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Estimation of Reliability in Multicomponent Stress-Strength based On Inverse Rayleigh Distribution

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Abstract: A multicomponent system of k components having strengths following k– independently and identically distributed random variables and each component experiencing a random stress Y is considered. The system is regarded as alive only if at least s out of k $(s < k)$ strengths exceed the stress. The reliability of such a system is obtained when strength, stress variates are given by inverse Rayleigh distribution with different scale parameters. The reliability is estimated using the Moment method and ML method of estimation when samples drawn from strength and stress distributions. The reliability estimators are compared asymptotically. The small sample comparison of the reliability estimates is made through Monte Carlo simulation.

Keywords: Reliability estimation, stress-strength, moment estimator, ML estimation, confidence intervals.

1 Introduction

Let X, Y are two independent random variables following inverse Rayleigh distribution with scale parameters respectively. Then the probability density function (pdf) and cumulative distribution function (cdf) of X and Y are respectively given by

$$f(x;\sigma_1) = \frac{2\sigma_1^2}{x^3} e^{-\left(\frac{\sigma_1}{x}\right)^2}; x \geq 0, \sigma_1 > 0, \tag{1}$$

$$F(x;\sigma_1) = e^{-\left(\frac{\sigma_1}{x}\right)^2}; x \geq 0, \sigma_1 > 0, \tag{2}$$

$$g(y;\sigma_2) = \frac{2\sigma_2^2}{y^3} e^{-\left(\frac{\sigma_2}{y}\right)^2}; y \geq 0, \sigma_2 > 0 \tag{3}$$

$$G(y;\sigma_2) = e^{-\left(\frac{\sigma_2}{y}\right)^2}; y \geq 0, \sigma_2 > 0. \tag{4}$$

Let the random samples $Y, X_1, X_2, \ldots, X_k$ being independent, $G(y)$ be the continuous distribution function of Y and $F(x)$ be the common continuous distribution function of $X_1, X_2, \ldots, X_k$. The reliability in a multicomponent stress-strength

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model developed by [2] is given by

$$\begin{aligned} \mathrm{R}_{s,k} &= P[\textit{at least } \mathrm{s} \textit{ of the } \mathrm{X}_1, X_2, \ldots, X_k \text{ exceed Y}] \\ &= \sum_{i=s}^{k} \binom{k}{i} \int_{-\infty}^{\infty} [1-G(y)]^i \, [G(y)]^{(k-i)} \, dF(y) r \end{aligned} \tag{5}$$

Where $X_1, X_2, \ldots, X_k$ are independently identically distributed (iid) with common distribution function $F(x)$,this system is subjected common random stress Y. Assuming that $F(.)$ and $G(.)$ are inverse Rayleigh distributions with unknown scale parameters σ_1, σ_2 and that independent random samples $X_1, X_2, \ldots, X_n$ and $Y_1, Y_2, \ldots, Y_m$ are available from $F(.)$ and $G(.)$, respectively. The reliability in multicomponent stress-strength for inverse Rayleigh distribution using (5) we get

$$\begin{aligned} R_{s,k} &= \sum_{i=s}^{k} \binom{k}{i} \int_{0}^{\infty} \left[1 - e^{\frac{-\sigma_2^2}{y^2}}\right]^i \left[e^{\frac{-\sigma_2^2}{y^2}}\right]^{k-i} \frac{2\sigma_1^2}{y^3} e^{\frac{-\sigma_1^2}{y^2}} dy \\ &= \sum_{i=s}^{k} \binom{k}{i} \int_{0}^{1} [1 - t^{1/\lambda^2}]^i \, [t^{1/\lambda^2}]^{k-i} dt \quad \text{where} \quad t = e^{\frac{-\sigma_1^2}{y^2}} \quad \text{and} \quad \lambda = \frac{\sigma_1^2}{\sigma_2^2} \\ &= \lambda^2 \sum_{i=s}^{k} \binom{k}{i} \int_{0}^{1} [1-z]^i \, [z]^{(k-i+\lambda^2-1)} dz \quad \text{if} \quad z = t^{\frac{1}{\lambda^2}} \\ &= \lambda^2 \sum_{i=s}^{k} \binom{k}{i} B\left(k-i+\lambda^2, i+1\right) \end{aligned}$$

