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Modeling of a Power Amplifier for Digital Pre-distortion Applications using Simplified Complex Memory Polynomial

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Abstract: With the advent of spectrally efficient wireless communication systems employing modulation schemes with varying amplitude of the communication signal, linearization techniques for nonlinear microwave power amplifiers have gained significant interest. The linearization task can be divided into two parts: exact modeling of the power amplifier (PA) to be linearized and development of linearization technique. In this paper PA modeling using simplified complex memory polynomial has been presented. The proposed model has been tested using two-tone method.

Keywords: Complex Memory Polynomial, Memory Effects, Power Amplifier, Thermal Effects.

1 Introduction

Orthogonal frequency division multiple access (OFDM) systems allow the transmission of high data rates over broadband radio channels without need of powerful channel equalizer. By using special modulation schemes, OFDM systems do not require a channel estimator. Thus OFDM systems are less complex as compared with a single carrier transmission system. But major disadvantage of OFDM signals is that the have very large peak-to-average power ratio (PAPR). This high PAPR derives the power amplifier (PA) into non linear region and hence causes inter-modulation distortion (IMD), which causes spreading of power both within the band and in the adjacent frequency bands. Various PA linearization techniques have been discussed in literature [1,2] to reduce IMD while maintaining the efficiency of the PA. Linearization task can't be accomplished successfully until the PA to be linearized is exactly modeled. This paper focuses on the discussion of the characteristics and modeling of a PA with thermal and memory effects. The proposed PA model has already used in development of Low Complexity Look Up Table based Adaptive Digital Pre-distorter with Low Memory Requirements [7].The organization of the paper is as follows: section I is brief introduction to the background of the problem, In section II and its subsections various performance indices on which the performance of a PA can be evaluated has been discussed, in section III, PA modeling with thermal and memory effects has been proposed and section IV is conclusion.

2 Performance Indices of a Power Amplifier

Modeling of PA has been a subject of intense research for the last few years. But the modeling task of PA will not be accomplished until the parameters on which its performance can be evaluated are not known. This section gives the insight into the basic parameters of the PA like IMD Products, adjacent channel leakage ratio (ACLR), Efficiency and error vector magnitude (EVM), which can be used to evaluate its performance.

An amplifier is said to be linear if its output voltage is simply a constant times the input voltage

$$V_{out}(t) = G.V_{in}(t) \tag{1}$$

But in reality they have non-linearities that make the output voltage a function of higher order terms of the

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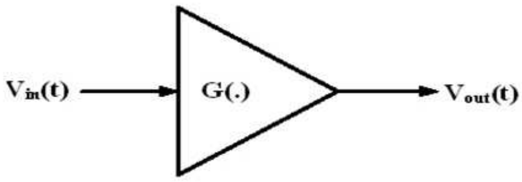

Fig. 1: Power Amplifier input-output diagram

input voltage. The output voltage $V_{out}(t)$ for such non-linear amplifier can be expressed mathematically as a Taylor series [8].

$$V_{out}(t) = G_1 V_{in}(t) + G_2 V_{in}^2(t) + G_3 V_{in}^3(t) ..... + G_n V_{in}^n(t) \quad (2)$$

Thus due to its non-linear characteristics, the PA is the main contributor for distortion products in a transmitter (TX) chain. With Multi carrier modulation schemes like OFDM used in W-CDMA systems, even harmonic components could be filtered out, but 3^{rd} and 5^{th} order distortion component will exist with the fundamental channel. Thus equation 2 can be written as [9]

$$V_{out}(t) = G_1 V_{in}(t) + G_3 V_{in}^3(t) + G_5 V_{in}^5(t) ..... + G_{2n-1} V_{in}^{2n-1}(t) \quad (3)$$

