Mathematical Sciences Letters
An International Journal

http://dx.doi.org/10.18576/msl/050206

Ulam-J Rassias Stability of Additive Functional Equation in Digital Logic Circuits

Narasimman Pasupathi[1,*] *and Amuda Rangappan*[2]

[1] Department of Mathematics, Thiruvalluvar University College of Arts and Science, Gajalnaickanpatti, Tirupattur-635 901, TamilNadu, India
[2] Department of Mathematics, Jeppiaar Institute of Technology, Kunnam, Sriperumbudur, Chennai-631 604, TamilNadu, India

Received: 11 Apr. 2015, Revised: 21 Sep. 2015, Accepted: 23 Sep. 2015
Published online: 1 May 2016

Abstract: In this paper, we obtain the general solution for the new additive functional equation and we prove its Hyers-Ulam-Rassias stability and Ulam-J Rassias stability in fuzzy normed space using the fixed point method. Also, the new additive functional equation is realized by Digital Logic Circuits(DLC).

Keywords: Additive functioanl equation, Super-Stability, Hyers - Ulam stability, Ulam-J Rassias stability, Fuzzy normed space, Fixed point method, Digital Logic Circuits

1 Introduction

Katsaras [14] introduced an idea of fuzzy norm on a linear space in 1984, In 1991, Biswas [3] defined and studied fuzzy inner product spaces in linear space. In 1992, Felbin [11] introduced an alternative definition of a fuzzy norm on a linear topological structures of a fuzzy normed linear spaces. In 1994, Cheng and Mordeson [8] introduced a definition of fuzzy norm on a linear space. In 2003, Bag and Samanta [2] modified the definition of Cheng and Mordeson [8] by removing a regular condition. Further, Xiao and Zhu [25] improved a bit the Felbin?s definition of fuzzy norm of a linear operator between FNSs.

In 2003, Cadariu and Radu [6,7] used the fixed-point method to the investigation of the Jensen functional equation for the first time. By using fixed point methods, the stability problems of several functional equations have been extensively investigated by a number of authors.

In 1940, Ulam[23] raised the following fundamental question in the theory of functional equations concerning the group homomorphism:

"When is it true that a function, which approximately satisfies a functional equation must be close to an exact solution of the equation?"

In 1941, Hyers [13] gave an affirmative solution to the above problem concerning the Banach space. The result of Hyers was generalized by Aoki[1] for approximate additive mappings and by Rassias[19] for approximate linear mappings by allowing the difference Cauchy equation $\|f(x+y)-f(x)-f(y)\|$ to be controlled by $\varepsilon(\|x\|^p+\|y\|^p)$. The stability phenomenon that was proved by Rassias[19] is called the Hyers-Ulam-Rassias stability.

In 1994, a generalization of Rassias theorem was obtained by Gavruta[12], who replaced $\varepsilon(\|x\|^p+\|y\|^p)$ by a general control function $\phi(x,y)$. Since then general stability problems of various functional equations have been investigated by a number of authors[4,5,9,15,16,17,18,20].

Now we present some basic definitions of a fuzzy norm [2], fuzzy normed space and we recall a fundamental result in fixed point theory which will be useful for proving our main theorems.

Definition 1. *Let X be a real linear space. A function $N : X\times\mathbb{R}\to[0,1]$ is said to be fuzzy norm on X if for all $x,y\in X$ and all $s,t\in\mathbb{R}$:*

(N_1) $N(x,c)=0$ *for* $c\le 0$;
(N_2) $x=0$ *if and only if* $N(x,c)=1$ *for all* $c>0$;
(N_3) $N(cx,t)=N\left(x,\frac{t}{|c|}\right)$ *if* $c\neq 0$;
(N_4) $N(x+y,s+t)\ge\min\{N(x,s),N(y,t)\}$;

* Corresponding author e-mail: drpnarasimman81@gmail.com

(N_5)$N(x,.)$ is a non-decreasing function on $\mathbb{R}$ and $\lim_{t\to\infty} N(x,t) = 1$;
(N_6)For $x \neq 0$, $N(x,.)$ is (upper semi) continuous on $\mathbb{R}$.

The pair (X,N) is called a fuzzy normed linear space. One may regard $N(x,t)$ as the truth value of the statement 'the norm of x is less than or equal to the real number t'.

*Example 1.*Let $(X,\|\cdot\|)$ be a normed linear space. One can be easily verify that for each $k > 0$

$$N_k(x,t) = \begin{cases} \frac{t}{t+k\|x\|} & ,t > 0 \ ,x \in X \\ 0 & ,t \leq 0 \ ,x \in X \end{cases}$$

defines a fuzzy norm on X.