After the simplification we get

$$R_{s,k} = \lambda^2 \sum_{i=s}^{k} \frac{k!}{(k-i)!} \left[\prod_{j=0}^{i} (k+\lambda^2-j)\right]^{-1} \quad \text{since } k \text{ and } i \text{ are integers} \tag{6}$$

The probability in (6) is called reliability in a multicomponent stress-strength model [2]. The survival probability of a single component stress-strength versions have been considered by several authors assuming various lifetime distributions for the stress-strength random variates (see e.g.[5], [4], [1], [11], [12], [19], [17], [8,9], [18], [10]). The reliability in a multicomponent stress- strength were developed by [2], [13] and the references therein cover the study of estimating in many standard distributions assigned to one or both of stress, strength variates. Recently [16] studied estimation of reliability in multicomponent stress-strength for log-logistic distribution and [15] developed an estimation of reliability in multicomponent stress-strength based on generalized exponential distribution.

Suppose a system, with k identical components, functions if s $(1 \leq s \leq k)$ or more of the components simultaneously operate. In its operating environment, the system is subjected to a stress Y which is a random variable with distribution function $G(.)$. The strengths of the components, that is the minimum stresses to cause failure, are independent and identically distributed random variables with distribution function $F(.)$. Then the system reliability, which is the probability that the system does not fail, is the function $R_{s,k}$ given in (5). The estimation of survival probability in a multicomponent stress-strength system when the stress, strength variates are following inverse Rayleigh distribution is not paid much attention. Therefore, an attempt is made here to study the estimation of reliability in multicomponent stress-strength model with reference to inverse Rayleigh distribution. In Section 2, we derive the expression for $R_{s,k}$ using different methods of estimation. The MLE are employed to obtain the asymptotic distribution and confidence intervals for $R_{s,k}$. The small sample comparisons made through Monte Carlo simulations are made in Section 3. Finally, the conclusion and comments are provided in Section 4.

2 Different Methods of Estimation of Parameters

If σ_1, σ_2 are not known, it is necessary to estimate σ_1, σ_2 to estimate $R_{s,k}$. In this paper we estimate σ_1, σ_2 by ML method and Method of moment thus giving rise to two estimates. The estimates are substituted in to get an estimate of using equation (6). The theory of methods of estimation is explained below. It is well known that the method of maximum likelihood estimation (MLE) has invariance property. When the method of estimation of parameter is changed from ML to any other traditional method, this invariance principle does not hold good to estimate the parametric function. However, such an adoption of invariance property for other optimal estimators of the parameters to estimate a parametric function is attempted in different situations by different authors. [20],[6] and the references therein are a few such instances. In this direction, we have proposed some estimators for the reliability of multicomponent stress-strength model by considering the estimators of the parameters of stress, strength distributions by standard methods of estimation in inverse Rayleigh distribution.

2.1 Method of Maximum Likelihood Estimation(MLE)

Let $X_1, X_2, \ldots, X_n$ and $Y_1, Y_2, \ldots, Y_m$ be two ordered random samples of size n, m respectively on strength, stress variates each following inverse Rayleigh distribution with scale parameters σ_1, σ_2. The log-likelihood function of the observed sample is

$$\begin{aligned} \ln L(\sigma_1,\sigma_2) = (m+n)\ln 2 + 2n\ln\sigma_1 + 2m\ln\sigma_2 - \sigma_1^2\sum_{i=1}^{n}\frac{1}{x_i^2} \\ -\sigma_2^2\sum_{j=1}^{m}\frac{1}{y_j^2} - \sum_{i=1}^{n}\ln x_i^3 - \sum_{j=1}^{m}\ln y_j^3 \end{aligned} \tag{7}$$

The MLEs of σ_1 and σ_2, say $\hat{\sigma}_1$ and $\hat{\sigma}_2$ respectively can be obtained as

$$\frac{\partial \ln L}{\partial \sigma_1} = 0 \Rightarrow \hat{\sigma}_1 = \sqrt{\frac{n}{\sum_{i=1}^{n}\frac{1}{x_i^2}}} \tag{8}$$

$$\frac{\partial \ln L}{\partial \sigma_2} = 0 \Rightarrow \hat{\sigma}_2 = \sqrt{\frac{m}{\sum_{j=1}^{m}\frac{1}{y_j^2}}} \tag{9}$$

The asymptotic variance of the MLE is given by

$$\left[-E\left(\frac{\partial^2 \ln L}{\partial \sigma_i}\right)\right]^{-1} = \frac{\sigma_i^2}{4n} \quad when \quad m=n \tag{10}$$

The MLE of survival probability of multicomponent stress-strength model is given by $R_{s,k}^{(1)}$ with λ is replaced by $\lambda^{(1)} = \frac{\sigma_1^{(1)}}{\sigma_2^{(1)}}$ in (6).

