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An Implicit Scheme for the Numerical Solution of the Generalized Burgers-Huxley Equation

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Abstract: In this study, numerical solutions to the generalized Burgers-Huxley problem are obtained utilizing a new approach. The Implicit logarithmic finite difference method (I-LFDM). The effectiveness of the suggested method is demonstrated by a numerical example for various parameter cases, demonstrating that the obtained results are in excellent agreement with the exact solutions and better than numerical results obtained by other methods in the literature. The method was analyzed with the von-Neumann stability analysis method and it was shown that the method was unconditionally stable.

Keywords: Implicit logarithmic finite difference method, Generalized Burgers-Huxley equation; von Neumann Stability analysis.

1 Introduction

Nonlinear partial differential equations are often used to model most of the problems in various fields such as physics, chemistry, biology, mathematics, and engineering. One of these nonlinear partial differential equations is generalized Burgers-Huxley equation.
The generalized Burgers-Huxley equation

$$\frac{\partial u}{\partial t}+\alpha u^{\delta}\frac{\partial u}{\partial x}-\frac{\partial^2 u}{\partial x^2}=\beta u\left(1-u^{\delta}\right)\left(u^{\delta}-\gamma\right), \qquad (1)$$

$a<x<b, t>0$
with initial condition

$$u(x,0)=q(x), \;\; a<x<b$$

and boundary conditions

$$u(a,t)=w_1(t),\, u(b,t)=w_2(t),\, t>0$$

shows a prototype model for describing the interaction between reaction mechanisms, convection effects and diffusion transports [1]. Where $q(x)$, $w_1(t)$ and $w_2(t)$ are known functions, $\alpha,\delta,\beta,\gamma$ are given parameters that $\beta\geq 0$, $\delta>0$ and $\gamma\in(0.1)$.

In order to solve the generalized Burgers-Huxley equation numerically, many researchers have used various numerical methods. Wazwaz [2] and Deng [3] studied the raveling wave solutions of equation. Hashim et. al. [4] solved the equation numerically by using the Adomian decomposition method. Pseudospectral method and spectral collocation method were used to acquire the numerical solutions of equation by Javidi [5,6]. Variational iteration method was applied to the equation by Batiha et. al. [7]. Spectral collocation method and Darvishi's Preconditionings used by Darvishi et. al. [8] to acquire the numerical solutions of equation. Khattak [9] used a numerical technique based on collocation method using Radial basis functions. Differential Quadrature Method was used by Sari and Gürarslan [10] for numerical solutions of equation. Javidi and Golbabai [11] used the spectral collocation method using Chebyshev polynomials for spatial derivatives and fourth order Runge-Kutta method for the integration to solve the equation numerically. Tomasiello [12] used the iterative differential quadrature method to acquire the numerical solutions of equation. Biazar and Mohammadi [13] applied the differential transform method to the equation. A fourth order finite difference scheme in a two time level recurrence relation was proposed for numerical solutions of equations by Bratsos [14]. Celik [15,16] used the haar wavelet method and Chebyshev wavelet collocation method for solving the equation. Duan et. al. [17] developed a lattice Boltzman model for the equation.

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El-Kady et. al. [18] used the Galerkin method to acquire the numerical solutions of equation. Mittal and Tripathi [19] used a numerical scheme based on collocation of modified cubic B-spline functions. An implicit exponential finite difference method was used to acquire the numerical solutions of equation by Inan and Bahadır [20]. Also, Inan [21] used the explicit exponential finite difference method to acquire the numerical solutions of equation. Singh et. al. [22] used the modified cubic B-spline quadrature method to acquire the numerical solutions of equation.

In this study, we have shown how to resolve the generalized Burgers-Huxley equation numerically using the implicit logarithmic finite difference method.

2 Model Problem and Numerical Method

2.1 Model Problem

Think the generalized Burgers-Huxley equation in the form of equation (1) for $0 \le x \le 1$, $t > 0$ with initial condition

$$u(x,0) = \left[\frac{\gamma}{2} + \frac{\gamma}{2}\tanh(A_1 x)\right]^{\frac{1}{\delta}}$$

and boundary conditions

$$u(0,t) = \left[\frac{\gamma}{2} + \frac{\gamma}{2}\tanh(-A_1 A_2 t)\right]^{\frac{1}{\delta}},$$

$$u(1,t) = \left[\frac{\gamma}{2} + \frac{\gamma}{2}\tanh(A_1(1 - A_2 t))\right]^{\frac{1}{\delta}}.$$