If a two tone signal $V_{in}(t) = V_1 \cos\omega_1 t + V_2 \cos\omega_2 t$ is applied to the PA, then 3^{rd} order term will produce$G_3 \left(V_1 \cos\omega_1 t + V_2 \cos\omega_2 t\right)^3$, which is equal to

$$G_3 \left(V_1^3 \cos^3\omega_1 t + V_2^3 \cos^3\omega_2 t + 3V_1^2 V_2 cos^2\omega_1 t . \cos\omega_2 t + 3V_1 V_2^2 cos\omega_1 t . \cos^2\omega_2 t\right) \quad (4)$$

Let us expand each term of equation 4 one by one

Now $V_1^3 \cos^3\omega_1 t = V_1^3 (\cos^2\omega_1 t . \cos\omega_1 t) = V_1^3 \left[\left(\frac{1}{2} + \frac{1}{2}\cos 2\omega_1 t\right)\cos\omega_1 t\right] = \frac{1}{2}V_1^3 \cos\omega_1 t + \frac{1}{2}V_1^3 \cos 2\omega_1 t . \cos\omega_1 t$

$$= \frac{1}{2}V_1^3 \cos\omega_1 t + \frac{1}{4}V_1^3 \left[\cos(2\omega_1 t - \omega_1 t) + \cos(2\omega_1 t + \omega_1 t)\right]$$

$$= \frac{1}{2}V_1^3 \cos\omega_1 t + \frac{1}{4}V_1^3 \left[\cos(\omega_1 t) + \cos(3\omega_1 t)\right]$$

$$\frac{3}{4}V_1^3 \cos\omega_1 t + \frac{1}{2}V_1^3 \cos 3\omega_1 t \quad (5)$$

Similarly

$$V_2^3 \cos^3\omega_2 t = \frac{3}{4}V_2^3 \cos\omega_2 t + \frac{1}{2}V_2^3 \cos 3\omega_2 t \quad (6)$$

Also
$3V_1^2 V_2 \cos^2\omega_1 t . \cos\omega_2 t = \left(3V_1^2 V_2 \cos\omega_2 t\right)\left(\frac{1+\cos 2\omega_1 t}{2}\right)$

$$= \frac{3}{2}V_1^2 V_2 . \cos\omega_2 t + \frac{3}{2}V_1^2 V_2 . \cos\omega_2 t . \cos 2\omega_1 t$$

$$\frac{3}{2}V_1^2 V_2 . \cos\omega_2 t + \frac{3}{4}V_1^2 V_2 \left[\cos(2\omega_1 t - \omega_2 t) + \cos(2\omega_1 t + \omega_2 t)\right] \quad (7)$$

Similarly
$3V_1 V_2^2 \cos\omega_1 t \cos^2\omega_2 t =$

$$\frac{3}{2}V_1 V_2^2 . \cos\omega_1 t + \frac{3}{4}V_1 V_2^2 \left[\cos(2\omega_2 t - \omega_1 t) + \cos(2\omega_2 t + \omega_1 t)\right] \quad (8)$$

From equations 5 to 8 see that 3^{rd} order term of equation 4 results in the frequency components $3f_1, 3f_2, 2f_1 - f_2, 2f_2 - f_1, 2f_1 + f_2, 2f_2 + f_1$ at the output of PA.

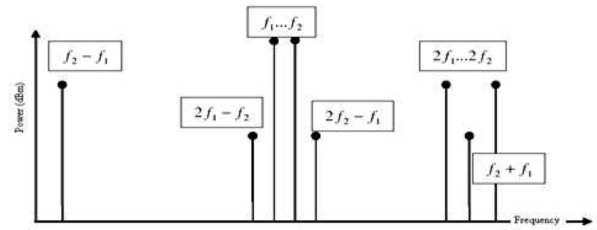

Fig. 2: Two Tone Test Spectrum Results of a non-linear Power Amplifier

As seen from Fig. 2, the components $2f_2 - f_1$ and $2f_1 - f_2$ are in-band components which contributes for the in-band distortion at the output of PA. In the following subsections various performance indices of have been discussed.