2.2 Method of moment estimation (MOM)

We know that,if $\bar{x}, \bar{y}$ are the sample mean of samples on strength,stress variates then moment estimators (MOM) of σ_1, σ_2 are $\hat{\sigma}_1^2 = \frac{\bar{x}}{\sqrt{\pi}}$ and $\hat{\sigma}_2^2 = \frac{\bar{y}}{\sqrt{\pi}}$ respectively. The second estimator, we propose here is $R_{s,k}^{(2)}$ with λ is replaced by $\lambda^{(2)} = \frac{\sigma_1^{(2)}}{\sigma_2^{(2)}}$ in (6). Thus for a given pair of samples on stress, strength variates we get two estimates of $R_{s,k}$ by the above two different methods. The asymptotic variance (AV) of an estimate of $R_{s,k}$ which a function of two independent statistics (say) t_1, t_2 is given by [14].

$$AV(\hat{R}_{s,k}) = AV(t_1)\left(\frac{\partial R_{s,k}}{\partial \sigma_1}\right)^2 + AV(t_2)\left(\frac{\partial R_{s,k}}{\partial \sigma_2}\right)^2 \tag{11}$$

Where t_1, t_2 are to be taken in two different ways namely, exact MLE and moment estimators. Unfortunately, we can not find the variance of inverse Rayleigh distribution, the asymptotic variance of $R_{s,k}$ is obtained using MLE only. From the asymptotic optimum properties of MLEs ([7]) and of linear unbiased estimators ([3]), we know that MLEs are asymptotically equally efficient having the Cramer-Rao lower bound as their asymptotic variance as given in (10). Thus from Equation (11), the asymptotic variance of $\hat{R}_{s,k}$ can be obtained when (t_1, t_2) are replaced by MLE . To avoid the difficulty of derivation of $R_{s,k}$, we obtain the derivatives of $R_{s,k}$ for (s,k)=(1,3) and (2,4) respectively, they are given by

$$\frac{\partial R_{1,3}}{\partial \sigma_1} = \frac{-6\lambda}{\sigma_2\,(3+\lambda^2)^2} \quad \text{and} \quad \frac{\partial R_{1,3}}{\partial \sigma_2} = \frac{6\lambda}{\sigma_2\,(3+\lambda^2)^2}.$$

$$\frac{\partial R_{2,4}}{\partial \sigma_1} = \frac{-24\lambda\left(7+2\lambda^2\right)}{\sigma_2\left[(3+\lambda^2)(4+\lambda^2)\right]^2} \quad \text{and} \quad \frac{\partial R_{2,4}}{\partial \sigma_2} = \frac{24\lambda\left(7+2\lambda^2\right)}{\sigma_2\left[(3+\lambda^2)(4+\lambda^2)\right]^2}.$$

Where $\lambda = \hat{\lambda}^{(1)} = \frac{\sigma_1^{(1)}}{\sigma_2^{(1)}}$

$$\text{Thus} \quad AV(\hat{R}_{1,3}) = \frac{18\lambda^4}{n\,(3+\lambda^2)^4} \quad \textit{when} \quad m=n.$$

$$AV(\hat{R}_{2,4}) = \frac{288\lambda^4\left(7+2\lambda^2\right)}{n\left[(3+\lambda^2)(4+\lambda^2)\right]^4} \quad \textit{when} \quad m=n.$$

$$\text{as} \quad n\to\infty, \frac{\hat{R}_{s,k}-R_{s,k}}{AV(\hat{R_{s,k}})} \xrightarrow{d} N(0,1).$$

and the asymptotic confidence 95 % confidence interval for $R_{s,k}$ is given by

$$\hat{R}_{s,k} \pm 1.96\sqrt{AV(\hat{R}_{s,k})}.$$

The asymptotic confidence 95% confidence interval for $R_{1,3}$ is given by

$$\hat{R}_{1,3} \pm 1.96\frac{3\lambda^2}{(3+\lambda^2)^2}\sqrt{\left(\frac{2}{n}\right)} \quad \textit{when} \quad m=n \quad \text{and} \quad \lambda = \hat{\lambda}^{(1)} = \frac{\sigma_1^{(1)}}{\sigma_2^{(1)}}.$$