This problem's exact solution is

$$u(x,t) = \left[\frac{\gamma}{2} + \frac{\gamma}{2}\tanh(A_1(x - A_2 t))\right]^{\frac{1}{\delta}}$$

where

$$A_1 = \frac{-\alpha\delta + \delta\sqrt{\alpha^2 + 4\beta(1+\delta)}}{4(1+\delta)}\gamma,$$

$$A_2 = \frac{\gamma\alpha}{1+\delta} - \frac{(1+\delta-\gamma)\left(-\alpha + \sqrt{\alpha^2 + 4\beta(1+\delta)}\right)}{2(1+\delta)}.$$

2.2 Implicit Logarithmic Finite Difference Method

We demonstrate the finite difference approximation of $u(x,t)$at the node point (x_i,t_n) by u_i^n in which $x_i = ih (i = 0,1,\ldots,N)$, $t_n = t_0 + nk (n = 0,1,2,\ldots)$, $h = \frac{b-a}{N}$ is the node size in x direction and $k$is the time step.

We reorganize Equation (1) to acquire

$$\frac{\partial u}{\partial t} = \beta u\left(1 - u^\delta\right)\left(u^\delta - \gamma\right) - \alpha u^\delta \frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial x^2}. \tag{2}$$

Multiplying equation (2) by e^u, we acquire the following equation:

$$\frac{\partial e^u}{\partial t} = e^u\left(\beta u\left(1 - u^\delta\right)\left(u^\delta - \gamma\right) - \alpha u^\delta \frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial x^2}\right) \tag{3}$$

using the finite difference approximations for derivatives in Equation (3) the following implicit logarithmic finite difference scheme is obtained

I-LFDM

$$u_i^{n+1} = u_i^n + \ln\left\{1 + k\left[\begin{array}{l}\beta u_i^n\left(1 - (u_i^n)^\delta\right)\left((u_i^n)^\delta - \gamma\right) \\ -\alpha (u_i^n)^\delta \frac{u_{i+1}^{n+1} - u_{i-1}^{n+1}}{2h} + \frac{u_{i+1}^{n+1} - 2u_i^{n+1} + u_{i-1}^{n+1}}{h^2}\end{array}\right]\right\} \tag{4}$$

where $1 \le i \le N-1$.

Equation (4) is a system of nonlinear difference equations. We assume this nonlinear system of equations in the form

$$G(V) = 0 \tag{5}$$

where $G = [g_1, g_2, \ldots, g_{N-1}]^T$ and $V = \left[u_1^{n+1}, u_2^{n+1}, \ldots, u_{N-1}^{n+1}\right]^T$. Newton's iterative method is used to linearize the nonlinear Equation (6) results in the following iteration:

1) Set $V^{(0)}$, an initial guess.

2) For $m = 0,1,2,\ldots$until convergence do:

Solve $J\left(V^{(m)}\right)\delta^{(m)} = -G\left(V^{(m)}\right)$;

Set $V^{(m+1)} = V^{(m)} + \delta^{(m)}$ where $J(V^{(m)})$ is the Jacobian matrix which is appraised analytically. The solution at the previous time-step is taken as the initial estimate. The Newton's iteration at each time-step is stopped when $\left\|G(V^{(m)})\right\|_\infty \le 10^{-5}$.

2.3 Local Truncation Error and Consistency

In order to analyze the local truncation errors of the numerical scheme (4), the nonlinear term of the scheme has been linearized by replacing the quantity $(u_i^n)^\delta$ by local constant $\tilde{U}$. Hence the numerical scheme (4), convert into

$$u_i^{n+1} = u_i^n + \ln\left\{1 + k\left[\begin{array}{l}\beta u_i^n\left(1 - \tilde{U}\right)\left(\tilde{U} - \gamma\right) \\ -\alpha\tilde{U}\frac{u_{i+1}^{n+1} - u_{i-1}^{n+1}}{2h} + \frac{u_{i+1}^{n+1} - 2u_i^{n+1} + u_{i-1}^{n+1}}{h^2}\end{array}\right]\right\} \tag{6}$$

