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Computing the Median and Range for Power Function Distributions

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Abstract: Median and range of a random sample are measures based on order statistics which are descriptive of the central tendency and dispersion of the population, respectively. In this paper, we obtain the median and range for order statistics from non-identical standard power distributions functions. Then, the median and range for identical standard power functions distributions and uniform distributions functions are given. Finally numerical results of the median and range are presented.

Keywords: Order statistics, median, range, standard power distributions, uniform distributions

1 Introduction

Arnold and Villaseñor [1] presented some preliminary results relating to the Lorenz ordering on sample medians and means. Gibbons and Chakraborti [2] at first consider the uniform distribution over (0,1) and then obtained the median when all X_i's are independent and identically distributed. Beg [3] generalized results by obtained Joshi and Balakrishnan [4] when the variables were independent but not assumed to be identically distributed using permanents. Barakat and Abdelkader [5] established new representations, identities and recurrence relations of order statistics arising from general independent non-identically distributed random variables. To keep abreast of recent developments in order statistics, see [6,7,8,9,10,11,12,13,14,15,16].

In this paper, we derive expressions for the median and range when all X_i's are independent but not assumed to be identically distributed. Then the median and range for order statistics from non-identical standard power function distributions are obtained. Also numerical results of the median and range for independent and identically distributed standard power function distributions and uniform distributions are presented.

Let $X_1, X_2, ..., X_n$ be n independent and non-identically distributed random variables with probability density functions (pdf's) $f_1(x), f_2(x), ..., f_n(x)$ and cumulative distribution functions (cdf's) $F_1(x), F_2(x), ..., F_n(x)$ and let $X_{1:n} \leq X_{2:n} \leq .... \leq X_{n:n}$ be the corresponding order statistics. David [17] has shown that the pdf's of the rth order statistics $X_{r:n}$ $1 \leq r \leq n$ is

$$f_{r:n}(x) = \frac{1}{(r-1)!(n-r)!} \sum_p \left[\prod_{a=1}^{r-1} F_{i_a}(x) \right] f_{i_r}(x) \cdot \left[\prod_{b=r+1}^{n} (1 - F_{i_b}(x)) \right], \tag{1}$$

where $-\infty < x < \infty$ and $\sum_p$ denotes the summation over all $n!$ permutations $(i_1, i_2, ..., i_n)$ of $(1, 2, ..., n)$. Moreover Childs and Balakrishnan [18] have shown that the joint pdf's of the rth and sth order statistics $X_{r:n}$ and $X_{s:n}$ $1 \leq r < s \leq n$ is

$$f_{r,s:n}(x,y) = \frac{1}{(r-1)!(s-r-1)!(n-s)!} \sum_p \left[\prod_{a=1}^{r-1} F_{i_a}(x) \right] \cdot f_{i_r}(x) \left[\prod_{b=r+1}^{s-1} (F_{i_b}(y) - F_{i_b}(x)) \right] \cdot f_{i_s}(y) \left[\prod_{c=s+1}^{n} (1 - F_{i_c}(y)) \right], \tag{2}$$

where $-\infty < x < y < \infty$. Alternatively, the densities in Eq. (1) and Eq. (2) can be written in terms of permanents of matrices (see Vaughan and Venables [19]).

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Suppose that the random variable $X_i (i = 1, 2, ..., n)$ has the cdf's given by

$$F_i(x) = \begin{cases} 0 & x < a, \\ e^{\frac{1}{\lambda_i [h(b) - h(x)]}} & a \leq x < b, \\ 1 & x \geq b. \end{cases} \tag{3}$$

where λ_i is a positive parameter, $h(x)$ is assumed to be decreasing, continuous and differentiable function on (a, b) with $h(b) \geq 0$ and $h_{a_+} = \infty$. Different choices of $h(x)$ lead to distributions that are important in life testing as well as other areas of statistics (see more details Ahmad [20]). We note that the cdf's Eq. (3) satisfies the differential equation