2.1 Intermodulation Products

As discussed above, for two tone (f_1, f_2) input, the 3^{rd} order term has resulted in $2f_1 - f_2, 2f_2 - f_1, 2f_1 + f_2, 2f_2 + f_1$ harmonic components at the output of PA. So, in general if we consider higher order terms also, we will get $mf_1 \pm nf_2$ harmonic components at the output of PA, where m and n are integers. These harmonic components are also known as IMD products. But out of these IMD's, the most serious is 3^{rd} order IMD which is very close to the desired frequency as shown in Fig. 2 and is not easy to filter out. This product is usually characterized by the third order intercept point refereeing either to the input or the output (IIP_3 or OIP_3). This is best defined by looking at Fig. 3. It can be shown that the slope of the linear gain for input and output powers in dBs is unity, likewise the slope of the third gain of the third order IMD component is 3 [10], the point where the third order line intersects with the linear gain line is the third order intercept point.

Another figure of merit to characterize non-linearity is the 1dB compression point. For a non-linear device, the

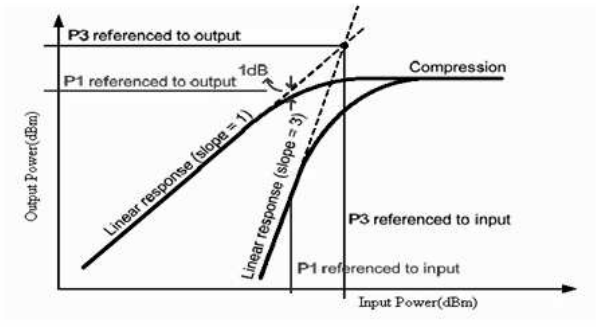

Fig. 3: Illustration of the first and third order intercepts points

1dB compression point is defined as the point where the difference between the non-linear device's output and the linear output is exactly 1 dB. 1dB compression point can be calculated as [11]

$$Y_{1dB} = G_1V_1 + G_3\left[\frac{3}{4}V_1^3 + \frac{3}{2}V_1V_2^2\right] \quad (9)$$

The 1 dB compression point is typically 12 to 15 dB less than the 3rd order intercept point assuming that they are referenced at the same point.

2.2 Adjacent Channel Leakage Ratio

As discussed in previous sections, PA produces unwanted signal emission to adjacent channels, which are outlined in the related standards for W-CDMA and other modulation schemes. The term ACLR is a measure of adjacent channel emission and is very often faced as regulatory parameter [12]. For two tone input, ACLR is defined as [13]

$$ACLR_{dB} = IMR_2 - 6 + 10\log_{10}\frac{n^3}{4\left(\frac{2n^3-3n^2-2n}{24}+\frac{mod\left(\frac{n}{2}\right)}{8}\right)+\left(\frac{n^2-mod\left(\frac{n}{2}\right)}{4}\right)} \quad (10)$$

Where $IMR_2$is Two Tone Inter Modulation Ratio (dB), defined as [13]

$$IMR_2 = \frac{\frac{3}{4}G_3V_1^2V_2}{G_1V_1} \quad (11)$$

and n is number of tones for the band of interest.

The idea behind this formula is to estimate ACLR for multi carrier modulation schemas, by expanding two tones inter modulation IMR to given number of in-band carrier. Permutation of each tones interaction with all of the others will shape the spectral growth depending on $G_3, G_5 \ldots . G_{2n-1}$odd order non-linearity coefficients. Also total power due to IMD components can be calculated as [13]

$$P_{IMD} = \frac{\left[\frac{3}{4}G_3V_1V_2^2\right]^2}{2} \quad (12)$$

2.3 Error Vector Magnitude

The modulation accuracy of the W-CDMA signals is measured by EVM (EVM). EVM is a measure for the difference between the theoretical waveform and modified version of the measured waveform. EVM can be defined as the distance between the desired and actual signal vectors (error vector), normalized to a fraction of the signal amplitude.

