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Robustness Study of the Sequential Testing Procedures for the New Weibull-Pareto Distribution

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Abstract: Sequential testing procedures are developed for testing the hypotheses regarding the parameters of the New Weibull-Pareto Distribution (NWPD). Theoretical expression for the operating characteristics (OC) and average sample number (ASN) functions are derived for the scale parameters of the distribution. The robustness of the SPRT'S in respect of OC and ASN functions is studied, when the distribution under study has undergone a change. The results are presented through Tables and Graphs, so that one can see the numerical evaluated departures in OC and ASN functions.

Keywords: New Weibull-Pareto Distribution, Sequential probability ratio test, Operating characteristics, Average Sample Number, Robustness and Acceptance and rejection region.

1 Introduction

Wald (1947), is the first who developed the concept of sequential testing of statistical hypotheses for testing between two simple hypotheses. The concept of sequential testing is heavily dominated by the sequential probability ratio test (SPRT). He derived the theoretical expressions for the operating characteristics (OC) and average sample number (ASN) functions, to study the performance of the SPRT'S.

The SPRT has been applied by various authors, to deal with testing problems, for references, Oakland (1950) developed SPRT for testing the simple vs. simple hypothesis concerning the mean of the negative binomial distribution, Epstein and Sobel (1955) dealt the testing of simple hypothesis problem regarding the mean of one parameter exponential distribution through SPRT, Johnson (1966) applied SPRT for testing the hypothesis for the scale parameter of the weibull distribution when the shape parameter is known, Phatarford (1971) dealt the problem of testing the composite hypothesis for the shape parameter of the gamma distribution through SPRT, when the scale parameter is unknown, Bain and Engelhardt (1982) applied SPRT for testing the hypothesis for the shape parameter of a non-homogenous Poisson process and Chaturvedi et al. (2000) developed SPRT for testing simple and composite hypothesis regarding the parameters of a class of distributions representing various life-testing models. Sevil and Demirhan (2008) developed a group sequential test when response variable has an inverse Gaussian distribution with known parameter.

The robustness of the SPRT in respect of OC and ASN functions has been studied by several authors, when the distribution under consideration has undergone a change, while dealing with various probabilistic models. For references, Harter and Moore (1976) gives sampling plans for reliability tests under the assumption of a constant failure rate and by using Monte Carlo techniques the robustness of the exponential SPRT is studied, when the underlying distribution is a weibull distribution, Montagne and Singpurwalla (1985) investigated the robustness of the sequential life-testing procedure with respect to the risks and the expected sample sizes for the exponential distribution when the life length is not exponential, Hubbard and Allen (1991) applied SPRT on the mean of the negative binomial distribution when the dispersion parameter is known and the robustness of the test to the misspecification of dispersion parameter is studied. Chaturvedi et al. (1998) considered a family of life-testing models and studied the robustness of the SPRT'S for various parameters involved in the model and also generalised the results of Montagne and Singpurwalla (1985).

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2 Set-up of the problem

In this article, we consider the NWPD proposed by Nasiru and Lugnterah (2015) with probability density function (pdf) given by

$$f(x;\beta,\partial,\theta)=\frac{\beta\partial}{\theta}\left(\frac{x}{\theta}\right)^{\beta-1}\exp\left\{-\partial\left(\frac{x}{\theta}\right)^{\beta}\right\};\quad x>0 \tag{1}$$

and cumulative distribution function (cdf)

$$F(x;\beta,\partial,\theta)=1-\exp\left\{-\partial\left(\frac{x}{\theta}\right)^{\beta}\right\};\quad x>0 \tag{2}$$

where β is a shape and ∂, θ are the scale parameters, respectively. Weibull and Exponential distributions are the specific cases of (1) for $\partial=1$ and for $\partial=1$, $\beta=1$, respectively.

In Sections 3, 4, 5 and 6, respectively, we develop SPRT'S for testing the simple null hypotheses for the parameters ∂ and θ involved in the model (1). The robustness of the SPRT'S in respect of OC and ASN functions is studied [see Remarks 3.1, 4.1, 5.1 and 6.1]. In Section 7, the acceptance and rejection regions for H_0 vs. H_1 in case of $'\theta'$ are derived and plotted in Figure 7.1. Finally, in Section 8, the results and findings are presented through Tables and Figures.

3 SPRT for testing the hypothesis regarding $'\partial'$ for known values of θ and β

The SPRT for testing the simple null hypothesis $H_0:\partial=\partial_0$ against the simple alternative $H_1:\partial=\partial_1(\partial_1>\partial_0)$ is defined as

$$Z_i=\ln\left[\frac{f(x_i;\beta,\partial_1,\theta)}{f(x_i;\beta,\partial_0,\theta)}\right] \tag{3}$$

$$Z_i=\ln\left(\frac{\partial_1}{\partial_0}\right)-(\partial_1-\partial_0)\left(\frac{x_i}{\theta}\right)^{\beta} \tag{4}$$

or,

$$e^{Z_i}=\left(\frac{\partial_1}{\partial_0}\right)\exp\left\{-(\partial_1-\partial_0)\left(\frac{x_i}{\theta}\right)^{\beta}\right\} \tag{5}$$

Now, we choose two numbers A and B such that $0<B<1<A$. At the n^{th} stage, accept H_0 if $\sum_{i=1}^{n} Z_i\le\ln B$, reject H_0 if $\sum_{i=1}^{n} Z_i\ge\ln A$, otherwise continue sampling by taking the $(n+1)^{th}$ observation. If $\alpha\in(0,1)$ and $\beta\in(0,1)$ are Type I and Type II errors, respectively, then according to Wald (1947), A and B are approximately given by

$$A\approx\frac{1-\beta}{\alpha}\text{ and }B\approx\frac{\beta}{1-\alpha} \tag{6}$$

The OC function $L(\theta)$ is given by

$$L(\theta)=\frac{A^h-1}{A^h-B^h} \tag{7}$$

where 'h' is the non-zero solution of

$$E[e^{Z_i}]^h=1 \tag{8}$$

or,

$$\int_0^{\infty}\left[\frac{f(x_i;\beta,\partial_1,\theta)}{f(x_i;\beta,\partial_0,\theta)}\right]^h f(x_i;\beta,\partial,\theta)dx=1 \tag{9}$$

From (1) and (5), we obtain

$$E[e^{Z_i}]^h=\frac{\partial\left(\frac{\partial_1}{\partial_0}\right)^h}{h(\partial_1-\partial_0)+\partial} \tag{10}$$

On substituting (10) in (8), we get

$$\partial = \frac{h(\partial_1 - \partial_0)}{\left(\frac{\partial_1}{\partial_0}\right)^h - 1} \tag{11}$$

The expression (11) is not very useful for finding the values of OC and ASN functions, hence, we will further evaluate (11) in the following manner to obtain the desired results.

$$h \ln\left(\frac{\partial_1}{\partial_0}\right) = \ln\left[1 + h\left(\frac{\partial_1 - \partial_0}{\partial}\right)\right] \tag{12}$$

Using the expansion of $\ln(1+x),\ -1 < x < 1$ in (12), retaining the terms up to third degree in 'h' and on simplifying, we obtain the real roots of 'h' from (13)

$$\left\{\frac{1}{3}\left(\frac{\partial_1 - \partial_0}{\partial}\right)^3\right\} h^2 - \left\{\frac{1}{2}\left(\frac{\partial_1 - \partial_0}{\partial}\right)^2\right\} h + \left\{\left(\frac{\partial_1 - \partial_0}{\partial}\right) - \ln\left(\frac{\partial_1}{\partial_0}\right)\right\} = 0 \tag{13}$$

