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Form and Periodicity of Solutions of Some Systems of Higher-Order Difference Equations

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Abstract: The paper deals with form and periodicity of solutions of the system

$$x_{n+1}=\frac{1}{1-y_{n-k}},\quad y_{n+1}=\frac{1}{1-x_{n-k}},\quad n,k\in\mathbb{N}_0 \tag{1}$$

where $\mathbb{N}_0=\mathbb{N}\cup\{0\}$ and the initial conditions $x_{-k},x_{-k+1},\ldots,x_0,y_{-k},y_{-k+1},\ldots,y_0$ are non zero real numbers.

Keywords: System of difference equations, general solution, periodicity.

1 Introduction

There has been a great interest in studying difference equations and systems. Solvable difference equations attract attention of mathematicians for a long time. Recently, there has been an increasing interest in the topic (see [1]-[15] and the related references therein). Difference equations usually describe the evolution of certain phenomena over the course of time. Indeed difference equations have been applied in several mathematical models in biology, economics, genetics, population dynamics, medicines and so forth.

In this paper and motivated by [2], we deal with the form of the solutions of the following systems of rational difference equations

$$x_{n+1}=\frac{1}{1-y_{n-k}},\quad y_{n+1}=\frac{1}{1-x_{n-k}},\quad n,k\in\mathbb{N}_0$$

where $\mathbb{N}_0=\mathbb{N}\cup\{0\}$ with arbitrary nonzero initial conditions.

2 Main result

We start-off this section by giving the periodicity of the solutions of the system (1).

2.1 Periodicity of the solutions

Theorem 1. *Every solution $\{x_n,y_n\}_{n\ge -k}$ of system (1) is periodic of period $6k+6$, that is*

$$x_{n+(6k+6)}=x_n,\quad y_{n+(6k+6)}=y_n,$$

where $n=-k,-k+1,\ldots$ for some natural number k.

Proof. We have

$$\begin{aligned}
x_{n+(6k+6)} &= \frac{1}{1-y_{n+5k+5}}=\frac{1}{1-\frac{1}{1-x_{n+4k+4}}}\\
&=\frac{-1+x_{n+4k+4}}{x_{n+4k+4}}=\frac{-1+\frac{1}{1-y_{n+3k+3}}}{\frac{1}{1-y_{n+3k+3}}}\\
&=y_{n+3k+3}=\frac{1}{1-x_{n+2k+2}}\\
&=\frac{1}{1-\frac{1}{1-y_{n+k+1}}}=\frac{-1+y_{n+k+1}}{y_{n+k+1}}\\
&=\frac{-1+\frac{1}{1-x_n}}{\frac{1}{1-x_n}}=x_n.
\end{aligned}$$

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Similarly, we have

$$y_{n+(6k+6)} = \frac{1}{1-x_{n+5k+5}} = \frac{1}{1-\frac{1}{1-y_{n+4k+4}}}$$
$$= \frac{-1+y_{n+4k+4}}{y_{n+4k+4}} = \frac{-1+\frac{1}{1-x_{n+3k+3}}}{\frac{1}{1-x_{n+3k+3}}}$$
$$= x_{n+3k+3} = \frac{1}{1-y_{n+2k+2}}$$
$$= \frac{1}{1-\frac{1}{1-x_{n+k+1}}} = \frac{-1+x_{n+k+1}}{x_{n+k+1}}$$
$$= \frac{-1+\frac{1}{1-y_n}}{\frac{1}{1-y_n}} = y_n.$$

2.2 Form of the solutions

In the following theorem we give explicit formulas for the solutions of system (1).

