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Constrained optimization of a newsboy problem with return policy

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Abstract: A newsboy model in which a vendor has a limited budget to procure the required items is developed. It is assumed that the manufacturer will either sell the items to the vendor outright or offer the items to the vendor with return policy. In the latter case, the manufacturer buys back from the vendor the unsold items at the end of the selling season. This study considers one item with budget constraints. The vendor has the option of purchasing the goods outright, obtaining the goods with return policy, or a combination of both. By using Kuhn-Tucker Conditions and inductive method, our analysis proposes an optimal inventory policy theorem. The purpose of this study is to investigate how the vendor should replenish the items with return policy under limited budget and changing prices. We show that there exists a set of conditions under which the vendor's optimal strategy will change based on the amount of available budget for product procurement. If the purchase price with return policy is greater than the price of outright purchase and the vendor has a small amount of available budget, only outright purchase will be optimal. However, if the budget reaches a certain threshold amount, mixed strategies where items are obtained by outright purchase and with return policy are used. We also show that when the purchase price with return policy is not greater than the outright purchase, the vendor will only obtain the items with return policy.

Keywords: Return Policy, Kuhn-Tucker Conditions, Ordering policy.

1 Introduction

This study investigates a single period problem in which a vendor has the option of purchasing the item outright and/or obtaining the item through a return-policy agreement with the manufacturer. The vendor has limited budget. A return policy allows a vendor to return the unsold products for a full or partial refund. This will entice the vendor to order a larger quantity, resulting in an increase in the joint profit. Some products like the catalogue or style goods are examples where return policies are used [1,2]. The "catalogue goods" are sold to customers through catalogue advertisement with fixed price during a particular selling season.

Pasternack [3] modeled a return policy and derived a global optimization in a single period with uncertain demand. He demonstrated that a return policy where a manufacturer offers the vendors partial credits for all unsold products could achieve channel coordination. Padmanabhan and Png [4] illustrated that the useful return policy can increase a manufacturer's profit and increase the vendor competition.

Emmons and Gilbert [1] studied the effect of return policy on both the manufacturer and the vendor. Such policy is to maximize the manufacturer's profit by inducing the vendor to place larger order when demand is uncertain.

The importance of the single period problem increases due to the shortening of the product life cycle in recent years. Many extensions of the single period problem have been studied [5]. Two major extensions are the unconstrained, single-item single-period problem, and the constrained, multi-item single-period problem. Hadley and Whitin [6] derived a constrained multi-item problem in a single period. Jucker and Rosenblatt [7] considered an unconstrained model with three types of quantity discounts: all-units quantity discount, incremental quantity discounts and Carload-lot discounts. Gerchak and Parlar [8] developed an unconstrained model in which the vendor decides the price and order. Lau and Lau [9] modeled a newsboy problem with price-dependent distribution demand. Khouja [10] developed a newsboy model in which multiple discounts are used to sell excess inventory. Khouja and Mehrez [11] extended Khouja's model [10] to the multi-item case. This model dealt with a newsboy selling many items under a budget constraint. Lau and Lau [12] derived a capacitated multiple-product single period inventory model. Pasternack [13] developed a capacitated single-item newsboy model with revenue sharing. Vlachos and Dekker [14] derived order quantity for single period products with return policy. Arcelus et al. [15] evaluated manufacturer's buyback policy under price-dependent stochastic demand. Chiu et al. [16] addressed returns supply contract for coordinating supply chains with price-dependent demands.

This study considers one item with budget constraints. The vendor has the option of purchasing the goods outright, obtaining the goods with return policy, or a combination of both. By using Kuhn-Tucker Conditions, our analysis proposes an optimal inventory policy theorem. We discuss the conditions for obtaining goods through outright purchase, return-policy purchase, or a combinatory purchase of both.

In Section 2 we present a general model and a theorem for three cases. In Section 3 four numerical examples are given to illustrate the theorem. The first three examples demonstrate the various optimal strategies with changing budget. The last example demonstrates the strategies when the purchase price through return policy changes for a fixed limited budget. The concluding remark is given in the last section.

2 Mathematical Modeling and Analysis

The mathematical model is developed based on the following assumptions:

(a) The demand is uncertain.

(b) An item with single order period, short selling season and long production lead-time is considered (an example of this type of product is the catalogue or style product).