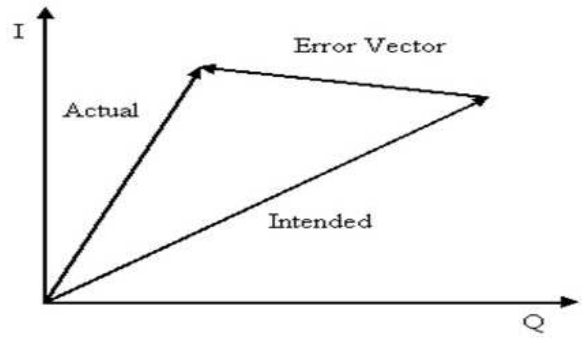

Fig. 4: Error Vector Magnitude

Mathematically, the error vector can be written as

$$e = y - x \quad (13)$$

Where y is the modified measured signal and x is the ideal transmitted signal.

The EVM can be defined as the square root of the ratio of the mean error vector power to the mean reference signal power expressed as

$$EVM_{RMS} = \sqrt{\frac{E\left[e^2\right]}{E\left[x^2\right]}} \quad (14)$$

Also for each symbol k , EVM can be defined

$$EVM(k) = \frac{|E(k)|}{\sqrt{\frac{1}{N}\sum_{k=1}^{N}|S(k)|^2}} \quad (15)$$

where E(k) is the error vector for symbol k, S(k) is the ideal signal vector of the symbol k and N is the number of symbols. Root-mean-square (RMS) value of EVM for a number of symbols is a widely used measure of system linearity and it can be defined as

$$EVM_{RMS} = \frac{\sqrt{\sum_{k=1}^{N}|E(k)|^2}}{\sqrt{\sum_{k=1}^{N}|S(k)|^2}} \quad (16)$$

EVM is an in-band distortion causing high bit error rates during reception of the transmitted data. Therefore EVM specifications must also be fulfilled in order to have proper communication.

2.4 Modeling of PA with Thermal And Memory effects

Any PA will show some dynamic deviations from its static characteristics. Such deviations have become known as memory and thermal effects [3,4]. These effects are very troublesome for the process of pre-distortion. Thus a PA model design simulations should be able to predict such memory and thermal effects and the designer must be bothered to include sufficient details of the bias circuitry, as well as the RF circuit, in the simulation file.

The Memory Polynomial model given by equation 17 can be used to incorporate both memory and thermal effects. The Memory Polynomial consists of several delay taps and non-linear static functions. This model is a truncation of the general Volterra series, which consists of only the diagonal terms in the Volterra kernels. Thus, the number of parameters is significantly reduced compared to general Volterra series. The Memory Polynomial model can be described as

$$y(n) = \sum_{q=0}^{Q}\sum_{k=1}^{K} c_{2k-1,q}\,|x(n-q)|^{2(k-1)}\,x(n-q) \qquad (17)$$

where
$x(n)$ is the input complex base band signal
$y(n)$ is the output complex base band signal
$c_{k,q}$ are complex valued parameters
Q is the memory depth
K is the order of the polynomial

The even order terms are usually outside of the operational bandwidth of the signal and can be easily filtered out. This model considers polynomials with orders up to$2k-1$, where K is a design parameter. For simplicity of implementation, the equation 17 can be rewritten as follows

$$y(n) = \sum_{q=0}^{Q} F_q(n-q)$$
$$= F_0(n) + F_1(n-1) + F_2(n-2) + \ldots.F_Q(n-Q) \qquad (18)$$

In its expanded form equation 18 can be written as

$$F_q(n) = c_{1,q}x(n) + c_{3,q}|x(n)|^2 x(n) + c_{5,q}|x(n)|^4 x(n) + \ldots. + c_{2K-1,q}|x(n)|^{2(K-1)} x(n) \qquad (19)$$

equation 17 can be implemented as [5].