The ASN function is approximately given by

$$E(N|\partial) = \frac{L(\partial)\ln B + [1 - L(\partial)]\ln A}{E(Z)} \tag{14}$$

provided that $E(Z) \neq 0$, where

$$E(Z) = \ln\left(\frac{\partial_1}{\partial_0}\right) - \left(\frac{\partial_1 - \partial_0}{\partial}\right) \tag{15}$$

From (14) ASN function under H_0 and H_1 are given by

$$E_0(N) = \frac{(1-\alpha)\ln B + \alpha \ln A}{\ln\left(\frac{\partial_1}{\partial_0}\right) - \left(\frac{\partial_1 - \partial_0}{\partial}\right)} \tag{16}$$

and

$$E_1(N) = \frac{\beta \ln B + (1-\beta)\ln A}{\ln\left(\frac{\partial_1}{\partial_0}\right) - \left(\frac{\partial_1 - \partial_0}{\partial}\right)} \tag{17}$$

Remarks 3.1: Let us consider the problem of testing the simple null hypothesis $H_0 : \partial = 13$ against the simple alternative hypothesis $H_1 : \partial = 15$, for $\alpha = \beta = 0.05$. The numerical values of OC and ASN functions are shown in Table 3.1 and their curves are plotted in Figure 3.1(a) and 3.1(b), respectively. It is evident from the Table and Figures that the approximation gives satisfactorily results.

4 Robustness of the SPRT for $'\partial'$ when $'\theta'$ has undergone a change

Let us suppose that the parameter $'\theta'$ has undergone a change to θ^* and then the probability distribution in (1) becomes $f(x;\beta,\partial,\theta^*)$. In order to study the robustness of SPRT developed in Section 3 with respect to OC and ASN functions, the values of $'h'$ are obtained by solving the following equation

$$\int_0^\infty \left[\frac{f(x_i;\beta,\partial_1,\theta)}{f(x_i;\beta,\partial_0,\theta)}\right]^h f(x_i;\beta,\partial,\theta^*)dx = 1 \tag{18}$$

$$\left(\frac{\partial_1}{\partial_0}\right)^h \frac{\beta\partial}{\theta^*} \int_0^\infty \left(\frac{x}{\theta^\beta}\right)^{\beta-1} \exp\left[-\left\{\frac{h(\partial_1 - \partial_0)}{\theta^\beta} + \frac{\partial}{\theta^{*\beta}}\right\} x^\beta\right] dx = 1$$

$$\frac{\left(\frac{\partial_1}{\partial_0}\right)^h \partial}{\left\{\frac{h(\partial_1 - \partial_0)}{\theta^\beta} + \frac{\partial}{\theta^{*\beta}}\right\} \theta^{*\beta}} = 1$$

$$\frac{\left(\frac{\partial_1}{\partial_0}\right)^h \partial}{\left(\frac{\theta^*}{\theta}\right)^\beta h(\partial_1-\partial_0)+\partial}=1$$

Finally, we get

$$\partial=\frac{(p)^\beta h(\partial_1-\partial_0)}{\left(\frac{\partial_1}{\partial_0}\right)^h-1} \tag{19}$$

where $p=\frac{\theta^*}{\theta}$

The expression (19) is not of much use for calculating the numerical values of OC and ASN functions. In order to handle the situation, we rewrite (19) as

$$h\ln\left(\frac{\partial_1}{\partial_0}\right)=\ln\left[1+h\left(\frac{\partial_1-\partial_0}{\partial}\right)\left(\frac{\theta^*}{\theta}\right)^\beta\right] \tag{20}$$

Using the expansion of $\ln(1+x)$, $-1<x<1$ in (20) and retaining the terms up to third degree in $'h'$ and on simplifying, we obtain the following quadratic equation in $'h'$

$$\left\{\frac{p^{3\beta}}{3}\left(\frac{\partial_1-\partial_0}{\partial}\right)^3\right\}h^2-\left\{\frac{p^{2\beta}}{2}\left(\frac{\partial_1-\partial_0}{\partial}\right)^2\right\}h+\left\{p^\beta\left(\frac{\partial_1-\partial_0}{\partial}\right)-\ln\left(\frac{\partial_1}{\partial_0}\right)\right\}=0 \tag{21}$$

where $p=\frac{\theta^*}{\theta}$

The Robustness of the SPRT with respect to ASN is studied by replacing the denominator of (14) by

$$E_{\theta^*}(Z)=\int_0^\infty z f(x;\beta,\partial,\theta^*)dx$$

or,

$$E_{\theta^*}(Z)=E\left[\ln\left(\frac{\partial_1}{\partial_0}\right)-(\partial_1-\partial_0)\frac{x^\beta}{\theta^\beta}\right]$$

$$=\ln\left(\frac{\partial_1}{\partial_0}\right)-\frac{(\partial_1-\partial_0)}{\theta^\beta}E(x^\beta)$$

$$=\ln\left(\frac{\partial_1}{\partial_0}\right)-\frac{(\partial_1-\partial_0)}{\theta^\beta}\frac{\theta^{*\beta}}{\partial}$$

$$=\ln\left(\frac{\partial_1}{\partial_0}\right)-\frac{(\partial_1-\partial_0)}{\partial}\left(\frac{\theta^*}{\theta}\right)^\beta$$

$$=\ln\left(\frac{\partial_1}{\partial_0}\right)-\frac{(\partial_1-\partial_0)}{\partial}(p)^\beta$$

where $p=\frac{\theta^*}{\theta}$.

Remarks 4.1: Let us consider the example of testing null hypothesis $H_0:\partial=13$ vs. $H_1:\partial=15$, for $\alpha=\beta=0.05$. The numerical values of OC and ASN functions are obtained for $p=1$, $p>1$ and $p<1$, in order to study robustness of the SPRT and are presented in Table 4.1(a) and 4.1(b), respectively. The OC and ASN curves are plotted in Figure 4.1(a) and 4.1(b), respectively. It follows from Figure 4.1(a) that the OC function curve shifts to left (right) for $p<1(p>1)$ of the curve corresponding to $p=1$ and the similar pattern is followed by the ASN function curve in Figure 4.1(b). It is evident from both the curves that the SPRT is highly sensitive for changes in $'\theta'$.

5 SPRT for testing the hypothesis regarding $'\theta'$, when $'\partial'$ is known

The SPRT for testing the simple null hypothesis $H_0 : \theta = \theta_0$ against the simple alternative $H_1 : \theta = \theta_1 (\theta_1 > \theta_0)$ is defined as

$$Z_i = \ln \left[\frac{f(x_i; \beta, \partial, \theta_1)}{f(x_i; \beta, \partial, \theta_0)} \right] \tag{22}$$

or,

$$Z_i = \ln \left(\frac{\theta_0}{\theta_1} \right)^{\beta} - \partial \left(\frac{1}{{\theta_1}^{\beta}} - \frac{1}{{\theta_0}^{\beta}} \right) {x_i}^{\beta} \tag{23}$$

or,

$$e^{Z_i} = \left(\frac{\theta_0}{\theta_1} \right)^{\beta} \exp \left\{ -\partial \left(\frac{1}{{\theta_1}^{\beta}} - \frac{1}{{\theta_0}^{\beta}} \right) {x_i}^{\beta} \right\} \tag{24}$$

