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On Bessel-Maitland Function and m-Parameter Mittag-Leffler Function Associated with Fractional Calculus Operators and its Applications

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Abstract: Fractional calculus is of utmost importance because of its extensive use in various fields of inequality theory, applied mathematics, science and engineering. In this paper, our aim is to discuss some image formulas involving various fractional operators that has kernel as the Appell function and the Saigo operators with Bessel-Maitland function and m-parameter Mittag-Leffler function. The results are then represented in the form of the generalized Wright-hypergeometric function. Further, some special and particular cases involving the Mittag-Leffler function, the Wiman function, the Prabhakar function among other generalizations of the Mittag-Leffler function are induced.

Keywords: Fractional calculus, Image formula, m-parameter Mittag-Leffler function, Bessel-Maitland function.

1 Introduction and Pre-requisites

Fractional Calculus is concerned with the study of real or complex orders of derivatives and integrals. It has numerous applications in the field of science and engineering such as fluid flow, viscoelasticity, optics, oscillation, diffusion, electrochemistry, wave propagation and various others. Various researchers and scientists have studied derivatives and integrals of arbitrary order to investigate various types of results (see, [1,2,3,4,5,6,7]).

Many authors have introduced the generalization of different special functions (see, e.g. [8,9,10,11]). The Mittag-Leffler function has major applications in the problems of physics and engineering. It also has many applications in the field of fractional calculus (see, e.g. [12,13,14,15,16,17,18]). It is majorly used in fractal kinetics (see, e.g. [19,20]), medical science (see, e.g. [21, 22,23]), fractional and fractal calculus and its applications (see, e.g. [24,25]) and complex systems.

The new generalization of the Mittag-Leffler function invloving m-parameters [10] is

$$\begin{aligned} &E_{\varpi,\tau;\varkappa_1,\kappa_1;\varkappa_2,\kappa_2;\ldots;\varkappa_{p'},\kappa_{p'}}^{\zeta_1,\nu_1;\zeta_2,\nu_2;\ldots;\zeta_{r'},\nu_{r'}}(t) \\ &= E_{\varpi,\tau;(\boldsymbol{\varkappa},\boldsymbol{\kappa})_{p'}}^{(\boldsymbol{\zeta},\boldsymbol{\nu})_{r'}}(t) \\ &= \sum_{n=0}^{\infty} \frac{(\zeta_1)_{\nu_1 n}(\zeta_2)_{\nu_2 n}\ldots(\zeta_{r'})_{\nu_{r'} n}}{\Gamma(\varpi n+\tau)(\varkappa_1)_{\kappa_1 n}\ldots(\varkappa_p)_{\kappa_p n}} t^n, \end{aligned} \tag{1}$$

with a positive integer m, $r'+p'=m-2$, complex variable t, $(\boldsymbol{\zeta},\boldsymbol{\nu})_{r'} = [\zeta_1,\nu_1;\zeta_2,\nu_2;\ldots;\zeta_{r'},\nu_{r'}]$, $(\boldsymbol{\varkappa},\boldsymbol{\kappa})_{p'} = [\varkappa_1,\kappa_1;\varkappa_2,\kappa_2;\ldots;\varkappa_{p'},\kappa_{p'}]$, $\zeta_\iota,\nu_\iota,\varpi,\tau,\varkappa_\ell,\kappa_\ell \in \mathbb{C}$, where $\min\Re\{\varpi,\tau,\zeta_\iota,\nu_\iota,\varkappa_\ell,\kappa_\ell\} > 0$ for $\iota=1,2,\ldots,r';\ell=1,2,\ldots,p'$.

(a)When $r',p'=0$ and $\tau=1$, then (1) gets reduced to the Mittag-Leffler function [26]

$$E_\varpi(t) = \sum_{n=0}^{\infty} \frac{t^n}{\Gamma(\varpi n+1)}, \quad \varpi,t\in\mathbb{C} \quad \text{and} \quad \mathrm{Re}(\varpi)>0, \tag{2}$$

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where E_{ϖ} is an entire function and the gamma function is denoted by Γ [27]

(b) When $r', p' = 0$, then (1) gets reduced to the Wiman function [28, p.191]

$$E_{\varpi,\tau}(t) = \sum_{n=0}^{\infty} \frac{t^n}{\Gamma(\varpi n+\tau)}, \tag{3}$$

where ϖ, τ and t are in $\mathbb{C}$, $\mathrm{Re}(\varpi), \mathrm{Re}(\tau) > 0$.

(c) When $r' = p' = 1$ and $\nu = \varkappa = \kappa = 1$, our equation (1) gets reduced to the Prabhakar function [29]

$$E^{\zeta}_{\varpi,\tau}(t) = \sum_{n=0}^{\infty} \frac{(\zeta)_n t^n}{\Gamma(\varpi n+\tau) n!}, \tag{4}$$

where $\varpi, \tau, \zeta, t \in \mathbb{C}, min\{\mathrm{Re}(\varpi), \mathrm{Re}(\tau), \mathrm{Re}(\zeta)\} > 0$ and $(\zeta')_n$ is the Pochhammer symbol [27], $\zeta \neq 0, -1, -2, \ldots$

$$(\zeta')_n = \frac{\Gamma(\zeta'+n)}{\Gamma(\zeta')} = \begin{cases} \prod_{r'=1}^{n} (\zeta'+r'-1), & n \in \mathbb{N} \\ 1, & n = 0 \end{cases}$$

(d) When $r' = p' = 1$ and $\varkappa = \kappa = 1$, our equation (1) gets reduced to the Shukla and Prajapati's generalization of the Prabhakar function [30, Eq. (1.4), p.798]

$$E^{\zeta,\nu}_{\varpi,\tau}(t) = \sum_{n=0}^{\infty} \frac{(\zeta)_{\nu n} t^n}{\Gamma(\varpi n+\tau) n!}, \tag{5}$$

with $minimum\{\mathrm{Re}(\varpi), \mathrm{Re}(\tau), \mathrm{Re}(\zeta)\} > 0$, $\varpi, \tau, \zeta, \nu, t \in \mathbb{C}$ and $(\zeta)_{\nu n} = \frac{\Gamma(\zeta+\nu n)}{\Gamma(\zeta)}$ is the generalized Pochhammer symbol.

(e) When $r', p' = 1$ and $\kappa = 1$, then (1) gets reduced to Khan and Ahmed's generalization of the Mittag-Leffler function [31]

$$E^{\zeta,\nu}_{\varpi,\tau;\varkappa}(t) = \sum_{n=0}^{\infty} \frac{(\zeta)_{\nu n} t^n}{\Gamma(\varpi n+\tau)(\varkappa)_n}, \tag{6}$$

where $\varpi, \tau, \zeta, \nu, \varkappa, t \in \mathbb{C}$, with $minimum\{\mathrm{Re}(\varpi), \mathrm{Re}(\tau), \mathrm{Re}(\varkappa), \mathrm{Re}(\zeta)\} > 0$ $\nu \in (0,1) \cup \mathbb{N}$.

(f) When $r' = p' = 2$, equation (1) gets reduced to Khan and Ahmed's further generalization of the Mittag-Leffler function [31, Eq. (1.9), p.2]

$$E^{\zeta_1,\nu_1;\zeta_2,\nu_2}_{\varpi,\tau;\varkappa_1,\kappa_1;\varkappa_2,\kappa_2}(t) = \sum_{n=0}^{\infty} \frac{(\zeta_1)_{\nu_1 n}(\zeta_2)_{\nu_2 n} t^n}{\Gamma(\varpi n+\tau)(\varkappa_1)_{\kappa_1 n}(\varkappa_2)_{\kappa_2 n}}, \tag{7}$$

where $t, \varpi, \tau, \zeta_1, \nu_1, \zeta_2, \varkappa_1, \kappa_1, \varkappa_2 \in \mathbb{C}$, $\nu_2, \kappa_2 > 0$, $\mathrm{Re}(\varpi) + \kappa_2 \geq \nu_2$, $minimum(\mathrm{Re}(\varpi), \mathrm{Re}(\tau), \mathrm{Re}(\zeta_1), \mathrm{Re}(\nu_1),$ $\mathrm{Re}(\zeta_2), \mathrm{Re}(\varkappa_1), \mathrm{Re}(\kappa_1), \mathrm{Re}(\varkappa_2))$ > 0.

The Wright generalized hypergeometric function is [32]

$$\begin{aligned} &{}_r\Psi_s\left[\begin{matrix}(\varpi_1,A_1), & (\varpi_2,A_2), & \ldots,(\varpi_r,A_r) \\ (\varkappa_1,B_1), & (\varkappa_2,B_2), & \ldots,(\varkappa_s,B_s)\end{matrix};t\right] \\ &= \sum_{n=0}^{\infty}\left(\frac{\prod_{\iota=1}^{r}\Gamma(\varpi_\iota+A_\iota n)}{\prod_{\ell=1}^{s}\Gamma(\varkappa_\ell+B_\ell n)}\frac{t^n}{n!}\right), \end{aligned} \tag{8}$$

where $r,s \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}, \varpi_\iota, \varkappa_\ell \in \mathbb{C}, A_\iota, B_\ell \in \mathbb{R}, A_\iota, B_\ell \neq 0$ and $\iota = 1,\ldots,r, \ell = 1,\ldots,s.$

The Bessel-Maitland function [33] is

$$\mathfrak{I}^{\upsilon}_{\tau}(z) = \sum_{n=0}^{\infty} \frac{(-z)^n}{\Gamma(\tau n+\upsilon+1)n!}. \tag{9}$$

where $\tau, \upsilon \in \mathbb{C}, \mathrm{Re}(\tau) \geq 0, \mathrm{Re}(\upsilon) > -1$.

In our manuscript, we will discuss some image formulas for the Bessel-Maitland function and the m-parameter Mittag-Leffler function engaged with various fractional calculus operators. The operators involved are the fractional operator with the Appell function in the kernel and Saigo fractional operator. Some special particular cases that involves the Mittag-Leffler function, the Wiman function, the Prabhakar function and other different generalizations of the Mittag-Leffler function are also discussed. The special cases represent the broad generalization of the main results. These cases clearly shows the huge difference between the already existing functions in the literature and our manuscript's main results, hence proving the novelty of our main results.

2 Image formulas using the fractional integral operator with the Appell function in the Kernel

We will discuss the image formulas for the Bessel-Maitland and the m-parameter Mittag-Leffler function using the fractional integral operator with the Appell function $F_3(.)$ in the kernel.

Definition 1. *The fractional integral operator with the Appell function $F_3(.)$ in the kernel [34] is*

$$\begin{aligned} &\left(\mathfrak{I}^{\gamma,\alpha,\varepsilon,\beta,\eth}_{0+} f\right)(y) \\ &= \frac{y^{-\gamma}}{\Gamma(\eth)} \\ &\times \int_0^y (y-t)^{\eth-1} t^{-\alpha} F_3\left(\gamma,\alpha,\varepsilon,\beta;\eth;1-\frac{t}{y},1-\frac{y}{t}\right) f(t)dt, \end{aligned} \tag{10}$$

$$\left(\Im_{0^-}^{\gamma,\alpha,\varepsilon,\beta,\eth} f\right)(y) = \frac{y^{-\alpha}}{\Gamma(\eth)} \times \int_y^{\infty} (t-y)^{\eth-1} t^{-\gamma} F_3\left(\gamma,\alpha,\varepsilon,\beta;\eth;1-\frac{t}{y},1-\frac{y}{t}\right) f(t)dt, \tag{11}$$

where $\gamma,\alpha,\varepsilon,\beta,\eth \in \mathbb{C}, \mathrm{Re}(\eth) > 0$, $y \in \mathbb{R}^+$ *and* $f(t)$ *is a real valued continuous function.*

We will use the following Lemmas [35] in our main result.

Lemma 1. *Let* $\gamma,\alpha,\varepsilon,\beta,\eth,\xi \in \mathbb{C}$ *with* $\mathrm{Re}(\eth) > 0$ *and* $\mathrm{Re}(\gamma) > \max\{0, \mathrm{Re}(\gamma-\alpha-\varepsilon-\eth), \mathrm{Re}(\alpha-\beta)\}$, *then*

$$\left(\Im_{0^+}^{\gamma,\alpha,\varepsilon,\beta,\eth} t^{\xi-1}\right)(y) = \frac{\Gamma(\xi)\Gamma(\xi+\eth-\gamma-\alpha-\varepsilon)\Gamma(\xi+\beta-\alpha)}{\Gamma(\xi+\beta)\Gamma(\xi+\eth-\gamma-\alpha)\Gamma(\xi+\eth-\alpha-\varepsilon)} y^{\xi-\gamma-\alpha+\eth-1}. \tag{12}$$

Lemma 2. *Let* $\gamma,\alpha,\varepsilon,\beta,\eth,\xi \in \mathbb{C}$ *with* $\mathrm{Re}(\eth) > 0$ *and* $\mathrm{Re}(\xi) > \max\{\mathrm{Re}(\varepsilon), \mathrm{Re}(-\gamma-\alpha+\eth), \mathrm{Re}(-\gamma-\beta+\eth)\}$, *then*

$$\left(\Im_{0^-}^{\gamma,\alpha,\varepsilon,\beta,\eth} t^{-\xi}\right)(y) = \frac{\Gamma(-\varepsilon+\xi)\Gamma(\gamma+\alpha-\eth+\xi)\Gamma(\gamma+\beta-\eth+\xi)}{\Gamma(\xi)\Gamma(\gamma-\varepsilon+\xi)\Gamma(\gamma+\alpha+\beta-\eth+\xi)} y^{-\gamma-\alpha+\eth-\xi}. \tag{13}$$

Theorem 1. *Let* $\gamma,\alpha,\varepsilon,\beta,\zeta_\iota,\nu_\iota,\varpi,\tau,\varkappa_\ell,\kappa_\ell \in \mathbb{C}$, $\min \mathrm{Re}\{\varpi,\tau,\zeta_\iota,\nu_\iota,\varkappa_\ell,\kappa_\ell\} > 0$ *with* $\iota = 1,\ldots,r'$; *and* $\ell = 1,\ldots,p'$, $\mathrm{Re}(\eth) > 0, \mathrm{Re}(\gamma) > \max\{0, \mathrm{Re}(\gamma-\alpha-\varepsilon-\eth), \mathrm{Re}(\alpha-\beta)\}$. *Then*

$$\left(\Im_{0^+}^{\gamma,\alpha,\varepsilon,\beta,\eth} t^{\xi-1} J_\rho^\omega(t) E_{\varpi,\tau;(\boldsymbol{\varkappa},\boldsymbol{\kappa})_{p'}}^{(\boldsymbol{\zeta},\boldsymbol{\nu})_{r'}}(t)\right)(y) = y^{\xi-\gamma-\alpha+\eth-1} \frac{\Gamma(\varkappa_1)\Gamma(\varkappa_2)\ldots\Gamma(\varkappa_{p'})}{\Gamma(\zeta_1)\Gamma(\zeta_2)\ldots\Gamma(\zeta_{r'})} J_\rho^\omega(y) \times {}_{r'+4}\Psi_{p'+4}\left[\begin{matrix} (\xi+m,1), \\ (\xi+m+\eth-\gamma-\alpha-\varepsilon,1),(\xi+m+\beta-\alpha,1), \\ (1,1),(\zeta_1,\nu_1),\ldots,(\zeta_{r'},\nu_{r'}) \\ \\ (\xi+m+\beta,1),(\xi+m+\eth-\gamma-\alpha,1) \\ ,(\xi+m+\eth-\alpha-\varepsilon,1),(\tau,\varpi), \\ (\varkappa_1,\kappa_1),\ldots,(\varkappa_{p'},\kappa_{p'}) \end{matrix} ;y\right]. \tag{14}$$