In modern communications applications, the actual modulation system is in use for testing a PA model. Obviously, such testing is essential, both during development and in production, for determining specification compliance on a product. But due to its simplicity in implementation, the two-carrier or two-tone test is still a convenient method for testing. So, in the present work the modeled PA has been tested using two carriers placed at 1.950 GHz and 1.955 GHz. The

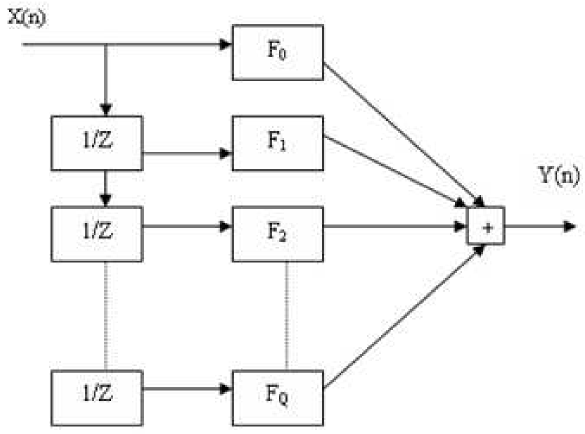

Fig. 5: Implementation of Memory Polynomial given by equation 19

memory depth, $q = 3$ and polynomial of the 5th degree,$k = 3$ has been used. The input power has been varied from 0 to 20 dB.The set up shown in Figure 6 has been used for measurements. Due its ability to find unique global optimum point and computational easiness, the LSE technique [6] is used to find the coefficients.

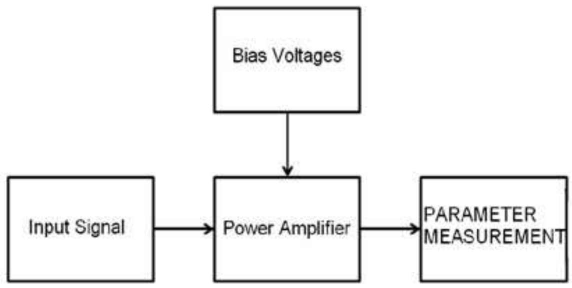

Fig. 6: Parameter measurement of PA

Figure 6 to 12 shows the comparison of Gain Compression, Phase Characteristics, Input Characteristics, Output Characteristics and constellations, 3^{rd} order IMD Gain reconstruction and 3^{rd} order IMD Phase reconstruction of actual and modeled PA respectively. From these figures we see that our modeled PA shows characteristics which are very similar to the characteristics of the actual PA. Measurements also show that Lower channel ACLR for the actual amplifier is -16.627 whereas for modeled amplifier, its value as -16.626. Similarly upper channel ACLR for the actual amplifier has been calculated as -19.175 whereas for modeled amplifier, its value as -19.176. Also value of EVM has been calculated as 8.13% and 8.124% for the