From (1) and (24), we get

$$E[e^{Z_i}]^h = \frac{\left(\frac{\theta_0}{\theta_1} \right)^{\beta h}}{1 + h\theta^{\beta} \left(\frac{1}{{\theta_1}^{\beta}} - \frac{1}{{\theta_0}^{\beta}} \right)} \tag{25}$$

We get from (25) that

$$\theta = \left[\frac{\left(\frac{\theta_0}{\theta_1} \right)^{\beta h} - 1}{h \left(\frac{1}{{\theta_1}^{\beta}} - \frac{1}{{\theta_0}^{\beta}} \right)} \right]^{1/\beta} \tag{26}$$

The expression (26) is not of much use in calculating the numerical values of OC and ASN functions. Again we may rewrite (26) as

$$\beta h \ln \left(\frac{\theta_0}{\theta_1} \right) = \ln \left[1 + h\theta^{\beta} \left(\frac{1}{{\theta_1}^{\beta}} - \frac{1}{{\theta_0}^{\beta}} \right) \right] \tag{27}$$

Using the expansion for $\ln(1+x)$, $-1 < x < 1$ in (27), retaining the terms up to third degree in $'h'$ and on simplifying, we obtain the following quadratic equation in $'h'$

$$\left\{ \frac{\theta^{3\beta}}{3} \left(\frac{1}{{\theta_1}^{\beta}} - \frac{1}{{\theta_0}^{\beta}} \right)^3 \right\} h^2 - \left\{ \frac{\theta^{2\beta}}{2} \left(\frac{1}{{\theta_1}^{\beta}} - \frac{1}{{\theta_0}^{\beta}} \right)^2 \right\} h + \left\{ \theta^{\beta} \left(\frac{1}{{\theta_1}^{\beta}} - \frac{1}{{\theta_0}^{\beta}} \right) - \beta \ln \left(\frac{\theta_0}{\theta_1} \right) \right\} = 0 \tag{28}$$

The ASN function is approximately given by

$$E(N|\theta) = \frac{L(\theta) \ln B + [1 - L(\theta)] \ln A}{E(Z)} \tag{29}$$

provided that $E(Z) \neq 0$, where

$$E(Z) = \ln \left(\frac{\theta_0}{\theta_1} \right)^{\beta} - \theta^{\beta} \left(\frac{1}{{\theta_1}^{\beta}} - \frac{1}{{\theta_0}^{\beta}} \right) \tag{30}$$

From (29) ASN function under H_0 and H_1 are given by

$$E_0(N) = \frac{(1-\alpha) \ln B + \alpha \ln A}{\ln \left(\frac{\theta_0}{\theta_1} \right)^{\beta} - \theta^{\beta} \left(\frac{1}{{\theta_1}^{\beta}} - \frac{1}{{\theta_0}^{\beta}} \right)} \tag{31}$$

and

$$E_1(N) = \frac{\beta \ln B + (1-\beta) \ln A}{\ln \left(\frac{\theta_0}{\theta_1} \right)^{\beta} - \theta^{\beta} \left(\frac{1}{{\theta_1}^{\beta}} - \frac{1}{{\theta_0}^{\beta}} \right)} \tag{32}$$

Remarks 5.1: Let us consider the problem of testing the simple null hypothesis $H_0 : \theta = 12$ against the simple alternative hypothesis $H_1 : \theta = 15$, for $\alpha = \beta = 0.05$. The numerical values of OC and ASN functions are shown in Table (5.1) and their curves are plotted in Figure 5.1(a) and 5.1(b), respectively. It is evident from the Table (5.1) and Figures 5.1(a) and 5.1(b) that the approximation gives satisfactorily results.

6 Robustness of SPRT for $'\theta'$ when ∂ has undergone a change

Let us suppose that the parameter $'\partial'$ has undergone a change then the probability distribution in (2.1) becomes $f(x;\beta,\partial^*,\theta)$. To study the robustness of the SPRT developed in Section 5 with respect to OC and ASN functions, the values of $'h'$ are obtained from the following equation

$$\int_0^\infty \left[\frac{f(x_i;\beta,\partial,\theta_1)}{f(x_i;\beta,\partial,\theta_0)}\right]^h f(x_i;\beta,\partial^*,\theta)dx = 1 \tag{33}$$

$$\left(\frac{\theta_0}{\theta_1}\right)^h \frac{\beta\partial^*}{\theta^\beta}\int_0^\infty x^{\beta-1}\exp\left[-\left\{\partial h\left(\frac{1}{\theta_1}-\frac{1}{\theta_0}\right)+\frac{\partial^*}{\theta^\beta}\right\}x^\beta\right]dx = 1$$

$$\frac{\left(\frac{\theta_0}{\theta_1}\right)^{\beta h}\frac{\partial}{\theta^\beta}}{\left\{\partial h\left(\frac{1}{\theta_1^\beta}-\frac{1}{\theta_0^\beta}\right)+\frac{\partial^*}{\theta^\beta}\right\}} = 1$$

$$\theta^\beta = \frac{\left\{\left(\frac{\theta_0}{\theta_1}\right)^{\beta h}-1\right\}\left(\frac{\partial^*}{\partial}\right)}{h\left(\frac{1}{\theta_1^\beta}-\frac{1}{\theta_0^\beta}\right)}$$

$$\theta = \left[\frac{\left\{\left(\frac{\theta_0}{\theta_1}\right)^{\beta h}-1\right\}\left(\frac{\partial^*}{\partial}\right)}{h\left(\frac{1}{\theta_1^\beta}-\frac{1}{\theta_0^\beta}\right)}\right]^{1/\beta} \tag{34}$$

The expression (34) is not of much use in calculating the numerical values of OC and ASN functions. Rewrite (34) to obtain the real roots of $'h'$.

$$\beta h\ln\left(\frac{\theta_0}{\theta_1}\right) = \ln\left[1+h\theta^\beta\left(\frac{\partial}{\partial^*}\right)\left(\frac{1}{\theta_1^\beta}-\frac{1}{\theta_0^\beta}\right)\right] \tag{35}$$

Using the expansion of $\ln(1+x)$, $-1<x<1$ in (35), retaining the terms up to third degree in $'h'$ and on simplifying, we obtain the real roots for $'h'$ from the following equation

$$\left\{\frac{\phi^3\theta^{3\beta}}{3}\left(\frac{1}{{\theta_1}^\beta}-\frac{1}{{\theta_0}^\beta}\right)^3\right\}h^2-\left\{\frac{\phi^2\theta^{2\beta}}{2}\left(\frac{1}{{\theta_1}^\beta}-\frac{1}{{\theta_0}^\beta}\right)^2\right\}h+\left\{\phi\theta^\beta\left(\frac{1}{{\theta_1}^\beta}-\frac{1}{{\theta_0}^\beta}\right)-\beta\ln\left(\frac{\theta_0}{\theta_1}\right)\right\}=0 \tag{36}$$

where $\phi = \dfrac{\partial}{\partial^*}$

The Robustness of the SPRT with respect to ASN can be studied by replacing the denominator of (29) by

$$E_{\partial^*}(Z) = \int_0^\infty zf(x;\beta,\partial^*,\theta)dx$$

or,

$$E_{\partial^*}(Z) = E\left[\ln\left(\frac{\theta_0}{\theta_1}\right)^\beta - \partial\left(\frac{1}{{\theta_1}^\beta}-\frac{1}{{\theta_0}^\beta}\right)x^\beta\right]$$

$$= \ln\left(\frac{\theta_0}{\theta_1}\right)^\beta - \partial\left(\frac{1}{{\theta_1}^\beta}-\frac{1}{{\theta_0}^\beta}\right)E[x^\beta]$$

$$= \ln\left(\frac{\theta_0}{\theta_1}\right)^\beta - \partial\left(\frac{1}{{\theta_1}^\beta}-\frac{1}{{\theta_0}^\beta}\right)\frac{\theta^\beta}{\partial^*}$$

$$= \ln\left(\frac{\theta_0}{\theta_1}\right)^{\beta} - \frac{\partial}{\partial^*}\left(\frac{1}{\theta_1{}^{\beta}} - \frac{1}{\theta_0{}^{\beta}}\right)\theta^{\beta}$$

$$= \ln\left(\frac{\theta_0}{\theta_1}\right)^{\beta} - \phi\left(\frac{1}{\theta_1{}^{\beta}} - \frac{1}{\theta_0{}^{\beta}}\right)\theta^{\beta} \tag{37}$$

where $\phi = \dfrac{\partial}{\partial^*}$.