Proof. Consider

$$\left(\Im_{0^+}^{\gamma,\alpha,\varepsilon,\beta,\eth} t^{\xi-1} J_\rho^\omega(t) E_{\varpi,\tau;(\boldsymbol{\varkappa},\boldsymbol{\kappa})_{p'}}^{(\boldsymbol{\zeta},\boldsymbol{\nu})_{r'}}(t)\right)(y) = \Im_{0^+}^{\gamma,\alpha,\varepsilon,\beta,\eth} t^{\xi-1} \sum_{m=0}^{\infty} \frac{(-1)^m t^m}{\Gamma(\rho m+\omega+1)} \sum_{n=0}^{\infty} \frac{(\zeta_1)_{\nu_1 n}(\zeta_2)_{\nu_2 n}\ldots(\zeta_{r'})_{\nu_{r'} n}}{\Gamma(\varpi n+\tau)(\varkappa_1)_{\kappa_1 n}\ldots(\varkappa_{p'})_{\kappa_{p'} n}} t^n(y)$$

Swapping the integral and summation orders using the valid conditions mentioned in the theorem, we get

$$\left(\Im_{0^+}^{\gamma,\alpha,\varepsilon,\beta,\eth} t^{\xi-1} J_\rho^\omega(t) E_{\varpi,\tau;(\boldsymbol{\varkappa},\boldsymbol{\kappa})_{p'}}^{(\boldsymbol{\zeta},\boldsymbol{\nu})_{r'}}(t)\right)(y) = \sum_{m=0}^{\infty} \frac{(-1)^m}{\Gamma(\rho m+\omega+1)} \times \sum_{n=0}^{\infty} \frac{(\zeta_1)_{\nu_1 n}(\zeta_2)_{\nu_2 n}\ldots(\zeta_{r'})_{\nu_{r'} n}}{\Gamma(\varpi n+\tau)(\varkappa_1)_{\kappa_1 n}\ldots(\varkappa_{p'})_{\kappa_{p'} n}} \left(\Im_{0^+}^{\gamma,\alpha,\varepsilon,\beta,\eth} t^{\xi+m+n-1}\right)(y). \tag{15}$$

Using Lemma 1, the above equation (15) reduces to

$$\left(\Im_{0^+}^{\gamma,\alpha,\varepsilon,\beta,\eth} t^{\xi-1} J_\rho^\omega(t) E_{\varpi,\tau;(\boldsymbol{\varkappa},\boldsymbol{\kappa})_{p'}}^{(\boldsymbol{\zeta},\boldsymbol{\nu})_{r'}}(t)\right)(y) = \sum_{m=0}^{\infty} \frac{(-1)^m}{\Gamma(\rho m+\omega+1)} \frac{\Gamma(\varkappa_1)\Gamma(\varkappa_2)\ldots\Gamma(\varkappa_{p'})}{\Gamma(\zeta_1)\Gamma(\zeta_2)\ldots\Gamma(\zeta_{r'})} \sum_{n=0}^{\infty} \frac{\Gamma(\zeta_1+\nu_1 n)\Gamma(\zeta_2+\nu_2 n)\ldots\Gamma(\zeta_{r'}+\nu_{r'} n)}{\Gamma(\varpi n+\tau)\Gamma(\varkappa_1+\kappa_1 n)\ldots\Gamma(\varkappa_{p'}+\kappa_{p'} n)} \frac{\Gamma(\xi+m+n)}{\Gamma(\xi+m+n+\beta)} \frac{\Gamma(\xi+m+n+\eth-\gamma-\alpha-\varepsilon)\Gamma(\xi+m+n+\beta-\alpha)}{\Gamma(\xi+m+n+\eth-\gamma-\alpha)\Gamma(\xi+m+n+\eth-\alpha-\varepsilon)} y^{\xi+m+n-\gamma-\alpha+\eth-1}$$

$$= y^{\xi-\gamma-\alpha+\eth-1} \frac{\Gamma(\varkappa_1)\Gamma(\varkappa_2)\ldots\Gamma(\varkappa_{p'})}{\Gamma(\zeta_1)\Gamma(\zeta_2)\ldots\Gamma(\zeta_{r'})} \sum_{m=0}^{\infty} \frac{(-y)^m}{\Gamma(\rho m+\omega+1)} \times \sum_{n=0}^{\infty} \frac{\Gamma(\zeta_1+\nu_1 n)\Gamma(\zeta_2+\nu_2 n)\ldots\Gamma(\zeta_{r'}+\nu_{r'} n)}{\Gamma(\varpi n+\tau)\Gamma(\varkappa_1+\kappa_1 n)\ldots\Gamma(\varkappa_{p'}+\kappa_{p'} n)} \times \frac{\Gamma(\xi+m+n)\Gamma(\xi+m+n+\eth-\gamma-\alpha-\varepsilon)}{\Gamma(\xi+m+n+\beta)\Gamma(\xi+m+n+\eth-\gamma-\alpha)} \times \frac{\Gamma(\xi+m+n+\beta-\alpha)}{\Gamma(\xi+m+n+\eth-\alpha-\varepsilon)} \frac{\Gamma(n+1)}{n!} y^n$$

$$=y^{\xi-\gamma-\alpha+\eth-1}\frac{\Gamma(\varkappa_1)\Gamma(\varkappa_2)\ldots\Gamma(\varkappa_{p'})}{\Gamma(\zeta_1)\Gamma(\zeta_2)\ldots\Gamma(\zeta_{r'})}J_\rho^\omega(y)$$
$$\times\,{}_{r'+4}\Psi_{p'+4}\left[\begin{matrix}(\xi+m,1),\\(\xi+m+\eth-\gamma-\alpha-\varepsilon,1),(\xi+m+\beta-\alpha,1),\\(1,1),(\zeta_1,\nu_1),\ldots,(\zeta'_r,\nu'_r)\\ \\(\xi+m+\beta,1),(\xi+m+\eth-\gamma-\alpha,1)\\,(\xi+m+\eth-\alpha-\varepsilon,1),(\tau,\varpi),\\(\varkappa_1,\kappa_1),\ldots,(\varkappa_{p'},\kappa_{p'})\end{matrix};y\right]. \tag{16}$$

Hence, we get the result (14).

Theorem 2.*Let* $\gamma,\alpha,\varepsilon,\beta,\zeta_\iota,\nu_\iota,\varpi,\tau,\varkappa_\ell,\kappa_\ell\in\mathbb{C}$, $\min\mathrm{Re}\{\varpi,\tau,\zeta_\iota,\nu_\iota,\varkappa_\ell,\kappa_\ell\}>0$ *with* $\iota=1,\ldots,r'$ *and* $\ell=1,\ldots,p'$ *and* $\mathrm{Re}(\eth)>0$ *and* $\mathrm{Re}(\xi)>\max\{\mathrm{Re}(\varepsilon),\mathrm{Re}(-\gamma-\alpha+\eth),\mathrm{Re}(-\gamma-\beta+\eth)\}$. *Then*

$$\left(\Im_{0-}^{\gamma,\alpha,\varepsilon,\beta,\eth}t^{-\xi}J_\rho^\omega(t)E_{\varpi,\tau;(\boldsymbol{\varkappa},\boldsymbol{\kappa})_{p'}}^{(\boldsymbol{\zeta},\boldsymbol{\nu})_{r'}}\left(\frac{1}{t}\right)\right)(y)$$
$$=y^{-\gamma-\alpha+\eth-\xi}\frac{\Gamma(\varkappa_1)\Gamma(\varkappa_2)\ldots\Gamma(\varkappa_{p'})}{\Gamma(\zeta_1)\Gamma(\zeta_2)\ldots\Gamma(\zeta_{r'})}J_\rho^\omega(y)$$
$$\times\,{}_{r'+4}\Psi_{p'+4}\left[\begin{matrix}(\xi-\varepsilon-m,1),(\gamma+\alpha-\eth+\xi-m,1),\\(\gamma+\beta-\eth+\xi-m,1),(1,1)\\(\zeta_1,\nu_1),\ldots,(\zeta_{r'},\nu_{r'})\\ \\(\xi-m,1),(\gamma-\varepsilon+\xi-m,1),\\(\gamma+\alpha+\beta-\eth+\xi-m,1),(\tau,\varpi)\\(\varkappa_1,\kappa_1),\ldots,(\varkappa_{p'},\kappa_{p'})\end{matrix};\frac{1}{y}\right]. \tag{17}$$

*Proof.*Consider

$$\left(\Im_{0-}^{\gamma,\alpha,\varepsilon,\beta,\eth}t^{-\xi}J_\rho^\omega(t)E_{\varpi,\tau;(\boldsymbol{\varkappa},\boldsymbol{\kappa})_{p'}}^{(\boldsymbol{\zeta},\boldsymbol{\nu})_{r'}}\left(\frac{1}{t}\right)\right)(y)$$
$$=\Im_{0-}^{\gamma,\alpha,\varepsilon,\beta,\eth}t^{-\xi}\sum_{m=0}^{\infty}\frac{(-1)^m t^m}{\Gamma(\rho m+\omega+1)}$$
$$\sum_{n=0}^{\infty}\frac{(\zeta_1)_{\nu_1 n}(\zeta_2)_{\nu_2 n}\ldots(\zeta_{r'})_{\nu_{r'}n}}{\Gamma(\varpi n+\tau)(\varkappa_1)_{\kappa_1 n}\ldots(\varkappa_{p'})_{\kappa_{p'}n}}t^{-n}(y).$$

Swapping the integral and summation orders using the valid conditions mentioned in the theorem, we get

$$\left(\Im_{0-}^{\gamma,\alpha,\varepsilon,\beta,\eth}t^{-\xi}J_\rho^\omega(t)E_{\varpi,\tau;(\boldsymbol{\varkappa},\boldsymbol{\kappa})_{p'}}^{(\boldsymbol{\zeta},\boldsymbol{\nu})_{r'}}\left(\frac{1}{t}\right)\right)(y)$$
$$=\sum_{m=0}^{\infty}\frac{(-1)^m}{\Gamma(\rho m+\omega+1)} \tag{18}$$
$$\sum_{n=0}^{\infty}\frac{(\zeta_1)_{\nu_1 n}(\zeta_2)_{\nu_2 n}\ldots(\zeta_{r'})_{\nu_{r'}n}}{\Gamma(\varpi n+\tau)(\varkappa_1)_{\kappa_1 n}\ldots(\varkappa_{p'})_{\kappa_{p'}n}}$$
$$\times\left(\Im_{0-}^{\gamma,\alpha,\varepsilon,\beta,\eth}t^{-(\xi+n-m)}\right)(y). \tag{19}$$

Using Lemma 2, the above equation (19) reduces to

$$\left(\Im_{0-}^{\gamma,\alpha,\varepsilon,\beta,\eth}t^{-\xi}J_\rho^\omega(t)E_{\varpi,\tau;(\boldsymbol{\varkappa},\boldsymbol{\kappa})_{p'}}^{(\boldsymbol{\zeta},\boldsymbol{\nu})_{r'}}\left(\frac{1}{t}\right)\right)(y)$$
$$=\sum_{m=0}^{\infty}\frac{(-1)^m}{\Gamma(\rho m+\omega+1)}\frac{\Gamma(\varkappa_1)\Gamma(\varkappa_2)\ldots\Gamma(\varkappa_{p'})}{\Gamma(\zeta_1)\Gamma(\zeta_2)\ldots\Gamma(\zeta_{r'})}$$
$$\sum_{n=0}^{\infty}\frac{\Gamma(\zeta_1+\nu_1 n)\Gamma(\zeta_2+\nu_2 n)\ldots\Gamma(\zeta_{r'}+\nu_{r'}n)}{\Gamma(\varpi n+\tau)\Gamma(\varkappa_1+\kappa_1 n)\ldots\Gamma(\varkappa_{p'}+\kappa_{p'}n)}$$
$$\frac{\Gamma(-\varepsilon+\xi-m+n)}{\Gamma(\xi-m+n)}$$
$$\frac{\Gamma(\gamma+\alpha-\eth+\xi+n-m)\Gamma(\gamma+\beta-\eth+\xi+n-m)}{\Gamma(\gamma-\varepsilon+\xi+n-m)\Gamma(\gamma+\alpha+\beta-\eth+\xi+n-m)}$$
$$\times y^{-\gamma-\alpha+\eth-\xi-n+m}$$
$$=y^{-\gamma-\alpha+\eth-\xi}\frac{\Gamma(\varkappa_1)\Gamma(\varkappa_2)\ldots\Gamma(\varkappa_{p'})}{\Gamma(\zeta_1)\Gamma(\zeta_2)\ldots\Gamma(\zeta_{r'})}J_\rho^\omega(y)$$
$$\times\,{}_{r'+4}\Psi_{p'+4}\left[\begin{matrix}(\xi-\varepsilon-m,1),(\gamma+\alpha-\eth+\xi-m,1),\\(\gamma+\beta-\eth+\xi-m,1),(1,1)\\(\zeta_1,\nu_1),\ldots,(\zeta_{r'},\nu_{r'})\\ \\(\xi-m,1),(\gamma-\varepsilon+\xi-m,1),\\(\gamma+\alpha+\beta-\eth+\xi-m,1),(\tau,\varpi)\\(\varkappa_1,\kappa_1),\ldots,(\varkappa_{p'},\kappa_{p'})\end{matrix};\frac{1}{y}\right]. \tag{20}$$

Hence, we get the desired result (17).

2.1 Particular Cases

Theorem 1 and 2 give the following image formulas for the Mittag-Leffler function (2) if $r'=p'=0$ and $\tau=1$:

Corollary 1.*Let* $\gamma,\alpha,\varepsilon,\beta\in\mathbb{C}$, $\varpi>0$ *and* $\mathrm{Re}(\eth)>0,\mathrm{Re}(\gamma)>\max\{0,\mathrm{Re}(\gamma-\alpha-\varepsilon-\eth),\mathrm{Re}(\alpha-\beta)\}$. *Then*

$$\left(\Im_{0+}^{\gamma,\alpha,\varepsilon,\beta,\eth}t^{\xi-1}J_\rho^\omega(t)E_\varpi(t)\right)(y)$$
$$=y^{\xi-\gamma-\alpha+\eth-1}J_\rho^\omega(y)$$
$$\times\,{}_4\Psi_4\left[\begin{matrix}(\xi+m,1),(\xi+m+\eth-\gamma-\alpha-\varepsilon,1),\\(\xi+m+\beta-\alpha,1),(1,1)\\ \\(\xi+m+\beta,1),(\xi+m+\eth-\gamma-\alpha,1),\\(\xi+m+\eth-\alpha-\varepsilon,1),(1,\varpi)\end{matrix};y\right]. \tag{21}$$

Corollary 2.*Let* $\gamma,\alpha,\varepsilon,\beta\in\mathbb{C},\varpi>0$ *and* $\mathrm{Re}(\eth)>0$ *and* $\mathrm{Re}(\xi)>\max\{\mathrm{Re}(\varepsilon),\mathrm{Re}(-\gamma-\alpha+\eth),\mathrm{Re}(-\gamma-\beta+\eth)\}$.