*Example 2.*Let $(X,\|\cdot\|)$ be a normed linear space. Then

$$N(x,t) = \begin{cases} 0 & ,t \leq \|x\| \ ,x \in X \\ 1 & ,t > \|x\| \ ,x \in X \end{cases}$$

is a fuzzy norm on X.

Definition 2.*Let (X,N) be a fuzzy normed linear space. Let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ is said to be convergent if there exists $x \in X$ such that $\lim_{n\to\infty} N(x_n - x,t) = 1$ for all $t > 0$. In that case, x is called the limit of the sequence $\{x_n\}$ and we denote it by $N - \lim_{n\to\infty} x_n = x$.*

Definition 3.*A sequence $\{x_n\}$ in X is called cauchy if for each $\varepsilon > 0$ and $t > 0$ there exists $n_0 \in N$ such that for all $n \geq n_0$ and all $p > 0$ we have*

$$N(x_{n+p} - x_n,t) > 1 - \varepsilon.$$

It is known that every convergent sequence in a fuzzy normed space is Cauchy. If each Cauchy sequence is convergent, then the fuzzy norm is said to be complete and the fuzzy normed space is called a fuzzy Banach space.

Theorem 1.*(Banach's Contraction principle). Let (X,d) be a complete generalized metric space and consider a mapping $J : X \to X$ be a strictly contractive mapping, that is*

$$d(Jx,Jy) \leq Ld(x,y), \quad \forall x,y \in X$$

for some (Lipschitz constant) $L < 1$. Then
(i) The mapping J has one and only one fixed point $x^* = J(x^*)$;
(ii) The fixed point x^* is globally attractive, that is $\lim_{n\to\infty} J^n x = x^*$, for any starting point $x \in X$;
(iii) One has the following estimation inequalities for all $x \in X$ and $n \geq 0$:

$$d(J^n x,x^*) \leq L^n d(x,x^*) \tag{1}$$

$$d(J^n x,x^*) \leq \frac{1}{1-L} d(J^n x,J^{n+1} x) \tag{2}$$

$$d(x,x^*) \leq \frac{1}{1-L} d(x,Jx). \tag{3}$$

Theorem 2.*(The alternative of fixed point)[10]. Suppose we are given a complete generalized metric space (X,d) and a strictly contractive mapping $J : X \to X$, with Lipschitz constant L. Then, for each given element $x \in X$, either*

$$d(J^n x,J^{n+1} x) = +\infty, \forall n \geq 0 \tag{4}$$

or

$$d(J^n x,J^{n+1} x) < +\infty, \forall n \geq n_0 \tag{5}$$

for some natural number n_0. Moreover, if the second alternative holds then
(i) The sequence $(J^n x)$ is convergent to a fixed point y^* of J;
(ii) y^* is the unique fixed point of J in the set $Y = y \in X, d(J^{n_0} x,y) < +\infty$
(iii) $d(\gamma,\gamma^*) \leq \frac{1}{1-L} d(\gamma,J\gamma), \quad \gamma \in Y$.

In this paper, we obtain the general solution for the following new additive functional equation of the form

$$\begin{aligned} &3f(x+3y) - f(3x+y) \\ &\quad = 12[f(x+y) + f(x-y)] - 24f(x) + 8f(y) \end{aligned} \tag{6}$$

for all $x,y \in X$, and using the fixed point technique, we prove the Hyers-Ulam-Rassias stability and Ulam-J Rassias stability in fuzzy normed space. Also, the new additive functional equation (6) is realized by Digital Logic Circuits(DLC).

The functional equation (6) is called the additive functional equation, since the function $f(x) = cx$ is its solution. Every solution of the additive functional equation is said to be a additive mapping.

The paper is organized as follows: In Section-2, we obtain the general solution of the new functional equation (6) and in Section-3 and in Section-4, using the fixed point technique, we investigate the Hyers-Ulam-Rassias stability and Ulam-J Rassias stability of the functional equation (6) in fuzzy normed space. Also, in Section-5, we study the application of functional equation (6) in digital logic circuits.

2 The General Solution of the Functional Equation (6)

Let X and Y be a linear spaces. In this section we will find out the general solution of (6).

Theorem 3.*A mapping $f : X \to Y$ is additive if and only if f satisfies the functional equation*

$$\begin{aligned} &3f(x+3y) - f(3x+y) \\ &\quad = 12[f(x+y) + f(x-y)] - 24f(x) + 8f(y) \end{aligned} \tag{7}$$

forall $x,y \in X$.