Since the scheme (6) is logarithmic, the examination will be improved by expanding the logarithmic term of the scheme into a Taylor's series. Hilal et al. [23] applied the same procedure to calculate the local truncation error of exponential finite difference schemes and examine their stability. If the scheme's logarithmic term is expanded to a Taylor series and the first term is used, the scheme can be expressed as:

$$u_i^{n+1} = u_i^n + k\beta u_i^n \left(1-\tilde{U}\right)\left(\tilde{U}-\gamma\right) - \alpha k\tilde{U}\left[\frac{u_{i+1}^{n+1}-u_{i-1}^{n+1}}{2h}\right] + k\left[\frac{u_{i+1}^{n+1}-2u_i^{n+1}+u_{i-1}^{n+1}}{h^2}\right] \tag{7}$$

Expansion of the terms u_i^{n+1}, u_{i+1}^{n+1} and u_{i-1}^{n+1}about the point (x_i,t_n) by Taylor' s series and substitution into

$$T_i^n = u_i^{n+1} - u_i^n - k\beta u_i^n \left(1-\tilde{U}\right)\left(\tilde{U}-\gamma\right) + \alpha k\tilde{U}\left[\frac{u_{i+1}^{n+1}-u_{i-1}^{n+1}}{2h}\right] - k\left[\frac{u_{i+1}^{n+1}-2u_i^{n+1}+u_{i-1}^{n+1}}{h^2}\right]$$

leads to

$$T_i^n = \left[\frac{\partial u}{\partial t} - \beta u\left(1-\tilde{U}\right)\left(\tilde{U}-\gamma\right) + \alpha\tilde{U}\frac{\partial u}{\partial x} - \frac{\partial^2 u}{\partial x^2}\right]_i^n + \frac{k}{2}\left(\frac{\partial^2 u}{\partial t^2}\right)_i^n + \tilde{U}\frac{h^2}{6}\left(\frac{\partial^3 u}{\partial x^3}\right)_i^n - \frac{h^2}{12}\left(\frac{\partial^4 u}{\partial x^4}\right)_i^n + \ldots$$

Therefore the principal part of the local truncation error is as follows:

$$\frac{k}{2}\left(\frac{\partial^2 u}{\partial t^2}\right)_i^n + \tilde{U}\frac{h^2}{6}\left(\frac{\partial^3 u}{\partial x^3}\right)_i^n$$

Hence the local truncation error is $T_i^n = O(k) + O\left(h^2\right)$

Since $\lim_{h,k\to 0}\left[O(k)+O\left(h^2\right)\right] = 0$ presented scheme is consistent. And the scheme is first order in time and second order in space.

2.4 Stability Analysis

We will utilize the von Neumann stability analysis to analyze the scheme's stability, where the growth factor of a characteristic Fourier mode is specified as follows:

$$u_i^n = \varepsilon^n e^{I\phi ih},\ I = \sqrt{-1}. \tag{8}$$

von Neumann stability analysis is used to analyze the stability of finite difference schemes applied to linear partial differential equations. So we will investigate the stability of linear form of the scheme. By substituting the (8) equality into the (7) linear form of the scheme, we get the growth factors as follows:

$$\varepsilon = \frac{1+k\beta\left(1-\tilde{U}\right)\left(\tilde{U}-\gamma\right)}{1+\frac{2k}{h^2}\sin^2\frac{\phi h}{2} + I\frac{\alpha k\tilde{U}}{h}\sin(\phi h)}.$$

Stability condition in von-Neumann method is $|\varepsilon| \le 1$ $|\varepsilon| \le 1$ since $\beta \ge 0$ and $\gamma \in (0.1)$. Therefore I-LFDM generalized Burgers-Huxley equation is unconditionally stable.

3 Numerical Results and Discussion

Implicit logarithmic finite difference method is used to acquire the numerical solutions of the generalized Burgers-Huxley equation. To demonstrate that the results are correct error norms L_2, L_∞ and absolute error:

$$A.E. = \left|U(x_i,t_n) - u(x_i,t_n)\right|,$$

$$L_\infty = \|U-u_N\|_\infty = \max_j\left|U_j - (u_N)_j\right|,$$

$$L_2 = \|U-u_N\|_2 = \sqrt{h\sum_{j=0}^{N}\left|U_j-(u_N)_j\right|^2}$$

are used, where $u$and U indicate computed numerical solutions and exact solutions, respectively. In all numerical computations we took as $h = 0.01$ and $k = 0.00001$. The absolute errors obtained by I-LFDM and by some other methods [4,7] in literature are compared in Table 1-3. The comparisons for the case $\delta = 1$, $\beta = 1$, $\alpha = 1$and $\gamma = 0.001$ are shown in Table 1 while the comparisons for the case $\delta = 2$, $\beta = 1$, $\alpha = 1$and $\gamma = 0.01$ are given in Table 2 and for the case $\delta = 4$, $\beta = 1$, $\alpha = 1$and $\gamma = 0.01$ are shown in Table 3. As it can be seen from the tables, the absolute errors obtained by the I-LFDM are less than the absolute errors obtained by some other methods in the literature. The error norms L_2 and L_∞ for the case $\delta = 1$, $\alpha = 1$, $\gamma = 0.01$ and various values of β are presented in Table 4. The error normsL_2 and L_∞ for the case $\delta = 1$, $\alpha = 1$,$\beta = 1$ and various values of γ are presented in Table 5. Table 6 presents L_2 and L_∞ error norms for the case $\alpha = 1$, $\beta = 1$, $\gamma = 0.001$ and varied values of δ when the error norms L_2 and L_∞ for the case $\alpha = -1$, $\beta = 1$, $\gamma = 0.001$ and various values of $\delta$are presented in Table 7. As it can be seen from the tables, the L_2 and L_∞ error norms obtained by the I-LFDM are quite small in all cases.

Table 1: Absolute errors for the case $\delta = 1$, $\beta = 1$, $\alpha = 1$ and $\gamma = 0.001$.

x	t	I-LFDM	ADM [4]	VIM [7]
0.1	0.05	7.72651 e-9	1.87406 e-8	1.87405 e-8
	0.1	1.12961 e-8	3.74812 e-8	3.74813 e-8
	1	1.68646 e-8	3.74812 e-7	3.74812 e-7
0.5	0.05	1.73514 e-8	1.87406 e-8	1.87405 e-8
	0.1	2.88279 e-8	3.74812 e-8	1.37481 e-8
	1	4.68487 e-8	3.74812 e-7	3.74813 e-7
0.9	0.05	7.72706 e-9	1.87406 e-8	1.87405 e-8
	0.1	1.12972 e-8	3.74812 e-8	3.74813 e-8
	1	1.68668 e-8	3.74812 e-7	3.74813 e-7

Table 2: Absolute errors for the case $\delta = 2$, $\beta = 1$, $\alpha = 1$ and $\gamma = 0.01$.

x	t	I-LFDM	ADM [4]	VIM [7]
0.1	0.1	1.66094 e-5	5.51554 e-5	5.51580 e-5
	0.2	2.17398 e-5	1.10342 e-4	1.10310 e-4
	0.3	2.36486 e-5	1.65529 e-4	1.65457 e-4
	0.4	2.43561 e-5	2.20708 e-4	2.20598 e-4
	0.5	2.46155 e-5	2.75950 e-4	2.75734 e-4
0.3	0.1	3.64284 e-5	5.51381 e-5	5.51340 e-5
	0.2	4.98679 e-5	1.10293 e-4	1.10262 e-4
	0.3	5.48697 e-5	1.65458 e-4	1.65385 e-4
	0.4	5.67248 e-5	2.20635 e-4	2.20502 e-4
	0.5	5.74063 e-5	2.75832 e-4	2.75614 e-4
0.5	0.1	4.23912 e-5	5.51134 e-5	5.51099 e-5
	0.2	5.90116 e-5	1.10243 e-4	1.10214 e-4
	0.3	6.51979 e-5	1.65402 e-4	1.65313 e-4
	0.4	6.74928 e-5	2.20543 e-4	2.20406 e-4
	0.5	6.83364 e-5	2.75716 e-4	2.75493 e-4

Table 3: Absolute errors for the case $\delta = 4$, $\beta = 1$, $\alpha = 1$ and $\gamma = 0.01$.

x	t	I-LFDM	ADM [4]	VIM [7]
0.1	0.1	6.55123 e-5	2.17787 e-4	2.17687 e-4
	0.2	8.57138 e-5	4.35690 e-4	4.35293 e-4
	0.3	9.31953 e-5	6.53711 e-4	6.52817 e-4
	0.4	9.59290 e-5	8.71847 e-4	8.70258 e-4
	0.5	9.68902 e-5	1.09010 e-3	1.08762 e-3
0.3	0.1	1.43654 e-4	2.17552 e-4	2.17453 e-4
	0.2	1.96592 e-4	4.35222 e-4	4.34824 e-4
	0.3	2.16215 e-4	6.53008 e-4	6.52113 e-4
	0.4	2.23403 e-4	8.70910 e-4	8.69320 e-4
	0.5	2.25948 e-4	1.08893 e-3	1.08644 e-3
0.5	0.1	1.67118 e-4	2.17318 e-4	2.17218 e-4
	0.2	2.32590 e-4	4.34753 e-4	4.34354 e-4
	0.3	2.56868 e-4	6.52304 e-4	6.51408 e-4
	0.4	2.65767 e-4	8.69972 e-4	8.68380 e-4
	0.5	2.68925 e-4	1.08776 e-3	1.08527 e-3

Table 4: The error norms L_2 and L_∞ for the case $\delta = 1$, $\alpha = 1$ and $\gamma = 0.01$.