Then

$$\left(\Im_{0^-}^{\gamma,\alpha,\varepsilon,\beta,\eth} t^{-\xi} J_\rho^\omega(t) E_\varpi\left(\frac{1}{t}\right)\right)(y)$$
$$= y^{-\gamma-\alpha+\eth-\xi} J_\rho^\omega(y) \times {}_4\Psi_4\left[\begin{matrix}(\xi-\varepsilon-m,1),(\gamma+\alpha-\eth+\xi-m,1),\\(\gamma+\beta-\eth+\xi-m,1),(1,1)\\ \\(\xi-m,1),(\gamma-\varepsilon+\xi-m,1),\\(\gamma+\alpha+\beta-\eth+\xi-m,1),(1,\varpi)\end{matrix};\frac{1}{y}\right]. \quad (22)$$

Theorem 1 and 2 give the following image formulas for the Wiman function (3), if $r' = p' = 0$:

Corollary 3. *Let* $\gamma,\alpha,\varepsilon,\beta \in \mathbb{C}$, $\mathrm{Re}(\varpi),\mathrm{Re}(\tau) > 0$ *and* $\mathrm{Re}(\eth) > 0, \mathrm{Re}(\gamma) > \max\{0,\mathrm{Re}(\gamma-\alpha-\varepsilon-\eth),\mathrm{Re}(\alpha-\beta)\}$. *Then*

$$\left(\Im_{0^+}^{\gamma,\alpha,\varepsilon,\beta,\eth} t^{\xi-1} J_\rho^\omega(t) E_{\varpi,\tau}(t)\right)(y)$$
$$= y^{\xi-\gamma-\alpha+\eth-1} J_\rho^\omega(y) \times {}_4\Psi_4\left[\begin{matrix}(\xi+m,1),(\xi+m+\eth-\gamma-\alpha-\varepsilon,1),\\(\xi+m+\beta-\alpha,1),(1,1)\\ \\(\xi+m+\beta,1),(\xi+m+\eth-\gamma-\alpha,1),\\(\xi+m+\eth-\alpha-\varepsilon,1),(\tau,\varpi)\end{matrix};y\right]. \quad (23)$$

Corollary 4. *Let* $\gamma,\alpha,\varepsilon,\beta \in \mathbb{C}$, $\mathrm{Re}(\varpi),\mathrm{Re}(\tau) > 0$ *and* $\mathrm{Re}(\eth) > 0$ *and* $\mathrm{Re}(\xi) > \max\{\mathrm{Re}(\varepsilon),\mathrm{Re}(-\gamma-\alpha+\eth),\mathrm{Re}(-\gamma-\beta+\eth)\}$. *Then*

$$\left(\Im_{0^-}^{\gamma,\alpha,\varepsilon,\beta,\eth} t^{-\xi} J_\rho^\omega(t) E_{\varpi,\tau}\left(\frac{1}{t}\right)\right)(y)$$
$$= y^{-\gamma-\alpha+\eth-\xi} J_\rho^\omega(y) \times {}_4\Psi_4\left[\begin{matrix}(\xi-\varepsilon-m,1),(\gamma+\alpha-\eth+\xi-m,1),\\(\gamma+\beta-\eth+\xi-m,1),(1,1)\\ \\(\xi-m,1),(\gamma-\varepsilon+\xi-m,1),\\(\gamma+\alpha+\beta-\eth+\xi-m,1),(\tau,\varpi)\end{matrix};\frac{1}{y}\right]. \quad (24)$$

Theorem 1 and 2 give the following image formulas for the Prabhakar function (4), if $r' = p' = 1$ and $\nu = \varkappa = \kappa = 1$,:

Corollary 5. *Let* $\gamma,\alpha,\varepsilon,\beta \in \mathbb{C}$, $\mathrm{Re}(\varpi),\mathrm{Re}(\tau) > 0, \zeta > 0$ *and* $\mathrm{Re}(\eth) > 0, \mathrm{Re}(\gamma) > \max\{0,\mathrm{Re}(\gamma-\alpha-\varepsilon-\eth),\mathrm{Re}(\alpha-\beta)\}$. *Then*

$$\left(\Im_{0^+}^{\gamma,\alpha,\varepsilon,\beta,\eth} t^{\xi-1} J_\rho^\omega(t) E_{\varpi,\tau}^{\zeta}(t)\right)(y)$$
$$= \frac{y^{\xi-\gamma-\alpha+\eth-1}}{\Gamma(\zeta)} J_\rho^\omega(y) \times {}_5\Psi_5\left[\begin{matrix}(\xi+m,1),(\xi+m-\eth-\gamma-\alpha-\varepsilon,1),\\(\xi+m+\beta-\alpha,1),(1,1),(\zeta,1)\\ \\(\xi+m+\beta,1),(\xi+m+\eth-\gamma-\alpha,1),\\(\xi+m+\eth-\alpha-\varepsilon,1),(\tau,\varpi),(1,1)\end{matrix};y\right]. \quad (25)$$

Corollary 6. *Let* $\gamma,\alpha,\varepsilon,\beta \in \mathbb{C}$, $\mathrm{Re}(\varpi),\mathrm{Re}(\tau) > 0, \zeta > 0$ *and* $\mathrm{Re}(\eth) > 0$ *and* $\mathrm{Re}(\xi) > \max\{\mathrm{Re}(\varepsilon),\mathrm{Re}(-\gamma-\alpha+\eth),\mathrm{Re}(-\gamma-\beta+\eth)\}$. *Then*

$$\left(\Im_{0^-}^{\gamma,\alpha,\varepsilon,\beta,\eth} t^{-\xi} J_\rho^\omega(t) E_{\varpi,\tau}^{\zeta}\left(\frac{1}{t}\right)\right)(y)$$
$$= y^{-\gamma-\alpha+\eth-\xi} J_\rho^\omega(y) \times {}_5\Psi_5\left[\begin{matrix}(\xi-\varepsilon-m,1),(\gamma+\alpha-\eth+\xi-m,1),\\(\gamma+\beta-\eth+\xi-m,1),(1,1),(\zeta,1)\\ \\(\xi-m,1),(\gamma-\varepsilon+\xi-m,1),\\(\gamma+\alpha+\beta-\eth+\xi-m,1),(\tau,\varpi),(1,1)\end{matrix};\frac{1}{y}\right]. \quad (26)$$

Theorem 1 and 2 give the following image formulas for Shukla and Prajapati's generalization of the Prabhakar function (5) if $r' = p' = 1$ and $\varkappa = \kappa = 1$:

Corollary 7. *Let* $\gamma,\alpha,\varepsilon,\beta \in \mathbb{C}$, *min* $\{\mathrm{Re}(\varpi),\mathrm{Re}(\tau),\mathrm{Re}(\zeta)\} > 0$, $\varpi,\tau,\zeta,\nu \in \mathbb{C}$, $(\zeta)_{\nu n} = \frac{\Gamma(\zeta+\nu n)}{\Gamma(\zeta)}$ *is the generalized Pochhammer symbol and* $\mathrm{Re}(\eth) > 0, \mathrm{Re}(\gamma) > \max\{0,\mathrm{Re}(\gamma-\alpha-\varepsilon-\eth),\mathrm{Re}(\alpha-\beta)\}$. *Then*

$$\left(\Im_{0^+}^{\gamma,\alpha,\varepsilon,\beta,\eth} t^{\xi-1} J_\rho^\omega(t) E_{\varpi,\tau}^{\zeta,\nu}(t)\right)(y)$$
$$= \frac{y^{\xi-\gamma-\alpha+\eth-1}}{\Gamma(\zeta)} J_\rho^\omega(y) \times {}_5\Psi_5\left[\begin{matrix}(\xi+m,1),(\xi+m-\eth-\gamma-\alpha-\varepsilon,1),\\(\xi+m+\beta-\alpha,1),(1,1),(\zeta,\nu)\\ \\(\xi+m+\beta,1),(\xi+m+\eth-\gamma-\alpha,1),\\(\xi+m+\eth-\alpha-\varepsilon,1),(\tau,\varpi),(1,1)\end{matrix};y\right]. \quad (27)$$

Corollary 8. *Let* $\gamma,\alpha,\varepsilon,\beta \in \mathbb{C}$, *min* $\{\mathrm{Re}(\varpi),\mathrm{Re}(\tau),\mathrm{Re}(\zeta)\} > 0$, $\varpi,\tau,\zeta,\nu \in \mathbb{C}$, $(\zeta)_{\nu n} = \frac{\Gamma(\zeta+\nu n)}{\Gamma(\zeta)}$ *is the generalized Pochhammer symbol,* $\mathrm{Re}(\eth) > 0$ *and* $\mathrm{Re}(\xi) > \max\{\mathrm{Re}(\varepsilon),\mathrm{Re}(-\gamma-\alpha+\eth),\mathrm{Re}(-\gamma-\beta+\eth)\}$.

Then

$$\left(\mathfrak{I}_{0-}^{\gamma,\alpha,\varepsilon,\beta,\eth}t^{-\xi}J_{\rho}^{\omega}(t)E_{\varpi,\tau}^{\zeta,\nu}\left(\frac{1}{t}\right)\right)(y)$$
$$=\frac{y^{-\gamma-\alpha+\eth-\xi}}{\Gamma(\zeta)}J_{\rho}^{\omega}(y)$$
$$\times {}_5\Psi_5\left[\begin{matrix}(\xi-\varepsilon-m,1),(\gamma+\alpha-\eth+\xi-m,1),\\(\gamma+\beta-\eth+\xi-m,1),(1,1),(\zeta,\nu)\\ \\(\xi-m,1),(\gamma-\varepsilon+\xi-m,1),\\(\gamma+\alpha+\beta-\eth+\xi-m,1),(\tau,\varpi),(1,1)\end{matrix};\frac{1}{y}\right]. \quad (28)$$

Theorem 1 and 2 give the following image formulas for Khan and Ahmed's generalization of the Mittag-Leffler function (6) if $r'=p'=1$ and $\kappa=1$:

Corollary 9. *Let* $\gamma,\alpha,\varepsilon,\beta\in\mathbb{C}$, $\min\{\mathrm{Re}(\varpi),\mathrm{Re}(\tau),\mathrm{Re}(\varkappa),\mathrm{Re}(\zeta)\}>0$, $\nu\in(0,1)\cup\mathbb{N}$ *and* $\mathrm{Re}(\eth)>0,\mathrm{Re}(\gamma)>\max\{0,\mathrm{Re}(\gamma-\alpha-\varepsilon-\eth),\mathrm{Re}(\alpha-\beta)\}$. *Then*

$$\left(\mathfrak{I}_{0+}^{\gamma,\alpha,\varepsilon,\beta,\eth}t^{\xi-1}J_{\rho}^{\omega}(t)E_{\varpi,\tau,\varkappa}^{\zeta,\nu}(t)\right)(y)$$
$$=y^{\xi-\gamma-\alpha+\eth-1}\frac{\Gamma(\varkappa)}{\Gamma(\zeta)}J_{\rho}^{\omega}(y)$$
$$\times {}_5\Psi_5\left[\begin{matrix}(\xi+m,1),(\xi+m-\eth-\gamma-\alpha-\varepsilon,1),\\(\xi+m+\beta-\alpha,1),(1,1),(\zeta,\nu)\\ \\(\xi+m+\beta,1),(\xi+m+\eth-\gamma-\alpha,1),\\(\xi+m+\eth-\alpha-\varepsilon,1),(\tau,\varpi),(\varkappa,1)\end{matrix};y\right]. \quad (29)$$

Corollary 10. *Let* $\gamma,\alpha,\varepsilon,\beta\in\mathbb{C}$, $\min\{\mathrm{Re}(\varpi),\mathrm{Re}(\tau),\mathrm{Re}(\varkappa),\mathrm{Re}(\zeta)\}>0$, $\nu\in(0,1)\cup\mathbb{N}$, $\mathrm{Re}(\eth)>0$ *and* $\mathrm{Re}(\xi)>\max\{\mathrm{Re}(\varepsilon),\mathrm{Re}(-\gamma-\alpha+\eth),\mathrm{Re}(-\gamma-\beta+\eth)\}$. *Then*

$$\left(\mathfrak{I}_{0-}^{\gamma,\alpha,\varepsilon,\beta,\eth}t^{-\xi}J_{\rho}^{\omega}(t)E_{\varpi,\tau,\varkappa}^{\zeta,\nu}\left(\frac{1}{t}\right)\right)(y)$$
$$=y^{-\gamma-\alpha+\eth-\xi}\frac{\Gamma(\varkappa)}{\Gamma(\zeta)}J_{\rho}^{\omega}(y)$$
$$\times {}_5\Psi_5\left[\begin{matrix}(\xi-\varepsilon-m,1),(\gamma+\alpha-\eth+\xi-m,1),\\(\gamma+\beta-\eth+\xi-m,1),(1,1),(\zeta,\nu)\\ \\(\xi-m,1),(\gamma-\varepsilon+\xi-m,1),\\(\gamma+\alpha+\beta-\eth+\xi-m,1),(\tau,\varpi),(\varkappa,1)\end{matrix};\frac{1}{y}\right]. \quad (30)$$

Theorem 1 and 2 give the following image formulas for Khan and Ahmed's further generalization of the Mittag-Leffler function (7) if $r'=p'=2$, :

Corollary 11. *Let* $\gamma,\alpha,\varepsilon,\beta\in\mathbb{C}$, $\varpi,\tau,\zeta_1,\nu_1,\zeta_2,\varkappa_1,\kappa_1,\varkappa_2\in\mathbb{C}$, $\nu_2,\kappa_2>0$, $\min(\mathrm{Re}(\varpi),\mathrm{Re}(\tau),\mathrm{Re}(\zeta_1),\mathrm{Re}(\nu_1),\mathrm{Re}(\zeta_2),\mathrm{Re}(\varkappa_1),\mathrm{Re}(\kappa_1),\mathrm{Re}(\varkappa_2))>0$, $\mathrm{Re}(\varpi)+\kappa_2\geq\nu_2$ *and* $\mathrm{Re}(\eth)>0,\mathrm{Re}(\gamma)>\max\{0,\mathrm{Re}(\gamma-\alpha-\varepsilon-\eth),\mathrm{Re}(\alpha-\beta)\}$. *Then*

$$\left(\mathfrak{I}_{0+}^{\gamma,\alpha,\varepsilon,\beta,\eth}t^{\xi-1}J_{\rho}^{\omega}(t)E_{\varpi,\tau;\varkappa_1,\kappa_1;\varkappa_2,\kappa_2}^{\zeta_1,\nu_1;\zeta_2,\nu_2}(t)\right)(y)$$
$$=y^{\xi-\gamma-\alpha+\eth-1}\frac{\Gamma(\varkappa_1)\Gamma(\varkappa_2)}{\Gamma(\zeta_1)\Gamma(\zeta_2)}J_{\rho}^{\omega}(y)$$
$$\times {}_6\Psi_6\left[\begin{matrix}(\xi+m,1),(\xi+m-\eth-\gamma-\alpha-\varepsilon,1),\\(\xi+m+\beta-\alpha,1),(1,1),\\(\zeta_1,\nu_1),(\zeta_2,\nu_2)\\ \\(\xi+m+\beta,1),(\xi+m+\eth-\gamma-\alpha,1),\\(\xi+m+\eth-\alpha-\varepsilon,1),(\tau,\varpi),\\(\varkappa_1,\kappa_1),(\varkappa_2,\kappa_2)\end{matrix};y\right]. \quad (31)$$

Corollary 12. *Let* $\gamma,\alpha,\varepsilon,\beta\in\mathbb{C}$, $\varpi,\tau,\zeta_1,\nu_1,\zeta_2,\varkappa_1,\kappa_1,\varkappa_2\in\mathbb{C}$, $\nu_2,\kappa_2>0$, $\min(\mathrm{Re}(\varpi),\mathrm{Re}(\tau),\mathrm{Re}(\zeta_1),\mathrm{Re}(\nu_1),\mathrm{Re}(\zeta_2),\mathrm{Re}(\varkappa_1),\mathrm{Re}(\kappa_1),\mathrm{Re}(\varkappa_2))>0$, $\mathrm{Re}(\varpi)+\kappa_2\geq\nu_2$, $\mathrm{Re}(\eth)>0$ *and* $\mathrm{Re}(\xi)>\max\{\mathrm{Re}(\varepsilon),\mathrm{Re}(-\gamma-\alpha+\eth),\mathrm{Re}(-\gamma-\beta+\eth)\}$. *Then*

$$\left(\mathfrak{I}_{0-}^{\gamma,\alpha,\varepsilon,\beta,\eth}t^{-\xi}J_{\rho}^{\omega}(t)E_{\varpi,\tau;\varkappa_1,\kappa_1;\varkappa_2,\kappa_2}^{\zeta_1,\nu_1;\zeta_2,\nu_2}\left(\frac{1}{t}\right)\right)(y)$$
$$=y^{-\gamma-\alpha+\eth-\xi}\frac{\Gamma(\varkappa_1)\Gamma(\varkappa_2)}{\Gamma(\zeta_1)\Gamma(\zeta_2)}J_{\rho}^{\omega}(y)$$
$$\times {}_6\Psi_6\left[\begin{matrix}(\xi-\varepsilon-m,1),(\gamma+\alpha-\eth+\xi-m,1),\\(\gamma+\beta-\eth+\xi-m,1),(1,1),\\(\zeta_1,\nu_1),(\zeta_2,\nu_2)\\ \\(\xi-m,1),(\gamma-\varepsilon+\xi-m,1),\\(\gamma+\alpha+\beta-\eth+\xi-m,1),(\tau,\varpi),\\(\varkappa_1,\kappa_1),(\varkappa_2,\kappa_2)\end{matrix};\frac{1}{y}\right]. \quad (32)$$

3 Image formulas using the fractional differential operator with the Appell function in the Kernel

Here, we will discuss the image formulas for the Bessel-Maitland function and the m-parameter Mittag-Leffler function using the fractional differential operator with the Appell function $F_3(.)$ in the kernel.

