

Decomposition of Multi-Objective Evolutionary Algorithm based on Estimation of Distribution

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Abstract: Decomposition of multi-objective evolutionary algorithm has better distribution, but the number of groups will increase dramatically as the target number increases, seriously affecting the efficiency of the algorithm. This paper presents a decomposition of multi-objective evolutionary algorithm based on estimation of distribution, the basic idea of which is: to decompose multiple objectives into several single objective first and then to establish the probability model for every single objective based on the idea of estimation of distribution, generating the solution by sampling. Numerical analysis and experiments show that the solution of the new algorithm not only has better diversity and uniformity, but also the computational complexity of the algorithm is significantly lower than the decomposition of multi-objective evolutionary algorithm, especially for optimization of three goals.

Keywords: Multi-objective optimization, evolutionary algorithm, decomposition strategy, estimation of distribution.

1 Introduction

Since 1985, Schaffer made the first multi-objective evolutionary algorithms Vector Evaluated Genetic Algorithms (VEGA) [1], scholars of all Nations have given different evolutionary multi-objective optimization algorithm one after another, among which Multi-objective Genetic Algorithm (MOGA) [2] by Fonseca and Fleming, Non-Dominated Sorting Genetic Algorithm (NSGA, NSGAI) [3] by Srinivas and Deb, Strength Pareto Evolutionary Algorithm (SPEA, SPEA2) [4] by Zitzler and Thiele, Niche Pareto Genetic Algorithm (NPGA, NPGA2) [5] by Horn and Nafpliotis are typical.

From 2003 up to now, new characteristics of research on evolutionary multi-objective algorithm could have been seen. Some new dominant mechanisms and evolutionary mechanisms have emerged, which is distinct from traditional Pareto-dominated mechanisms. Laumanns and Deb put forward Pareto-Adaptive ϵ dominance [6], and Alfredo and Coello Coello Pareto-Adaptive ϵ dominance [7], Zhang and Zhou Regularity Model Based Multi-Objective Estimation of Distribution Algorithm (RM-MEDA) [8], Zhang and Li Multi-Objective Evolutionary Algorithm Based on Decomposition (MOEA/D) [9] by combining evolutionary algorithms with traditional mathematical

programming methods. At present, the new dominant mechanisms, new evolution mechanism, high-dimensional multi-objective optimization and multi-objective optimization test have been the focus of research on evolutionary multi-objective optimization algorithm.

This paper researched the principle of RM-MEDA & MOEA/D, proposed Regularity Model Based Multi-Objective Estimation of Distribution Algorithm and Decomposition Algorithm, use performance analysis and numerical simulation to prove the suppose.

2 Decomposition of multi-objective evolutionary algorithm and estimation of distribution algorithms

The basic idea of solving multi-objective optimization by traditional optimization algorithms is to transform various sub-goals after a weighted combination into single-objective optimization. Multi-objective evolutionary algorithms is to regard all targets as a whole so as to find as many representative evenly distributed Pareto optimal solutions as possible by appropriate evolutionary approach.

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Zhang and Li introduced the idea of traditional multi-objective optimization algorithm into the multi-objective evolution algorithm, proposing MOEA/D. Decomposition of multi-objective evolutionary algorithm is to decompose the multi-objective optimization into several simple-target optimization, and at the same time evolve them as a community, each generation of community of the evolution being composed by the current optimal solutions of each sub target. In MOEA/D, optimization of each sub target only needs individual information of its neighbor. The neighbor relation among sub targets is decided by the distance between each weight vector of objective function. The more close the distance of weight vector between two sub-goals is, the more approximate their solutions will inevitably be. Thus it can be seen that whether weight vector of each objective function will fill the entire space and whether the distribution is even is the key problem in MOEA/D.