Theorem 2. *Let $\{x_n, y_n\}_{n\geq -k}$ be a solution of system (1). Then for $n = 0,1,...,$ we have*

$$x_{6(k+1)n+i} = \frac{1}{1-y_{-k+i-1}}, \quad i = 1,...,k+1, \tag{2}$$
$$y_{6(k+1)n+i} = \frac{1}{1-x_{-k+i-1}}, \quad i = 1,...,k+1, \tag{3}$$
$$x_{6(k+1)n+i} = \frac{-1+x_{-k+i-1}}{x_{-k+i-1}}, \quad i = k+2,...,2k+2, \tag{4}$$
$$y_{6(k+1)n+i} = \frac{-1+y_{-k+i-1}}{y_{-k+i-1}}, \quad i = k+2,...,2k+2, \tag{5}$$
$$x_{6(k+1)n+i} = y_{-k+i-1}, \quad i = 2k+3,...,3k+3, \tag{6}$$
$$y_{6(k+1)n+i} = x_{-k+i-1}, \quad i = 2k+3,...,3k+3, \tag{7}$$
$$x_{6(k+1)n+i} = \frac{1}{1-x_{-k+i-1}}, \quad i = 3k+4,...,4k+4, \tag{8}$$
$$y_{6(k+1)n+i} = \frac{1}{1-y_{-k+i-1}}, \quad i = 3k+4,...,4k+4, \tag{9}$$
$$x_{6(k+1)n+i} = \frac{-1+y_{-k+i-1}}{y_{-k+i-1}}, \quad i = 4k+5,...,5k+5, \tag{10}$$
$$y_{6(k+1)n+i} = \frac{-1+x_{-k+i-1}}{x_{-k+i-1}}, \quad i = 4k+5,...,5k+5, \tag{11}$$
$$x_{6(k+1)n+i} = x_{-k+i-1}, \quad i = 5k+6,...,6k+6, \tag{12}$$
$$y_{6(k+1)n+i} = y_{-k+i-1}, \quad i = 5k+6,...,6k+6, \tag{13}$$

where the initial values are arbitrary nonzero real numbers with $x_{-k}, x_{-k+1}, \ldots, x_0 \neq 1$ and $y_{-k}, y_{-k+1}, \ldots, y_0 \neq 1$.

Proof. **1)** Let $n = 0,1,\ldots,k$. We get from system (1)

$$x_1 = \frac{1}{1-y_{-k}},$$
$$y_1 = \frac{1}{1-x_{-k}},$$
$$x_2 = \frac{1}{1-y_{-k+1}},$$
$$y_2 = \frac{1}{1-x_{-k+1}},$$
$$\vdots$$
$$x_{k+1} = \frac{1}{1-y_0},$$
$$y_{k+1} = \frac{1}{1-x_0}.$$

From Theorem (1) we get

$$x_1 = x_{6(k+1)+1} = x_{6(k+1)2+1} = \cdots = \frac{1}{1-y_{-k}},$$
$$y_1 = y_{6(k+1)+1} = y_{6(k+1)2+1} = \cdots = \frac{1}{1-x_{-k}},$$
$$x_2 = x_{6(k+1)+2} = x_{6(k+1)2+2} = \cdots = \frac{1}{1-y_{-k+1}},$$
$$y_2 = y_{6(k+1)+2} = y_{6(k+1)2+2} = \cdots = \frac{1}{1-x_{-k+1}},$$
$$\vdots$$
$$x_{k+1} = x_{6(k+1)+k+1} = x_{6(k+1)2+k+1} = \cdots \frac{1}{1-y_0},$$
$$y_{k+1} = y_{6(k+1)+k+1} = y_{6(k+1)2+k+1} = \cdots \frac{1}{1-x_0}.$$

Hence we have the formulas (2) and (3).