Definition 2. *The fractional differential operator with the Appell function $F_3(.)$ in the kernel [34] are*

$$\left(\mathfrak{D}_{0-}^{\gamma,\alpha,\varepsilon,\beta,\eth}f\right)(y)$$
$$=\left(-\frac{d}{dy}\right)^{[\mathrm{Re}(\eth)]+1}$$
$$\left(\mathfrak{I}_{0-}^{-\alpha,-\gamma,-\beta,-\varepsilon+[\mathrm{Re}(\eth)]+1,-\eth+[\mathrm{Re}(\eth)]+1}f\right)(y) \quad (33)$$

and

$$\left(\mathfrak{D}_{0+}^{\gamma,\alpha,\varepsilon,\beta,\eth}f\right)(y)=\left(\frac{d}{dy}\right)^{[\mathrm{Re}(\eth)]+1}\left(\mathfrak{I}_{0+}^{-\alpha,-\gamma,-\beta+[\mathrm{Re}(\eth)]+1,-\varepsilon,-\eth+[\mathrm{Re}(\eth)]+1}f\right)(y), \tag{34}$$

where $\gamma,\alpha,\varepsilon,\beta,\eth\in\mathbb{C},\mathrm{Re}(\eth)>0$, $y\in\mathbb{R}^{+}$ *and* f *is a real valued continuous function.*

We will use the following Lemmas [35] in our main result.

Lemma 3.*Let* $\gamma,\alpha,\varepsilon,\beta,\eth,\xi\in\mathbb{C}$ *such that* $\mathrm{Re}(\xi)>\max\{0,\mathrm{Re}(-\gamma+\beta),\mathrm{Re}(-\gamma-\alpha-\varepsilon+\eth)\}$*, then*

$$\left(\mathfrak{D}_{0+}^{\gamma,\alpha,\varepsilon,\beta,\eth}t^{\xi-1}\right)(y)=\frac{\Gamma(\xi)\Gamma(-\varepsilon+\gamma+\xi)\Gamma(\gamma+\alpha+\beta-\eth+\xi)}{\Gamma(-\varepsilon+\xi)\Gamma(\gamma+\alpha-\eth+\xi)\Gamma(\gamma+\beta-\eth+\xi)}y^{\gamma+\alpha-\eth+\xi-1}. \tag{35}$$

Lemma 4.*Let* $\gamma,\alpha,\varepsilon,\beta,\eth,\xi\in\mathbb{C}$ *such that* $\mathrm{Re}(\xi)>\max\{\mathrm{Re}(-\beta),\mathrm{Re}(\alpha+\varepsilon-\eth),\mathrm{Re}(\gamma+\alpha-\eth)+[\mathrm{Re}(\eth)]+1\}$*, then*

$$\left(\mathfrak{D}_{0-}^{\gamma,\alpha,\varepsilon,\beta,\eth}t^{-\xi}\right)(y)=\frac{\Gamma(\beta+\xi)\Gamma(-\gamma-\alpha+\eth+\xi)\Gamma(-\alpha-\varepsilon+\eth+\xi)}{\Gamma(\xi)\Gamma(-\alpha+\beta+\xi)\Gamma(-\gamma-\alpha-\varepsilon+\eth+\xi)}y^{\gamma+\alpha-\eth-\xi}. \tag{36}$$

Theorem 3.*Let* $\gamma,\alpha,\varepsilon,\beta,\zeta_\iota,\nu_\iota,\varpi,\tau,\varkappa_\ell,\kappa_\ell\in\mathbb{C}$, $\min\mathrm{Re}\{\varpi,\tau,\zeta_\iota,\nu_\iota,\varkappa_\ell,\kappa_\ell\}>0$ *for* $\iota=1,\ldots,r'$ *and* $\ell=1,\ldots,p'$, $\mathrm{Re}(\xi)>\max\{0,\mathrm{Re}(-\gamma+\beta),\mathrm{Re}(-\gamma-\alpha-\varepsilon+\eth)\}$*. Then*

$$\left(\mathfrak{D}_{0+}^{\gamma,\alpha,\varepsilon,\beta,\eth}t^{\xi-1}J_\rho^\omega(t)E_{\varpi,\tau;(\boldsymbol{\varkappa},\boldsymbol{\kappa})_{p'}}^{(\boldsymbol{\zeta},\boldsymbol{\nu})_{r'}}(t)\right)(y)$$
$$=y^{\gamma+\alpha+\xi-\eth-1}\frac{\Gamma(\varkappa_1)\Gamma(\varkappa_2)\ldots\Gamma(\varkappa_{p'})}{\Gamma(\zeta_1)\Gamma(\zeta_2)\ldots\Gamma(\zeta_{r'})}J_\rho^\omega(y)$$
$$\times\,{}_{r'+4}\Psi_{p'+4}\left[\begin{matrix}(\xi+m,1),(-\varepsilon+\gamma+\xi+m,1),\\(\gamma+\alpha+\beta-\eth+\xi+m,1),(1,1)\\(\zeta_1,\nu_1),\ldots,(\zeta_{r'},\nu_{r'})\\ \\(-\varepsilon+\xi+m,1),(\gamma+\alpha-\eth+\xi+m,1),\\(\gamma+\beta-\eth+\xi+m,1),(\tau,\varpi)\\(\varkappa_1,\kappa_1),\ldots,(\varkappa_{p'},\kappa_{p'})\end{matrix};y\right]. \tag{37}$$

*Proof.*Consider

$$\left(\mathfrak{D}_{0+}^{\gamma,\alpha,\varepsilon,\beta,\eth}t^{\xi-1}J_\rho^\omega(t)E_{\varpi,\tau;(\boldsymbol{\varkappa},\boldsymbol{\kappa})_{p'}}^{(\boldsymbol{\zeta},\boldsymbol{\nu})_{r'}}(t)\right)(y)$$
$$=\mathfrak{D}_{0+}^{\gamma,\alpha,\varepsilon,\beta,\eth}t^{\xi-1}\sum_{m=0}^{\infty}\frac{(-1)^m t^m}{\Gamma(\rho m+\omega+1)}\sum_{n=0}^{\infty}\frac{(\zeta_1)_{\nu_1 n}(\zeta_2)_{\nu_2 n}\ldots(\zeta_{r'})_{\nu_{r'}n}}{\Gamma(\varpi n+\tau)(\varkappa_1)_{\kappa_1 n}\ldots(\varkappa_{p'})_{\kappa_{p'}n}}t^n(y)$$

Switching the integral and summation orders using the valid conditions given in the theorem, we get

$$\left(\mathfrak{D}_{0+}^{\gamma,\alpha,\varepsilon,\beta,\eth}t^{\xi-1}J_\rho^\omega(t)E_{\varpi,\tau;(\boldsymbol{\varkappa},\boldsymbol{\kappa})_{p'}}^{(\boldsymbol{\zeta},\boldsymbol{\nu})_{r'}}(t)\right)(y)$$
$$=\sum_{m=0}^{\infty}\frac{(-1)^m}{\Gamma(\rho m+\omega+1)}\times\sum_{n=0}^{\infty}\frac{(\zeta_1)_{\nu_1 n}(\zeta_2)_{\nu_2 n}\ldots(\zeta_{r'})_{\nu_{r'}n}}{\Gamma(\varpi n+\tau)(\varkappa_1)_{\kappa_1 n}\ldots(\varkappa_{p'})_{\kappa_{p'}n}}\times\left(\mathfrak{D}_{0+}^{\gamma,\alpha,\varepsilon,\beta,\eth}t^{\xi+m+n-1}\right)(y). \tag{38}$$

Using Lemma 3, the above equation (38) reduces to

$$\left(\mathfrak{D}_{0+}^{\gamma,\alpha,\varepsilon,\beta,\eth}t^{\xi-1}J_\rho^\omega(t)E_{\varpi,\tau;(\boldsymbol{\varkappa},\boldsymbol{\kappa})_{p'}}^{(\boldsymbol{\zeta},\boldsymbol{\nu})_{r'}}(t)\right)(y)$$
$$=y^{\gamma+\alpha+\xi-\eth-1}\frac{\Gamma(\varkappa_1)\Gamma(\varkappa_2)\ldots\Gamma(\varkappa'_p)}{\Gamma(\zeta_1)\Gamma(\zeta_2)\ldots\Gamma(\zeta_{r'})}\sum_{m=0}^{\infty}\frac{(-y)^m}{\Gamma(\rho m+\omega+1)}\sum_{n=0}^{\infty}\frac{\Gamma(\zeta_1+\nu_1 n)\Gamma(\zeta_2+\nu_2 n)\ldots\Gamma(\zeta_{r'}+\nu_{r'}n)}{\Gamma(\varpi n+\tau)\Gamma(\varkappa_1+\kappa_1 n)\ldots\Gamma(\varkappa_{p'}+\kappa_{p'}n)}$$
$$\times\frac{\Gamma(\xi+m+n)\Gamma(-\varepsilon+\gamma+\xi+m+n)}{\Gamma(-\varepsilon+\xi+m+n)\Gamma(\gamma+\alpha-\eth+\xi+m+n)}\times\frac{\Gamma(\gamma+\alpha+\beta-\eth+\xi+m+n)}{\Gamma(\gamma+\beta-\eth+\xi+m+n)}\frac{\Gamma(n+1)}{n!}y^n$$
$$=y^{\gamma+\alpha+\xi-\eth-1}\frac{\Gamma(\varkappa_1)\Gamma(\varkappa_2)\ldots\Gamma(\varkappa_{p'})}{\Gamma(\zeta_1)\Gamma(\zeta_2)\ldots\Gamma(\zeta_{r'})}J_\rho^\omega(y)$$
$$\times\,{}_{r'+4}\Psi_{p'+4}\left[\begin{matrix}(\xi+m,1),(-\varepsilon+\gamma+\xi+m,1),\\(\gamma+\alpha+\beta-\eth+\xi+m,1),(1,1)\\(\zeta_1,\nu_1),\ldots,(\zeta'_r,\nu'_r)\\ \\(-\varepsilon+\xi+m,1),(\gamma+\alpha-\eth+\xi+m,1),\\(\gamma+\beta-\eth+\xi+m,1),(\tau,\varpi)\\(\varkappa_1,\kappa_1),\ldots,(\varkappa_{p'},\kappa_{p'})\end{matrix};y\right]. \tag{39}$$

Hence, we get the desired result (37).

Theorem 4.*Let* $\gamma,\alpha,\varepsilon,\beta,\zeta_\iota,\nu_\iota,\varpi,\tau,\varkappa_\ell,\kappa_\ell\in\mathbb{C}$, $\min\mathrm{Re}\{\varpi,\tau,\zeta_\iota,\nu_\iota,\varkappa_\ell,\kappa_\ell\}>0$ *with* $\iota=1,\ldots,r'$ *and* $\ell=1,\ldots,p'$, $\mathrm{Re}(\xi)>\max\{\mathrm{Re}(-\beta),\mathrm{Re}(\alpha+\varepsilon-$

$\eth), \mathrm{Re}(\gamma+\alpha-\eth)+[\mathrm{Re}(\eth)]+1\}$. *Then*

$$\left(\mathfrak{D}_{0-}^{\gamma,\alpha,\varepsilon,\beta,\eth} t^{-\xi} J_\rho^\omega(t) E_{\varpi,\tau;(\varkappa,\kappa)_{p'}}^{(\zeta,\nu)_{r'}}\left(\frac{1}{t}\right)\right)(y) = y^{\gamma+\alpha-\eth-\xi} \frac{\Gamma(\varkappa_1)\Gamma(\varkappa_2)\ldots\Gamma(\varkappa_{p'})}{\Gamma(\zeta_1)\Gamma(\zeta_2)\ldots\Gamma(\zeta_{r'})} J_\rho^\omega(y) \times {}_{r'+4}\Psi_{p'+4}\left[\begin{matrix}(\beta+\xi-m,1),(-\gamma-\alpha+\eth+\xi-m,1),\\(-\alpha-\varepsilon+\eth+\xi-m,1),(1,1)\\(\zeta_1,\nu_1),\ldots,(\zeta_{r'},\nu_{r'})\\ \\(\xi-m,1),(-\alpha+\beta+\xi-m,1),\\(-\gamma-\alpha-\varepsilon+\eth+\xi-m,1),(\tau,\varpi)\\(\varkappa_1,\kappa_1),\ldots,(\varkappa_{p'},\kappa_{p'})\end{matrix};\frac{1}{y}\right]. \quad (40)$$

*Proof.*Consider

$$\left(\mathfrak{D}_{0-}^{\gamma,\alpha,\varepsilon,\beta,\eth} t^{-\xi} J_\rho^\omega(t) E_{\varpi,\tau;(\varkappa,\kappa)_{p'}}^{(\zeta,\nu)_{r'}}\left(\frac{1}{t}\right)\right)(y) = \mathfrak{D}_{0-}^{\gamma,\alpha,\varepsilon,\beta,\eth} t^{-\xi} \sum_{m=0}^{\infty} \frac{(-1)^m t^m}{\Gamma(\rho m+\omega+1)} \times \sum_{n=0}^{\infty} \frac{(\zeta_1)_{\nu_1 n}(\zeta_2)_{\nu_2 n}\ldots(\zeta_{r'})_{\nu_{r'} n}}{\Gamma(\varpi n+\tau)(\varkappa_1)_{\kappa_1 n}\ldots(\varkappa_{p'})_{\kappa_{p'} n}} t^{-n}(y)$$

Switching the integral and summation orders under the valid conditions given in the theorem, we get

$$\left(\mathfrak{D}_{0-}^{\gamma,\alpha,\varepsilon,\beta,\eth} t^{-\xi} J_\rho^\omega(t) E_{\varpi,\tau;(\varkappa,\kappa)_{p'}}^{(\zeta,\nu)_{r'}}\left(\frac{1}{t}\right)\right)(y) = \sum_{m=0}^{\infty} \frac{(-1)^m}{\Gamma(\rho m+\omega+1)} \sum_{n=0}^{\infty} \frac{(\zeta_1)_{\nu_1 n}(\zeta_2)_{\nu_2 n}\ldots(\zeta_{r'})_{\nu_{r'} n}}{\Gamma(\varpi n+\tau)(\varkappa_1)_{\kappa_1 n}\ldots(\varkappa_{p'})_{\kappa_{p'} n}} \times \left(\mathfrak{D}_{0-}^{\gamma,\alpha,\varepsilon,\beta,\eth} t^{-(\xi-m+n)}\right)(y). \quad (41)$$