L_2			
t	$\beta = 1$	$\beta = 10$	$\beta = 100$
0.01	3.294669 e-7	3.953494 e-6	4.241487 e-5
0.1	2.138861 e-6	2.566565 e-5	2.751631 e-4
1	3.405947 e-6	4.081875 e-5	3.792732 e-4
10	3.403986 e-6	3.514660 e-5	1.987002 e-7
L_∞			
t	$\beta = 1$	$\beta = 10$	$\beta = 100$
0.01	3.730857 e-7	4.476926 e-6	4.803344 e-5
0.1	2.869806 e-6	3.443684 e-5	3.692235 e-4
1	4.663776 e-6	5.589440 e-5	5.204188 e-4
10	4.661103 e-6	4.813608 e-5	2.736617 e-7

Table 5: The error norms L_2 and L_∞ for the case $\delta = 1$, $\alpha = 1$ and $\beta = 1$.

L_2			
t	$\gamma = 0.01$	$\gamma = 0.001$	$\gamma = 0.0001$
0.01	3.294669 e-7	3.309566 e-9	3.309889 e-11
0.1	2.138861 e-6	2.148537 e-8	2.148569 e-10
1	3.405947 e-6	3.421355 e-8	3.426528 e-10
10	3.403986 e-6	3.421474 e-8	3.426510 e-10
L_∞			
t	$\gamma = 0.01$	$\gamma = 0.001$	$\gamma = 0.0001$
0.01	3.730857 e-7	3.747715 e-9	5.284751 e-12
0.1	2.869807 e-6	2.882789 e-8	4.065019 e-11
1	4.663776 e-6	4.684870 e-8	6.615971 e-11
10	4.661103 e-6	4.685045 e-8	6.615937 e-11

Table 6: The error norms L_2 and L_∞ for the case $\alpha = 1$, $\beta = 1$, $\gamma = 0.001$.

L_2			
t	$\delta = 1$	$\delta = 2$	$\delta = 4$
0.01	3.309566 e-9	1.545048 e-7	1.083361 e-6
0.1	2.148537 e-8	1.003030 e-6	7.033081 e-6
1	3.421355 e-8	1.596914 e-6	1.119156 e-5
10	3.421474 e-8	1.592902 e-6	1.109450 e-5
L_∞			
t	$\delta = 1$	$\delta = 2$	$\delta = 4$
0.01	3.747715 e-9	1.749599 e-7	1.226791 e-6
0.1	2.882789 e-8	1.345812 e-6	9.436640 e-6
1	4.684870 e-8	2.186666 e-6	1.532476 e-5
10	4.685045 e-8	2.181175 e-6	1.519189 e-5

Table 7: L_2 and L_∞ error norms for the case $\alpha = -1$, $\beta = 1$, $\gamma = 0.001$.

L_2			
t	$\delta = 1$	$\delta = 2$	$\delta = 4$
0.01	6.619141 e-9	2.730778 e-7	1.687891 e-6
0.1	4.297074 e-8	1.772737 e-6	1.095683 e-5
1	6.842730 e-8	2.820927 e-6	1.741510 e-5
10	6.842493 e-8	2.798075 e-6	1.704327 e-5
L_∞			
t	$\delta = 1$	$\delta = 2$	$\delta = 4$
0.01	7.495449 e-9	3.092310 e-7	1.911362 e-6
0.1	5.765578 e-8	2.378567 e-6	1.470136 e-5
1	9.369772 e-8	3.862722 e-6	2.384683 e-5
10	9.369468 e-8	3.831438 e-6	2.333772 e-5

4 Conclusions

In this study, implicit logarithmic finite difference method is used to obtain the numerical solutions of the generalized Burgers-Huxley equation. Tables compare the absolute errors obtained by the presented method to those obtained by earlier studies in the literature. It is clear from the tables that the results obtained by I-LFDM are better than the results obtained by some other methods in literature. The present method is an effective method to find numerical solutions of various kinds of nonlinear problems.

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