re-examination of the basic postulates of thermo- mechanics, Proceedings of Royal Society of London: A, 432,171 -194,1991.
[6] A. E. Green, P. M. Naghdi, On undamped heat waves in an elastic solid , Journal of Thermal Stresses, 15, 253–264,1992.
[7] A. E. Green, P. M. Naghdi, Thermoelasticity without energy dissipation, Journal of Elasticity, 31,189-209,1993.
[8] M. I. A. Othman , S. Y. Atwa , R. M. Farouk, The effect of diffusion on two dimensional problem of generalized thermoelasticity with Green-Naghdi theory, International Communications in Heat and Mass Transfer, 36,857-864,2009.
[9] M. I. A. Othman , S. Y. Atwa, Thermoelastic plane waves for an elastic solid half space under hydrostatic initial stress of type III, Meccanica, 47,1337-1347,2012.

[10] M. I. A. Othman, S. Y. Atwa, A. W. Elwan, Generalized thermoelasticity problem with temperature dependent properties without energy dissipation in two and three dimensions, SYLWAN, 159 ,175-195,2015.

[11] M. I. A. Othman, S. Y. Atwa, A. W. Elwan, Effect of thermal loading due to laser pulse on 3-D problem on micropolar thermoelastic solid with energy dissipation, Mechanics and Mechanical Engineering, 21,679-701,2017.

[12] P. J. Chen, M. E. Gurtin, On a theory of heat conduction involving two temperatures, Zeitschrift für angewandte Mathematik und Physik, 19, 614-627,1968.

[13] P. J. Chen, M. E. Gurtin and W. O. Williams, A note on non simple heat conduction, Zeitschrift für angewandte Mathematik und Physik, 19,969- 970,1968.

[14] P. J. Chen, M. E. Gurtin, W. O. Williams, On the thermodynamicof non-simple elastic materials with two temperatures, Zeitschrift für angewandte Mathematik und Physik, 20,107-112,1969.

[15] B. Singh and K. Bala, Propagation of waves in a two-temperature rotating thermoelastic solid half-space without energy dissipation, Applied Mathematics,3, 1903-1909,2012.

[16] S. Y. Atwa, Generalized magneto- thermoelasticity with two temperature and initial stress under Green-Naghdi theory, Applied Mathematical Modling, 38 , 5217- 5230,2014.

[17] S .Y. Atwa, A. Jahangir, Two temperature effects on plane waves in generalized Thermo Microstretch Elastic Solid, International Journal of Thermophysics, 35,175-193,2014.

[18] S. K. Roychoudhuri, On thermoelastic three phase lag model, Journal of Thermal Stresses, 30, 231-238,2007.

[19] R. Quintanilla, R. Racke, A note on stability in three-phase-lag heat Conduction, International Journal of Heat and Mass Transfer, 51, 24-29,2008.

[20] R. Kumar, V. Chawla, I. A. Abbas, Effect of viscosity on wave propagation in anisotropic thermoelastic medium with three phase-lag model, Journal of Theoritecal and Applied Mechanics, 39 ,313-341,2012.

[21] M. I. A. Othman, W. M. Hasona, Nehal T. Mansour, Effect of rotation and initial stress on generalized stress on generalized thermoelastic medium with two- temperature under three- phase- lag model, International Journal of Innovative Research in Science Engineering and Technology, 4 ,2327-2337,2015.

[22] M. I. A. Othman, W. M. Hasona, Nehal T. Mansour, The effect of magnetic field on generalized thermoelastic medium with two temperature under three phase lag model, Multidiscipline Modeling in Materials and Structures, 11, 544-557,2015.

[23] M. I. A. Othman, S. Y. Atwa, R. M. Farouk, Generalized magneto- thermoviscoelastic plane waves under the effect of rotation without energy dissipation, International Journal of Engineering Science, 46 ,639-653,2008.

[24] P. Ailawila, G. Khurana, S. Kumar, Effect of rotation in a generalized thermoelastic medium with two-temperature under the influence of gravity, Archives of Thermodynamics, 5,99-116,2009.

[25] M. I. A. Othman, M. E. M. Zidan, M. I. M. Hilal, Effect of magnetic field on a rotating thermoelastic medium with voids under thermal loading due to laser pulse with energy dissipation, Canadian Journal of Physics, 2015, 1-13,2014.

[26] M. I. A. Othman, S. Y. Atwa, A. Jahangir, A. Khan, The effect of rotation on plane waves in generalized thermo-microstretch elastic solid for a mode- I crack under Green Naghdi theory, Journal of Computational and Theoretical Nanoscience, 12,1-10,2015.

[27] M. I. A. Othman, S. Y. Atwa, A. Jahangir, A. Khan, The effect of gravity on plane waves in a rotating thermo- microstretch elastic solid for a mode- I crack with energy dissipation, Mechanics of Advanced Materials and Structures, 22 , 945-955,2015.

[28] M. I. A. Othman, S. Y. Atwa, 2- D problem of anistropic rotating thermoelastic half- space under Green- Naghdi theory, Journal of Comutational and Theortical Nanoscience, 12 ,1363- 3270,2015.

[29] W. Nowacki, Dynamical problems of thermoelastic diffusion in solids, Bulletin of Polish Academy of Sciences Series, Science and Technology, 22,55-64,1974.

[30] W. Nowacki, Dynamical problems of thermoelastic diffusion in solids II, Bulletin of Polish Academy of Sciences Series, Science and Technology, 22,129-135,1974.

[31] W. Nowacki, Dynamical problems of thermoelastic diffusion in solids III, Bulletin of Polish Academy of Sciences Series, Science and Technology, 22,257-266,1974.

[32] S. Y. Atwa, Effect of fractional parameter on plane waves of generalized thermoelastic diffusion with temperature-dependent elastic medium, Journal of Materials and Chemical Engineering, 1,55-74,2013.

[33] J .J. Tripathi, G. D. Kedar , K.C. Deshmukh, Generalized thermoelastic diffusion problem in a thick circular plate with axisymmetric heat supply, Acta Mechanica, 226,2121-2134,2015.

[34] R. Kumar, Wave propagation in a microstretch thermoelastic diffusion solid VERSITA, 23,127-196,2015.

[35] M. I. A. Othman, Y. Q. Song, Effect of rotation on plane waves of the generalized electro magneto- thermo-viscoelasticity with two relaxation times, Applied Mathematical Modeeling, 32,811-825,2008.

[36] M. I. A. Othman, S. M. Said, " The effect of rotation on two- dimensional problem of a fiber- reinforced thermoelastic with one relaxation time, International Journal of Thermophysics, 33,160-171,2012.

[37] M. Zakaria, Effects of hall current and rotation on magneto- micropolar generalized thermoelasticity due to ramp-type

heating, International Journal of Electromagnetics and Application, 2 , 24–32,2012.
[38] J. D. Elmaklizi, M. I. A. Othman, The effect of rotation on a thermoelastic diffusion with temperature dependent elastic moduli comparison of different theories, Journal of Thermoelasticity, 1, 6-15,2013.
[39] M. I. A. Othman, W. M. Hasona, E. M. Abd-Elaziz, Effect of rotation on micro-polar generalized thermoelasticity with two temperature using a dual- phase-lag model, Candian Journal of Physics, 92, 149-158, 2014.