2) Let $n = k+1, k+2, ..., 2k+1$. From (1) we have

$$x_{n+1} = \frac{1}{1-y_{(n-k-1)+1}} = \frac{1}{1-\frac{1}{1-x_{n-k-1-k}}} = \frac{-1+x_{n-2k-1}}{x_{n-2k-1}}, \tag{14}$$

and

$$y_{n+1} = \frac{1}{1-x_{(n-k-1)+1}} = \frac{1}{1-\frac{1}{1-y_{n-k-1-k}}} = \frac{-1+y_{n-2k-1}}{y_{n-2k-1}}. \tag{15}$$

Now from (14) and (15), we get

$$x_{k+2} = \frac{-1+x_{-k}}{x_{-k}},$$
$$y_{k+2} = \frac{-1+y_{-k}}{x_{-k}},$$
$$x_{k+3} = \frac{-1+x_{-k+1}}{x_{-k}},$$
$$y_{k+3} = \frac{-1+y_{-k}}{x_{-k+1}},$$
$$\vdots$$
$$x_{2k+2} = \frac{-1+x_0}{x_0},$$
$$y_{2k+2} = \frac{-1+y_0}{x_0}.$$

From Theorem (1), we get

$$x_{k+2} = x_{6(k+1)+k+2} = x_{6(k+1)2+k+2} = \cdots = \frac{-1+x_{-k}}{x_{-k}},$$
$$y_{k+2} = y_{6(k+1)+k+2} = y_{6(k+1)2+k+2} = \cdots = \frac{-1+x_{-k}}{x_{-k}},$$
$$x_{k+3} = x_{6(k+1)+k+3} = x_{6(k+1)2+k+3} = \cdots = \frac{-1+x_{-k+1}}{x_{-k+1}},$$
$$y_{k+3} = y_{6(k+1)+k+3} = y_{6(k+1)2+k+3} = \cdots = \frac{-1+x_{-k+1}}{x_{-k+1}},$$
$$\vdots$$
$$x_{2k+2} = x_{6(k+1)+2k+2} = x_{6(k+1)2+2k+2} = \cdots = \frac{-1+x_0}{x_0},$$
$$y_{2k+2} = y_{6(k+1)+2k+2} = y_{6(k+1)2+2k+2} = \cdots = \frac{-1+y_0}{y_0}.$$

This complete the proof of formulas (4) and (5).

3) Let $n = 2k+2, 2k+3, ..., 3k+2$. From (1), (14) and (15) we get

$$x_{n+1} = \frac{-1+\frac{1}{1-y_{n-2k-2-k}}}{\frac{1}{1-y_{n-2k-2-k}}} = \frac{\frac{y_{n-3k-2}}{1-y_{n-3k-2}}}{\frac{1}{1-y_{n-3k-2}}} = y_{n-3k-2}, \quad (16)$$

and

$$y_{n+1} = \frac{-1+\frac{1}{1-x_{n-2k-2-k}}}{\frac{1}{1-x_{n-2k-2-k}}} = \frac{\frac{x_{n-3k-2}}{1-x_{n-3k-2}}}{\frac{1}{1-x_{n-3k-2}}} = x_{n-3k-2}. \quad (17)$$

Using (16) and (17) we obtain

$$x_{2k+3} = y_{-k},$$
$$y_{2k+3} = x_{-k},$$
$$x_{2k+4} = y_{-k+1},$$
$$y_{2k+4} = x_{-k+1},$$
$$\vdots$$
$$x_{3k+3} = y_0,$$
$$y_{3k+3} = x_0.$$

Using the fact that $\{x_n\}$ and $\{y_n\}$ are periodic with period $6(k+1)$, we get formulas (6) and (7). That is

$$x_{2k+3} = x_{6(k+1)+2k+3} = x_{6(k+1)2+2k+3} = \cdots = y_{-k},$$
$$y_{2k+3} = y_{6(k+1)+2k+3} = y_{6(k+1)2+2k+3} = \cdots = x_{-k},$$
$$x_{2k+4} = x_{6(k+1)+2k+4} = x_{6(k+1)2+2k+4} = \cdots = y_{-k+1},$$
$$y_{2k+4} = y_{6(k+1)+2k+4} = y_{6(k+1)2+2k+4} = \cdots = x_{-k+1},$$
$$\vdots$$
$$x_{3k+3} = x_{6(k+1)+3k+3} = x_{6(k+1)2+3k+3} = \cdots = y_0,$$
$$y_{3k+3} = y_{6(k+1)+3k+3} = y_{6(k+1)2+3k+3} = \cdots = x_0.$$