(c) A vendor has the option of purchasing the item outright and/or obtaining the item through a return-policy agreement with the manufacturer.

(d) The products purchased through return policy begin selling only after the outright purchase items are sold out.

The decision variables are:

- Q_1 vendor's lot size obtained from the manufacturer through outright purchase
- Q_2 vendor's lot size obtained from the manufacturer through return policy

The known parameters are:

- $f(x)$ probability density function of uncertain demand x
- $F(x)$ cumulative distribution function of the probability density function $f(x)$
- C_1 vendor's purchase price through outright purchase
- C_2 vendor's purchase price through return policy
- P unit retail price
- S vendor's shortage cost per unit if the item is out of stock
- R vendor's return price per unit if the item is unsold
- T total amount of funds the vendor has for obtaining the item through either outright purchase or return policy, or both.
- EP vendor's expected profit

The vendor's expected profit can be expressed as

$$EP = P\left(\int_0^{Q_1+Q_2} xf(x)dx + \int_{Q_1+Q_2}^{\infty} (Q_1+Q_2) f(x)dx\right) + R\int_0^{Q_1} Q_2 f(x)dx + R\int_{Q_1}^{Q_1+Q_2} (Q_1+Q_2-x) f(x)dx - S\int_{Q_1+Q_2}^{\infty} (x-Q_1-Q_2) f(x)dx - C_1Q_1 - C_2Q_2 \quad (1)$$

The first two terms in the right side of (1) are the expected sales revenue. The second two terms are the return revenue for the unsold units. The last three terms are the expected shortage cost and the purchase costs. The problem faced by the vendor is a nonlinear programming with constraints as follows:

Maximize EP

Subject to:

$$C_1Q_1 + C_2Q_2 \le T$$
$$-Q_1 \le 0$$
$$-Q_2 \le 0$$

Looking at the partial second derivatives for *EP*, one has:

$$\frac{\partial^2 EP}{\partial Q_1^2} = -f(Q_1+Q_2)(P+S-R) - Rf(Q_1), \quad (2)$$

$$\frac{\partial^2 EP}{\partial Q_2^2} = -f(Q_1+Q_2)(P+S-R), \quad (3)$$

$$\frac{\partial^2 EP}{\partial Q_1 \partial Q_2} = -f(Q_1+Q_2)(P+S-R), \quad (4)$$

Hence, if $P+S-R \geq 0$, EP is concave.

The following Kuhn-Tucker conditions are required for optimality:

1. $u_1(T - C_1Q_1 - C_2Q_2) = 0$
2. $u_2Q_1 = 0$
3. $u_3Q_2 = 0$
4. $P+S-C_1(1+u_1)+u_2-F(Q_1+Q_2)(P+S-R) - F(Q_1)R = 0$
5. $P+S-C_2(1+u_1)+u_3-F(Q_1+Q_2)(P+S-R)=0$
6. $u_1 \geq 0, u_2 \geq 0, u_3 \geq 0, Q_1 \geq 0, Q_2 \geq 0$

Three cases of solution are discussed. The first case is $Q_1 > 0, Q_2 = 0$. The second case is $Q_1 = 0, Q_2 > 0$. The last case is $Q_1 > 0, Q_2 > 0$.

Case 1: $Q_1 > 0, Q_2 = 0$ **(**$u_2 = 0, Q_1 = \frac{T}{C_1}$**)**

From Kuhn-Tucker conditions 4 and 5, one has

$$P+S-C_1(1+u_1)-F(Q_1)(P+S)=0 \quad (5)$$

and

$$P+S-C_2(1+u_1)+u_3-F(Q_1)(P+S-R)=0 \quad (6)$$

From (5), it is

$$1+u_1 = \frac{1}{C_1}[P+S-F(Q_1)(P+S)] \geq 1 \quad (7)$$

Then,

$$F(\frac{T}{C_1}) \leq \frac{P+S-C_1}{P+S}. \quad (8)$$

Substituting (7) into (6), one has

$$C_1u_3 = -F(Q_1)[(P+S)(C_2-C_1)+C_1R]+(P+S)(C_2-C_1) \geq 0$$

. (9)