Remarks 6.1: Let us consider the problem of testing null hypothesis $H_0 : \theta = 12$ vs. $H_1 : \theta = 15$ for $\alpha = \beta = 0.05$. In order to study the robustness of the SPRT, the numerical values of OC and ASN functions are obtained for $\phi = 1$, $\phi > 1$ and $\phi < 1$ and are given in Table 6.1(a) and 6.1(b), respectively. The OC and ASN curves are plotted in Figure 6.1(a) and 6.1(b), respectively. It follows from Figure 6.1(a) that the OC function curve shifts to right (left) for $\phi < 1(\phi > 1)$ of the curve corresponding to $\phi = 1$ and the similar pattern is followed by the ASN function curve in Figure 6.1(b). It is evident from both the curves that the SPRT is highly sensitive for changes in $'\partial'$.

7 Implementation of New Weibull Pareto Distribution (NWPD)

The nature of SPRT in case of NWPD is described as, let $X_1, X_2, X_3, \ldots$ be (iid) random variables from NWPD where $\theta > 0$. We wish to test the simple null hypothesis $H_0 : \theta = \theta_0$ vs. simple alternative hypothesis $H_1 : \theta = \theta_1(\theta_1 > \theta_0)$ having a pre-assigned $\alpha > 0, \beta < 1$. Let A and B be approximately given by $A \approx \dfrac{1-\beta}{\alpha}$ and $B \approx \dfrac{\beta}{1-\alpha}$ and Z_i is defined as

$$Z_i = \ln\left(\frac{\theta_0}{\theta_1}\right)^{\beta} - \partial\left(\frac{1}{\theta_1{}^{\beta}} - \frac{1}{\theta_0{}^{\beta}}\right)x_i{}^{\beta}$$

where $i = 1, 2, 3\ldots$. Let $n(\geq 1)$, the SPRT given at (23) can simplify as:

Let us define, $Y(n) = \sum_{i=1}^{n} X_i$ and N=first integer $n(\geq 1)$, for which the inequality $Y(n) \leq c_1 + dn$ or $Y(n) \geq c_2 + dn$ holds with the constants

$$c_1 = \frac{\ln B}{\partial\left(\frac{1}{\theta_1} - \frac{1}{\theta_0}\right)},\ c_2 = \frac{\ln A}{\partial\left(\frac{1}{\theta_1} - \frac{1}{\theta_0}\right)} \text{ and } d = \frac{\ln\left(\frac{\theta_0}{\theta_1}\right)}{\partial\left(\frac{1}{\theta_1} - \frac{1}{\theta_0}\right)} \tag{38}$$

At the stopping stage, if $Y(N) \leq c_1 + dN$, we accept H_0 and if $Y(N) \geq c_2 + dN$, we reject H_0 for different values of N, where A and B are the fixed quantities. Figure 7.1 shows the acceptance and rejection region for H_0 under the case when $H_0 : \theta = 12$ vs. $H_1 : \theta = 15$, $\partial = 2$ and $\alpha = \beta = 0.05$. From (38), values of the constants are $c_1 = 0.33316$, $c_2 = -0.33316$ and $d = 6.694$, respectively. Thus, if $Y(N) \leq 0.33316 + 6.694N$, we accept H_0 and if $Y(N) \geq -0.3316 + 6.694N$, we accept H_1 and at the intermediate stage, we continue sampling.

8 Tables and Figures

Table 3.1: OC and ASN Function for $(H_0 : \partial = 13, H_1 : \partial = 15, \alpha = \beta = 0.05)$

∂	$L(\partial)$	$E(N)$	∂	$L(\partial)$	$E(N)$
12.4	0.99	159.810	14.2	0.34	413.371
12.6	0.99	183.473	14.4	0.22	384.705
12.8	0.97	212.476	14.6	0.14	346.856
13.0	0.95	247.619	14.8	0.08	307.783
13.2	0.91	288.857	15.0	0.05	272.012
13.4	0.85	334.095	15.2	0.03	241.208
13.6	0.75	377.757	15.4	0.02	215.434
13.8	0.63	410.695	15.6	0.01	194.095
14.0	0.48	423.680	15.8	0.01	176.433

Table 4.1(a): OC Function for different values of p
$(H_0 : \partial = 13, H_1 : \partial = 15, \alpha = \beta = 0.05)$

∂	$p = .96$	$p = .98$	$p = 1$	$p = 1.02$	$p = 1.04$
12.0	0.99	1.00	1.00	1.00	1.00
12.2	0.98	0.99	1.00	1.00	1.00
12.4	0.96	0.98	0.99	1.00	1.00
12.6	0.93	0.97	0.99	0.99	1.00
12.8	0.87	0.94	0.97	0.99	1.00
13.0	0.79	0.90	0.95	0.98	0.99
13.2	0.66	0.82	0.91	0.96	0.98
13.4	0.51	0.71	0.85	0.93	0.97
13.6	0.36	0.57	0.75	0.87	0.94
13.8	0.24	0.42	0.63	0.79	0.89
14.0	0.15	0.29	0.48	0.68	0.82
14.2	0.09	0.18	0.34	0.54	0.72
14.4	0.05	0.11	0.22	0.4	0.59
14.6	0.03	0.06	0.14	0.27	0.45
14.8	0.02	0.04	0.08	0.17	0.32
15.0	0.01	0.02	0.05	0.11	0.21
15.2		0.01	0.03	0.06	0.13
15.4		0.01	0.02	0.04	0.08
15.6			0.01	0.02	0.05
15.8			0.01	0.01	0.03
16.0				0.01	0.02
16.2					0.01

Table 4.1(b): ASN Function for different values of p
$(H_0 : \partial = 13, H_1 : \partial = 15, \alpha = \beta = 0.05)$

∂	$p = .96$	$p = .98$	$p = 1$	$p = 1.02$	$p = 1.04$
12.0	171.036	144.504	124.632	109.377	97.377
12.2	198.459	165.165	140.498	121.858	107.428
12.4	232.202	190.607	159.810	136.813	119.284
12.6	272.750	221.868	183.473	154.911	133.412
12.8	318.832	259.639	212.476	176.973	150.415
13.0	365.674	303.378	247.619	203.929	171.036
13.2	404.026	349.827	288.857	236.622	196.137
13.4	422.916	391.650	334.095	275.333	226.567
13.6	416.583	418.47	377.757	318.832	262.812
13.8	388.984	422.174	410.695	363.045	304.262
14.0	350.145	402.825	423.680	400.329	348.066
14.2	309.365	368.300	413.371	421.371	388.110
14.4	272.012	328.311	384.705	420.285	415.719
14.6	240.036	289.695	346.856	398.756	423.461
14.8	213.496	255.647	307.783	364.480	409.944
15.0	191.702	226.935	272.012	325.954	380.629
15.2	173.796	203.175	241.208	288.978	343.814
15.4	158.991	183.600	215.434	256.263	306.328
15.6	146.638	167.418	194.095	228.481	272.012
15.8	136.225	153.941	176.433	205.306	242.297
16.0	127.356	142.611	161.737	186.069	217.249
16.2	119.725	132.990	149.409	170.060	196.355
16.4	113.100	124.740	138.970	156.649	178.941
16.6	107.298	117.600	130.046	145.321	164.363