$$f_i(x) = -\frac{1}{\lambda_i} g'(x) F_i(x). \tag{4}$$

Put $h(b) = 0$, $h(x) = -\ln(\frac{x}{g})$, $x \in [0, g]$, $g > 0$ and $\lambda_i = \frac{1}{\alpha_i}$ (α_i are positive integers). Then the cdf's Eq. (3) and pdf's Eq. (4), respectively, reduce to the power function distributions in the form

$$F_i(x) = \left(\frac{x}{g}\right)^{\alpha_i}, x \in [0, g] \tag{5}$$

and

$$f_i(x) = \frac{\alpha_i}{g} \left(\frac{x}{g}\right)^{\alpha_i - 1}, x \in [0, g]. \tag{6}$$

2 Distributions of the Median and Range

The median and range of a random sample are measures based on order statistics which are descriptive of the central tendency and dispersion of the population, respectively. For n odd, the median of a sample has the pdf's Eq.(1) with $r = \frac{n+1}{2}$. If n is even and a unique value is desired for the sample median M, the usual definition is

$$M = \frac{X_{\frac{n}{2}:n} + X_{\frac{n}{2}+1:n}}{2}$$

so that the distribution of what must be derived from the joint pdf's of these two-order statistics. Letting $n = 2m$, from Eq.(2) we have for all $x < y$

$$f_{m,m+1:n}(x,y) = \frac{1}{[(m-1)!]^2} \sum_p \left[\prod_{a=1}^{m-1} F_{i_a}(x)\right] \cdot f_{i_m}(x) f_{i_{m+1}}(y) \left[\prod_{c=m+2}^{2m} (1 - F_{i_c}(y))\right],$$

Upon making the transformation $x = 2u - v$, $y = v$ and using the method of Jacobians, the pdf's of the median for $n = 2m$ is

$$f_M(u) = \frac{2}{[(m-1)!]^2} \sum_p \int_u^{\infty} \left[\prod_{a=1}^{m-1} F_{i_a}(2u - v)\right] \cdot f_{i_m}(2u - v) f_{i_{m+1}}(v) \left[\prod_{c=m+2}^{2m} (1 - F_{i_c}(v))\right] dv, \tag{7}$$

where $u \leq v$. A similar procedure can be used to obtain the distribution of the range, defined as $R = X_{n:n} - X_{1:n}$ [1, 17]. From Eq. (2) the joint pdf's of $X_{1:n}$ and $X_{n:n}$ is

$$f_{1:n,n}(x,y) = \frac{1}{[(n-2)!]} \sum_p f_{i_1}(x) \left[\prod_{b=2}^{n-1} (F_{i_b}(y) - F_{i_b}(x))\right] f_{i_n}(y),$$

where $x < y$. Upon making the transformation $x = v - u$, $y = v$ and by integration out of v, the pdf's of the range is

$$f_R(u) = \frac{1}{[(n-2)!]} \sum_p \int_{-\infty}^{\infty} (f_{i_1}(v-u)) \cdot \left[\prod_{b=2}^{n-1} (F_{i_b}(v) - F_{i_b}(v-u))\right] f_{i_n}(v) dv, \tag{8}$$

where $u < v$. Let $F_{i_l(x)}$ and $f_{i_l(x)}$ $(l = 1, 2, ..., n)$ be as in Eq. (5) and Eq. (6). Then the integrant in Eq. (7) is nonzero for the intersection of the regions $0 < 2u - v < g$ and $0 < v < g$. Therefore the region of integration is the intersection of the three regions $u < v$, $\frac{v}{2} < u < \frac{g+v}{2}$ and $0 < v < g$. Finally we see that the limits on the integral in Eq. (7) must be $u < v < 2u$ for $0 < u < \frac{g}{2}$ and $u < v < g$ for $\frac{g}{2} < u < g$. Similarly the integrant in Eq. (8) is $0 < u < v < g$ for $0 < v - u < g$ and $0 < v < g$.

Now we are ready to derive the median and range for order statistics from non-identical power function distributions. At first we derive pdf's of the median, respectively, for $0 < u \leq \frac{g}{2}$ and $\frac{g}{2} < u < g$. Then we derive pdf's of the range for $0 < u < v < a$.