If $C_2 < C_1$ and $(P+S)(C_2-C_1)+C_1R < 0$, then

$$F(\frac{T}{C_1}) \geq \frac{(P+S)(C_2-C_1)}{(P+S)(C_2-C_1)+C_1R} > 1 \quad \text{(Contradiction)}$$

(10)

If $C_2 < C_1$ and $(P+S)(C_2-C_1)+C_1R > 0$, then

$$F(\frac{T}{C_1}) \geq \frac{(P+S)(C_2-C_1)}{(P+S)(C_2-C_1)+C_1R} < 0 \quad \text{(Contradiction)}$$

(11)

If $C_2 = C_1$, then

$$C_1u_3 = -F(Q_1)C_1R \geq 0 \quad \text{(Contradiction)}$$

(12)

If $C_2 > C_1$, then

$$F(\frac{T}{C_1}) \leq \frac{(P+S)(C_2-C_1)}{(P+S)(C_2-C_1)+C_1R} \quad (13)$$

One can see that conditions (8) and (13) must be satisfied simultaneously for the case of $Q_1 > 0, Q_2 = 0$.

Case 2: $Q_1 = 0, Q_2 > 0$ **(**$u_3 = 0, Q_2 = \frac{T}{C_2}$**)**

Derived from Kuhn-Tucker conditions 4 and 5, one has

$$C_2u_1 = P+S-C_2-F(Q_2)(P+S-R) \geq 0 \quad (14)$$

$$C_2u_2 = (P+S)(C_1-C_2)-F(Q_2)(C_1-C_2)(P+S-R) \geq 0$$

(15)

From (14), one has

$$F(\frac{T}{C_2}) \leq \frac{P+S-C_2}{P+S-R}, \quad (16)$$

where $R < C_2$.

From (15), if $C_1 \leq C_2$, one has

$$F(\frac{T}{C_2}) \geq \frac{P+S}{P+S-R} > 1 \quad \text{(Contradiction)}$$

(17)

From (15), if $C_1 \geq C_2$, one has

$$F(\frac{T}{C_2}) \leq \frac{P+S}{P+S-R} \quad (18)$$

From (16) and (18), one can see that conditions $C_1 \geq C_2$ and $F(\frac{T}{C_2}) \leq \frac{P+S-C_2}{P+S-R}$ must be satisfied simultaneously for the case of $Q_1 = 0, Q_2 > 0$.

Case3: $Q_1 > 0, Q_2 > 0$
($u_2 = 0, u_3 = 0, C_1Q_1 + C_2Q_2 = T$**)**

From Kuhn-Tucker conditions 4 and 5, one has

$$P+S-C_1(1+u_1)-F(Q_1+Q_2)(P+S-R)-F(Q_1)R=0$$

(19)

$$P+S-C_2(1+u_1)-F(Q_1+Q_2)(P+S-R)=0 \quad (20)$$

The optimal solution of Q_1 and Q_2 must satisfy (19), (20) and $C_1Q_1+C_2Q_2=T$ simultaneously.

If the solution of Q_1 is positive, after substituting $(T-C_1Q_1)/C_2$ into Q_2, the first derivatives of *EP* with respect to Q_1 is greater than zero when Q_1 = 0, that is

$$(C_1-C_2)[F(\frac{T}{C_2})(P+S-R)-(P+S)]>0 \quad (21)$$

If $C_1>C_2$, one has

$$F(\frac{T}{C_2})>\frac{P+S}{P+S-R}>1 \quad \text{(Contradiction)} \quad (22)$$

If $C_2>C_1$, one has

$$F(\frac{T}{C_2})<\frac{P+S}{P+S-R} \quad (23)$$

If the solution of Q_2 is positive, then, after substituting $(T-C_2Q_2)/C_1$ into Q_1, the first derivatives of *EP* with respect to Q_2 is greater than zero when Q_2 =0, that is

$$F(\frac{T}{C_1})[(P+S)(C_2-C_1)+RC_1]-(P+S)(C_2-C_1)>0 \quad (24)$$

If $C_2>C_1$, one has

$$F(\frac{T}{C_1})>\frac{(P+S)((C_2-C_1)}{(P+S)(C_2-C_1)+RC_1} \quad (25)$$

If $C_2<C_1$ and $(P+S)(C_2-C_1)+RC_1<0$, one has

$$F(\frac{T}{C_1})<\frac{(P+S)(C_2-C_1)}{(P+S)(C_2-C_1)+RC_1} \quad \text{(Contradiction)} \quad (26)$$