Table 5.1: OC and ASN Function for
$(H_0 : \theta = 12, H_1 : \theta = 15, \alpha = \beta = 0.05)$

θ	$L(\theta)$	$E(N)$	θ	$L(\theta)$	$E(N)$
11.0	0.996	73.443	13.6	0.398	169.990
11.2	0.994	79.719	13.8	0.311	162.483
11.4	0.989	86.936	14.0	0.236	152.610
11.6	0.982	95.210	14.2	0.175	141.529
11.8	0.970	104.619	14.4	0.127	130.189
12.0	0.953	115.155	14.6	0.091	119.241
12.2	0.926	126.650	14.8	0.064	109.063
12.4	0.888	138.693	15.0	0.045	99.822
12.6	0.836	150.554	15.2	0.031	91.557
12.8	0.768	161.174	15.4	0.021	84.225
13.0	0.686	169.303	15.6	0.013	77.747
13.2	0.593	173.801	15.8	0.008	72.030
13.4	0.494	174.009			

Table 6.1(a): OC Function for different values of ϕ
$(H_0 : \theta = 12, H_1 : \theta = 15, \alpha = \beta = 0.05)$

θ	$\phi = .96$	$\phi = .98$	$\phi = 1$	$\phi = 1.02$	$\phi = 1.04$
11.0	0.999	0.998	0.996	0.993	0.988
11.2	0.998	0.997	0.994	0.989	0.980
11.4	0.997	0.994	0.989	0.981	0.966
11.6	0.995	0.99	0.982	0.968	0.945
11.8	0.991	0.984	0.970	0.949	0.914
12.0	0.985	0.973	0.953	0.919	0.869
12.2	0.976	0.957	0.926	0.878	0.808
12.4	0.962	0.933	0.888	0.821	0.731
12.6	0.941	0.899	0.836	0.748	0.639
12.8	0.911	0.852	0.768	0.661	0.538
13.0	0.869	0.790	0.686	0.563	0.436
13.2	0.814	0.714	0.593	0.463	0.341
13.4	0.744	0.625	0.494	0.367	0.258
13.6	0.661	0.530	0.398	0.282	0.191
13.8	0.569	0.434	0.311	0.211	0.138
14.0	0.475	0.344	0.236	0.154	0.098
14.2	0.384	0.266	0.175	0.111	0.068
14.4	0.301	0.200	0.127	0.078	0.047
14.6	0.231	0.148	0.091	0.055	0.032
14.8	0.173	0.107	0.064	0.037	0.021
15.0	0.127	0.077	0.045	0.025	0.013
15.2	0.092	0.054	0.031	0.017	0.008
15.4	0.066	0.038	0.021	0.010	0.005
15.6	0.047	0.026	0.013	0.006	0.002
15.8	0.033	0.017	0.008	0.003	0.001

Table 6.1(b): ASN Function for different values of ϕ
$(H_0 : \theta = 12, H_1 : \theta = 15, \alpha = \beta = 0.05)$

θ	$\phi = .96$	$\phi = .98$	$\phi = 1$	$\phi = 1.02$	$\phi = 1.04$
11.0	62.344	67.479	73.443	80.396	88.503
11.2	66.782	72.748	79.719	87.871	97.363
11.4	71.838	78.786	86.936	96.458	107.459
11.6	77.614	85.713	95.210	106.231	118.743
11.8	84.221	93.638	104.619	117.161	130.953
12.0	91.767	102.645	115.155	129.033	143.512
12.2	100.337	112.746	126.65	141.351	155.452
12.4	109.963	123.821	138.693	153.262	165.464
12.6	120.571	135.544	150.554	163.578	172.152
12.8	131.917	147.307	161.174	170.968	174.460
13.0	143.512	158.192	169.303	174.326	172.071
13.2	154.584	167.048	173.801	173.159	165.521
13.4	164.106	172.734	174.009	167.777	155.953
13.6	170.969	174.441	169.99	159.138	144.693
13.8	174.253	171.981	162.483	148.484	132.895
14.0	173.528	165.837	152.610	136.981	121.378
14.2	168.997	156.967	141.529	125.516	110.631
14.4	161.399	146.486	130.189	114.654	100.880
14.6	151.754	135.389	119.241	104.691	92.183
14.8	141.075	124.418	109.063	95.735	84.501
15.0	130.189	114.041	99.822	87.779	77.747
15.2	119.666	104.506	91.557	80.755	71.816
15.4	109.844	95.900	84.225	74.571	66.603
15.6	100.880	88.220	77.747	69.126	62.011
15.8	92.815	81.405	72.030	64.323	57.960

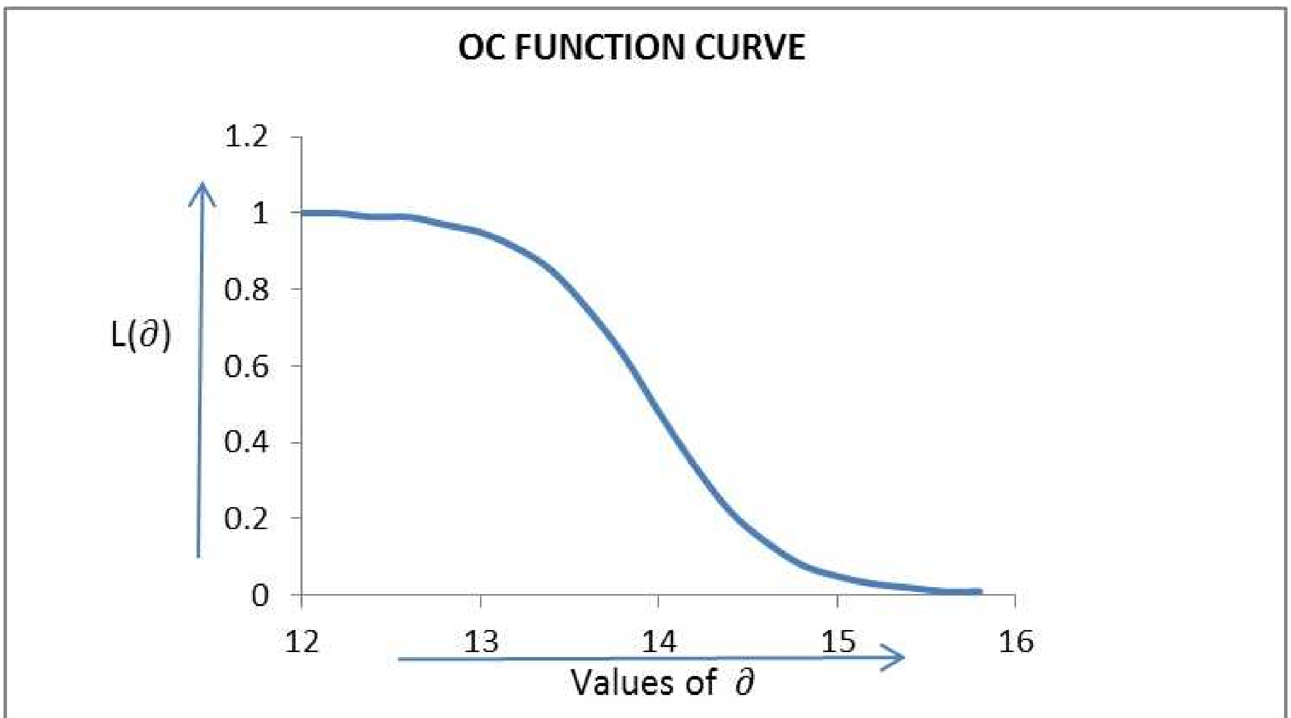

Figure :3.1(a)