Theorem 1.

For $0 < u \leq \frac{g}{2}$, we have

$$f_M(u) = \frac{1}{[(m-1)!]^2} \sum_p \frac{\alpha_{i_m} \alpha_{i_{m+1}}}{g^{\left(\sum_{a=1}^{2m} \alpha_{i_a}\right)}} \sum_{k_1=0}^{\lambda} \binom{\lambda}{k_1} \sum_{k_2=0}^{m-l} (-1)^{k_1+k-2} \cdot \sum_{\substack{n_\zeta = m-k_2-1 \\ n_{\zeta'} = k_2}} \frac{g^{\left(\sum_{c=1}^{m-k_2-1} \alpha_{\zeta_c}\right)} u^{\left(\sum_{a=1}^{m} \alpha_{i_a} + \sum_{c=1}^{k_2} \alpha_{\zeta'_c} + \alpha_{i_{m+1}} - 1\right)}}{k_1 + \sum_{c=1}^{k_2} \alpha_{\zeta'_c} + \alpha_{i_{m+1}}} \cdot \left[2^{\left(\sum_{a=1}^{m} \alpha_{i_a} + \sum_{c=1}^{k_2} \alpha_{\zeta'_c} + \alpha_{i_{m+1}}\right)} - 2^{\left(\sum_{a=1}^{m} \alpha_{i_a} - k_1\right)}\right] \tag{9}$$

where $\lambda = \sum_{a=1}^{m} \alpha_{i_a} - 1$, $\sum_{\substack{n_\zeta = m-k_2-1 \\ n_{\zeta'} = k_2}}$ denotes the sum over all $\binom{m-1}{m-k_2-1}$ subsets $\zeta = \{\zeta_1, \zeta_2, ..., \zeta_{m-k_2-1}\}$ and

$\zeta' = \{\zeta'_1, \zeta'_2, ..., \zeta'_{k_2}\}$ of $\{i_1, i_2, ..., i_{m-1}\}$.

Proof From using Eq. (5) and Eq. (6) in Eq. (7), we get

$$f_M(u) = \frac{2}{[(m-1)!]^2}\sum_p \frac{\alpha_{i_m}\alpha_{i_{m+1}}}{g^2 g^{\lambda} g^{(\alpha_{i_{m+1}}-1)} g^{(\sum_{c=m+2}^{2m}\alpha_{i_c})}} \cdot \int_u^{2u}(2u-v)^{\lambda} v^{\alpha_{i_{m+1}}-1}\prod_{c=m+2}^{2m}(g^{\alpha_{i_c}}-v^{\alpha_{i_c}})dv.$$

Upon using

$$(2u-v)^{\lambda} = \sum_{k_1=0}^{\lambda}\binom{\lambda}{k_1}(-1)^{k_1}(v)^{k_1}(2u)^{\lambda-k_1}$$

and

$$\prod_{c=m+2}^{2m}(g^{\alpha_{i_c}}-v^{\alpha_{i_c}}) = \sum_{k_2=0}^{m-1}(-1)^{k_2}\sum_{\substack{n_\zeta=m-k_2-1\\ n_{\zeta'}=k_2}}\prod_{c=1}^{m-k_2-1} g^{(\alpha_{\zeta_c})}\prod_{c=1}^{k_2} v^{(\alpha_{\zeta'_c})}$$

we get

$$f_M(u) = \frac{2}{[(m-1)!]^2}\sum_p \frac{\alpha_{i_m}\alpha_{i_{m+1}}}{g^{(\sum_{a=1}^{2m}\alpha_{i_a})}}\sum_{k_1=0}^{\lambda}\binom{\lambda}{k_1}(2u)^{\lambda-k_1}\sum_{k_2=0}^{m-l}(-1)^{k_1+k_2} \cdot \sum_{\substack{n_\zeta=m-k_2-1\\ n_{\zeta'}=k_2}} g^{\left(\sum_{c=1}^{m-k_2-1}\alpha_{\zeta_c}\right)}\int_u^{2u} v^{\left(k_1+\alpha_{i_{m+1}}-1+\sum_{c=1}^{k_2}\alpha_{\zeta'_c}\right)}dv. \quad (10)$$

The proof of Eq. (9) follows from computing the integrant in Eq. (10) and after simple manipulations.