Using Lemma 4, the above equation (41) reduces to

$$\left(\mathfrak{D}_{0-}^{\gamma,\alpha,\varepsilon,\beta,\eth} t^{-\xi} J_\rho^\omega(t) E_{\varpi,\tau;(\varkappa,\kappa)_{p'}}^{(\zeta,\nu)_{r'}}\left(\frac{1}{t}\right)\right)(y) = y^{\gamma+\alpha-\xi-\eth} \frac{\Gamma(\varkappa_1)\Gamma(\varkappa_2)\ldots\Gamma(\varkappa_{p'})}{\Gamma(\zeta_1)\Gamma(\zeta_2)\ldots\Gamma(\zeta_{r'})} \sum_{m=0}^{\infty} \frac{(-y)^m}{\Gamma(\rho m+\omega+1)} \sum_{n=0}^{\infty} \frac{\Gamma(\zeta_1+\nu_1 n)\Gamma(\zeta_2+\nu_2 n)\ldots\Gamma(\zeta_{r'}+\nu_{r'} n)}{\Gamma(\varpi n+\tau)\Gamma(\varkappa_1+\kappa_1 n)\ldots\Gamma(\varkappa_{p'}+\kappa_{p'} n)} \times \frac{\Gamma(\beta+\xi-m+n)\Gamma(-\gamma-\alpha+\eth+\xi-m+n)}{\Gamma(\xi-m+n)\Gamma(-\alpha+\beta+\xi-m+n)} \times \frac{\Gamma(-\alpha-\varepsilon+\eth+\xi-m+n)}{\Gamma(-\gamma-\alpha-\varepsilon+\eth+\xi-m+n)} \frac{\Gamma(n+1)}{n!} y^{-n}$$

$$= y^{\gamma+\alpha-\eth-\xi} \frac{\Gamma(\varkappa_1)\Gamma(\varkappa_2)\ldots\Gamma(\varkappa_{p'})}{\Gamma(\zeta_1)\Gamma(\zeta_2)\ldots\Gamma(\zeta_{r'})} J_\rho^\omega(y) \times {}_{r'+4}\Psi_{p'+4}\left[\begin{matrix}(\beta+\xi-m,1),(-\gamma-\alpha+\eth+\xi-m,1),\\(-\alpha-\varepsilon+\eth+\xi-m,1),(1,1)\\(\zeta_1,\nu_1),\ldots,(\zeta_{r'},\nu_{r'})\\ \\(\xi-m,1),(-\alpha+\beta+\xi-m,1),\\(-\gamma-\alpha-\varepsilon+\eth+\xi-m,1),(\tau,\varpi)\\(\varkappa_1,\kappa_1),\ldots,(\varkappa_{p'},\kappa_{p'})\end{matrix};\frac{1}{y}\right]. \quad (42)$$

Hence, we get the desired result (40).

3.1 Particular Cases

Theorem 3 and 4 give the following image formulas for the Mittag-Leffler function (3), if $r'=p'=0$ and $\tau=1$:

Corollary 13.*Let* $\gamma,\alpha,\varepsilon,\beta\in\mathbb{C}$, $\varpi>0$ *and* $\Re(\xi)>\max\{0,\mathrm{Re}(-\gamma+\beta),\mathrm{Re}(-\gamma-\alpha-\varepsilon+\eth)\}$. *Then*

$$\left(\mathfrak{D}_{0+}^{\gamma,\alpha,\varepsilon,\beta,\eth} t^{\xi-1} J_\rho^\omega(t) E_\varpi(t)\right)(y) = y^{\gamma+\alpha+\xi-\eth-1} J_\rho^\omega(y) \times {}_4\Psi_4\left[\begin{matrix}(\xi+m,1),(-\varepsilon+\gamma+\xi+m,1),\\(\gamma+\alpha+\beta-\eth+\xi+m,1),(1,1)\\ \\(-\varepsilon+\xi+m,1),(\gamma+\alpha-\eth+\xi+m,1),\\(\gamma+\beta-\eth+\xi+m,1),(1,\varpi)\end{matrix};y\right]. \quad (43)$$

Corollary 14.*Let* $\gamma,\alpha,\varepsilon,\beta\in\mathbb{C}$, $\varpi>0$ *and* $\mathrm{Re}(\xi)>\max\{\mathrm{Re}(-\beta),\mathrm{Re}(\alpha+\varepsilon-\eth),\mathrm{Re}(\gamma+\alpha-\eth)+[\mathrm{Re}(\eth)]+1\}$. *Then*

$$\left(\mathfrak{D}_{0-}^{\gamma,\alpha,\varepsilon,\beta,\eth} t^{-\xi} J_\rho^\omega(t) E_\varpi\left(\frac{1}{t}\right)\right)(y) = y^{\gamma+\alpha-\eth-\xi} J_\rho^\omega(y) \times {}_4\Psi_4\left[\begin{matrix}(\beta+\xi-m,1),(-\gamma-\alpha+\eth+\xi-m,1),\\(-\alpha-\varepsilon+\eth+\xi-m,1),(1,1)\\ \\(\xi-m,1),(-\alpha+\beta+\xi-m,1),\\(-\gamma-\alpha-\varepsilon+\eth+\xi-m,1),(1,\varpi)\end{matrix};\frac{1}{y}\right]. \quad (44)$$

Theorem 3 and 4 give the following image formulas for the Wiman function, if $r'=p'=0$,:

Corollary 15. *Let* $\gamma,\alpha,\varepsilon,\beta \in \mathbb{C}$, $\Re(\varpi), \mathrm{Re}(\tau) > 0$ *and* $\mathrm{Re}(\xi) > \max\{0, \mathrm{Re}(-\gamma+\beta), \mathrm{Re}(-\gamma-\alpha-\varepsilon+\eth)\}$. *Then*

$$\left(\mathfrak{D}_{0^+}^{\gamma,\alpha,\varepsilon,\beta,\eth} t^{\xi-1} J_\rho^\omega(t) E_{\varpi,\tau}(t)\right)(y)$$
$$= y^{\gamma+\alpha+\xi-\eth-1} J_\rho^\omega(y) \times {}_4\Psi_4\left[\begin{matrix}(\xi+m,1),(-\varepsilon+\gamma+\xi+m,1),\\(\gamma+\alpha+\beta-\eth+\xi+m,1),(1,1)\\ \\(-\varepsilon+\xi+m,1),(\gamma+\alpha-\eth+\xi+m,1),\\(\gamma+\beta-\eth+\xi+m,1),(\tau,\varpi)\end{matrix};y\right]. \quad (45)$$

Corollary 16. *Let* $\gamma,\alpha,\varepsilon,\beta \in \mathbb{C}$, $\mathrm{Re}(\varpi), \mathrm{Re}(\tau) > 0$ *and* $\mathrm{Re}(\xi) > \max\{\mathrm{Re}(-\beta), \mathrm{Re}(\alpha+\varepsilon-\eth), \mathrm{Re}(\gamma+\alpha-\eth)+[\mathrm{Re}(\eth)]+1\}$. *Then*

$$\left(\mathfrak{D}_{0^-}^{\gamma,\alpha,\varepsilon,\beta,\eth} t^{-\xi} J_\rho^\omega(t) E_{\varpi,\tau}\left(\frac{1}{t}\right)\right)(y)$$
$$= y^{\gamma+\alpha-\eth-\xi} J_\rho^\omega(y) \times {}_4\Psi_4\left[\begin{matrix}(\beta+\xi-m,1),(-\gamma-\alpha+\eth+\xi-m,1),\\(-\alpha-\varepsilon+\eth+\xi-m,1),(1,1)\\ \\(\xi-m,1),(-\alpha+\beta+\xi-m,1),\\(-\gamma-\alpha-\varepsilon+\eth+\xi-m,1),(\tau,\varpi)\end{matrix};\frac{1}{y}\right]. \quad (46)$$

Theorem 3 and 4 give the following image formulas for the Prabhakar function (4), if $r' = p' = 1$ and $\nu = \varkappa = \kappa = 1$,:

Corollary 17. *Let* $\gamma,\alpha,\varepsilon,\beta \in \mathbb{C}$, $\mathrm{Re}(\varpi), \mathrm{Re}(\tau) > 0, \zeta > 0$ *and* $\mathrm{Re}(\xi) > \max\{0, \mathrm{Re}(-\gamma+\beta), \mathrm{Re}(-\gamma-\alpha-\varepsilon+\eth)\}$. *Then*

$$\left(\mathfrak{D}_{0^+}^{\gamma,\alpha,\varepsilon,\beta,\eth} t^{\xi-1} J_\rho^\omega(t) E_{\varpi,\tau}^{\zeta}(t)\right)(y)$$
$$= \frac{y^{\gamma+\alpha+\xi-\eth-1}}{\Gamma(\zeta)} J_\rho^\omega(y) \times {}_5\Psi_5\left[\begin{matrix}(\xi+m,1),(-\varepsilon+\gamma+\xi+m,1),\\(\gamma+\alpha+\beta-\eth+\xi+m,1),(1,1),\\(\zeta_1,1)\\ \\(-\varepsilon+\xi+m,1),(\gamma+\alpha-\eth+\xi+m,1),\\(\gamma+\beta-\eth+\xi+m,1),(\tau,\varpi),\\(1,1)\end{matrix};y\right]. \quad (47)$$

Corollary 18. *Let* $\gamma,\alpha,\varepsilon,\beta \in \mathbb{C}$, $\mathrm{Re}(\varpi), \mathrm{Re}(\tau) > 0, \zeta > 0$ *and* $\mathrm{Re}(\xi) > \max\{\mathrm{Re}(-\beta), \mathrm{Re}(\alpha+\varepsilon-\eth), \mathrm{Re}(\gamma+\alpha-\eth)+[\mathrm{Re}(\eth)]+1\}$. *Then*

$$\left(\mathfrak{D}_{0^-}^{\gamma,\alpha,\varepsilon,\beta,\eth} t^{-\xi} J_\rho^\omega(t) E_{\varpi,\tau}^{\zeta}\left(\frac{1}{t}\right)\right)(y)$$
$$= \frac{y^{\gamma+\alpha-\eth-\xi}}{\Gamma(\zeta)} J_\rho^\omega(y) \times {}_5\Psi_5\left[\begin{matrix}(\beta+\xi-m,1),(-\gamma-\alpha+\eth+\xi-m,1),\\(-\alpha-\varepsilon+\eth+\xi-m,1),(1,1),\\(\zeta,1)\\ \\(\xi-m,1),(-\alpha+\beta+\xi-m,1),\\(-\gamma-\alpha-\varepsilon+\eth+\xi-m,1),(\tau,\varpi),\\(1,1)\end{matrix};\frac{1}{y}\right]. \quad (48)$$

Theorem 3 and 4 give the following image formulas for Shukla and Prajapati's generalization of the Prabhakar function (5), if $r' = p' = 1$ and $\varkappa = \kappa = 1$:

Corollary 19. *Let* $\gamma,\alpha,\varepsilon,\beta \in \mathbb{C}$, *min* $\{\Re(\varpi), \mathrm{Re}(\tau), \mathrm{Re}(\zeta_1)\} > 0$, $\varpi,\tau,\zeta,\nu \in \mathbb{C}$ *and* $\mathrm{Re}(\xi) > \max\{0, \mathrm{Re}(-\gamma+\beta), \mathrm{Re}(-\gamma-\alpha-\varepsilon+\eth)\}$. *Then*

$$\left(\mathfrak{D}_{0^+}^{\gamma,\alpha,\varepsilon,\beta,\eth} t^{\xi-1} J_\rho^\omega(t) E_{\varpi,\tau}^{\zeta,\nu}(t)\right)(y)$$
$$= \frac{y^{\gamma+\alpha+\xi-\eth-1}}{\Gamma(\zeta)} J_\rho^\omega(y) \times {}_5\Psi_5\left[\begin{matrix}(\xi+m,1),(-\varepsilon+\gamma+\xi+m,1),\\(\gamma+\alpha+\beta-\eth+\xi+m,1),(1,1),\\(\zeta,\nu)\\ \\(-\varepsilon+\xi+m,1),(\gamma+\alpha-\eth+\xi+m,1),\\(\gamma+\beta-\eth+\xi+m,1),(\tau,\varpi),\\(1,1)\end{matrix};y\right]. \quad (49)$$

Corollary 20. *Let* $\gamma,\alpha,\varepsilon,\beta \in \mathbb{C}$, *min* $\{\mathrm{Re}(\varpi), \mathrm{Re}(\tau), \mathrm{Re}(\zeta)\} > 0$, $\varpi,\tau,\zeta,\nu \in \mathbb{C}$ *and* $\mathrm{Re}(\xi) > \max\{\mathrm{Re}(-\beta), \mathrm{Re}(\alpha+\varepsilon-\eth), \mathrm{Re}(\gamma+\alpha-\eth)+[\mathrm{Re}(\eth)]+1\}$. *Then*

$$\left(\mathfrak{D}_{0^-}^{\gamma,\alpha,\varepsilon,\beta,\eth} t^{-\xi} J_\rho^\omega(t) E_{\varpi,\tau}^{\zeta,\nu}\left(\frac{1}{t}\right)\right)(y)$$
$$= \frac{y^{\gamma+\alpha-\eth-\xi}}{\Gamma(\zeta)} J_\rho^\omega(y) \times {}_5\Psi_5\left[\begin{matrix}(\beta+\xi-m,1),(-\gamma-\alpha+\eth+\xi-m,1),\\(-\alpha-\varepsilon+\eth+\xi-m,1),(1,1),\\(\zeta,\nu)\\ \\(\xi-m,1),(-\alpha+\beta+\xi-m,1),\\(-\gamma-\alpha-\varepsilon+\eth+\xi-m,1),(\tau,\varpi),\\(1,1)\end{matrix};\frac{1}{y}\right]. \quad (50)$$

Theorem 3 and 4 give the following image formulas for Khan and Ahmed's generalization of the Mittag-Leffler function (6), if $r' = p' = 1$ and $\kappa = 1$:

Corollary 21. *Let* $\gamma,\alpha,\varepsilon,\beta \in \mathbb{C}$, $\min\{\mathrm{Re}(\varpi),\mathrm{Re}(\tau),\mathrm{Re}(\varkappa),\mathrm{Re}(\zeta)\} > 0$, $\nu \in (0,1)\cup\mathbb{N}$ *and* $\mathrm{Re}(\xi) > \max\{0,\mathrm{Re}(-\gamma+\beta),\mathrm{Re}(-\gamma-\alpha-\varepsilon+\eth)\}$, *then*

$$\left(\mathfrak{D}_{0+}^{\gamma,\alpha,\varepsilon,\beta,\eth} t^{\xi-1} J_{\rho}^{\omega}(t) E_{\varpi,\tau,\varkappa}^{\zeta,\nu}(t)\right)(y) = y^{\gamma+\alpha+\xi-\eth-1}\frac{\Gamma(\varkappa)}{\Gamma(\zeta)} J_{\rho}^{\omega}(y) \times {}_5\Psi_5\left[\begin{matrix}(\xi+m,1),(-\varepsilon+\gamma+\xi+m,1),\\(\gamma+\alpha+\beta-\eth+\xi+m,1),(1,1),\\(\zeta,\nu)\\ \\(-\varepsilon+\xi+m,1),(\gamma+\alpha-\eth+\xi+m,1),\\(\gamma+\beta-\eth+\xi+m,1),(\tau,\varpi),\\(\varkappa,1)\end{matrix};y\right]. \tag{51}$$