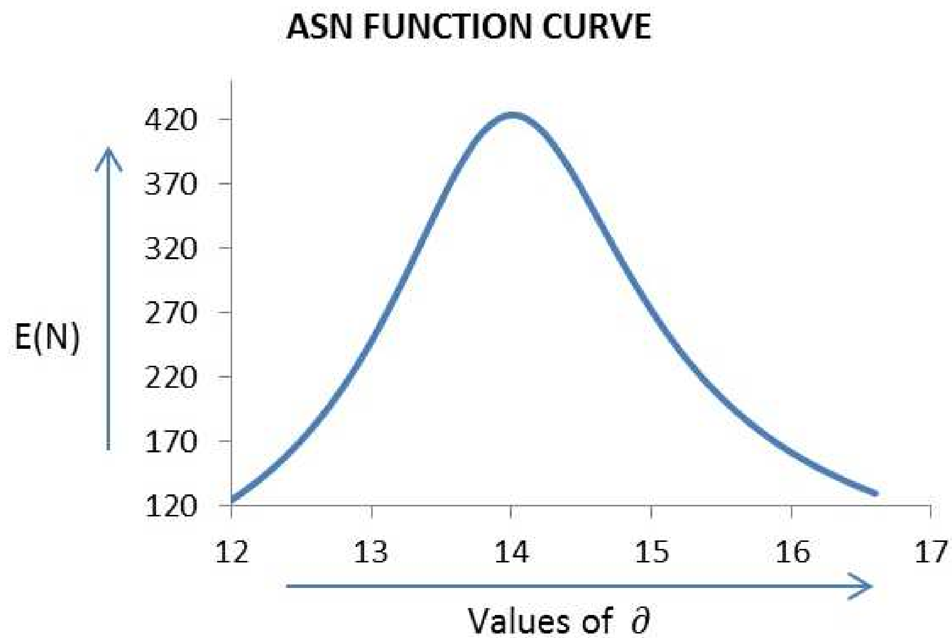

Figure : 3.1(b)

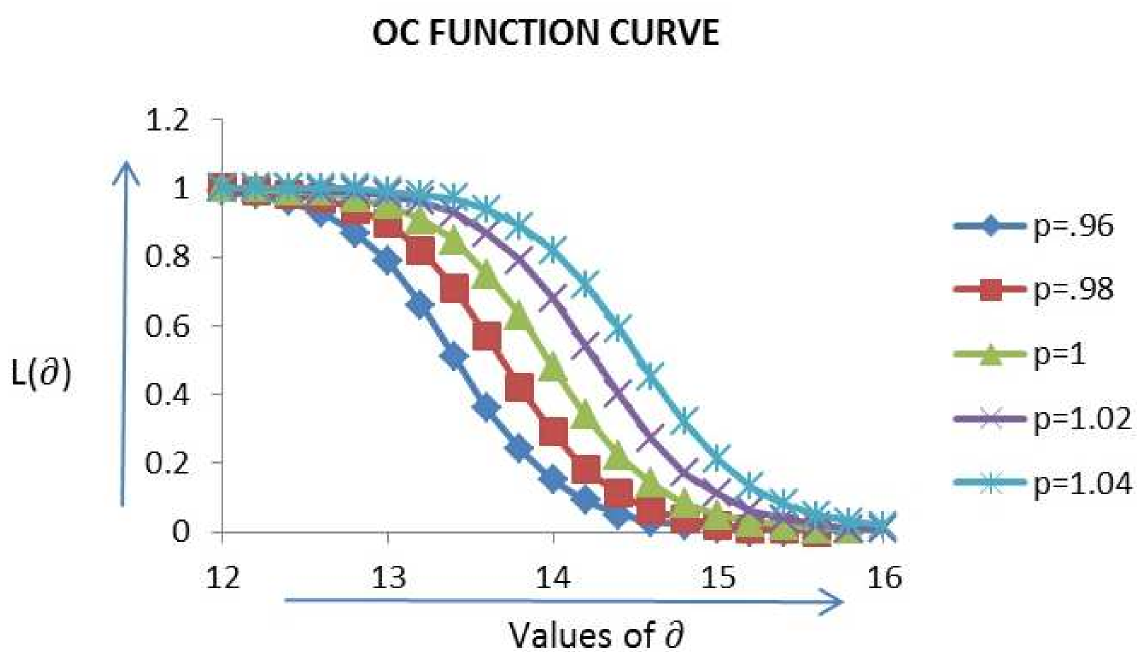

Figure: 4.1(a)

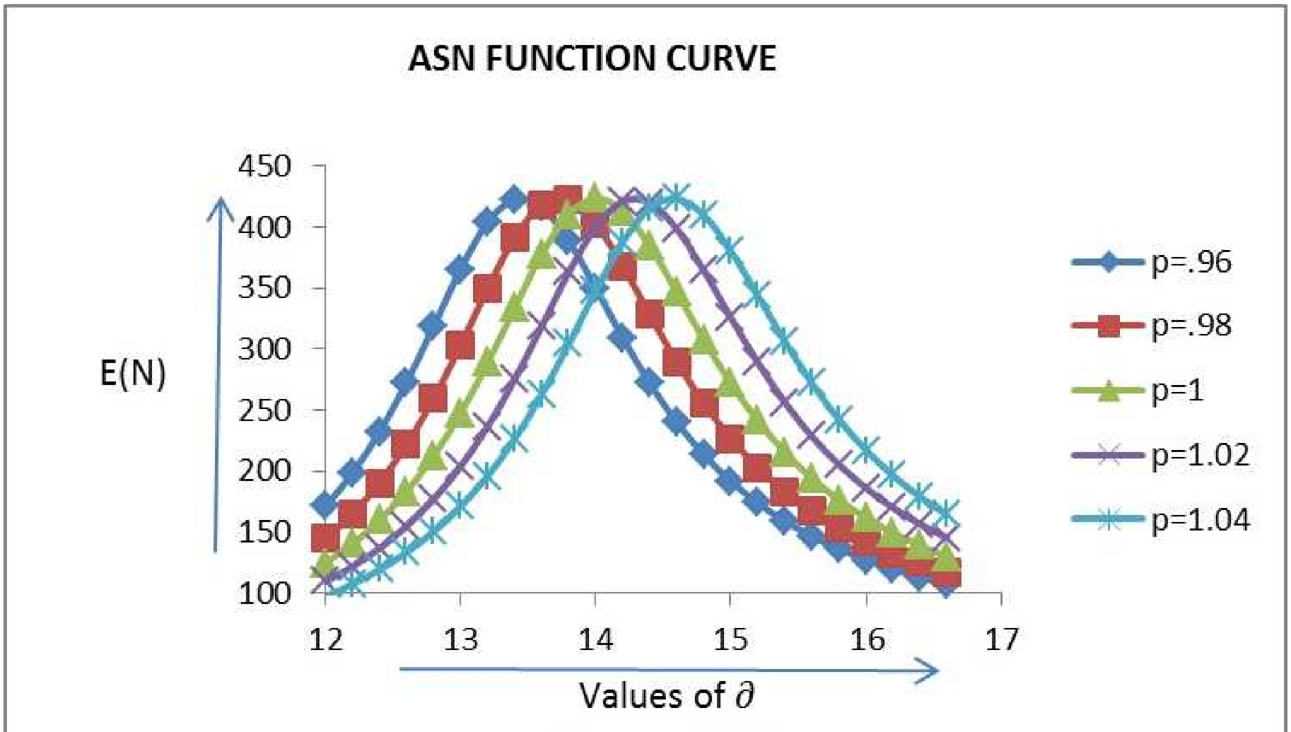

Figure : 4.1(b)