Theorem 2.

For $\frac{g}{2} < u < g$, we have

$$f_M(u) = \frac{1}{[(m-1)!]^2}\sum_p \frac{\alpha_{i_m}\alpha_{i_{m+1}}}{g^{(\sum_{a=1}^{2m}\alpha_{i_a})}}\sum_{k_1=0}^{\lambda}\binom{\lambda}{k_1}2^{(\sum_{a=1}^{m}\alpha_{i_a}-k_1)} \cdot \sum_{k_2=0}^{m-1}(-1)^{k_1+k_2}\sum_{\substack{n_\zeta=m-k_2-1\\ n_{\zeta'}=k_2}}\frac{g^{\left(\sum_{c=1}^{m-k_2-1}\alpha_{\zeta_c}+\sum_{c=1}^{k_2}\alpha_{\zeta'_c}+k_1+\alpha_{i_{m+1}}\right)}u^{(\lambda-k_1)}}{k_1+\sum_{c=1}^{k_2}\alpha_{\zeta'_c}+\alpha_{i_{m+1}}} - \frac{g^{\left(\sum_{c=1}^{m-k_2-1}\alpha_{\zeta_c}\right)}u^{\left(\sum_{a=1}^{m}\alpha_{i_1}+\sum_{c=1}^{k_2}\alpha_{\zeta'_c}+\alpha_{i_{m+1}}-1\right)}}{k_1+\sum_{c=1}^{k_2}\alpha_{\zeta'_c}+\alpha_{i_{m+1}}} \quad (11)$$

where λ and $\sum_{\substack{n_\zeta=m-k_2-1\\ n_{\zeta'}=k_2}}$ is as Eq.(9).

Proof. It can be obtained similarly as theorem 1.

Theorem 3.

For $0 < u < v < g$, we have

$$f_R(u) = \frac{1}{(n-2)!}\sum_p \frac{\alpha_{i_m}\alpha_{i_{m+1}}}{g^{(\sum_{a=1}^{n}\alpha_{i_a})}}\sum_{k_3=0}^{n-2}\sum_{\substack{n_\zeta=n-k_3-2\\ n_{\zeta'}=k_3}}\sum_{k_4=0}^{w}\binom{w}{k_4}(-1)^{k_3+k_4} \cdot \frac{u^{k_4}g^{\left(\sum_{b=1}^{n-k_3-2}\alpha_{\zeta_b}+\sum_{b=1}^{k_3}\alpha_{\zeta'_b}+\alpha_{i_1}+\alpha_{i_n}-k_4-1\right)} - u^{\left(\sum_{b=1}^{n-k_3-2}\alpha_{\zeta_b}+\sum_{b=1}^{k_3}\alpha_{\zeta'_b}+\alpha_{i_1}+\alpha_{i_n}-1\right)}}{\sum_{b=1}^{n-k_3-2}\alpha_{\zeta_b}+\sum_{b=1}^{k_3}\alpha_{\zeta'_b}+\alpha_{i_1}+\alpha_{i_n}-k_4-1} \quad (12)$$

where $w = \sum_{b=1}^{k_3}\alpha_{\zeta'_b}+\alpha_{i_1}-1$, $\sum_{\substack{n_\zeta=n-k_3-2\\ n_{\zeta'}=k_3}}$ denotes the sum over all $\binom{n-2}{n-k_3-2}$ subsets $\zeta = \{\zeta_1, \zeta_2, ..., \zeta_{n-k_3-2}\}$ and $\zeta' = \{\zeta'_1, \zeta'_2, ..., \zeta'_{k_3}\}$ of $\{i_1, i_2, ..., i_{n-2}\}$.