The asymptotic confidence 95% confidence interval for $R_{2,4}$ is given by

$$\hat{R}_{2,4} \pm 1.96\frac{12\lambda^2(7+2\lambda^2)}{\left[(3+\lambda^2)(4+\lambda^2)\right]^2}\sqrt{\left(\frac{2}{n}\right)}, \quad \textit{when} \quad m=n \quad \text{and} \quad \lambda = \hat{\lambda}^{(1)} = \frac{\sigma_1^{(1)}}{\sigma_2^{(1)}}.$$

The small sample comparisons are studied through simulation in Section 3.

3 Small Sample Comparison

In this sub section we present some results based on Monte Carlo simulations to compare the performance of the $R_{s,k}$ using for different sample sizes. 3000 random sample of size 5(5)35 each from stress and strength populations are generated for (σ_1,σ_2)=(1,1), (1,1.5), (1,2), (1,2.5), (1,3), (1.5,1),(2,1), (2.5,1) and (3,1) as proposed by [2]. The scale parameters σ_1 and σ_2 of the variates are estimated by ML estimators and MOM estimators are used in estimating λ. These two estimators of λ are used to get the multicomponent reliability for (s,k) =(1, 3),(2, 4). The sampling bias, mean square error (MSE) of the reliability estimates over the 3000 such samples are given in Table 1. Average confidence length and coverage probability of the simulated 95% confidence intervals of $R_{s,k}$ are given in Table 2. Thus the true value of reliability is increases as λ decreases and vice versa. Both bias and MSE are decreases as sample size increases for both methods of estimation in reliability. With respect to bias Moment estimator shows very close to exact MLE in most of the parametric and sample combinations. Also the bias is negative when $\sigma_1 \leq \sigma_2$ and other cases bias is positive in both situations (s,k). With respect to MSE also MLE shows first preference than moment method of estimation. The length of the confidence interval is also decreases as the sample size increases. The coverage probability is close to the nominal value in all cases for MLE. Overall, the performance of the confidence interval is quite good for MLE. The simulation results also show that there is no considerable difference in the average bias and average MSE for different choices of the parameters, whereas considerable difference in MLE and MOM. The same phenomenon is observed for the average lengths and coverage probabilities of the confidence intervals using MLE.

Table 1: Results of the simulation study of Bias and MSE for estimates of $R_{s,k}$