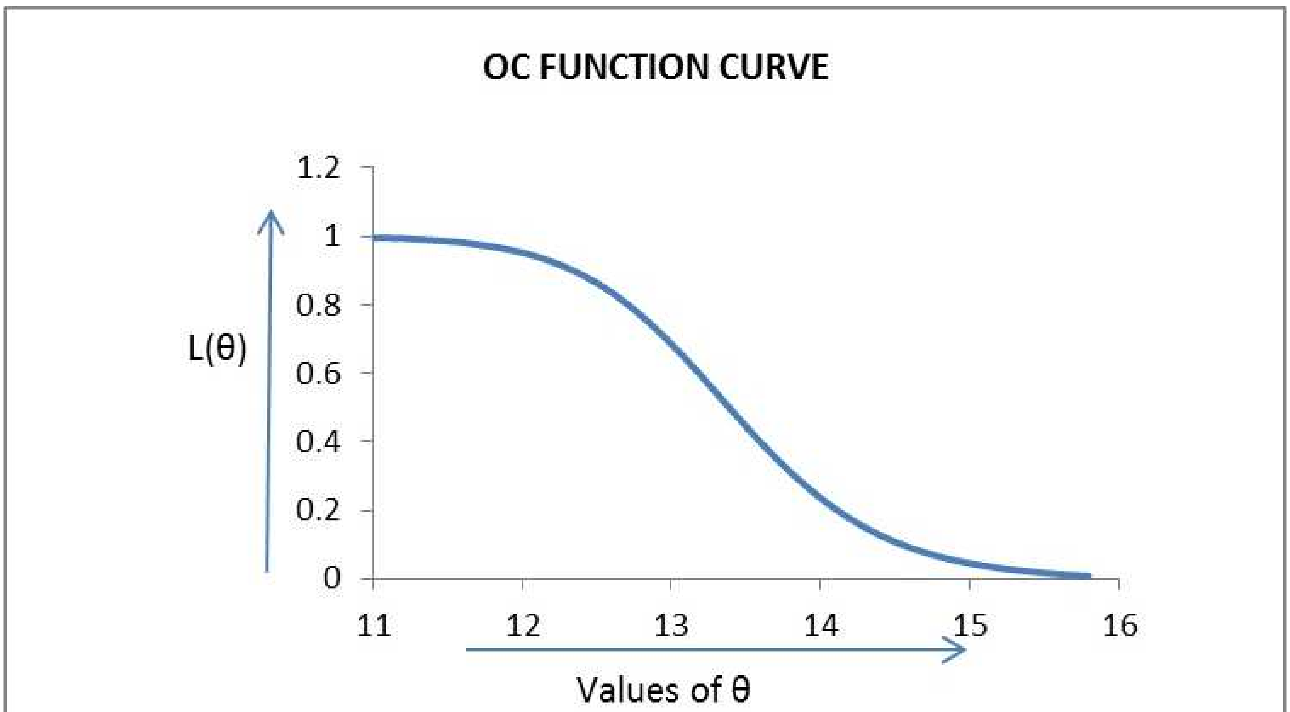

Figure : 5.1(a)

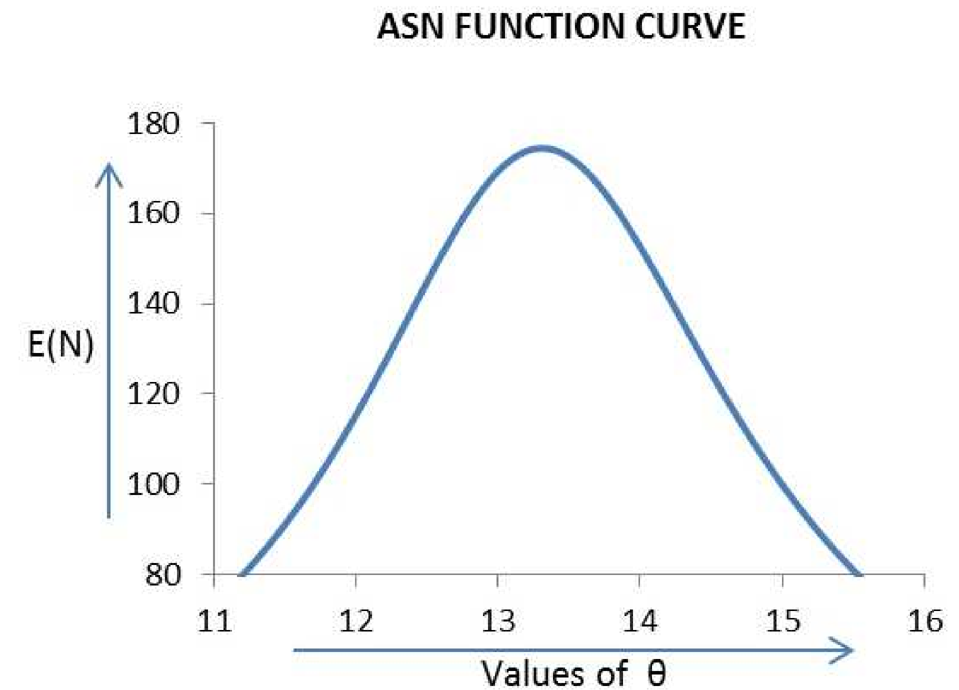

Figure : 5.1(b)