4) Let $n = 3k+3, 3k+4, ..., 4k+3$. From (1), (16) and (17), we obtain

$$x_{n+1} = \frac{1}{1-x_{n-4k-3}}, \quad (18)$$

and

$$y_{n+1} = \frac{1}{1-y_{n-4k-3}}. \quad (19)$$

Hence we have

$$x_{3k+4} = \frac{1}{1-x_{-k}},\ y_{3k+4} = \frac{1}{1-y_{-k}},$$
$$x_{3k+5} = \frac{1}{1-x_{-k+1}},\ y_{3k+5} = \frac{1}{1-y_{-k+1}},$$
$$\vdots$$
$$x_{4k+4} = \frac{1}{1-x_0},\ y_{4k+4} = \frac{1}{1-y_0}.$$

From Theorem (1) we get

$$x_{3k+4} = x_{6(k+1)+3k+4} = x_{6(k+1)2+3k+4} = \cdots = \frac{1}{1-x_{-k}},$$
$$y_{3k+4} = y_{6(k+1)+3k+4} = y_{6(k+1)2+3k+4} = \cdots = \frac{1}{1-y_{-k}},$$
$$x_{3k+5} = x_{6(k+1)+3k+5} = x_{6(k+1)2+3k+5} = \cdots = \frac{1}{1-x_{-k+1}},$$
$$y_{3k+5} = y_{6(k+1)+3k+5} = y_{6(k+1)2+3k+5} = \cdots = \frac{1}{1-y_{-k+1}},$$
$$\vdots$$
$$x_{4k+4} = x_{6(k+1)+4k+4} = x_{6(k+1)2+4k+4} = \cdots = \frac{1}{1-x_0},$$
$$y_{4k+4} = y_{6(k+1)+4k+4} = y_{6(k+1)2+4k+4} = \cdots = \frac{1}{1-y_0}.$$

This complete the proof of formulas (8) and (9).

5) Let $n = 4k+4, 4k+5, ..., 5k+4$. From (1), (18) and (19) we have

$$x_{n+1} = \frac{1}{1-\frac{1}{1-y_{n-4k-4-k}}} = \frac{-1+y_{n-5k-4}}{y_{n-5k-4}}, \quad (20)$$

and

$$y_{n+1}=\frac{1}{1-\frac{1}{1-x_{n-4k-4-k}}}=\frac{-1+x_{n-5k-4}}{x_{n-5k-4}}. \quad (21)$$

So, it follows that

$$\begin{aligned}x_{4k+5}&=\frac{-1}{1-y_{-k}},\\ y_{4k+5}&=\frac{1}{1-x_{-k}},\\ x_{4k+6}&=\frac{-1}{1-y_{-k+1}},\\ y_{4k+6}&=\frac{1}{1-x_{-k+1}},\\ &\vdots\\ x_{5k+5}&=\frac{1}{1-y_0},\\ y_{5k+5}&=\frac{1}{1-x_0}.\end{aligned}$$

Using Theorem (1) we obtain formulas in (10) and (11), that is

$$\begin{aligned}x_{4k+5}&=x_{6(k+1)+4k+5}=x_{6(k+1)2+4k+5}=\cdots=\frac{-1}{1-y_{-k}},\\ y_{4k+5}&=y_{6(k+1)+4k+5}=y_{6(k+1)2+4k+5}=\cdots=\frac{1}{1-x_{-k}},\\ x_{4k+6}&=x_{6(k+1)+4k+6}=x_{6(k+1)2+4k+6}=\cdots=\frac{-1}{1-y_{-k}},\\ y_{4k+6}&=y_{6(k+1)+4k+6}=y_{6(k+1)2+4k+6}=\cdots=\frac{1}{1-x_{-k}},\\ &\vdots\\ x_{5k+5}&=x_{6(k+1)+5k+5}=x_{6(k+1)2+5k+5}=\cdots=\frac{-1}{1-y_{-k}},\\ y_{5k+5}&=y_{6(k+1)+5k+5}=y_{6(k+1)2+5k+5}=\cdots=\frac{1}{1-x_{-k}}.\end{aligned}$$