Proof. From using Eq. (5) and Eq. (6) in Eq. (8), we get

$$f_R(u) = \frac{1}{(n-2)!}\sum_p \frac{\alpha_{i_1}\alpha_{i_n}}{g^{(\sum_{a=1}^{n}\alpha_{i_a})}}\int_u^g (v-u)^{\alpha_{i_1}-1} \cdot \prod_{b=2}^{n-1}\left[v^{\alpha_{i_b}}-(v-u)^{\alpha_{i_b}}\right]v^{\alpha_{i_n}-1}dv.$$

Upon using

$$\prod_{b=2}^{n-1}\left[v^{\alpha_{i_b}}-(v-u)^{\alpha_{i_b}}\right] = \sum_{k_3=0}^{n-2}(-1)^{k_3} \cdot \sum_{\substack{n_\zeta=n-k_3-2\\ n_{\zeta'}=k_3}}\prod_{b=1}^{n-k_3-2}(v)^{\alpha_{\zeta_b}}\prod_{b=1}^{k_3}(v-u)^{\alpha_{\zeta'_b}}$$

we get

$$f_R(u) = \frac{1}{(n-2)!}\sum_p \frac{\alpha_{i_1}\alpha_{i_n}}{g^{(\sum_{a=1}^{n}\alpha_{i_a})}}\sum_{k_3=0}^{n-2}(-1)^{k_3} \cdot \sum_{\substack{n_\zeta=n-k_3-2\\ n_{\zeta'}=k_3}}\int_u^g v^{\left(\sum_{b=1}^{n-k_3-2}\alpha_{\zeta_b}+\alpha_{i_n}-1\right)}(v-u)^w dv.$$

The proof of theorem 3.follows from the relation

$$(v-u)^w = \sum_{k_4=0}^{w}\binom{w}{k_4}(-1)^{k_4}(v)^{w-k_4}$$

and after simple manipulations.

Table 1: Numerical values of the median of order statistics from uniform distributions.

$u \setminus n$	2	4	6	8	10	12	14	16	18	20
0.1	0.4	0.208	0.09192	0.03820	0.01536	0.00605	0.00235	0.00090	0.00034	0.00013
0.2	0.8	0.704	0.54144	0.39546	0.28079	0.19578	0.13479	0.09194	0.06226	0.04191
0.3	0.12	1.296	1.27656	1.20942	1.11948	1.02009	0.91905	0.82095	0.72843	0.64285
0.4	0.16	1.792	1.96608	2.10698	2.21708	2.30147	2.3647	2.41042	2.44153	2.46038
0.5	0.20	2.000	2.25	2.5	2.73437	2.95312	3.1582	3.35156	3.53485	3.70941
0.6	0.16	1.792	1.96608	2.10698	2.21708	2.30147	2.3647	2.41042	2.44153	2.46038
0.7	0.12	1.296	1.27656	1.20942	1.11948	1.02009	0.91905	0.82095	0.72843	0.64285
0.8	0.8	0.704	0.54144	0.39546	0.28079	0.19578	0.13479	0.09194	0.06226	0.04191
0.9	0.4	0.208	0.09192	0.03820	0.01536	0.00605	0.00235	0.0009	0.00034	0.00013

Table 2: Numerical values of the range of order statistics from uniform distributions.