*Proof.*Suppose that f is additive, then consider the standard additive functional equation

$$f(x+y)=f(x)+f(y) \tag{8}$$

holds for all $x,y\in X$ .By putting $x=y=0$ in (8), we see that $f(0)=0$, and setting (x,y) by (x,x)in (8), we obtain

$$f(2x)=2f(x) \tag{9}$$

for all $x\in X$. Setting $(x,y)=(x,2x)$ in (8) and using (9), we get

$$f(3x)=3f(x) \tag{10}$$

for all $x\in X$. Setting $(x,y)=(x,-x)$ in (8),we get

$$f(-x)=-f(x)$$

for all $x\in X$. Therefore, f is odd. Setting $(x,y)=(x+y,x-y)$ in (8) and multiply the resultant by 12, we get

$$24f(x)=12[f(x+y)+f(x-y)] \tag{11}$$

for all $x\in X$. Setting $(x,y)=(x,3y)$ in (8) and multiply the resultant by 3, we get

$$3f(x+3y)=3f(x)+9f(y) \tag{12}$$

for all $x\in X$. Setting $(x,y)=(3x,y)$ in (8), we get

$$f(3x+y)=3f(x)+f(y) \tag{13}$$

for all $x\in X$. Subtracting (13) from (12), we get

$$3f(x+3y)-f(3x+y)=8f(y) \tag{14}$$

for all $x\in X$. Adding (11) and (14), we arrive (7).

Conversely, Assume that f satisfies the functional equation (7). Setting $(x,y)=(0,0)$ and $(x,0)$ in (7), we get

$$f(0)=0 \ \ and \ \ f(3x)=3f(x) \tag{15}$$

respectively for all $x\in X$. Setting $(x,y)=(0,x)$ in (7) and using (15), we obtain

$$f(-x)=-f(x) \tag{16}$$

for all $x\in X$. Thus f is an odd function. Setting $(x,y)=(-x,x)$ in (7) and using (16), we obtain

$$f(2x)=2f(x) \tag{17}$$

for all $x\in X$. Setting $(x,y)=(x-y,x+y)$ in (7), we get

$$\begin{aligned}&3f(4x+2y)-f(4x-2y)\\&=24[f(x)-f(y)]-24f(x-y)+8f(x+y)\end{aligned} \tag{18}$$

for all $x,y\in X$. Replacing y by $-y$ in (18), using oddness of f, adding the resultant equation with (18) and using (17), we arrive

$$\begin{aligned}&f(2x+y)+f(2x-y)\\&=12f(x)-4f(x+y)-4f(x-y)\end{aligned} \tag{19}$$

for all $x,y\in X$. Setting $(x,y)=(x,-2y)$ in (19) and using (17), we arrive

$$\begin{aligned}&f(x-y)+f(x+y)\\&=6f(x)-2f(x-2y)-2f(x+2y)\end{aligned} \tag{20}$$

for all $x,y\in X$. Replacing x by y, y by x in (20) and using oddness of f, we obtain

$$\begin{aligned}&-f(x-y)+f(x+y)\\&=6f(y)+2f(2x-y)-2f(2x+y)\end{aligned} \tag{21}$$

for all $x,y\in X$. Replacing y by $-y$ in (21), using oddness of f, we arrive

$$\begin{aligned}&2f(2x-y)\\&=2f(2x+y)-6f(y)-f(x-y)+f(x+y)\end{aligned} \tag{22}$$

for all $x,y\in X$. Using (22) in (19), we obtain

$$\begin{aligned}&4f(2x+y)\\&=-9f(x+y)-7f(x-y)+24f(x)+6f(y)\end{aligned} \tag{23}$$

for all $x,y\in X$. Replacing $x+y$ by y in (23), we get

$$\begin{aligned}&4f(x+x+y)=-9f(x+y)\\&-7f(2x-x-y)+24f(x)+6f(-(x-(x+y)))\end{aligned}$$

$$\begin{aligned}&7f(2x-y)\\&=-4f(x+y)-6f(x-y)-9f(y)+24f(x)\end{aligned} \tag{24}$$

for all $x,y\in X$. Adding (23) and (24), we arrive

$$\begin{aligned}&f(2x+y)+f(2x-y)\\&=-\frac{79}{28}f(x+y)-\frac{73}{28}f(x-y)+\frac{6}{28}f(y)+\frac{264}{28}f(x)\end{aligned} \tag{25}$$

for all $x,y\in X$. using (19) and (25), we arrive

$$-11f(x+y)-13f(x-y)=2f(y)-24f(x) \tag{26}$$

for all $x,y\in X$. Replacing x by $2x$ in (19) and in the resultant again using (19), we get

$$\begin{aligned}&f(4x+y)+f(4x-y)\\&=-24f(x)+16f(x+y)+16f(x-y)\end{aligned} \tag{27}$$