Estimation of distribution algorithms is an emerging branch in the field of evolutionary computation, which is the combination of evolutionary algorithms and statistical learning. The algorithms are to build individual distribution probability models in the solution space by statistical learning, and then to evolve the model with the idea of evolution. In Estimation of distribution algorithm there is no crossover and mutation operations, which is replaced by estimating probability model of solution space and generating the new group by sampling the probability model. Estimation of distribution algorithms grasps the direction of group evolution from a macro which can solve high-dimensional multi-objective optimization so as to reduce the time complexity effectively at the initial stages. In multi-objective optimization, it is impossible to optimize multiple goals at the same time, so the goal of optimization is to find a set of Pareto optimal solutions, which can be solved appropriately since estimation of distribution algorithm is suitable for solving such problems because of its inherent parallelism.

But basic difference between multi-objective optimization and the simple-target optimization lies in taking the influence of multiple goals into consideration in the meanwhile during the process of seeking optimization, thus causing the entire community to evolve in the direction that does not evolution which does not increase the function value of multiple goals. When solving simple-target optimization by estimation of distribution algorithms, the renewal of probability vector is carried on according to the distribution of the main body of the highest part of adaptation value, then the adaptation value of multiple goals should be considered simultaneously in the renewal of probability vector in multi-objective optimization. Therefore the order may be listed separately according to the adaptation value of each objective function to select multiple sub-communities as the best representatives of different goal compatibility separately, just as model worker delegation in various

trades, and then renew the probability vector according to various subgroups.

With the development of estimation of distribution algorithm and the advantages revealed when this algorithm is used to solve certain problems, some multi-objective optimization algorithms based on the idea of estimation of distribution have been proposed one after another. Khan combined selective strategies in NSGA-II and Bayesian optimization algorithm (BOA) together, proposed multiple-objective Bayesian optimization algorithm (MBOA) [10], and achieved better results than NSGA-II. Laumanns combines SPEA2 and BOA for solving multi-objective Knapsack problem [11]. Scholars such as Zhang and Zhou introduce RM-MEDA [8], a more classic algorithm for solving multi-objective optimization by estimation of distribution algorithm.

Algorithm flow chart of estimation of distribution algorithms and decomposition of multi-objective evolutionary algorithm are shown in Fig.1 and Fig.2.

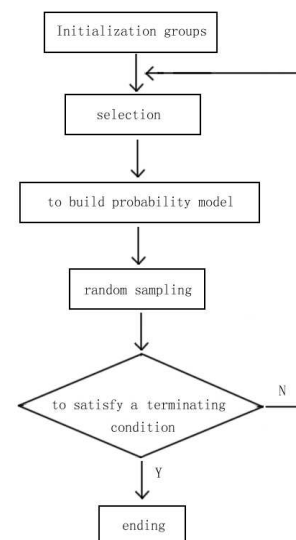


Fig. 1: Algorithm flow chart of estimation of distribution algorithms

3 Decomposition of multi-objective evolutionary algorithm based on estimation of distribution

In MOEA/D, community size N and the weight vector $\lambda^1, \dots, \lambda^N$ are controlled by parameter H , namely every weight value of each objective function in each weight vector is the got from $\{0/H, 1/H, \dots, H/H\}$, but the

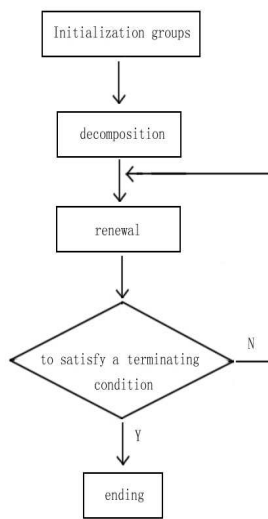


Fig. 2: decomposition of multi-objective evolutionary algorithm

number of weight vector $N = C_{H+m-1}^{m-1}$. When target figure m increases, N will increase rapidly; this will affect the operating efficiency of MOEA/D.

Considering that estimation of distribution algorithms can grasp the macro-group evolutionary direction, that there is no crossover and mutation operations, and that it can reduce time complexity very effectively during the initial phase, the paper presents DE-MOEA/D based on estimation of distribution, which uses probability models to evolve groups on the basis of the framework of MOEA/D.

There are a variety of methods that can transfer multi-objective optimization into a series of sub-optimization close to PF, such as boundary intersection method, Tchebycheff decomposition method, weights summation method. This algorithm is to use Tchebycheff decomposition method [12].