6) Let $n=5k+5,5k+6,...,6k+5.$ From (1), (20) and (21) we get

$$x_{n+1}=\frac{-1+\frac{1}{1-x_{n-6k-5}}}{\frac{1}{1-x_{n-6k-5}}}=\frac{\frac{x_{n-6k-5}}{1-x_{n-6k-5}}}{\frac{1}{1-x_{n-6k-5}}}=x_{n-6k-5},$$

and

$$y_{n+1}=\frac{-1+\frac{1}{1-y_{n-6k-5}}}{\frac{1}{1-y_{n-6k-5}}}=\frac{\frac{y_{n-6k-5}}{1-y_{n-6k-5}}}{\frac{1}{1-y_{n-6k-5}}}=y_{n-6k-5}.$$

From this it follows that

$$\begin{aligned}x_{5k+6}&=x_{-k},\\ y_{5k+6}&=y_{-k},\\ x_{5k+7}&=x_{-k+1},\\ y_{5k+7}&=y_{-k+1},\\ &\vdots\\ x_{6k+6}&=x_0,\\ y_{6k+6}&=y_0.\end{aligned}$$

Now by Theorem (1) we get

$$\begin{aligned}x_{5k+6}&=x_{6(k+1)+5k+6}=x_{6(k+1)2+5k+7}=\cdots=x_{-k},\\ y_{5k+6}&=y_{6(k+1)+5k+7}=y_{6(k+1)2+5k+7}=\cdots=y_{-k},\\ x_{5k+7}&=x_{6(k+1)+5k+7}=x_{6(k+1)2+5k+7}=\cdots=x_{-k},\\ y_{5k+7}&=y_{6(k+1)+5k+7}=y_{6(k+1)2+5k+7}=\cdots=y_{-k},\\ &\vdots\\ x_{6k+6}&=x_{6(k+1)+6k+6}=x_{6(k+1)2+6k+6}=\cdots=x_0,\\ y_{6k+6}&=y_{6(k+1)+6k+6}=y_{6(k+1)2+6k+6}=\cdots=y_0\end{aligned}$$

which are formulas in (12) and (13). The proof of the theorem is complete.

Example 1. For confirming the results of this section, we consider the following numerical example. Let $k=4$ in system (1), then we obtain the system

$$x_{n+1}=\frac{1}{1-y_{n-4}}, \quad y_{n+1}=\frac{1}{1-x_{n-4}}. \quad (22)$$

Assume $x_{-5}=1$, $x_{-4}=1.6$, $x_{-3}=3.4$, $x_{-2}=6.1$, $x_{-1}=2$, $x_0=1.3$, $y_{-5}=0.7$, $y_{-4}=4.2$, $y_{-3}=0.3$, $y_{-2}=2.4$, $y_{-1}=0.2$ and $y_0=5$. *(See Fig. (1)).*

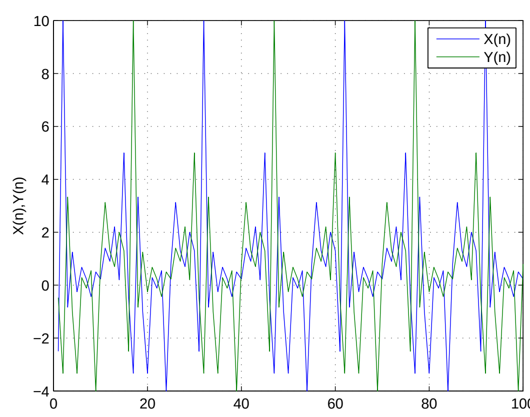

Fig. 1: This figure shows the periodicity of the solutions of system (22)