for all $x,y\in X$. Putting $2x+y$ instead of y in (19) and using oddness of f, we obtain

$$\begin{aligned}&f(4x+y)-f(y)\\&=12f(x)-4f(3x+y)+4f(x+y)\end{aligned} \tag{28}$$

for all $x,y\in X$. Replacing y by $-y$ in (28), using oddness of f and adding the resultant equation with (28), we arrive

$$\begin{aligned}&f(4x+y)+f(4x-y)=24f(x)\\&-4[f(3x+y)+f(3x-y)]+4[f(x+y)+f(x-y)]\end{aligned} \tag{29}$$

for all $x,y \in X$. Replacing y by $x+y$ in (19) and using oddness of f, we arrive

$$\begin{aligned} &f(3x+y)+f(x-y) \\ &\quad = 12f(x)-4f(2x+y)+4f(y) \end{aligned} \tag{30}$$

for all $x,y \in X$. Replacing y by $-y$ in (30) and combining the resultant equation with (30) and using (19), we get

$$\begin{aligned} &f(3x+y)+f(3x-y) \\ &\quad = -24f(x)+15f(x+y)+15f(x-y) \end{aligned} \tag{31}$$

for all $x,y \in X$. Using (31) in (29), we get

$$\begin{aligned} &f(4x+y)+f(4x-y) \\ &\quad = 120f(x)-56f(x+y)-56f(x-y) \end{aligned} \tag{32}$$

for all $x,y \in X$. Using equations (27) and (32), we arrive at

$$f(x-y) = 2f(x)-f(x+y) \tag{33}$$

for all $x,y \in X$. Using (33) in (26), we arrive (8). Therefore, f is additive function.

Let ϕ be a function from $X \times X$ to Z. A mapping $f : X \to Y$ is said to be ϕ-approximately additive function if

$$\begin{aligned} &N\big(3f(x+3y)-f(3x+y)-12[f(x+y)+f(x-y)] \\ &+24f(x)-8f(y),t\big) \geq N'(\phi(x,y),t) \end{aligned} \tag{34}$$

3 Fuzzy Stability of (6) using Fixed Point Alternative

The following lemma [22] which will be used in our main result.

Lemma 1.*Let (Z,N') be a fuzzy normed space and $\phi : X \to Z$ be a function. Let $E = \{g : X \to Y; g(0) = 0\}$ and define*

$$\begin{aligned} &d_M(g,h) \\ &= \inf\{a \in \mathbb{R}^+ : N(g(x)-h(x),at) \geq N'(\phi(x,0),t)\}, \end{aligned}$$

for all $x \in X$, $t > 0$ and $h \in E$. Then d_M is a complete generalized metric on E.

Theorem 4.*Let X be a linear space and (Z,N') be a FNS. Suppose that a function $\phi : X \times X \to Z$ satisfying $\phi(3x,3y) = \alpha\phi(x,y)$ for all $x,y \in X$ and $\alpha \neq 0$. Suppose that (Y,N) be a fuzzy Banach space and $f : X \to Y$ be a ϕ-approximately additive function. If for some $0 < \alpha < 3$*

$$N'(\phi(3x,0),t) \geq N'(\alpha\phi(x,0),t), \tag{35}$$

and

$$\lim_{n\to\infty} N'(\phi(3^n x,3^n y),3^n t) = 1, \tag{36}$$

for all $x,y \in X$ and $t > 0$. Then there exists a unique additive mapping $A : X \to Y$ such that

$$N(A(x)-f(x),t) \geq N'(\phi(x,0),(3-\alpha)t),$$

for all $x \in X$ and $t > 0$.

*Proof.*Put $y = 0$ in (34). we arrive

$$N\left(\frac{f(3x)}{3}-f(x),\frac{t}{3}\right) \geq N'(\phi(x,0),t), \tag{37}$$

for all $x \in X$ and $t > 0$. Consider the set $E = \{g : X \to Y; g(0) = 0\}$ together with the mapping d_M defined on $E \times E$ by

$$\begin{aligned} &d_M(g,h) \\ &= \inf\{a \in \mathbb{R}^+ : N(g(x)-h(x),at) \geq N'(\phi(x,0),t)\}. \end{aligned}$$

for all $x \in X$, $t > 0$ and $h \in E$. It is known that $d_M(g,h)$ complete generalized metric space by Lemma 1. Now, we define the linear mapping $J : E \to E$ such that

$$Jg(x) = \frac{1}{3}g(3x).$$

It is easy to see that J is a strictly contractive self-mapping of E with the Lipschitz constant $\frac{\alpha}{3}$. Indeed, let $g,h \to E$ be given such that $d_M(g,h) = \varepsilon$. Then