If $C_2<C_1$ and $(P+S)(C_2-C_1)+RC_1>0$, one has

$$F(\frac{T}{C_1})>\frac{(P+S)(C_2-C_1)}{(P+S)(C_2-C_1)+RC_1} \quad (27)$$

Solution for Q_1 > 0 and Q_2 > 0 is located at the intersection of (23) and {(25) or (27)}. One can find that the two conditions $C_2>C_1$ and $F(\frac{T}{C_1})>\frac{(P+S)((C_2-C_1)}{(P+S)(C_2-C_1)+RC_1}$ must be satisfied for the case of $Q_1>0, Q_2>0$.

The theorems resulted from the above discussion can be stated as follows:

For $T=C_1Q_1+C_2Q_2$ and $P+S-R\geq 0$, one has

Theorem (i) $Q_1>0$ and $Q_2=0$ if $C_2>C_1$, $F(\frac{T}{C_1})\leq\frac{(P+S)(C_2-C_1)}{(P+S)(C_2-C_1)+RC_1}$ and $F(\frac{T}{C_1})\leq\frac{P+S-C_1}{P+S}$.

Theorem (ii) $Q_1=0$ and $Q_2>0$ if $C_2\leq C_1$ and $F(\frac{T}{C_2})\leq\frac{P+S-C_2}{P+S-R}$.

Theorem (iii) $Q_1>0$ and $Q_2>0$ if $C_2>C_1$ and $F(\frac{T}{C_1})>\frac{(P+S)(C_2-C_1)}{(P+S)(C_2-C_1)+RC_1}$.

3 Numerical Example

The preceding theorems are illustrated by the following examples:

Example 1

This example illustrates Theorem (i). Suppose *f(x)*= *U*(0,600), *P* = 20, *S* = 10, *R* = 6, C_1 = 12 and C_2 = 18, when the available fund is unlimited, the solutions are Q_1 = 360, Q_2 = 0 and $C_1Q_1+C_2Q_2=4320$ (see Appendix A).

From $F(\frac{T}{C_1})\leq\frac{(P+S)(C_2-C_1)}{(P+S)(C_2-C_1)+RC_1}$ and $F(\frac{T}{C_1})\leq\frac{P+S-C_1}{P+S}$, one has

$T\leq 5143$ and $T\leq 4320$ respectively. The intersection of $T\leq 5143$ and $T\leq 4320$ is $T\leq 4320$. Therefore, when $T\leq 4320$, the solution is $Q_1>0$ (i.e., $Q_1=T/C_1$) and $Q_2=0$. Numerical data for Theorem (i) are given in Table 1 and illustrated in Fig. 1.

Example 2

This example illustrates Theorem (i) and (iii). Suppose *f(x)*= *U*(0,600), *P* = 20, *S* = 10, *R* = 6, C_1 = 12 and C_2 = 15, when the available fund is unlimited (see Appendix A), the solutions are Q_1 = 300, Q_2 = 75 and $C_1Q_1+C_2Q_2=4725$.

From $F(\frac{T}{C_1}) \le \frac{(P+S)(C_2-C_1)}{(P+S)(C_2-C_1)+RC_1}$ and $F(\frac{T}{C_1}) \le \frac{P+S-C_1}{P+S}$, one has

$T \le 4000$ and $T \le 4320$ respectively. Therefore, when $T \le 4000$, the solution is $Q_1 > 0$ (i.e., $Q_1 = \frac{T}{C_1}$) and $Q_2 = 0$; when $4000 < T < 4725$, the solution is $Q_1 > 0$ and $Q_2 > 0$. In the case when $T = 4600$, the solution of Q_1 and Q_2 is 306 and 62 respectively. It is noted that when $C_1 \le C_2$ and the available fund is small ($T \le 4000$), the optimal strategy is to buy Q_1 only; when the available fund is abundant ($4000 < T < 4725$), it is better to buy both Q_1 and Q_2. Numerical data for Theorem (i) and (iii) are given in Table 2 and illustrated in Fig. 2.