2.3 Other systems

Corollary 1. *Let* $\{x_n, y_n\}_{n\geq -k}$ *be a solution of system*

$$x_{n+1} = \frac{1}{1+y_{n-k}}, \quad y_{n+1} = \frac{1}{-1+x_{n-k}}, \quad n,k \in \mathbb{N}_0$$

where $\mathbb{N}_0 = \mathbb{N}\cup\{0\}$ *and the initial values are arbitrary nonzero real numbers with* $x_{-k}, x_{-k+1}, ..., x_0 \neq 1$ *and* $y_{-k}, y_{-k+1}, ..., y_0 \neq -1$*. Then for* $n = 0,1,...$*, we have*

$$\begin{aligned}
x_{6(k+1)n+i} &= \frac{1}{1+y_{-k+i-1}}, & i &= 1,...,k+1.\\
y_{6(k+1)n+i} &= \frac{1}{1-x_{-k+i-1}}, & i &= 1,...,k+1.\\
x_{6(k+1)n+i} &= \frac{-1+x_{-k+i-1}}{x_{-k+i-1}}, & i &= k+2,...,2k+2.\\
y_{6(k+1)n+i} &= \frac{1+y_{-k+i-1}}{y_{-k+i-1}}, & i &= k+2,...,2k+2.\\
x_{6(k+1)n+i} &= -y_{-k+i-1}, & i &= 2k+3,...,3k+3.\\
y_{6(k+1)n+i} &= x_{-k+i-1}, & i &= 2k+3,...,3k+3.\\
x_{6(k+1)n+i} &= \frac{1}{1-x_{-k+i-1}}, & i &= 3k+4,...,4k+4.\\
y_{6(k+1)n+i} &= \frac{1}{1+y_{-k+i-1}}, & i &= 3k+4,...,4k+4.\\
x_{6(k+1)n+i} &= \frac{1+y_{-k+i-1}}{y_{-k+i-1}}, & i &= 4k+5,...,5k+5.\\
y_{6(k+1)n+i} &= \frac{-1+x_{-k+i-1}}{x_{-k+i-1}}, & i &= 4k+5,...,5k+5.\\
x_{6(k+1)n+i} &= x_{-k+i-1}, & i &= 5k+6,...,6k+6.\\
y_{6(k+1)n+i} &= -y_{-k+i-1}, & i &= 5k+6,...,6k+6.
\end{aligned}$$

Proof. It follows from Theorem (2) by replacing y_n by $-y_n$.

Corollary 2. *Let* $\{x_n, y_n\}_{n\geq -k}$ *be a solution of system*

$$x_{n+1} = \frac{1}{-1+y_{n-k}}, \quad y_{n+1} = \frac{1}{1+x_{n-k}}, \quad n,k \in \mathbb{N}_0$$

where $\mathbb{N}_0 = \mathbb{N}\cup\{0\}$ *and the initial values are arbitrary nonzero real numbers with* $x_{-k}, x_{-k+1}, ..., x_0 \neq -1$ *and* $y_{-k}, y_{-k+1}, ..., y_0 \neq 1$*. Then for* $n = 0,1,...,$ *we have*

$$\begin{aligned}
x_{6(k+1)n+i} &= \frac{1}{1-y_{-k+i-1}}, & i &= 1,...,k+1.\\
y_{6(k+1)n+i} &= \frac{1}{1+x_{-k+i-1}}, & i &= 1,...,k+1.\\
x_{6(k+1)n+i} &= \frac{1+x_{-k+i-1}}{x_{-k+i-1}}, & i &= k+2,...,2k+2.\\
y_{6(k+1)n+i} &= \frac{-1+y_{-k+i-1}}{y_{-k+i-1}}, & i &= k+2,...,2k+2.\\
x_{6(k+1)n+i} &= y_{-k+i-1}, & i &= 2k+3,...,3k+3.\\
y_{6(k+1)n+i} &= -x_{-k+i-1}, & i &= 2k+3,...,3k+3.\\
x_{6(k+1)n+i} &= \frac{1}{1+x_{-k+i-1}}, & i &= 3k+4,...,4k+4.\\
y_{6(k+1)n+i} &= \frac{1}{1-y_{-k+i-1}}, & i &= 3k+4,...,4k+4.\\
x_{6(k+1)n+i} &= \frac{-1+y_{-k+i-1}}{y_{-k+i-1}}, & i &= 4k+5,...,5k+5.\\
y_{6(k+1)n+i} &= \frac{1+x_{-k+i-1}}{x_{-k+i-1}}, & i &= 4k+5,...,5k+5.\\
x_{6(k+1)n+i} &= -x_{-k+i-1}, & i &= 5k+6,...,6k+6.\\
y_{6(k+1)n+i} &= y_{-k+i-1}, & i &= 5k+6,...,6k+6.
\end{aligned}$$