$$N(g(x)-h(x),\varepsilon t) \geq N'(\phi(x,0),t),$$

for all $x \in X$ and $t > 0$. Thus

$$\begin{aligned} &N\left(Jg(x) = Jh(x),\frac{\alpha}{3}\varepsilon t\right) \\ &= N\left(\frac{g(3x)}{3}-\frac{h(3x)}{3},\frac{\alpha}{3}\varepsilon t\right) \\ &= N(g(3x)-h(3x),\alpha\varepsilon t) \geq N'(\phi(3x,0),\alpha t), \end{aligned} \tag{38}$$

for all $x \in X$ and $t > 0$. Using (35) in (38), we obtain

$$\begin{aligned} &N\left(Jg(x)-Jh(x),\frac{\alpha}{3}\varepsilon t\right) \\ &\qquad \geq N'(\alpha\phi(x,0),\alpha t) = N'(\phi(x,0),t), \end{aligned}$$

for all $x \in X$ and $t > 0$. Therefore

$$d_M(g,h) = \varepsilon \Rightarrow d_M(Jg,Jh) \leq \frac{\alpha\varepsilon}{3}.$$

This means that

$$d_M(Jg,Jh) \leq \frac{\alpha}{3}d_M(g,h),$$

for all $g,h \in E$. Next from (37), we have $d_M(f,Jf) \leq \frac{1}{3}$. Using the fixed point alternative we deduce the existence of a fixed point of J, that is, the existence of a mapping $A : X \to Y$ such that $A(3x) = 3A(x)$, for all $x \in X$. Moreover, we have $d_M(J^n f,A) \to 0$, which implies

$$N - lim_n \frac{f(3^n x)}{3^n} = A(x),$$

for every $x \in X$. Also

$$d_M(f,A) \leq \frac{1}{1-L}d_M(f,Jf)$$

implies

$$d_M(f,A) \le \frac{1}{3\left(1-\frac{\alpha}{3}\right)} = \frac{1}{3-\alpha}.$$

This implies that

$$N\left(A(x)-f(x),\frac{1}{3-\alpha}t\right) \ge N'\left(\phi(x,0),t\right), \tag{39}$$

for all $x\in X$ and $t>0$. Replacing t by $(3-\alpha)t$ in (39), we obtain

$$N\left(A(x)-f(x),t\right) \ge N'\left(\phi(x,0),(3-\alpha)t\right),$$

for all $x\in X$ and $t>0$. Let $x,y\in X$. Then

$$N\big(3A(x+3y)-A(3x+y)-12[A(x+y)+A(x-y)] \tag{40}$$
$$+24A(x)-8A(y),t\big) \ge N'\left(\phi(x,y),t\right),$$

for all $x\in X$ and $t>0$. Replacing (x,y) by $(3^n x,3^n y)$ in (40), we get

$$N\big(\frac{1}{3^n}\left(3A\left(3^n(x+3y)\right)-A\left(3^n(3x+y)\right)\right.$$
$$-12[A(3^n x+3^n y)+A(3^n x-3^n y)]$$
$$\left.+24A(3^n x)-8A(3^n y)\right),t\big) \ge N'\left(\phi(3^n x,3^n y),t\right),$$

for all $x,y\in X$ and $t>0$. By (36), we conclude that A fulfills (6).

The uniqueness of A follows from the fact that A is the unique fixed point of J with the following property that there exists $u\in(0,\infty)$ such that

$$N\left(A(x)-f(x),ut\right) \ge N'\left(\phi(x,0),t\right),$$

for all $x\in X$ and $t>0$. This completes the proof of the theorem.

By a modification in the proof of Theorem 4, one can prove the following:

Theorem 5.*Let X be a linear space and (Z,N') be a FNS. Suppose that a function $\phi: X\times X\to Z$ satisfying $\phi(\frac{x}{3},\frac{y}{3}) = \frac{1}{\alpha}\phi(x,y)$ for all $x,y\in X$ and $\alpha\neq 0$. Suppose that (Y,N) be a fuzzy Banach space and $f:X\to Y$ be a ϕ-approximately additive function. If for some $\alpha>3$*

$$N'\left(\phi(\frac{x}{3},0),t\right) \ge N'\left(\phi(x,0),\alpha t\right), \tag{41}$$

and

$$lim_{n\to\infty}N'\left(\phi(3^{-n}x,3^{-n}y),3^{-n}t\right)=1, \tag{42}$$

for all $x,y\in X$ and $t>0$. Then there exists a unique additive mapping $A:X\to Y$ such that

$$N\left(A(x)-f(x),t\right) \ge N'\left(\phi(x,0),(\alpha-3)t\right),$$

for all $x\in X$ and $t>0$.