Corollary 22. *Let* $\gamma,\alpha,\varepsilon,\beta \in \mathbb{C}$, $\min\{\mathrm{Re}(\varpi),\mathrm{Re}(\tau),\mathrm{Re}(\varkappa),\mathrm{Re}(\zeta)\} > 0$, $\nu \in (0,1)\cup\mathbb{N}$ *and* $\mathrm{Re}(\xi) > \max\{\mathrm{Re}(-\beta),\mathrm{Re}(\alpha+\varepsilon-\eth),\mathrm{Re}(\gamma+\alpha-\eth)+[\mathrm{Re}(\eth)]+1\}$. *Then*

$$\left(\mathfrak{D}_{0-}^{\gamma,\alpha,\varepsilon,\beta,\eth} t^{-\xi} J_{\rho}^{\omega}(t) E_{\varpi,\tau,\varkappa}^{\zeta,\nu}\left(\frac{1}{t}\right)\right)(y) = y^{\gamma+\alpha-\eth-\xi}\frac{\Gamma(\varkappa)}{\Gamma(\zeta)} J_{\rho}^{\omega}(y) \times {}_5\Psi_5\left[\begin{matrix}(\beta+\xi-m,1),(-\gamma-\alpha+\eth+\xi-m,1),\\(-\alpha-\varepsilon+\eth+\xi-m,1),(1,1),\\(\zeta,\nu)\\ \\(\xi-m,1),(-\alpha+\beta+\xi-m,1),\\(-\gamma-\alpha-\varepsilon+\eth+\xi-m,1),(\tau,\varpi),\\(\varkappa,1)\end{matrix};\frac{1}{y}\right]. \tag{52}$$

Theorem 3 and 4 give the following image formulas for Khan and Ahmed's further generalization of the Mittag-Leffler function (7), if $r' = p' = 2$:

Corollary 23. *Let* $\gamma,\alpha,\varepsilon,\beta \in \mathbb{C}$, $\nu_2,\kappa_2 > 0$, $\varpi,\tau,\zeta_1,\nu_1,\zeta_2,\varkappa_1,\kappa_1,\varkappa_2 \in \mathbb{C}$, $\mathrm{Re}(\varpi)+\kappa_2 \geq \nu_2$, $\min(\mathrm{Re}(\varpi),\mathrm{Re}(\tau),\mathrm{Re}(\zeta_1),\mathrm{Re}(\nu_1),\mathrm{Re}(\zeta_2),\mathrm{Re}(\varkappa_1),\mathrm{Re}(\kappa_1),\mathrm{Re}(\varkappa_2)) > 0$, $\mathrm{Re}(\xi) > \max\{0,\mathrm{Re}(-\gamma+\beta),\mathrm{Re}(-\gamma-\alpha-\varepsilon+\eth)\}$. *Then*

$$\left(\mathfrak{D}_{0+}^{\gamma,\alpha,\varepsilon,\beta,\eth} t^{\xi-1} J_{\rho}^{\omega}(t) E_{\varpi,\tau;\varkappa_1,\kappa_1;\varkappa_2,\kappa_2}^{\zeta_1,\nu_1;\zeta_2,\nu_2}(t)\right)(y) = y^{\gamma+\alpha+\xi-\eth-1}\frac{\Gamma(\varkappa_1)\Gamma(\varkappa_2)}{\Gamma(\zeta_1)\Gamma(\zeta_2)} J_{\rho}^{\omega}(y) \times {}_6\Psi_6\left[\begin{matrix}(\xi+m,1),(-\varepsilon+\gamma+\xi+m,1),\\(\gamma+\alpha+\beta-\eth+\xi+m,1),(1,1),\\(\zeta_1,\nu_1),(\zeta_2,\nu_2)\\ \\(-\varepsilon+\xi+m,1),(\gamma+\alpha-\eth+\xi+m,1),\\(\gamma+\beta-\eth+\xi+m,1),(\tau,\varpi),\\(\varkappa_1,\kappa_1),(\varkappa_2,\kappa_2)\end{matrix};y\right]. \tag{53}$$

Corollary 24. *Let* $\gamma,\alpha,\varepsilon,\beta \in \mathbb{C}$, $\varpi,\tau,\zeta_1,\nu_1,\zeta_2,\varkappa_1,\kappa_1,\varkappa_2 \in \mathbb{C}$, $\nu_2,\kappa_2 > 0$, $\mathrm{Re}(\varpi)+\kappa_2 \geq \nu_2$ $\min(\mathrm{Re}(\varpi),\mathrm{Re}(\tau),\mathrm{Re}(\zeta_1),\mathrm{Re}(\nu_1),\mathrm{Re}(\zeta_2),\mathrm{Re}(\varkappa_1),\mathrm{Re}(\kappa_1),\mathrm{Re}(\varkappa_2)) > 0$, *and* $\mathrm{Re}(\xi) > \max\{\mathrm{Re}(-\beta),\mathrm{Re}(\alpha+\varepsilon-\eth),\mathrm{Re}(\gamma+\alpha-\eth)+[\mathrm{Re}(\eth)]+1\}$. *Then*

$$\left(\mathfrak{D}_{0-}^{\gamma,\alpha,\varepsilon,\beta,\eth} t^{-\xi} J_{\rho}^{\omega}(t) E_{\varpi,\tau;\varkappa_1,\kappa_1;\varkappa_2,\kappa_2}^{\zeta_1,\nu_1;\zeta_2,\nu_2}\left(\frac{1}{t}\right)\right)(y) = y^{\gamma+\alpha-\eth-\xi}\frac{\Gamma(\varkappa_1)\Gamma(\varkappa_2)}{\Gamma(\zeta_1)\Gamma(\zeta_2)} J_{\rho}^{\omega}(y) \times {}_6\Psi_6\left[\begin{matrix}(\beta+\xi-m,1),(-\gamma-\alpha+\eth+\xi-m,1),\\(-\alpha-\varepsilon+\eth+\xi-m,1),(1,1),\\(\zeta_1,\nu_1),(\zeta_2,\nu_2)\\ \\(\xi-m,1),(-\alpha+\beta+\xi-m,1),\\(-\gamma-\alpha-\varepsilon+\eth+\xi-m,1),(\tau,\varpi),\\(\varkappa_1,\kappa_1),(\varkappa_2,\kappa_2)\end{matrix};\frac{1}{y}\right]. \tag{54}$$

4 Image formulas using the Saigo fractional integral operator

We will discuss the image formulas for the Bessel-Maitland function and the m-parameter Mittag-Leffler function using the Saigo fractional integral operator.

Definition 3. *The Saigo fractional integral operators are defined as [36,37]*

$$\left(\mathfrak{I}_{0-}^{\gamma,\varepsilon,\eth} f\right)(y) = \frac{1}{\Gamma(\gamma)}\int_y^{\infty}(t-y)^{\gamma-1}t^{-\gamma-\varepsilon}{}_2F_1\left(\gamma+\varepsilon,-\eth;\gamma;1-\frac{x}{t}\right)f(t)dt. \tag{55}$$

and

$$\left(\mathfrak{I}_{0+}^{\gamma,\varepsilon,\eth} f\right)(y) = \frac{y^{-\gamma-\varepsilon}}{\Gamma(\gamma)}\int_0^{y}(y-t)^{\gamma-1}{}_2F_1\left(\gamma+\varepsilon,-\eth;\gamma;1-\frac{t}{x}\right)f(t)dt. \tag{56}$$

where $y > 0,\gamma,\varepsilon,\eth \in \mathbb{C},\mathrm{Re}(\gamma) > 0$, ${}_2F_1(.)$ *is the Gauss hypergeometric series and* $f(t)$ *is a real valued continuous function.*

We will make use of the following Lemmas [37] in proving our result:

Lemma 5. *Let* $\gamma,\varepsilon,\eth \in \mathbb{C}$ *and* $\mathrm{Re}(\gamma) > 0,\mathrm{Re}(\xi) > \max\{0,\mathrm{Re}(\varepsilon-\eth)\}$, *then*

$$\left(\mathfrak{I}_{0+}^{\gamma,\varepsilon,\eth} t^{\xi-1}\right)(y) = \frac{\Gamma(\xi)\Gamma(\xi+\eth-\varepsilon)}{\Gamma(\xi-\varepsilon)\Gamma(\xi+\gamma+\eth)}y^{\xi-\varepsilon-1}. \tag{57}$$

Lemma 6. *Let* $\gamma,\varepsilon,\eth \in \mathbb{C}$ *and* $\mathrm{Re}(\gamma)>0, \mathrm{Re}(\xi)<1+\min[\mathrm{Re}(\varepsilon),\mathrm{Re}(\eth)]$, *then*

$$\left(\Im_{0-}^{\gamma,\varepsilon,\eth} t^{\xi-1}\right)(y)=\frac{\Gamma(\varepsilon-\xi+1)\Gamma(\eth-\xi+1)}{\Gamma(1-\xi)\Gamma(\gamma+\varepsilon+\eth-\xi+1)}y^{\xi-\varepsilon-1}. \tag{58}$$

Theorem 5. *Let* $\gamma,\varepsilon,\zeta_\iota,\nu_\iota,\varpi,\tau,\varkappa_\ell,\kappa_\ell \in \mathbb{C}$, $\min\mathrm{Re}\{\varpi,\tau,\zeta_\iota,\nu_\iota,\varkappa_\ell,\kappa_\ell\}>0$ *with* $\iota=1,...,r'$ *and* $\ell=1,...,p'$, $\mathrm{Re}(\gamma)>0,\mathrm{Re}(\xi)>\max\{0,\mathrm{Re}(\varepsilon-\eth)\}$. *Then*

$$\begin{aligned}&\left(\Im_{0+}^{\gamma,\varepsilon,\eth} t^{\xi-1} J_\rho^\omega(t) E_{\varpi,\tau;(\boldsymbol{\varkappa},\boldsymbol{\kappa})_{p'}}^{(\boldsymbol{\zeta},\boldsymbol{\nu})_{r'}}(t)\right)(y)\\ &= y^{\xi-\varepsilon-1}\frac{\Gamma(\varkappa_1)\Gamma(\varkappa_2)\dots\Gamma(\varkappa_{p'})}{\Gamma(\zeta_1)\Gamma(\zeta_2)\dots\Gamma(\zeta_{r'})}J_\rho^\omega(y)\\ &\quad {}_{r'+3}\Psi_{p'+3}\left[\begin{matrix}(\xi+m,1),(\xi+m+\eth-\varepsilon,1),\\ (1,1),(\zeta_1,\nu_1),\dots,(\zeta_{r'},\nu_{r'})\\ \\ (\xi+m-\varepsilon,1),(\xi+m+\gamma+\eth,1),\\ (\tau,\varpi),(\varkappa_1,\kappa_1),\dots,(\varkappa_{p'},\kappa_{p'})\end{matrix};y\right].\end{aligned} \tag{59}$$

Proof. Consider

$$\begin{aligned}&\left(\Im_{0+}^{\gamma,\varepsilon,\eth} t^{\xi-1} J_\rho^\omega(t) E_{\varpi,\tau;(\boldsymbol{\varkappa},\boldsymbol{\kappa})_{p'}}^{(\boldsymbol{\zeta},\boldsymbol{\nu})_{r'}}(t)\right)(y)\\ &= \Im_{0+}^{\gamma,\varepsilon,\eth} t^{\xi-1}\sum_{m=0}^{\infty}\frac{(-1)^m t^m}{\Gamma(\rho m+\omega+1)}\\ &\times\sum_{n=0}^{\infty}\frac{(\zeta_1)_{\nu_1 n}(\zeta_2)_{\nu_2 n}\dots(\zeta_{r'})_{\nu_{r'}n}}{\Gamma(\varpi n+\tau)(\varkappa_1)_{\kappa_1 n}\dots(\varkappa_{p'})_{\kappa_{p'}n}}t^n(y)\end{aligned}$$

Swapping the integral and summation orders using the valid conditions mentioned in the theorem, we get

$$\begin{aligned}&\left(\Im_{0+}^{\gamma,\varepsilon,\eth} t^{\xi-1} J_\rho^\omega(t) E_{\varpi,\tau;(\boldsymbol{\varkappa},\boldsymbol{\kappa})_{p'}}^{(\boldsymbol{\zeta},\boldsymbol{\nu})_{r'}}(t)\right)(y)\\ &= \sum_{m=0}^{\infty}\frac{(-1)^m}{\Gamma(\rho m+\omega+1)}\sum_{n=0}^{\infty}\frac{(\zeta_1)_{\nu_1 n}(\zeta_2)_{\nu_2 n}\dots(\zeta_{r'})_{\nu_{r'}n}}{\Gamma(\varpi n+\tau)(\varkappa_1)_{\kappa_1 n}\dots(\varkappa_{p'})_{\kappa_{p'}n}}\\ &\times\left(\Im_{0+}^{\gamma,\varepsilon,\eth} t^{\xi+m+n-1}\right)(y)\end{aligned} \tag{60}$$

Using Lemma 5, the above equation (60) reduces to

$$\begin{aligned}&\left(\Im_{0+}^{\gamma,\varepsilon,\eth} t^{\xi-1} J_\rho^\omega(t) E_{\varpi,\tau;(\boldsymbol{\varkappa},\boldsymbol{\kappa})_{p'}}^{(\boldsymbol{\zeta},\boldsymbol{\nu})_{r'}}(t)\right)(y)\\ &= y^{\xi-\varepsilon-1}\frac{\Gamma(\varkappa_1)\Gamma(\varkappa_2)\dots\Gamma(\varkappa_{p'})}{\Gamma(\zeta_1)\Gamma(\zeta_2)\dots\Gamma(\zeta_{r'})}\sum_{m=0}^{\infty}\frac{(-y)^m}{\Gamma(\rho m+\omega+1)}\\ &\sum_{n=0}^{\infty}\frac{\Gamma(\zeta_1+\nu_1 n)\Gamma(\zeta_2+\nu_2 n)\dots\Gamma(\zeta_{r'}+\nu_{r'}n)}{\Gamma(\varpi n+\tau)\Gamma(\varkappa_1+\kappa_1 n)\dots\Gamma(\varkappa_{p'}+\kappa_{p'}n)}\\ &\times\frac{\Gamma(\xi+m+n)\Gamma(\eth+\xi+m+n-\varepsilon)}{\Gamma(\xi+m+n-\varepsilon)\Gamma(\xi+m+n+\gamma+\eth)}\frac{\Gamma(n+1)}{n!}y^n\\ &= y^{\xi-\varepsilon-1}\frac{\Gamma(\varkappa_1)\Gamma(\varkappa_2)\dots\Gamma(\varkappa_{p'})}{\Gamma(\zeta_1)\Gamma(\zeta_2)\dots\Gamma(\zeta_{r'})}J_\rho^\omega(y)\\ &\times {}_{r'+3}\Psi_{p'+3}\left[\begin{matrix}(\xi+m,1),(\xi+m+\eth-\varepsilon,1),\\ (1,1),(\zeta_1,\nu_1),\dots,(\zeta_{r'},\nu_{r'})\\ \\ (\xi+m-\varepsilon,1),(\xi+m+\gamma+\eth,1),\\ (\tau,\varpi),(\varkappa_1,\kappa_1),\dots,(\varkappa_{p'},\kappa_{p'})\end{matrix};y\right].\end{aligned} \tag{61}$$

Hence, we get the desired result (59).