Proof. It follows from Theorem (2) by replacing x_n by $-x_n$.

Corollary 3. *Let* $\{x_n, y_n\}_{n\geq -k}$ *be a solution of system*

$$x_{n+1} = \frac{1}{-1-y_{n-k}}, \quad y_{n+1} = \frac{1}{-1-x_{n-k}}, \quad n,k \in \mathbb{N}_0$$

where $\mathbb{N}_0 = \mathbb{N}\cup\{0\}$ *and the initial values are arbitrary nonzero real numbers with* x_{-k}, y_{-k}, x_{-k+1}, y_{-k+1},...,x_0, $y_0 \neq -1$. *Then for* $n = 0,1,...,$ *we have*

$$\begin{aligned}
x_{6(k+1)n+i} &= \frac{1}{1+y_{-k+i-1}}, & i &= 1,...,k+1.\\
y_{6(k+1)n+i} &= \frac{1}{1+x_{-k+i-1}}, & i &= 1,...,k+1.\\
x_{6(k+1)n+i} &= \frac{1+x_{-k+i-1}}{x_{-k+i-1}}, & i &= k+2,...,2k+2.\\
y_{6(k+1)n+i} &= \frac{1+y_{-k+i-1}}{y_{-k+i-1}}, & i &= k+2,...,2k+2.\\
x_{6(k+1)n+i} &= -y_{-k+i-1}, & i &= 2k+3,...,3k+3.\\
y_{6(k+1)n+i} &= -x_{-k+i-1}, & i &= 2k+3,...,3k+3.\\
x_{6(k+1)n+i} &= \frac{1}{1+x_{-k+i-1}}, & i &= 3k+4,...,4k+4.\\
y_{6(k+1)n+i} &= \frac{1}{1+y_{-k+i-1}}, & i &= 3k+4,...,4k+4.\\
x_{6(k+1)n+i} &= \frac{1+y_{-k+i-1}}{y_{-k+i-1}}, & i &= 4k+5,...,5k+5.\\
y_{6(k+1)n+i} &= \frac{1+x_{-k+i-1}}{x_{-k+i-1}}, & i &= 4k+5,...,5k+5.\\
x_{6(k+1)n+i} &= -x_{-k+i-1}, & i &= 5k+6,...,6k+6.\\
y_{6(k+1)n+i} &= -y_{-k+i-1}, & i &= 5k+6,...,6k+6.
\end{aligned}$$

Proof. It follows from Theorem (2) by replacing x_n by $-x_n$ and y_n by $-y_n$.

3 Conclusion

In this study, we mainly prove the periodicity and we obtained the forme of the solutions of the system of difference equations (1). The results in this paper can be extended to the following system of difference equations

$$x_{n+1} = \frac{\alpha}{\beta - \gamma y_{n-k}}, \quad y_{n+1} = \frac{\alpha}{\beta - \gamma x_{n-k}}, \quad n,k \in \mathbb{N}_0$$

where $\mathbb{N}_0 = \mathbb{N}\cup\{0\}$ and the initial conditions x_{-k}, $x_{-k+1}, \ldots, x_0$, y_{-k}, $y_{-k+1}, \ldots, y_0$, and α, β, γ are non zero real numbers.

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