The proof of the above theorem is similar to the proof of Theorem 4, hence omitted.

Corollary 1.*Let $0<\alpha<3$ or $0>3$. Let X be a linear space, (Z,N') an Fuzzy Normed Space, and (Y,N) be a fuzzy Banach space. Suppose $z_0\in Z$ If $f:X\to Y$ is a mapping such that for all $x,y\in X$, $t>0$,*

$$N\left(Df(x,y),t\right) \ge N'\left((\|x\|^p+\|y\|^p)z_0,t\right),$$

and $f(0)=0$, then there is a unique additive mapping $A: X\to Y$ such that

$$N\left(A(x)-f(x),t\right) \ge N'\left(\|x\|^p z_0,|3-\alpha|t\right),$$

for all $x\in X$ and $t>0$.

*Proof.*Let $\phi(x,y)=(\|x\|^p+\|y\|^p)z_0$ for all $x,y\in X$. The result follows from Theorem 4 and Theorem 5.

4 Ulam- J Rassias Stability of functional equation (6)

The following corollary gives the Ulam-J Rassias stability for the additive functional equation (6). This stability involving the sum of powers of norms and the product of powers of norms, i.e., it involves the mixed powers of norms and this was introduced by J.M. Rassias [21]. Hence the stability discussed in the following Corollaries 2 and 3 is known as Ulam-J Rassias stability involving the mixed product of sum of powers of norms.

Corollary 2.*Let $0<\alpha<3$. Let X be a linear space, (Z,N') an Fuzzy Normed Space, and (Y,N) be a fuzzy Banach space. Suppose $z_0\in Z$ If $f:X\to Y$ is a mapping such that for all $x,y\in X$, $t>0$,*

$$N\left(Df(x,y),t\right) \ge N'\left((\|x\|^p+\|y\|^p+\|x\|^p\|y\|^p)z_0,t\right),$$

and $f(0)=0$, then there is a unique additive mapping $A: X\to Y$ such that

$$N\left(A(x)-f(x),t\right) \ge N'\left(\|x\|^p z_0,(3-\alpha)t\right),$$

for all $x\in X$ and $t>0$.

*Proof.*Let $\phi(x,y)=(\|x\|^p+\|y\|^p+\|x\|^p\|y\|^p)z_0$ for all $x,y\in X$. The result follows from Theorem 4.

Corollary 3.*Let $\alpha>3$. Let X be a linear space, (Z,N') an Fuzzy Normed Space, and (Y,N) be a fuzzy Banach space. Suppose $z_0\in Z$ If $f:X\to Y$ is a mapping such that for all $x,y\in X$, $t>0$,*

$$N\left(Df(x,y),t\right) \ge N'\left((\|x\|^p+\|y\|^p+\|x\|^p\|y\|^p)z_0,t\right),$$

and $f(0)=0$, then there is a unique additive mapping $A: X\to Y$ such that

$$N\left(A(x)-f(x),t\right) \ge N'\left(\|x\|^p z_0,(\alpha-3)t\right),$$

for all $x\in X$ and $t>0$.

*Proof.*Let $\phi(x,y)=(\|x\|^p+\|y\|^p+\|x\|^p\|y\|^p)z_0$ for all $x,y\in X$. The result follows from Theorem 5.

5 Application of functional equation (6) in digital logic circuits

The basic elements of digital systems are logic circuits. The logic gate can have a minimum of two inputs (except NOT gate) and gives output based on the operations. The OR gate is only used for this property. Digital circuits are simpler, cheaper but equivalent, realizations of a circuit can reap huge payoff's in reducing the overall cost of the design.

All digital electronic circuits and microprocessor based systems are constructed from the basic building blocks of digital logic gates. Basic digital logic gates perform logical operations of AND, OR and NOT on binary numbers. In digital logic design only two voltage levels or states are allowed and these states are generally referred to as Logic 1 and Logic 0, High and Low, or True and False. These two states are represented in Boolean algebra and standard truth tables by the binary digits of 1 and 0 respectively.

A good example of a digital state is a simple light switch as it is either ON or OFF but not both at the same time. Then we can summarise the relationship between these various digital states as being:
Boolean Algebra: Logic 1 Logic 0, Boolean Logic: True(T), False (F) and Voltage State: High(H), Low (L).