In each generation of populations, the quantity which the DE-MOEA/D algorithm needs to preserve includes:

- (1) The population which has N individuals $x^1, \dots, x^N \in \Omega$, in which x_i is the current solution of the i th question;
- (2) FV^1, \dots, FV^N , in which FV^i is the function value of x_i , namely $FV^i = F(x_i), i = 1, \dots, N$;
- (3) $z = (z_1, \dots, z_m)^T$, in which z_i is the optimal solution to the objective function f_i which is found currently;
- (4) Exterior population (EP) is used to preserve the non-control solution found in search process.

Processes of DE - MOEA/D are as follows:

Step1: Initialization:

Step1.1 to set $EP = \emptyset$;

Step1.2 to calculate the Euclid distance between each two weight vectors and select the recent election T as its neighbors for each weight vector. That is, for each

$i = 1, \dots, N$, there is $B(i) = \{i_1, \dots, i_T\}$ in which $\lambda^{i_1}, \dots, \lambda^{i_T}$ is the nearest T weight vector to λ^i ;

Step1.3 to generate randomly the initial population $x^1, \dots, x^N \in \Omega$ and set $FV^i = F(x^i)$;

Step 1.4 to set the initial value $z = (z_1, \dots, z_m)^T$ according to the specific questions.

Step2: Renewal:

Step2.2 to calculate the different objective function values which are in correspondence with subgroups;

Step2.3 to establish a reasonable probability model according to the sub-group controlled by T closest weight vectors, and get new sample y based on this probability model;

Step2.4 renew z ; for each $j = 1, \dots, m$, if $z_j < f_j(y')$, $z_j = f_j(y')$;

Step2.5 renew the groups; if $g(y'|\lambda, z) \leq g(x^j|\lambda, z)$, for all $j \in B(i), x^j = y', F(v^j) = F(y')$;

Step2.6 renew EP; move all the solution controlled by $F(y')$ out of EP; if $F(y')$ is not dominated by any solution in EP, then move $F(y')$ into EP.

Step3: stop judgment; if the stopping criterion is satisfied it stops and output EP, otherwise return to Step2.

4 Numerical experiments

The following is numerical calculation on two standard test functions DTLZ1 and DTLZ2 by DE-MOEA/D and comparative analysis with NSGA-II to test the performance of DE-MOEA/D.

(1) DTLZ1 test function

$$\begin{aligned} \min f_1(X) &= \frac{1}{2}x_1x_2\dots x_{M-1}[1 + g(X_M)] \\ \min f_2(X) &= \frac{1}{2}x_1x_2\dots(1 - x_{M-1})[1 + g(X_M)] \\ &\vdots \\ \min f_{M-1}(X) &= \frac{1}{2}x_1(1 - x_2)[1 + g(X_M)] \\ \min f_M(X) &= \frac{1}{2}(1 - x_1)[1 + g(X_M)] \\ &0 \leq x_i \leq 1, i = 1, 2, \dots, n. \end{aligned}$$

In which

$$g(x_M) = 100[|x_M| + \sum_{x_i \in X_M} ((x_i - 0.5)^2 - \cos[20\pi(x_i - 0.5)])].$$

When the test function achieves Pareto optimal boundary, all the corresponding x_i belonging to x_m is 0.5, objective function value is on a linear hyper plane which satisfy $\sum_{m=1}^M f_m^* = 0.5$. It is difficult for this problem to converge to Pareto optimal boundary because the corresponding search space includes $11^k - 1 PF_{local}$, which will converge to the local optimal boundary generally. PF_{true} When function value $M = 3$ is shown in Fig.3.