Example 3

This example illustrates Theorem (ii). Suppose $f(x)= U(0,600)$, P = 20, S = 10, R = 6, C_1 = 12 and C_2 = 8, when the available fund is unlimited (see Appendix A), the solution is $Q_1 = 0$, $Q_2 = 550$ and $C_1Q_1 + C_2Q_2 = 4400$.

From $F(\frac{T}{C_2}) \le \frac{P+S-C_2}{P+S-R}$, one has $T \le 4400$.

Therefore, when $T \le 4400$, the solution is Q_1= 0 and $Q_2 > 0$ (i.e., $Q_2 = T/C_2$).

Example 4

This example illustrates the sensitivity analysis of the purchase price through return policy when the available limited fund is fixed at $T = 4000$. Letting $f(x)= U(0,600)$, P = 20, S = 10, R = 6 and C_1 = 12, the result of the sensitivity analysis is given in Table 3 and illustrated in Fig. 3 When $C_2 < 6.938$, the inventory fund needed when the available fund is unlimited is less than 4000. Therefore, when C_2 is not greater than C_1 (i.e., $6.938 \le C_2 \le 12$), the optimal strategy is to buy Q_2 only; when C_2 increases slightly (i.e., $12 < C_2 < 15$), the optimal strategy is to buy both Q_1 and Q_2. However, when C_2 increases significantly ($C_2 \ge 15$), it is more economical to buy Q_1 only.

Table 1: Some numerical data for Theorem (i) with C_2=18

T	Q_1	Q_2	Remark
≥ 4320	360	0	Unlimited budget
4300	358	0	$C_2 > C_1$, $F(\frac{T}{C_1}) \le \frac{P+S-C_1}{P+S}$
4000	333	0	
3500	292	0	
3000	250	0	
2000	167	0	(i.e., $T \le 4320$), $F(\frac{T}{C_1}) \le \frac{(P+S)(C_2-C_1)}{(P+S)(C_2-C_1)+RC_1}$ (i.e., $T \le 5143$)

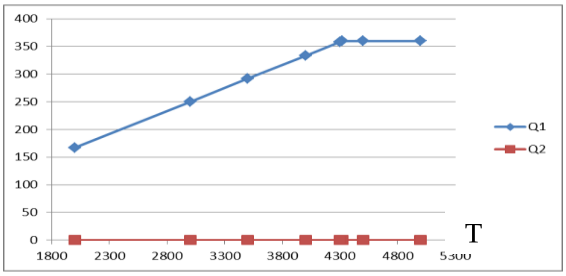

Figure 1: Vendor's lot sizes with various budgets when a higher price C_2=18

Table 2: Some numerical data for Theorem (i) and (iii) with C_2=15

T	Q_1	Q_2	Remark
≥ 4725	300	75	Unlimited budget
4700	301	72	$C_2 > C_1$, $F(\frac{T}{C_1}) > \frac{(P+S)(C_2-C_1)}{(P+S)(C_2-C_1)+RC_1}$ (i.e., $T > 4000$)
4600	306	62	
4500	310	52	
4000	333	0	$C_2 > C_1$, $F(\frac{T}{C_1}) \le \frac{P+S-C_1}{P+S}$ (i.e., $T \le 4320$) $F(\frac{T}{C_1}) \le \frac{(P+S)(C_2-C_1)}{(P+S)(C_2-C_1)+RC_1}$ (i.e., $T \le 4000$)
3500	292	0	
3000	250	0	
2500	208	0	

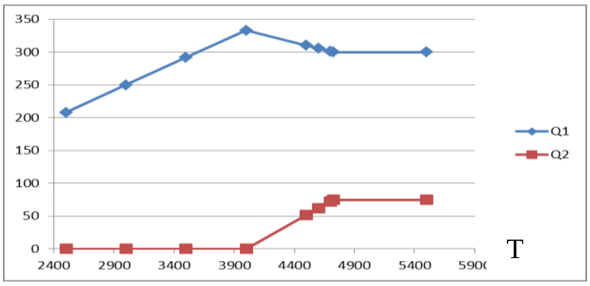

Figure 2: Vendor's lot sizes with various budgets when a lower price C_2=15

Table 3: Sensitivity analysis of the purchase price with return policy when $T = 4000$