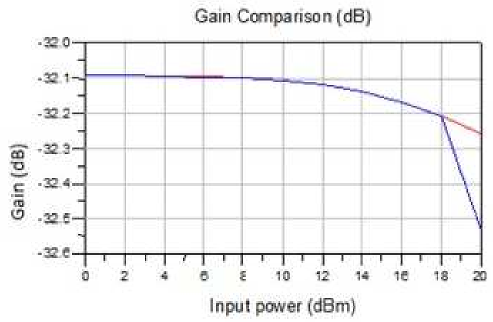

Fig. 7: Comparison of Gain Compression of actual and modeled PA

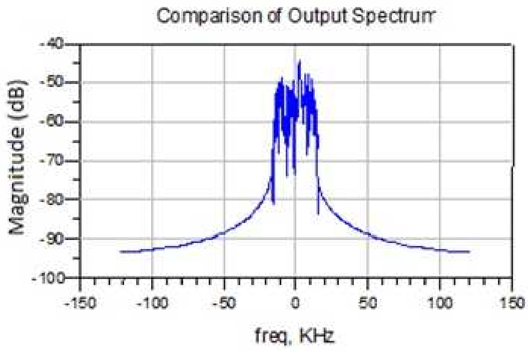

Fig. 10: Comparison of Output Characteristics of actual and modeled PA

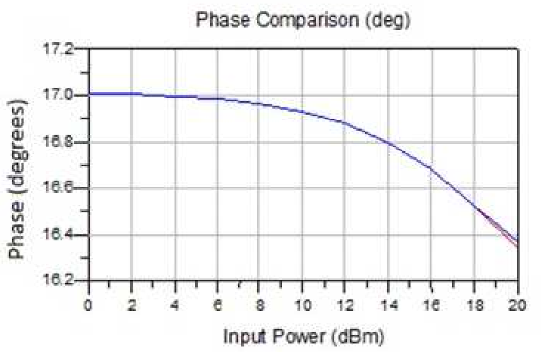

Fig. 8: Comparison of Phase Characteristics of actual and modeled PA

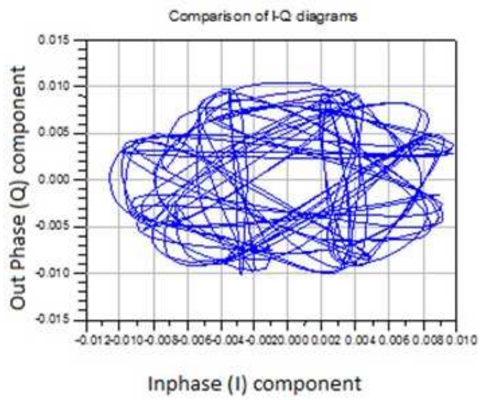

Fig. 11: Comparison of constellations of actual and modeled PA

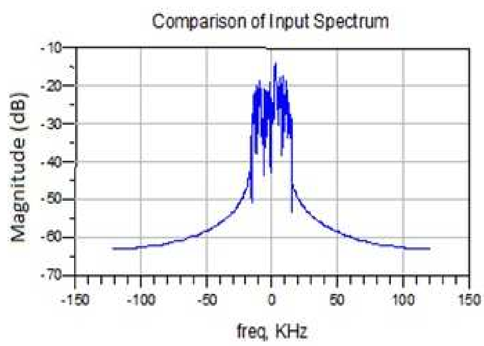

Fig. 9: Comparison of Input Characteristics of actual and modeled PA

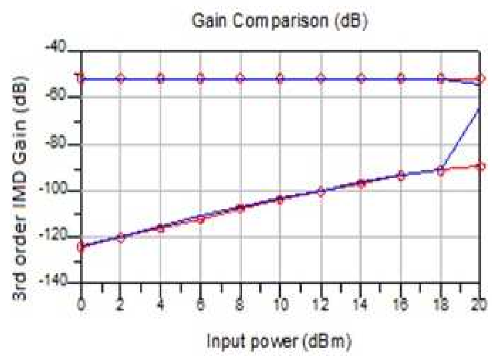

Fig. 12: 3^{rd} order IMD Gain reconstruction of actual and modeled PA

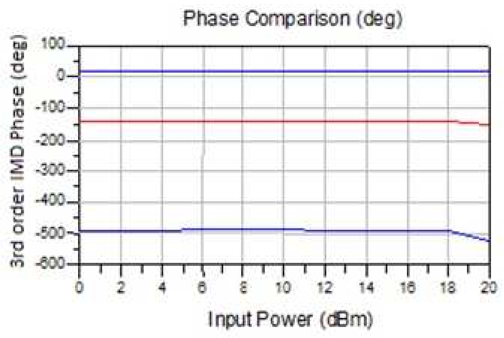

Fig. 13: 3^{rd} order IMD Phase reconstruction of actual and modeled PA

actual and modeled amplifier respectively. Thus actual amplifier and the modeled amplifier show almost similar characteristics, which show the validity of the modeled amplifier.

3 Conclusion

For accurate design and implementation of any PA linearization technique, exact modeling of PA is an important issue. In this paper modeling of PA using complex memory polynomial has been presented. The proposed memory polynomial is simple from implementation point of view. For validity of the proposed PA model, its various characteristic like Gain Compression, Phase Characteristics, Input Characteristics, Output Characteristics and constellations, 3^{rd} order IMD Gain reconstruction, 3^{rd} order IMD Phase reconstruction, ACLR and EVM have been compared with the actual PA.

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