Theorem 6. *Let* $\gamma,\varepsilon,\zeta_\iota,\nu_\iota,\varpi,\tau,\varkappa_\ell,\kappa_\ell \in \mathbb{C}$, $\min\mathrm{Re}\{\varpi,\tau,\zeta_\iota,\nu_\iota,\varkappa_\ell,\kappa_\ell\}>0$ *for* $\iota 1,...,r';\ell=1,...,p'$ *and* $\mathrm{Re}(\gamma)>0,\mathrm{Re}(\xi)<1+\min[\mathrm{Re}(\varepsilon),\mathrm{Re}(\eth)]$. *Then*

$$\begin{aligned}&\left(\Im_{0-}^{\gamma,\varepsilon,\eth} t^{\xi-1} J_\rho^\omega(t) E_{\varpi,\tau;(\boldsymbol{\varkappa},\boldsymbol{\kappa})_{p'}}^{(\boldsymbol{\zeta},\boldsymbol{\nu})_{r'}}\left(\frac{1}{t}\right)\right)(y)\\ &= y^{\xi-\varepsilon-1}\frac{\Gamma(\varkappa_1)\Gamma(\varkappa_2)\dots\Gamma(\varkappa_{p'})}{\Gamma(\zeta_1)\Gamma(\zeta_2)\dots\Gamma(\zeta_{r'})}J_\rho^\omega(y)\\ &{}_{r'+3}\Psi_{p'+3}\\ &\left[\begin{matrix}(\varepsilon-\xi-m+1,1),(\eth-\xi-m+1,1),\\ (1,1),(\zeta_1,\nu_1),\dots,(\zeta_{r'},\nu_{r'})\\ \\ (1-\xi-m,1),(\gamma+\varepsilon+\eth-\xi-m+1,1),\\ (\tau,\varpi),(\varkappa_1,\kappa_1),\dots,(\varkappa_{p'},\kappa_{p'})\end{matrix};\frac{1}{y}\right].\end{aligned} \tag{62}$$

Proof. Consider

$$\begin{aligned}&\left(\Im_{0-}^{\gamma,\varepsilon,\eth} t^{\xi-1} J_\rho^\omega(t) E_{\varpi,\tau;(\boldsymbol{\varkappa},\boldsymbol{\kappa})_{p'}}^{(\boldsymbol{\zeta},\boldsymbol{\nu})_{r'}}\left(\frac{1}{t}\right)\right)(y)\\ &= \Im_{0-}^{\gamma,\varepsilon,\eth} t^{\xi-1}\sum_{m=0}^{\infty}\frac{(-1)^m t^m}{\Gamma(\rho m+\omega+1)}\\ &\times\sum_{n=0}^{\infty}\frac{(\zeta_1)_{\nu_1 n}(\zeta_2)_{\nu_2 n}\dots(\zeta_{r'})_{\nu_{r'}n}}{\Gamma(\varpi n+\tau)(\varkappa_1)_{\kappa_1 n}\dots(\varkappa_{p'})_{\kappa_{p'}n}}t^{-n}(y)\end{aligned}$$

Swapping the integral and summation orders using the valid conditions mentioned in the theorem, we get

$$\begin{aligned}&\left(\Im_{0-}^{\gamma,\varepsilon,\eth} t^{\xi-1} J_\rho^\omega(t) E_{\varpi,\tau;(\boldsymbol{\varkappa},\boldsymbol{\kappa})_{p'}}^{(\boldsymbol{\zeta},\boldsymbol{\nu})_{r'}}\left(\frac{1}{t}\right)\right)(y)\\ &= \sum_{m=0}^{\infty}\frac{(-1)^m}{\Gamma(\rho m+\omega+1)}\sum_{n=0}^{\infty}\frac{(\zeta_1)_{\nu_1 n}(\zeta_2)_{\nu_2 n}\dots(\zeta_{r'})_{\nu_{r'}n}}{\Gamma(\varpi n+\tau)(\varkappa_1)_{\kappa_1 n}\dots(\varkappa_{p'})_{\kappa_{p'}n}}\\ &\times\left(\Im_{0+}^{\gamma,\varepsilon,\eth} t^{\xi+m-n-1}\right)(y)\end{aligned} \tag{63}$$

Using Lemma 6, the above equation (63) reduces to

$$\begin{aligned}
&\left(\mathfrak{I}^{\gamma,\varepsilon,\eth}_{0-}t^{-\xi}J^{\omega}_{\rho}(t)E^{(\zeta,\nu)_{r'}}_{\varpi,\tau;(\varkappa,\kappa)_{p'}}\left(\frac{1}{t}\right)\right)(y)\\
=&y^{\xi-\varepsilon-1}\frac{\Gamma(\varkappa_1)\Gamma(\varkappa_2)\dots\Gamma(\varkappa_p)}{\Gamma(\zeta_1)\Gamma(\zeta_2)\dots\Gamma(\zeta_{r'})}\sum_{m=0}^{\infty}\frac{(-y)^m}{\Gamma(\rho m+\omega+1)}\\
&\times\sum_{n=0}^{\infty}\frac{\Gamma(\zeta_1+\nu_1 n)\Gamma(\zeta_2+\nu_2 n)\dots\Gamma(\zeta_{r'}+\nu_{r'}n)}{\Gamma(\varpi n+\tau)\Gamma(\varkappa_1+\kappa_1 n)\dots\Gamma(\varkappa_{p'}+\kappa_{p'}n)}\\
&\times\frac{\Gamma(\varepsilon-\xi-m+n+1)\Gamma(\eth-\xi-m+n+1)}{\Gamma(1-\xi-m+n)\Gamma(\gamma+\varepsilon+\eth-\xi-m+n+1)}\\
&\frac{\Gamma(n+1)}{n!}y^n\\
=&y^{\xi-\varepsilon-1}\frac{\Gamma(\varkappa_1)\Gamma(\varkappa_2)\dots\Gamma(\varkappa_{p'})}{\Gamma(\zeta_1)\Gamma(\zeta_2)\dots\Gamma(\zeta_{r'})}J^{\omega}_{\rho}(y)\\
&{}_{r'+3}\Psi_{p'+3}\\
&\left[\begin{matrix}(\varepsilon-\xi-m+1,1),(\eth-\xi-m+1,1),\\(1,1),(\zeta_1,\nu_1),\dots,(\zeta_{r'},\nu_{r'})\\ \\(1-\xi-m,1),(\gamma+\varepsilon+\eth-\xi-m+1,1),\\(\tau,\varpi),(\varkappa_1,\kappa_1),\dots,(\varkappa_{p'},\kappa_{p'})\end{matrix};\frac{1}{y}\right].
\end{aligned}\tag{64}$$

Hence, we get the desired result (62).

4.1 Particular Cases

Theorem 5 and 6 give the following image formulas for the Mittag-Leffler function, if $r'=p'=0$ and $\tau=1$:

Corollary 25. *Let* $\gamma,\varepsilon\in\mathbb{C}$, $\varpi>0$ *and* $\mathrm{Re}(\gamma)>0,\mathrm{Re}(\xi)>\max\{0,\mathrm{Re}(\varepsilon-\eth)\}$. *Then*

$$\begin{aligned}
&\left(\mathfrak{I}^{\gamma,\varepsilon,\eth}_{0+}t^{\xi-1}J^{\omega}_{\rho}(t)E_{\varpi}(t)\right)(y)\\
=&y^{\xi-\varepsilon-1}J^{\omega}_{\rho}(y)\\
&{}_3\Psi_3\left[\begin{matrix}(\xi+m,1),(\xi+m+\eth-\varepsilon,1),\\(1,1)\\ \\(\xi+m-\varepsilon,1),(\xi+m+\gamma+\eth,-1),\\(1,\varpi)\end{matrix};y\right].
\end{aligned}\tag{65}$$

Corollary 26. *Let* $\gamma,\varepsilon\in\mathbb{C}$, $\varpi>0$ *and* $\mathrm{Re}(\gamma)>0,\mathrm{Re}(\xi)<1+\min[\mathrm{Re}(\varepsilon),\mathrm{Re}(\eth)]$. *Then*

$$\begin{aligned}
&\left(\mathfrak{I}^{\gamma,\varepsilon,\eth}_{0-}t^{\xi-1}J^{\omega}_{\rho}(t)E_{\varpi}\left(\frac{1}{t}\right)\right)(y)\\
=&y^{\xi-\varepsilon-1}J^{\omega}_{\rho}(y)\\
&{}_3\Psi_3\left[\begin{matrix}(\varepsilon-\xi-m+1,1),(\eth-\xi-m+1,1),\\(1,1)\\ \\(1-\xi-m,1),(\gamma+\varepsilon+\eth-\xi-m+1,1),\\(1,\varpi)\end{matrix};\frac{1}{y}\right].
\end{aligned}\tag{66}$$

Theorem 5 and 6 give the following image formulas for the Wiman function, if $r'=p'=0$:

Corollary 27. *Let* $\gamma,\varepsilon\in\mathbb{C}$, $\mathrm{Re}(\varpi),\mathrm{Re}(\tau)>0$ *and* $\mathrm{Re}(\gamma)>0,\mathrm{Re}(\xi)>\max\{0,\mathrm{Re}(\varepsilon-\eth)\}$. *Then*

$$\begin{aligned}
&\left(\mathfrak{I}^{\gamma,\varepsilon,\eth}_{0+}t^{\xi-1}J^{\omega}_{\rho}(t)E_{\varpi,\tau}(t)\right)(y)\\
&=y^{\xi-\varepsilon-1}J^{\omega}_{\rho}(y)\\
&{}_3\Psi_3\left[\begin{matrix}(\xi+m,1),(\xi+m+\eth-\varepsilon,1),(1,1)\\ \\(\xi+m-\varepsilon,1),(\xi+m+\gamma+\eth,1),(\tau,\varpi)\end{matrix};y\right].
\end{aligned}\tag{67}$$

Corollary 28. *Let* $\gamma,\varepsilon\in\mathbb{C}$, $\mathrm{Re}(\varpi),\mathrm{Re}(\tau)>0$ *and* $\mathrm{Re}(\gamma)>0,\mathrm{Re}(\xi)<1+\min[\mathrm{Re}(\varepsilon),\mathrm{Re}(\eth)]$. *Then*

$$\begin{aligned}
&\left(\mathfrak{I}^{\gamma,\varepsilon,\eth}_{0-}t^{\xi-1}J^{\omega}_{\rho}(t)E_{\varpi,\tau}\left(\frac{1}{t}\right)\right)(y)\\
&=y^{\xi-\varepsilon-1}J^{\omega}_{\rho}(y)\\
&{}_3\Psi_3\left[\begin{matrix}(\varepsilon-\xi-m+1,1),(\eth-\xi-m+1,1),\\(1,1)\\ \\(1-\xi-m,1),(\gamma+\varepsilon+\eth-\xi-m+1,1),\\(\tau,\varpi)\end{matrix};\frac{1}{y}\right].
\end{aligned}\tag{68}$$

Theorem 5 and 6 give the following image formulas for the Prabhakar function, if $r'=p'=1$ and $\nu=\varkappa=\kappa=1$:

Corollary 29. *Let* $\gamma,\varepsilon\in\mathbb{C}$, $\mathrm{Re}(\varpi),\mathrm{Re}(\tau)>0,\zeta>0$ *and* $\mathrm{Re}(\gamma)>0,\mathrm{Re}(\xi)>\max\{0,\mathrm{Re}(\varepsilon-\eth)\}$. *Then*

$$\begin{aligned}
&\left(\mathfrak{I}^{\gamma,\varepsilon,\eth}_{0+}t^{\xi-1}J^{\omega}_{\rho}(t)E^{\zeta}_{\varpi,\tau}(t)\right)(y)\\
=&\frac{y^{\xi-\varepsilon-1}}{\Gamma(\zeta)}J^{\omega}_{\rho}(y)\\
&{}_4\Psi_4\left[\begin{matrix}(\xi+m,1),(\xi+m+\eth-\varepsilon,1),(1,1),(\zeta,1)\\ \\(\xi+m-\varepsilon,1),(\xi+m+\gamma+\eth,1),(\tau,\varpi),(1,1)\end{matrix};y\right].
\end{aligned}\tag{69}$$

Corollary 30. *Let* $\gamma,\varepsilon\in\mathbb{C}$, $\mathrm{Re}(\varpi),\mathrm{Re}(\tau)>0,\zeta>0$ *and* $\mathrm{Re}(\gamma)>0,\mathrm{Re}(\xi)<1+\min[\mathrm{Re}(\varepsilon),\mathrm{Re}(\eth)]$, *then*

$$\begin{aligned}
&\left(\mathfrak{I}^{\gamma,\varepsilon,\eth}_{0-}t^{\xi-1}J^{\omega}_{\rho}(t)E^{\zeta}_{\varpi,\tau}\left(\frac{1}{t}\right)\right)(y)\\
=&\frac{y^{\xi-\varepsilon-1}}{\Gamma(\zeta)}J^{\omega}_{\rho}(y)\\
&{}_4\Psi_4\left[\begin{matrix}(\varepsilon-\xi-m+1,1),(\eth-\xi-m+1,1),\\(1,1),(\zeta,1)\\ \\(1-\xi-m,1),(\gamma+\varepsilon+\eth-\xi-m+1,1),\\(\tau,\varpi),(1,1)\end{matrix};\frac{1}{y}\right].
\end{aligned}\tag{70}$$

Theorem 5 and 6 give the image formulas for Shukla and Prajapati's generalization of the Prabhakar function, if $r'=p'=1$ and $\varkappa=\kappa=1$:

Corollary 31.*Let* $\gamma,\varepsilon \in \mathbb{C}$*,* $min\{\mathrm{Re}(\varpi),\mathrm{Re}(\tau),\mathrm{Re}(\zeta)\} > 0$*,* $\varpi,\tau,\zeta,\nu \in \mathbb{C}$ *and* $\mathrm{Re}(\gamma) > 0, \mathrm{Re}(\xi) > \max\{0, \mathrm{Re}(\varepsilon - \eth)\}$*, then*

$$\left(\mathfrak{I}_{0+}^{\gamma,\varepsilon,\eth} t^{\xi-1} J_\rho^\omega(t) E_{\varpi,\tau}^{\zeta,\nu}(t)\right)(y) = \frac{y^{\xi-\varepsilon-1}}{\Gamma(\zeta)} J_\rho^\omega(y)\, {}_4\Psi_4\left[\begin{matrix} (\xi+m,1),(\xi+m+\eth-\varepsilon,1),\\ (1,1),(\zeta,\nu) \\ \\ (\xi+m-\varepsilon,1),(\xi+m+\gamma+\eth,1),\\ (\tau,\varpi),(1,1)\end{matrix};y\right]. \quad (71)$$

Corollary 32.*Let* $\gamma,\varepsilon \in \mathbb{C}$*,* $min\{\mathrm{Re}(\varpi),\mathrm{Re}(\tau),\mathrm{Re}(\zeta)\} > 0$*,* $\varpi,\tau,\zeta,\nu \in \mathbb{C}$ *and* $\mathrm{Re}(\gamma) > 0, \mathrm{Re}(\xi) < 1 + \min[\mathrm{Re}(\varepsilon), \mathrm{Re}(\eth)]$*, then*

$$\left(\mathfrak{I}_{0-}^{\gamma,\varepsilon,\eth} t^{\xi-1} J_\rho^\omega(t) E_{\varpi,\tau}^{\zeta,\nu}\left(\frac{1}{t}\right)\right)(y) = \frac{y^{\xi-\varepsilon-1}}{\Gamma(\zeta)} J_\rho^\omega(y)\, {}_4\Psi_4\left[\begin{matrix} (\varepsilon-\xi-m+1,1),(\eth-\xi-m+1,1),\\ (1,1),(\zeta,\nu) \\ \\ (1-\xi-m,1),(\gamma+\varepsilon+\eth-\xi-m+1,1),\\ (\tau,\varpi),(1,1)\end{matrix};\frac{1}{y}\right]. \quad (72)$$

Theorem 5 and 6 give the image formulas for Khan and Ahmed's generalization of the Mittag-Leffler function, if $r' = p' = 1$ and $\kappa = 1$:

Corollary 33.*Let* $\gamma,\varepsilon \in \mathbb{C}$*,* $\min\{\mathrm{Re}(\varpi),\mathrm{Re}(\tau),\mathrm{Re}(\varkappa),\mathrm{Re}(\zeta)\} > 0$*,* $\nu \in (0,1)\cup\mathbb{N}$ *and* $\mathrm{Re}(\gamma) > 0, \mathrm{Re}(\xi) > \max\{0, \mathrm{Re}(\varepsilon - \eth)\}$*. Then*

$$\left(\mathfrak{I}_{0+}^{\gamma,\varepsilon,\eth} t^{\xi-1} J_\rho^\omega(t) E_{\varpi,\tau,\varkappa}^{\zeta,\nu}(t)\right)(y) = y^{\xi-\varepsilon-1}\frac{\Gamma(\varkappa)}{\Gamma(\zeta)} J_\rho^\omega(y)\, {}_4\Psi_4\left[\begin{matrix} (\xi+m,1),(\xi+m+\eth-\varepsilon,1),\\ (1,1),(\zeta,\nu) \\ \\ (\xi+m-\varepsilon,1),(\xi+m+\gamma+\eth,1),\\ (\tau,\varpi),(\varkappa,1)\end{matrix};y\right]. \quad (73)$$

Corollary 34.*Let* $\gamma,\varepsilon \in \mathbb{C}$*,* $\min\{\mathrm{Re}(\varpi),\mathrm{Re}(\tau),\mathrm{Re}(\varkappa),\mathrm{Re}(\zeta)\} > 0$*,* $\nu \in (0,1)\cup\mathbb{N}$ *and* $\mathrm{Re}(\gamma) > 0, \mathrm{Re}(\xi) < 1 + \min[\mathrm{Re}(\varepsilon), \mathrm{Re}(\eth)]$*. Then*

$$\left(\mathfrak{I}_{0-}^{\gamma,\varepsilon,\eth} t^{\xi-1} J_\rho^\omega(t) E_{\varpi,\tau,\varkappa}^{\zeta,\nu}\left(\frac{1}{t}\right)\right)(y) = y^{\xi-\varepsilon-1}\frac{\Gamma(\varkappa)}{\Gamma(\zeta)} J_\rho^\omega(y)\, {}_4\Psi_4\left[\begin{matrix} (\varepsilon-\xi-m+1,1),(\eth-\xi-m+1,1),\\ (1,1),(\zeta,\nu) \\ \\ (1-\xi-m,1),(\gamma+\varepsilon+\eth-\xi-m+1,1),\\ (\tau,\varpi),(\varkappa,1)\end{matrix};\frac{1}{y}\right]. \quad (74)$$

Theorem 5 and 6 give the image formulas for Khan and Ahmed's further generalization of the Mittag-Leffler function, if $r' = p' = 2$:

Corollary 35.*Let* $\gamma,\varepsilon,\varpi,\tau,\zeta_1,\nu_1,\zeta_2,\varkappa_1,\kappa_1,\varkappa_2 \in \mathbb{C}$*,* $\nu_2,\kappa_2 > 0$*,* $\min(\mathrm{Re}(\varpi),\mathrm{Re}(\tau),\mathrm{Re}(\zeta_1),\mathrm{Re}(\nu_1),\mathrm{Re}(\zeta_2),\mathrm{Re}(\varkappa_1),\mathrm{Re}(\kappa_1),\mathrm{Re}(\varkappa_2)) > 0$*,* $\mathrm{Re}(\varpi)+\kappa_2 \geq \nu_2$ *and* $\mathrm{Re}(\gamma) > 0, \mathrm{Re}(\xi) > \max\{0, \mathrm{Re}(\varepsilon-\eth)\}$*. Then*

$$\left(\mathfrak{I}_{0+}^{\gamma,\varepsilon,\eth} t^{\xi-1} J_\rho^\omega(t) E_{\varpi,\tau;\varkappa_1,\kappa_1;\varkappa_2,\kappa_2}^{\zeta_1,\nu_1;\zeta_2,\nu_2}(t)\right)(y) = y^{\xi-\varepsilon-1}\frac{\Gamma(\varkappa_1)\Gamma(\varkappa_2)}{\Gamma(\zeta_1)\Gamma(\zeta_2)} J_\rho^\omega(y)\, {}_5\Psi_5\left[\begin{matrix} (\xi+m,1),(\xi+m+\eth-\varepsilon,1),\\ (1,1),(\zeta_1,\nu_1),(\zeta_2,\nu_2) \\ \\ (\xi+m-\varepsilon,1),(\xi+m+\gamma+\eth,1),\\ (\tau,\varpi),(\varkappa_1,\kappa_1),(\varkappa_2,\kappa_2)\end{matrix};y\right]. \quad (75)$$

Corollary 36.*Let* $\gamma,\varepsilon,\varpi,\tau,\zeta_1,\nu_1,\zeta_2,\varkappa_1,\kappa_1,\varkappa_2 \in \mathbb{C}$*,* $\nu_2,\kappa_2 > 0$*,* $\min(\mathrm{Re}(\varpi),\mathrm{Re}(\tau),\mathrm{Re}(\zeta_1),\mathrm{Re}(\nu_1),\mathrm{Re}(\zeta_2),\mathrm{Re}(\varkappa_1),\mathrm{Re}(\kappa_1),\mathrm{Re}(\varkappa_2)) > 0$*,* $\mathrm{Re}(\varpi)+\kappa_2 \geq \nu_2$ *and* $\mathrm{Re}(\gamma) > 0, \mathrm{Re}(\xi) < 1 + \min[\mathrm{Re}(\varepsilon), \mathrm{Re}(\eth)]$*. Then*

$$\left(\mathfrak{I}_{0-}^{\gamma,\varepsilon,\eth} t^{\xi-1} J_\rho^\omega(t) E_{\varpi,\tau;\varkappa_1,\kappa_1;\varkappa_2,\kappa_2}^{\zeta_1,\nu_1;\zeta_2,\nu_2}\left(\frac{1}{t}\right)\right)(y) = y^{\xi-\varepsilon-1}\frac{\Gamma(\varkappa_1)\Gamma(\varkappa_2)}{\Gamma(\zeta_1)\Gamma(\zeta_2)} J_\rho^\omega(y) \times {}_5\Psi_5\left[\begin{matrix} (\varepsilon-\xi-m+1,1),(\eth-\xi-m+1,1),\\ (1,1),(\zeta_1,\nu_1),(\zeta_2,\nu_2) \\ \\ (1-\xi-m,1),(\gamma+\varepsilon+\eth-\xi-m+1,1),\\ (\tau,\varpi),(\varkappa_1,\kappa_1),(\varkappa_2,\kappa_2)\end{matrix};\frac{1}{y}\right]. \quad (76)$$

5 Conclusion

Many areas of research have been extensively using the concept of fractional calculus in recent years since it is used in generalizing mathematical models represented by

differential equations and provides more precise representations than integer order approaches. Fractional calculus operators applied to various special functions and combined with mathematical modeling is used by several authors for modeling child growth standards, cell cycle of a tumor cell, to study transmission dynamics of HIV/AIDS, to study the effects of various diseases such as malaria, epidemic model of some childhood disease, pandemics such as COVID-19 among various others.
Physical and electrical properties of heterogeneous-complex, with composite biomaterials can be described in a simpler way using fractional calculus models. Fractional calculus models propose new observations and measurements that can further explain biological system structure and dynamics. In our paper, we have introduced image formulas involving the fractional calculus operators with Appell function kernel and Saigo fractional operators for the product of Bessel-Maitland function and the m-parameter Mittag-Leffler function. These fractional operators have wide range of applications in solving kinetic equations, fractional diffusion, fractional reaction diffusion equations and many more. We have also obtained a number of image formulas involving the Mittag-Leffler function, the Wiman function, the Prabhakar function and various other generalizations of the Mittag-Leffler function by giving specific particular values to the parameters $r', p', \varpi, \tau, \zeta_\iota, v_\iota, \varkappa_\ell, \kappa_\ell$. We can extend the results obtained for other fractional operators of significance such as Caputo fractional derivative, Erdelyi Kober operators, Caputo-Fabrizio operators among various others. In future fractional operators applied to Bessel-Maitland function and m-parameter Mittag-Leffler function may find some applications in biology and other fields of mathematics, science and engineering.

References

[1] M.P Yadav, and R. Agarwal, Numerical investigation of fractional-fractal Boussinesq equation. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, **29**(1):013109 (2019).

[2] R. Agarwal, Kritika, and S.D Purohit, mathematical fractional model with non-singular kernel for thrombin receptor activation in calcium signalling. *Math. Meth. Appl. Sci.*, **42**:7160–7171 (2019).

[3] A. Chandola, R.M Pandey, R. Agarwal, and R.P Agarwal, Hadamard inequality for (k-r) Riemann-Liouville fractional integral operator via convexity. *Prog. Fract. Differ. Appl.*, **8**(2):205–215 (2022)

[4] A. Chandola, R. Agarwal, and R.M Pandey, Some new Hermite-Hadamard, Hermite-Hadamard fejer and weighted Hardy type inequalities involving (k-p) Riemann-Liouville fractional integral operator. *Applied Mathematics & Information Sciences*, **16**(2):287–297 (2022).

[5] K.S. Miller, and B Ross, *An introduction to the fractional calculus and fractional differential equations*. Wiley Interscience (1993)

[6] K.S. Nisar, D.L Suthar, R. Agarwal, and S.D Purohit, Fractional calculus operators with Appell function kernels applied to Srivastava polynomials and extended Mittag-Leffler function. *Adv. Differ. Equ*, **2020**(148). https://doi.org/10.1186/s13662-020-02610-3 (2020).

[7] R. Agarwal, M.P Yadav, R.P. Agarwal, and R. Goyal, Analytic solution of fractional advection dispersion equation with decay for contaminant transport in porous media. *Matematicki Vesnik*, **2019**(71):5–15 (2019).

[8] A. Chandola, R.M Pandey, R. Agarwal, and S.D Purohit, An extension of beta function, its statistical distribution and associated fractional operator. *Adv. Differ. Equ*, **2020**:684. https://doi.org/10.1186/s13662-020-03142-6 (2020).

[9] A. Chandola, R.M Pandey, and R. Agarwal, Bessel-maitland function of several variables and its properties related to integral transforms and fractional calculus. *Applications and Applied Mathematics: An International Journal*, **16**(1):Article 23. https://digitalcommons.pvamu.edu/aam/vol16/iss1/23 (2021)

[10] R. Agarwal, A. Chandola, R.M. Pandey, and K.S Nisar, m-parameter Mittag-Leffler function, its various properties, and relation with fractional calculus operators. *Math. Methods Appl. Sci.*, **44**(7):5365–5384 (2021). https://doi.org/10.1002/mma.7115.

[11] A. Chandola, R.M Pandey, and R. Agarwal, A study of integral transform of convolution type integrals involving k-hypergeometric functions. *Mathematics in Engineering, Science & Aerospace (MESA)*, **12**(3):795–810 (2021)

[12] D. Baleanu, D. Kumar, and S.D Purohit, Generalized fractional integrals of product of two H-functions and a general class of polynomials. *Int. J. Comput. Math.*, **93**:1320–1329 (2016).

[13] A. Chouhan, A.M Khan, and S. Saraswat, A note on Marichev-Saigo-Maeda fractional integral operator. *J. Fract. Calc. Appl.*, **5**:88–95 (2014).

[14] D. Kumar, S.D Purohit, and J Choi, Generalized fractional integrals involving product of multivariable H-function and a general class of polynomials. *J. Nonlinear Sci. Appl.*, **9**:8–21 (2016).

[15] S. Kumar, K.S Nisar, R. Kumar, C. Cattani, and B. Samet, A new Rabotnov fractional-exponential function based fractional derivative for diffusion equation under external force. *Math. Meth. Appl. Sci.*, pages 1–12. https://doi.org/10.1002/mma.6208 (2020).

[16] V.N Mishra, D.L Suthar, and S.D Purohit, Marichev-Saigo-Maeda fractional calculus operators, Srivastava polynomials and generalized Mittag-Leffler function. *Cogent Math.*, **4**. https://doi.org/10.1080/23311835.2017.1320830 (2017).

[17] S.R Mondal, and K.S Nisar, Marichev-Saigo-Maeda fractional integration operators involving generalized Bessel functions. *Math. Probl. Eng.*, **2014**(274093) (2014).

[18] S.D Purohit, D.L Suthar, and S.L Kalla, Marichev - Saigo -Maeda fractional integration operators of the Bessel function. *Matematiche (Catania)*, 67:21–32 (2012).

[19] A. Atangana, Fractal-fractional differentiation and integration: connecting fractal calculus and fractional calculus to predict complex system. *Chaos, Solitons & Fractals*, **102**:396–406 (2017)

[20] F. Brouers, The fractal (BSf) kinetics equation and its approximations. *J. Mod. Phys.*, **5**:1594–1601 (2014).

[21] B. Ghanbari, S. Kumar, and R. Kumar, A study of behaviour for immune and tumor cells in immunogenetic tumour model with non-singular fractional derivative. *Chaos, Solitons & Fractals*, **133** https://doi.org/10.1016/j.chaos.2020.109619 (2020).

[22] K. Kosmidis, and P Macheras, On the dilemma of fractal or fractional kinetics in drug release studies: a comparison between Weibull and Mittag-Leffler functions. *Int. J. Pharm*, **43**:269–273 (2018).

[23] S. Kumar, R. Kumar, J. Singh, K.S Nisar, and D. Kumar, An efficient numerical scheme for fractional model of HIV-1 infection of CD4+ T-cells with the effect of antiviral drug therapy. *Alex. Eng. J.* https://doi.org/10.1016/j.aej.2019.12.046 (2019).

[24] D.P Ahokposi, A. Atangana, and D.P Vermeulen, Modelling groundwater fractal flow with fractional differentiation via Mittag-Leffler law. *Eur. Phys. J. Plus*, **132**:165–175 (2017).

[25] W. Chen, and Y Liang, New methodologies in fractional and fractal derivatives modeling. *Chaos, Solitons & Fractals*, **102**:72–77 (2017).

[26] G. Mittag-Leffler, Sur la nouvelle fonction $E_{\varpi}(x)$. *C. R. Acad. Sci. Paris, (Ser. II)*, **137**:554–558 (1903).

[27] E.D Rainville, newblock *Special Functions*. Chelsea Publications, Bronx, New York (1971).

[28] A. Wiman, Über den fundamentalsatz in der teorie der funktionen $E_a(x)$. *Acta Math.*, **29**:191–201 (1905).

[29] T. R Prabhakar, A singular integral equation with a generalized Mittag-Leffler function in the kernel. *Yokohama Math. J.*, **19**:7–15 (1971).

[30] A.K Shukla, and J.C Prajapati, On a generalization of Mittag-Leffler function and its properties. *J. Math. Anal. Appl.*, **336**:797–811 (2007).

[31] M. Khan, and S Ahmed, On some properties of the generalized Mittag-Leffler function. *Springerplus*, **2**:337 (2013).

[32] H.M Srivastava, and H. L Manocha, *A Treatise on Generating functions*. John Wiley and Sons, New York (1984).

[33] O.I Marichev, *Handbook of integral transform and Higher transcendental functions*. Ellis, Harwood, chichester (John Wiley and Sons), New York (1983).

[34] O.I Marichev, Volterra equation of Mellin convolution type with a Horn function in the kernel. *Izvestiya Akademii Nauk BSSR. Seriya Fiziko-Matematicheskikh Nauk (Russian)*, **1**:128–129 (1974).

[35] M. Saigo, and N. Maeda, More generalization of fractional calculus. *Transform Methods & Special Functions, Bulgarian Academy of Sciences, Sofia*, **96**:386–400 (1998).

[36] M. Saigo, A remark on integral operators involving the gauss hypergeometric functions. *Mathematical reports of College of General Education, Kyushu University*, **11**:135–143 (1998).

[37] A. A Kilbas, and N. Sebastian, Generalized fractional integration of Bessel function of the first kind. *Integral Transforms Spec. Funct*, **19**:869–883 (2008).

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