C_2	Q_1	Q_2	Remark
< 6.938	0	>576	Unlimited budget
7	0	571	$C_2 \le C_1$, $F(\frac{T}{C_2}) \le \frac{P+S-C_2}{P+S-R}$ (i.e., $6.938 \le C_2 \le 23$)
9	0	444	
12	0	333	
12.1	14	317	$C_2 > C_1$,
13	132	185	

14	245	75	$F(\frac{T}{C_1}) > \frac{(P+S)(C_2-C_1)}{(P+S)(C_2-C_1)+RC_1}$ (i.e., $C_2 < 15$)
15	333	0	$C_2 > C_1$, $F(\frac{T}{C_1}) \le \frac{P+S-C_1}{P+S}$ (i.e., $10 \le C_1 \le 20$), $F(\frac{T}{C_1}) \le \frac{(P+S)(C_2-C_1)}{(P+S)(C_2-C_1)+RC_1}$ (i.e., $C_2 \ge 15$)
16	333	0	
17	333	0	
18	333	0	

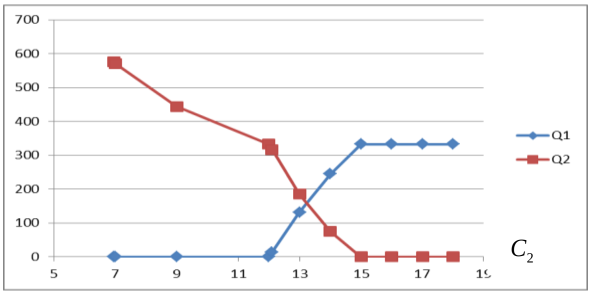

Figure 3: Vendor's lot sizes with various C_2 when $T = 4000$

4 Concluding Remark

This study analyzes the vendor's replenishment strategy in a newsboy model with limited budget. We show that there exists a set of conditions under which the vendor's optimal strategy will change based on the amount of available budget for product procurement. If the purchase price with return policy is greater than the price of outright purchase and the vendor has a small amount of available budget, only outright purchase will be optimal. However, if the budget reaches a certain threshold amount, mixed strategies where items are obtained by outright purchase and with return policy are used. We also show that when the purchase price with return policy is not greater than the outright purchase, the vendor will only obtain the items with return policy.

When the available budget is fixed, three conditions to obtain the optimal strategies are derived. The first condition is when the purchase price with return policy is not greater than the price with outright purchase; it is better to buy the items with return policy only. The second condition is when the purchase price with return policy is greater than the price of outright purchase over a certain threshold value; it is better to choose outright purchase only. The last condition is when the purchase price with return policy is between the two conditions, it is better to choose a mixture of items with outright purchase and return policy.

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Appendix A

Maximize EP

Subject to:

$-Q_1 \le 0$

$-Q_2 \le 0$

where *EP* is stated in (1)

Solution:

Equating the first derivatives of *EP* with respect to Q_1 and Q_2, one has

$$P+S-C_1-F(Q_1+Q_2)(P+S-R)-F(Q_1)R=0 \qquad \text{(A1)}$$

and

$$P+S-C_2-F(Q_1+Q_2)(P+S-R)=0 \qquad \text{(A2)}$$

Let the solution of the two simultaneous equations (A1) and (A2) be denoted as $(Q_1^{\#}, Q_2^{\#})$.

If $Q_1^{\#} > 0$ and $Q_2^{\#} > 0$, then the optimal solution, denoted by (Q_1 and Q_2), is $Q_1 = Q_1^{\#}$ and $Q_2 = Q_2^{\#}$.

If $Q_1^{\#} \le 0$ and $Q_2^{\#} > 0$, then the optimal solution is derived by the following steps:

(i) Let $Q_1 = 0$

(ii) After substituting $Q_1 = 0$ into (A2), one has

$$Q_2 = F^{-1}(\frac{P+S-C_2}{P+S-R}) \qquad \text{(A3)}$$

If $Q_1^{\#} > 0$ and $Q_2^{\#} \le 0$, the following steps derive the optimal solution:

(iii) Let $Q_2 = 0$

(iv) After substituting $Q_2 = 0$ into (A1), one has

$$Q_1 = F^{-1}(\frac{P+S-C_1}{P+S}) \qquad \text{(A4)}$$

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