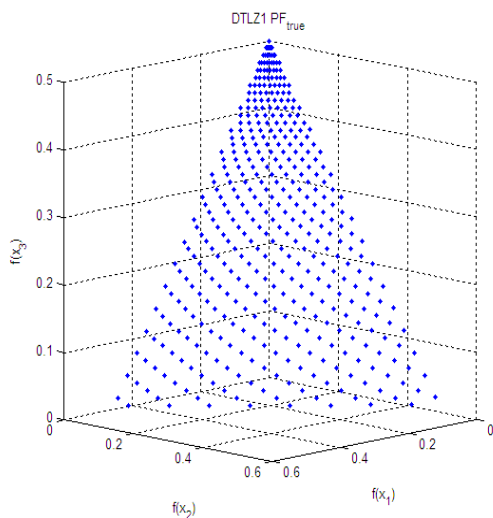


Fig. 3: DTLZ1 of PFtrue

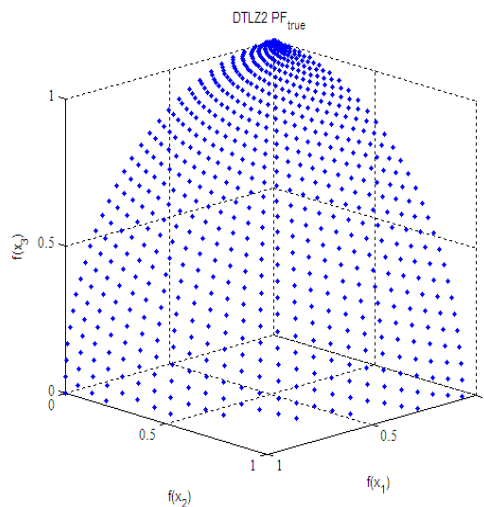


Fig. 4: DTLZ2 of PFtrue

(2) Test function in DTLZ2

$$\begin{aligned} \min f_1(X) &= [1 + g(X_M)] \cos(x_1 \pi/2) \dots \cos(x_{M-2} \pi/2) \cos(x_{M-1} \pi/2) \\ \min f_2(X) &= [1 + g(X_M)] \cos(x_1 \pi/2) \dots \cos(x_{M-2} \pi/2) \sin(x_{M-1} \pi/2) \\ \min f_3(X) &= [1 + g(X_M)] \cos(x_1 \pi/2) \dots \sin(x_{M-2} \pi/2) \\ &\vdots \\ \min f_M(X) &= [1 + g(X_M)] \sin(x_1 \pi/2) \\ 0 \leq x_i \leq 1, i &= 1, 2, \dots, n \end{aligned}$$

In which $g(x_M) = \sum_{x_i \in X_M} (x_i - 0.5)^2$

All x_i , belonging to x_M , to which Pareto optimal boundary of the test function corresponds is 0.5, and the objective function value satisfies $\sum_{m=1}^M f_m^2 = 1$. This problem can be used to test operating capability of one MOEA. PF_{true} when function value $M = 3$ is a unit sphere within the first octant, as is shown in Fig.4.

It can be seen clearly from Figs.6 to 8 that DE-MOEA/D is obviously superior to NSGA-II in both distribution and uniform of Pareto optimal solutions, which shows that DE-MOEA/D inherit fully the advantages of distribution and uniformity of MOEA/D.

In order to test the convergence properties of DE-MOEA/D, the average value of IGD data of the last generation when DE-MOEA/D and MOEA/D are operated for 20 times is given in Table 1.

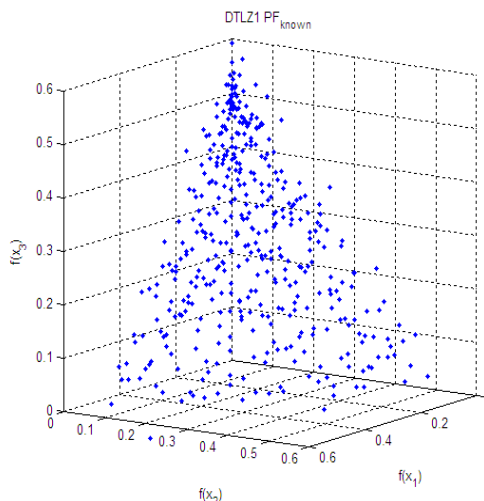


Fig. 5: DTLZ1s Pareto Optimal boundary with NSGA-II

The data in Table 1 clearly shows that convergence of DE-MOEA/D is faster than that of MOEA/D, that is, DE-MOEA/D is superior to MOEA/D in performance of calculation complexity.