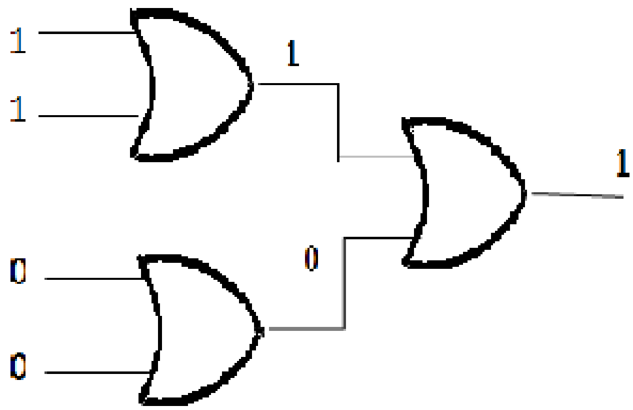
Most digital logic gates and digital logic systems use Positive logic, in which a logic level 0 or LOW is represented by a zero voltage, $0v$ or ground and a logic level 1 or HIGH is represented by a higher voltage such as $+5$ volts, with the switching from one voltage level to the other, from either a logic level 0 to a 1 or a 1 to a 0 being made as quickly as possible to prevent any faulty operation of the logic circuit.

There also exists a complementary Negative Logic system in which the values and the rules of a logic 0 and a logic 1 are reversed but in this tutorial section about digital logic gates we shall only refer to the positive logic convention as it is the most commonly used.

Consider the left hand side of functional equation (6).

$$\begin{aligned}
&3f(x+3y)-f(3x+y)\\
&\Rightarrow 3\left(f(x)+3f(y)\right)-3f(x)-f(y)\\
&\Rightarrow 3\left(f(x)+3f(y)\right)+3\overline{f}(x)+f(\overline{y}).
\end{aligned}$$

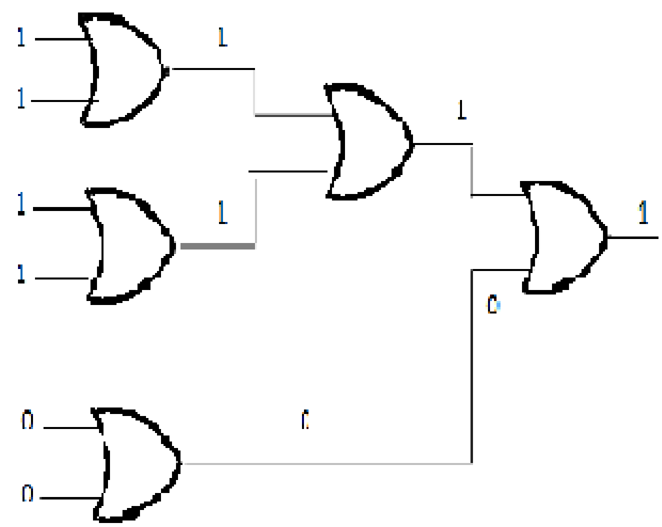
Assume $3f(x)$ is 1-level and $3\overline{f}(x)$ is 0-level, we get



Consider the Right hand side of functional equation (6)

$$\begin{aligned}
&12[f(x+y)+f(x-y)]-24f(x)+8f(y)\\
&\Rightarrow 12f(x)+12f(y)+12f(x)-12f(y)-24f(x)+8f(y)\\
&\Rightarrow 12f(x)+12f(y)+12f(x)+8f(y)+12\overline{f}(y)+24\overline{f}(x).
\end{aligned}$$

We get



6 Conclusion

In this paper, we studied the general solution for the new additive functional equation and investigated its Hyers-Ulam-Rassias stability and Ulam-J Rassias stability in fuzzy normed space using the fixed point technique. This method is easier and gives better results than the stability problem solved by other fuzzy settings. Also, we studied the application of functional equation (6) in digital circuits. The additive functional equation (6) with digital logic circuits are used for many industrial applications, such as electrical drives, locomotive, control systems and traction, etc.,

Acknowledgement

The author is grateful to the referees and the Managing Editor for their careful checking and useful comments to bring up this article substantially.

References

[1] **T. Aoki**, On the stability of the linear transformation in Banach spaces, J. Math . Soc.Japan,2 (1950), 64-66.

[2] **T. Bag and S .K. Samanta**, Finite dimensional fuzzy normed linear spaces, J. Fuzzy Math. 11 (3) (2003) 687-705.

[3] **R. Biswas**, Fuzzy inner product space and fuzzy norm functions, Inform. Sci. 53 (1991) 185-190.

[4] **A.Bodaghi**, Stability of a mixed type additive and quartic function equation, Filomat, 28:8 (2014).

[5] **A. Bodaghi and S. O. Kim**, Ulam's type stability of a functional equation deriving from quadratic and additive functions, J. Math. Ineq., 9, No. 1 (2015), 73-84.

[6] **I. Cadariu and V. Radu**, Fixed points and the stability of Jensen's functional equation, Journal of Inequalities in Pure and Applied Mathematics, vol. 4, pp. 1-7, 2003.