		(s,k)=(1,3)					**(s,k)=(2,4)**				
			Bias		MSE			Bias		MSE	
σ_1, σ_2	(n,m)	True	MOM	MLE	MOM	MLE	True	MOM	MLE	MOM	MLE
(1,1)	(5,5)		-0.0402	-0.0120	0.0389	0.0148		-0.0289	-0.0067	0.0580	0.0265
	(10,10)		-0.0262	-0.0091	0.0246	0.0079		-0.0198	-0.0076	0.0409	0.0151
	(15,15)	0.750	-0.0166	-0.0055	0.0208	0.0050	0.600	-0.0087	-0.0044	0.0338	0.0097
	(20,20)		-0.0191	-0.0022	0.0171	0.0037		-0.0152	-0.0006	0.0294	0.0073
	(25,25)		-0.0164	-0.0048	0.0152	0.0028		-0.0127	-0.0050	0.0267	0.0056
(1, 1.5)	(5,5)		-0.0437	-0.0136	0.0248	0.0069		-0.0525	-0.0173	0.0436	0.0153
	(10,10)		-0.0271	-0.0087	0.0131	0.0033		-0.0345	-0.0118	0.0266	0.0078
	(15,15)	0.871	-0.0198	-0.0053	0.0118	0.0020	0.784	-0.0237	-0.0072	0.0225	0.0048
	(20,20)		-0.0191	-0.0028	0.0086	0.0014		-0.0249	-0.0035	0.0181	0.0035
	(25,25)		-0.0166	-0.0040	0.0074	0.0011		-0.0217	-0.0057	0.0160	0.0027
(1, 2)	(5,5)		-0.0367	-0.0108	0.0158	0.0033		-0.0504	-0.0159	0.0304	0.0082
	(10,10)		-0.0215	-0.0065	0.0071	0.0014		-0.0313	-0.0099	0.0158	0.0037
	(15,15)	0.923	-0.0166	-0.0039	0.0068	0.0008	0.869	-0.0232	-0.0060	0.0142	0.0022
	(20,20)		-0.0149	-0.0022	0.0044	0.0006		-0.0220	-0.0032	0.0103	0.0016
	(25,25)		-0.0128	-0.0028	0.0036	0.0004		-0.0191	-0.0044	0.0087	0.0012
(1, 2.5)	(5,5)	0.949	-0.0296	-0.0082	0.0105	0.0017	0.913	-0.0432	-0.0129	0.0212	0.0045
	(10,10)		-0.0165	-0.0048	0.0040	0.0007		-0.0255	-0.0077	0.0096	0.0019
	(15,15)		-0.0133	-0.0029	0.0042	0.0004		-0.0197	-0.0046	0.0092	0.0011
	(20,20)		-0.0113	-0.0016	0.0024	0.0003		-0.0176	-0.0026	0.0060	0.0008
	(25,25)		-0.0097	-0.0020	0.0019	0.0002		-0.0152	-0.0033	0.0049	0.0006
(1, 3)	(5,5)		-0.0239	-0.006	0.0072	0.0010		-0.0362	-0.0102	0.0152	0.0026
	(10,10)		-0.0128	-0.0036	0.0024	0.0004		-0.0204	-0.0059	0.0060	0.0010
	(15,15)	0.964	-0.0106	-0.0022	0.0028	0.0002	0.938	-0.0163	-0.0036	0.0063	0.0006
	(20,20)		-0.0087	-0.0012	0.0014	0.0001		-0.0140	-0.0020	0.0037	0.0004
	(25,25)		-0.0073	-0.0015	0.0011	0.0001		-0.0119	-0.0025	0.0028	0.0003
(1.5, 1)	(5,5)		-0.0127	0.0011	0.0478	0.0216		0.0241	0.0186	0.0594	0.0286
	(10,10)		-0.0075	-0.0026	0.0334	0.0122		0.0182	0.0066	0.0434	0.0163
	(15,15)	0.571	0.0006	-0.0012	0.0277	0.0079	0.366	0.0231	0.0048	0.0356	0.0106
	(20,20)		-0.0060	0.0016	0.0240	0.0060		0.0122	0.0067	0.0314	0.0083
	(25,25)		-0.0044	-0.0028	0.0216	0.0046		0.0122	0.0003	0.0280	0.0062
(2,1)	(5,5)		0.0148	0.0137	0.0482	0.0219		0.0560	0.0321	0.0521	0.0218
	(10,10)		0.0119	0.0044	0.0341	0.0122		0.0411	0.0150	0.0360	0.0114
	(15,15)	0.429	0.0166	0.0033	0.0280	0.0078	0.214	0.0401	0.0101	0.0289	0.0071
	(20,20)		0.0079	0.0051	0.0244	0.0061		0.0289	0.0102	0.0252	0.0056
	(25,25)		0.0081	0.0002	0.0217	0.0046		0.0267	0.0039	0.0216	0.0041
(2.5, 1)	(5,5)		0.0328	0.0209	0.0449	0.0190		0.0658	0.0335	0.0422	0.0142
	(10,10)		0.0244	0.0087	0.0310	0.0102		0.0471	0.0163	0.0271	0.0067
	(15,15)	0.324	0.0260	0.0060	0.0251	0.0064	0.127	0.0428	0.0107	0.0210	0.0039
	(20,20)		0.0169	0.0069	0.0219	0.0050		0.0330	0.0100	0.0183	0.0031
	(25,25)		0.0158	0.0016	0.0190	0.0037		0.0297	0.0048	0.0149	0.0022
(3, 1)	(5,5)		0.0423	0.0236	0.0400	0.0153		0.0636	0.0294	0.0328	0.0087
	(10,10)		0.0306	0.0105	0.0267	0.0079		0.0443	0.0143	0.0194	0.0037
	(15,15)	0.250	0.0301	0.0070	0.0212	0.0048	0.077	0.0387	0.0092	0.0146	0.0020
	(20,20)		0.0212	0.0074	0.0185	0.0038		0.0308	0.0083	0.0128	0.0016
	(25,25)		0.0193	0.0025	0.0156	0.0028		0.0270	0.0043	0.0098	0.0011