5 Conclusion

This paper improves MOEA/D in view of the defect that calculation complexity increases rapidly when the number of objectives is rather large, proposes a new decomposition of multi-objective evolutionary algorithm based on estimation of distribution, and makes a

Table 1: DE-MOEA/D and MOEA/Ds average IGD data

Algorithm	DTLZ1			DTLZ2		
	Average	Minimum	Standard deviation	Average	Minimum	Standard deviation
DE-MOEA/D	0.0254	0.0201	0.0018	0.0265	0.0259	0.0008
MOEA/D	0.0350	0.0288	0.0042	0.0389	0.0358	0.0011

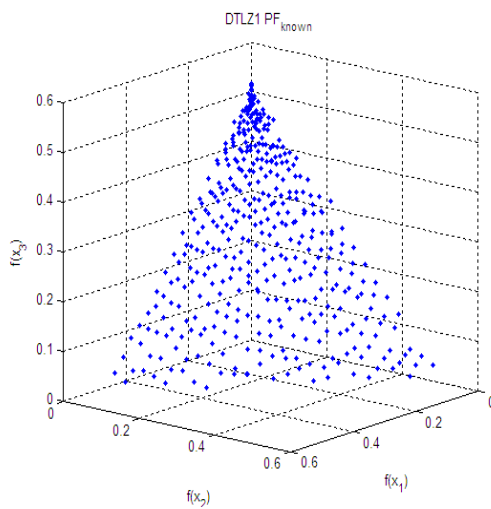


Fig. 6: DTLZ1s Pareto Optimal boundary with DE-MOEA/D

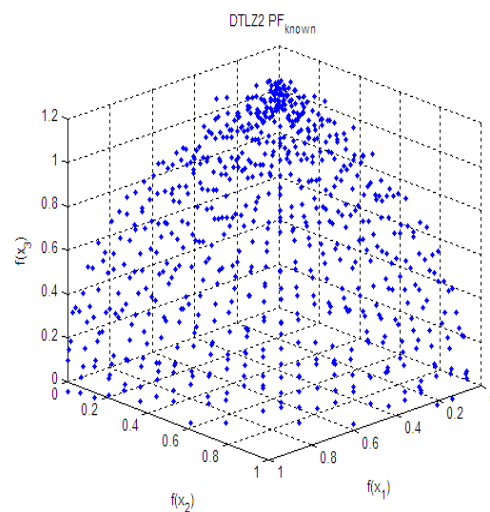


Fig. 8: DTLZ2s Pareto Optimal boundary with DE-MOEA/D

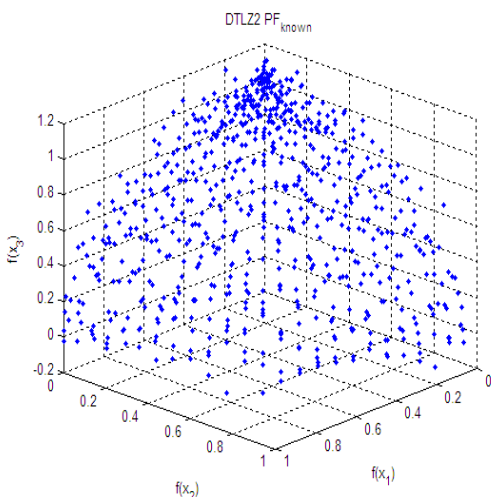


Fig. 7: DTLZ2s Pareto Optimal boundary with NSGA-II

comparative analysis with the current most outstanding NSGA-II.

Numerical analysis and experimental results show that the new algorithm is equivalent with MOEA/D but significantly superior to NSGA-II in distribution and uniformity of Pareto solution; As for the three-objective optimization, calculation complexity of the new algorithm is lower than that of MOEA/D because there is no traditional crossover and mutation operation in the new algorithm but a probability model which is used to generate evolutionary solution.

But some problems come up when the new algorithm is used in optimization of four objectives and the effect of optimization is less ideal. How to further improve the

performance of decomposition of multi-objective evolutionary algorithm based on estimation of distribution to enable it to solve the higher dimensional multi-objective optimization will be our further research.

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Quantum information.

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