$u \setminus n$	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0.1	1.8	0.54	0.108	0.018	0.0027	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.2	1.6	0.96	0.384	0.128	0.0384	0.0107	0.0028	0.0007	0.0001	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000
0.3	1.4	1.26	0.756	0.378	0.1701	0.0714	0.0285	0.011	0.0041	0.0015	0.0005	0.0001	0.0001	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000
0.4	1.2	1.44	1.152	0.768	0.4608	0.258	0.1376	0.7077	0.0353	0.0173	0.0083	0.0039	0.0018	0.0008	0.0003	0.0001	0.0000	0.0000	0.0000
0.5	1.0	1.5	1.5	1.25	0.9375	0.6562	0.4375	0.2812	0.1757	0.1074	0.0644	0.038	0.0222	0.0128	0.0007	0.0041	0.0023	0.0013	0.0007
0.6	0.8	1.44	1.728	1.728	1.5552	1.3063	1.045	0.8062	0.6046	0.4434	0.3192	0.2263	0.1584	0.1097	0.0752	0.0511	0.0345	0.0231	0.0154
0.7	0.6	1.26	1.764	2.058	2.1609	2.1176	1.9765	1.7788	1.5564	1.3316	1.1186	0.9253	0.7557	0.6104	0.4883	0.3874	0.3050	0.2386	0.1856
0.8	0.4	0.96	1.536	2.048	2.4576	2.7525	2.936	3.0198	3.0198	2.9527	2.8346	2.68	2.5013	2.3089	2.111	1.914	1.7226	1.5402	1.369
0.9	0.2	0.54	0.972	1.458	1.9683	2.48	2.976	3.4437	3.8742	4.2616	4.6025	4.8954	5.1402	5.3379	5.4904	5.6002	5.6702	5.7035	5.7035

3 Numerical Results

In this section, at first we obtain the median and range for identical power function distributions by setting $g=1$ and $\alpha_{i_1}=\alpha_{i_2}=\ldots=\alpha_{i_n}=\alpha$ and then we obtain the median and range for uniform distributions by setting $g=1$ and $\alpha=1$ finally we compute them for uniform distributions for n=2,4, ...,20.

Result 1. By setting $g=1$ and $\alpha_{i_1}=\alpha_{i_2}=\ldots=\alpha_{i_n}=\alpha$ in Eq. (9) for $0<u\leq\frac{1}{2}$, we have

$$f_M(u)=\frac{n!}{[(m-1)!]^2}\alpha^2\sum_{k_1=0}^{m\alpha-1}\binom{m\alpha-1}{k_1}\sum_{k_2=0}^{m-1}(-1)^{k_1+k_2}\cdot\binom{m-1}{k_2}\frac{u^{\alpha(m+k_2+1)-1}}{k_1+\alpha(k_2+1)}\left[2^{\alpha(m+k_2+1)}-2^{m\alpha-k_1}\right] \tag{13}$$

Result 2. By setting $g=1$ and $\alpha_{i_1}=\alpha_{i_2}=\ldots=\alpha_{i_n}=\alpha$ in Eq. (11) for $\frac{1}{2}<u\leq 1$, we have

$$f_M(u)=\frac{n!}{[(m-1)!]^2}\alpha^2\sum_{k_1=0}^{m\alpha-1}\binom{m\alpha-1}{k_1}2^{\alpha m-k_1}\sum_{k_2=0}^{m-1}(-1)^{k_1+k_2}\cdot\binom{m-1}{k_2}\left[\frac{u^{m\alpha+k_1-1}-u^{\alpha(m+k_2+1)-1}}{k_1+\alpha(k_2+1)}\right] \tag{14}$$

Result 3. By setting $g=1$ and $\alpha_{i_1}=\alpha_{i_2}=\ldots=\alpha_{i_n}=\alpha$ in Eq. (12) for $0<u<v<1$, we have

$$f_R(u)=n(n-1)\alpha^2\sum_{k_3=0}^{n-2}\binom{n-2}{k_3}\sum_{k_4=0}^{\alpha(k_3+1)-1}\binom{\alpha(k_3+1)-1}{k_4}\cdot(-1)^{k_3+k_4}\left[\frac{u^{k_4}-u^{n\alpha-1}}{n\alpha-k_4-1}\right] \tag{15}$$

4 Conclusion

Remarks

1.Putting $\alpha=1$ in Eq. (13) for $0<u\leq\frac{1}{2}$, we obtain the median for uniform distributions.
2.Putting $\alpha=1$ in Eq. (14) for $\frac{1}{2}<u<1$, we obtain the median for uniform distributions.
3.Putting $\alpha=1$ in Eq. (15) for $0<u<v<1$, we obtain the range for uniform distributions.

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