Table 2: Average confidence length and coverage probability of the simulated 95% confidence intervals of $R_{s,k}$ using MLE

		(s,k)=(1,3)		**(s,k)=(2,4)**	
σ_1,σ_2	(n,m)	average confidence length	coverage probability	average confidence length	coverage probability
(1,1)	(5,5)	0.44710	0.9470	0.61153	0.9463
	(10,10)	0.32180	0.9457	0.45127	0.9433
	(15,15)	0.26696	0.9483	0.37643	0.9483
	(20,20)	0.23108	0.9463	0.32790	0.9473
	(25,25)	0.20659	0.9490	0.29421	0.9520
(1,1.5)	(5,5)	0.29058	0.9423	0.44463	0.9440
	(10,10)	0.20109	0.9440	0.31467	0.9460
	(15,15)	0.16525	0.9500	0.26002	0.9500
	(20,20)	0.14166	0.9500	0.22396	0.9483
	(25,25)	0.12597	0.9493	0.19963	0.9483
(1,2)	(5,5)	0.19354	0.9413	0.31136	0.9413
	(10,10)	0.13033	0.9430	0.21336	0.9437
	(15,15)	0.10637	0.9503	0.17486	0.9487
	(20,20)	0.09065	0.9500	0.14953	0.9500
	(25,25)	0.08038	0.9500	0.13279	0.9503
(1,2.5)	(5,5)	0.13539	0.9413	0.22371	0.9413
	(10,10)	0.08956	0.9417	0.14998	0.9423
	(15,15)	0.07278	0.9503	0.12225	0.9503
	(20,20)	0.06181	0.9510	0.10408	0.9507
	(25,25)	0.05472	0.9497	0.09224	0.9497
(1,3)	(5,5)	0.09907	0.9413	0.16630	0.9413
	(10,10)	0.06476	0.9410	0.10985	0.9417
	(15,15)	0.05248	0.9503	0.08923	0.9503
	(20,20)	0.04448	0.9510	0.07576	0.9510
	(25,25)	0.03934	0.9490	0.06706	0.9497
(1.5,1)	(5,5)	0.55454	0.9483	0.63405	0.9457
	(10,10)	0.40908	0.9440	0.47469	0.9510
	(15,15)	0.34022	0.9480	0.39440	0.9467
	(20,20)	0.29669	0.9457	0.34567	0.9520
	(25,25)	0.26653	0.9507	0.31163	0.9510
(2,1)	(5,5)	0.55611	0.9413	0.53166	0.9400
	(10,10)	0.41050	0.9493	0.39039	0.9510
	(15,15)	0.33943	0.9450	0.31893	0.9487
	(20,20)	0.29646	0.9510	0.27889	0.9517
	(25,25)	0.26668	0.9520	0.25118	0.9563
(2.5,1)	(5,5)	0.51087	0.9377	0.41080	0.9307
	(10,10)	0.37338	0.9493	0.29141	0.9477
	(15,15)	0.30609	0.9447	0.23308	0.9473
	(20,20)	0.26709	0.9500	0.20265	0.9473
	(25,25)	0.24011	0.9537	0.18176	0.9570
(3,1)	(5,5)	0.45178	0.9413	0.30794	0.9287
	(10,10)	0.32572	0.9483	0.20993	0.9443
	(15,15)	0.26471	0.9470	0.16441	0.9447
	(20,20)	0.23056	0.9500	0.14204	0.9450
	(25,25)	0.20699	0.9547	0.12672	0.9547

4 Conclusions

In this paper, we have studied the multicomponent stress-strength reliability for inverse Rayleigh distribution when both of stress, strength variates follows the same population. Also, we have estimated asymptotic confidence interval for multicomponent stress-strength reliability. The simulation results indicates that in order to estimate the multicomponent stress-strength reliability for inverse Rayleigh distribution the ML method of estimation is preferable than the moment method of estimation. The length of the confidence interval is also decreases as the sample size increases and coverage probability is close to the nominal value in all cases for MLE.

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