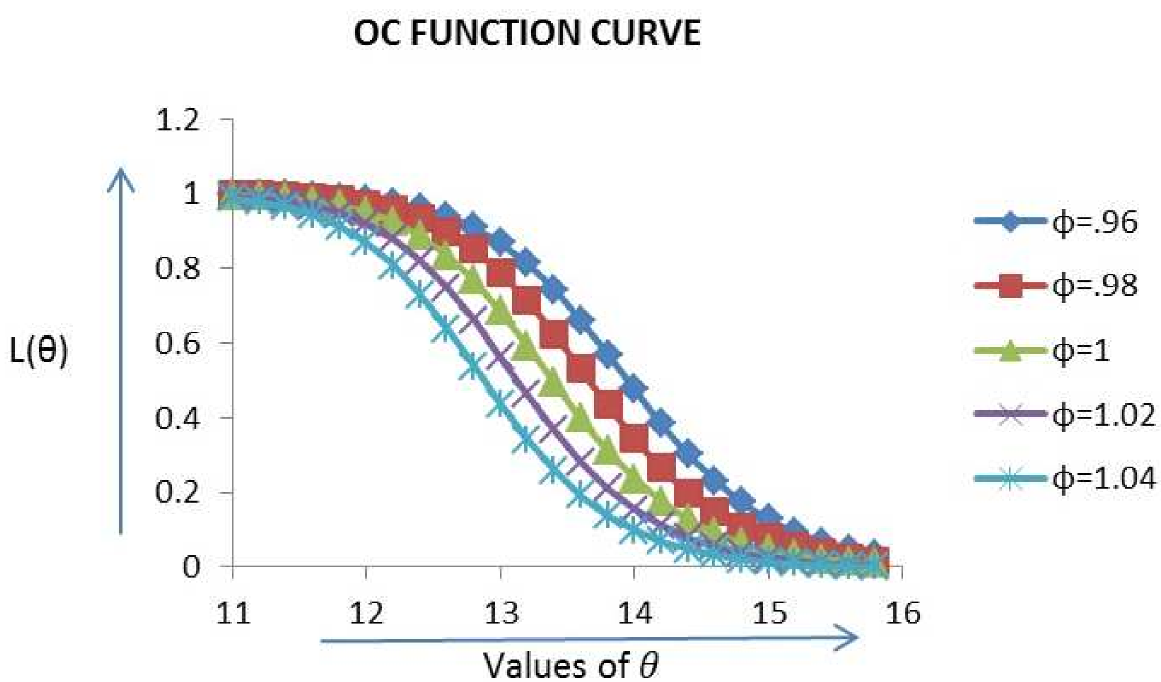

Figure : 6.1(a)

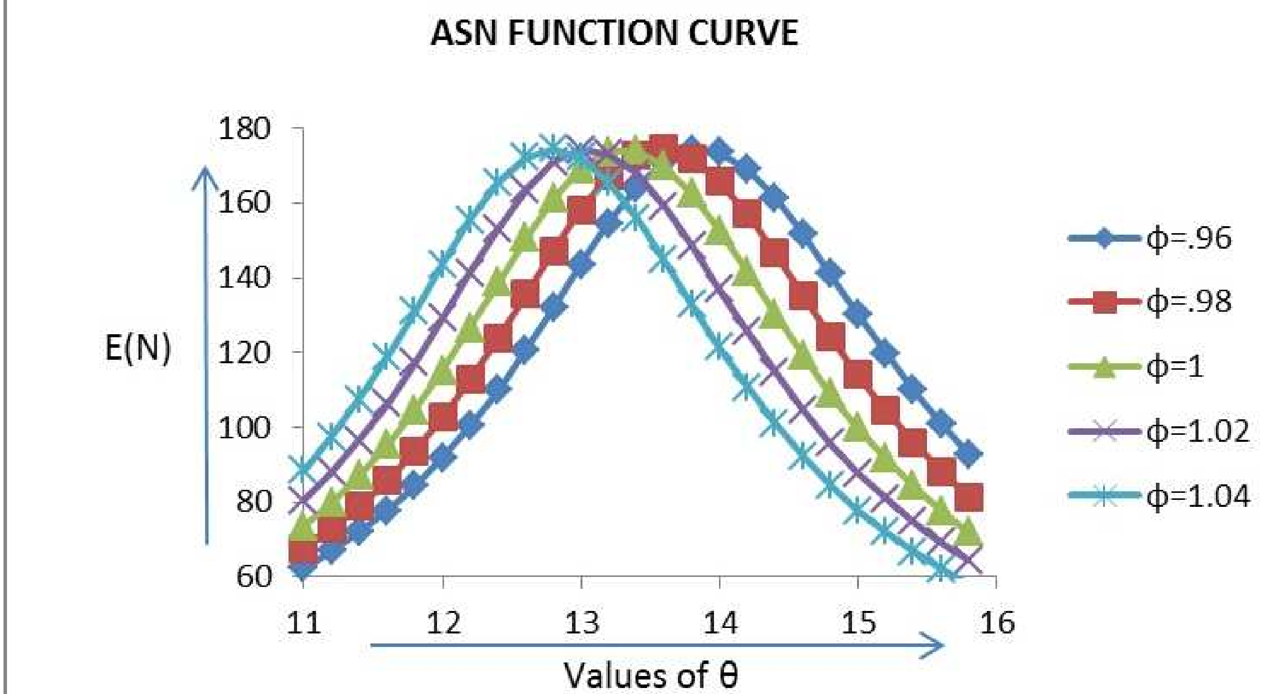

Figure : 6.1(b)

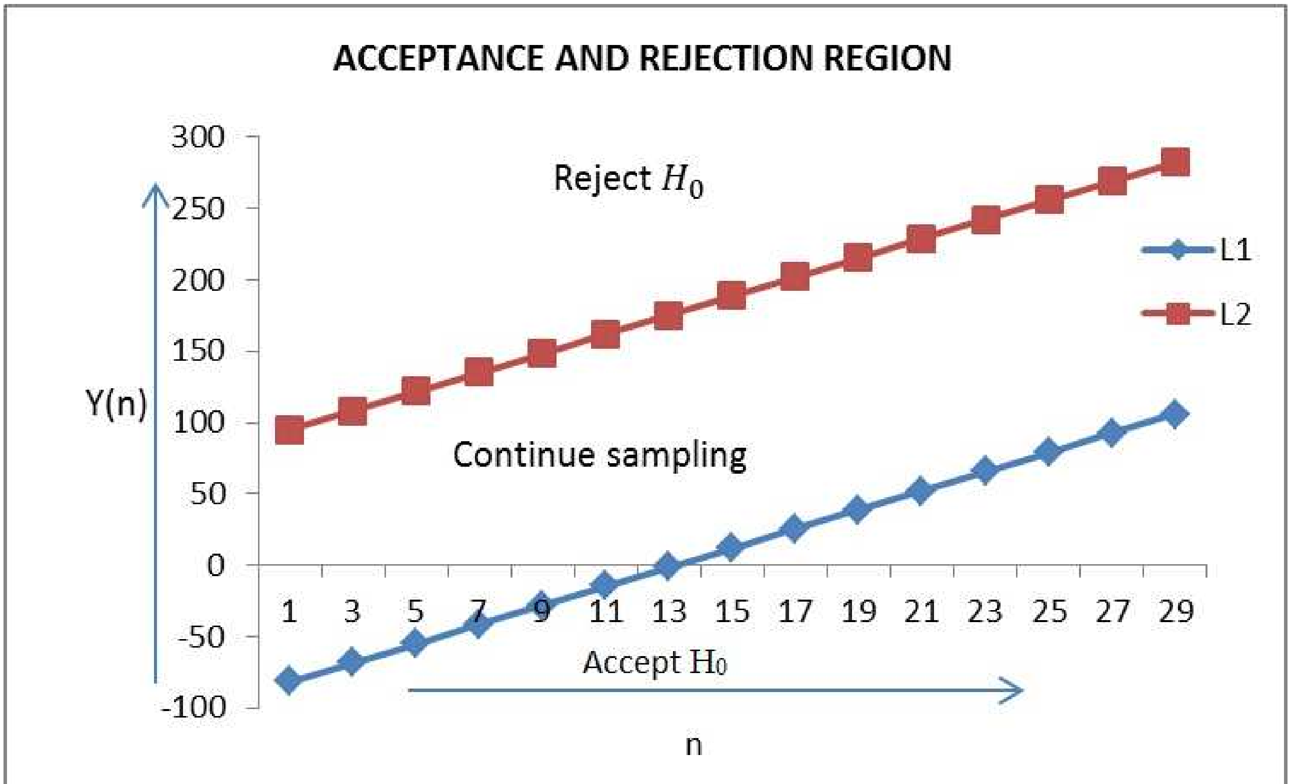

Figure : 7.1

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