[7] **L. Cadariu and V. Radu**, On the stability of the Cauchy functional equation: a fixed point approach, in Iteration Theory (ECIT 02), Karl-Franzens-Universitat Graz, Austria, Grazer Mathematische Berichte, vol. 346, pp. 43-52, 2004.

[8] **S.C. Cheng and J.N.Mordeson**, Fuzzy linear operator and fuzzy normed linear spaces, Bull. Calcuta Math. Soc. 86 (1994) 429-436.

[9] **S. Czerwik**, On the stability of the quadratic mapping in normed spaces, Abh. Math. Sem. Univ.Hamburg 62 (1992), 59-64.

[10] **JB.Diaz, B.Margolis,** A fixed point theorem of the alternative for contractions on generalized complete metric space, Bull Am Math Soc. 126(74):305-309 (1968).

[11] **C. Felbin**, Finite dimensional fuzzy normed linear space, Fuzzy Sets Syst. 48 (1992) 239-248.

[12] **P.Gavruta**, A generalization of the Hyers-Ulam-Rassias stability of approximately additive mappings, J. Math. Anal. Appl. 184 (1994), 431-436.

[13] **D. H. Hyers**, On the stability of the linear functional equation, Proc. Natl. Acad. Sci. 27 (1941), 222-224.

[14] **A.K.Katsaras**, Fuzzy topological vector spaces II, Fuzzy Sets Syst. 12 (1984) 143-154.

[15] **Matina J. Rassias, M. Arunkumar, S. Ramamoorthi**, Stability of the Leibniz additive-quadratic functional equation in Quasi-Beta normed space: Direct and fixed point methods, Journal of Concrete and Applicable Mathematics (JCAAM), Vol. 14 No. 1-2, (2014), 22-46.

[16] **J.M. Rassias**, on approximately of approximately linear mappings by linear mappings, J. Funct. Anal. USA, 46, (1982) 126-130. http://dx.doi.org/10.1016/0022-1236(82)90048-9.

[17] **J.M. Rassias, H.M. Kim**, Generalized Hyers-Ulam stability for general additive functional equations in quasi- -normed spaces, J. Math. Anal. Appl., 356 (2009), no. 1, 302-309.

[18] **Hark-Mahn Kim and Soon Lee**, Refined Stability of an Additive Functional Equation with Several Variables, Applied Mathematical Sciences, Vol. 7, 2013, no. 126, 6251 - 6261.

[19] **Th. M. Rassias**, On the stability of the linear mapping in Banacb spaces, Proc. Amer. Math. Soc. 72 (1978), 297-300.

[20] **K.Ravi, M.Arunkumar, P.Narasimman**, Fuzzy stability of an additive functional equation, International Journal of Mathematics and Statistics, Autumn 2011, Volume 9, Number A11, pp.88-105.

[21] **K.Ravi, M. Arunkumar and J.M. Rassias**, On the Ulam stability for the orthogonally general Euler-Lagrange type functional equation, International Journal of Mathematical Sciences, Autumn 2008 Vol.3, No. 08, 36 - 47.

[22] **Syed Abdul Mohiuddine and Abdullah Alotaibi**, Fuzzy stability of a additive functional equation via fixed point technique, Advances in Difference Equations 2012:48, http://www.advances in difference equations.com/content/2012/1/48.

[23] **S.M. Ulam**, A Colloection of the Mathematical Problems, Interscience Publ., New York, 1960.

[24] **C.Wu and Jinxuan Fang**, Fuzzy generalization of Klomogoroffs theorem, J.Harbin Inst. Technol. 1 (1984) 1-7.

[25] **JZ.Xiao, XH.Zhu** Fuzzy normed spaces of operators and its completeness. Fuzzy Sets Syst. 133(3) 135-146 (2003).

Pasupathi Narasimman was conferred Doctorate in Mathematics for his work on Stability of Quadratic, Cubic and Mixed type Functional Equations at Bharathiar University. He is an Assistant Professor in the Department of Mathematics at Thiruvalluvar University College of Arts and Science, Tirupattur. His research interests are applied mathematics and functional equations. He has published many research articles in reputed international journals of mathematics and engineering sciences.

R. Amuda is an Associate Professor and Head of Mathematics Department at Jeppiaar Institute of Technology, Chennai. She has a teaching experience of more than 25 years. She has guided 25 candidates for their M. Phil. She is an expert in boundary value problems and she has published 5 papers in reputed international journals. She is passionate about Mathematics and shares her vast knowledge of the same with her students. Her areas of specialisation are dynamical systems, game theory, differential geometry and its applications.