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# Transmuted Janardan Distribution: A Generalization of the Janardan Distribution

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**Abstract:** In this paper, a continuous distribution so-called transmuted Janardan distribution (TJD) is suggested and studied. The quadratic rank transmutation map suggested by Shaw and Buckley (2009) is used in generating the TJD. Various structural properties of the TJD including explicit expressions for the reliability and hazard rate functions, order statistics, the $r$th moment and the moment generating function are derived. Also, the skewness, kurtosis, coefficient of variation are derived and calculated for some values of the TJD parameters. The maximum likelihood method is used to estimate the unknown parameters of the TJD for complete sample and the entropy is studied and proved.

**Keywords:** Transmuted Janardan distribution, Moments, Entropy, Order statistics.

## 1 Introduction

[1] suggested the transmutation map method, which is used to derive a new model as a generalization of the Janardan Distribution. The new distribution is called the transmuted Janardan distribution (TJD).

The transmuted quasi Lindley distribution is obtained by [2]. The transmuted Gumbel distribution and its application in climate data is suggested by [3]. [4] suggested transmuted Two-Parameter Lindley distribution. [5] suggested transmuted exponentiated Frechet distribution. [6] introduced transmuted exponentiated inverse Rayleigh distribution. [7] proposed transmuted exponentiated gamma distribution as a generalization of the exponentiated gamma probability distribution. [8] proposed transmuted exponential-Weibull distribution with some applications. [9] proposed transmuted inverse Rayleigh distribution as a modification of the inverse Rayleigh distribution.

The cumulative distribution function (cdf) technique based on the quadratic rank transmutation map satisfies the following general form

$$\Phi_2(x) = (1+\lambda)\Phi_1(x) - \lambda[\Phi_1(x)]^2, \tag{1}$$

with pdf given by

$$\psi_2(x) = \psi_1(x)[1+\lambda-2\lambda\Phi_1(x)], \ \ -1 \le \lambda \le 1, \tag{2}$$

where $\psi_1(x)$ and $\psi_2(x)$ are the corresponding probability density functions (pdf) of $\Phi_1(x)$ and $\Phi_2(x)$, respectively. Note that at $\lambda = 0$ , we have $\psi_1(x) = \psi_2(x)$ .

**Definition:** A random variable $X$ is said to have a transmuted probability distribution with cdf $\Phi(x)$ if

$$\Phi(x) = (1+\lambda)G(x) - \lambda[G(x)]^2, \ \ |\lambda| \le 1, \tag{3}$$

where $G(x)$ is the cdf of the base distribution.

The rest of this paper is organized as follows: In Section 2 we demonstrate the pdf and cdf of the transmuted Janardan distribution. The reliability and hazard rate functions of the subject model are presented in Section 3. In Section 4, the distributions of order statistics are summarized. In Section 5, the statistical properties including the rth moment, variance,

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skewness, kurtosis, coefficient of variation and the moment generating function are derived. We demonstrate the maximum likelihood estimates (MLE) of the distribution unknown parameters and the Renyi entropy in Section 6. The conclusion is given in Section 7.

## 2 Transmuted Janardan distribution

[10] suggested the Janardan distribution with parameters $\alpha$ and $\theta$ with cumulative distribution function defined as

$$\Phi(x;\alpha,\theta)=1-\frac{\alpha\left(\theta+\alpha^2\right)+\theta\alpha^2x}{\alpha\left(\theta+\alpha^2\right)}e^{-\frac{\theta}{\alpha}x};x>0,\ \theta>0,\ \alpha>0. \tag{4}$$

The corresponding pdf is derived as

$$\psi(x;\alpha,\theta)=\frac{\theta^2}{\alpha\left(\theta+\alpha^2\right)}(1+\alpha x)e^{-\frac{\theta}{\alpha}x};\ x>0,\ \theta>0,\ \alpha>0. \tag{5}$$

The suggested transmuted Janardan distribution using Equations (1), (3) and (4) has the cdf given by

$$\begin{aligned}\Phi_{TJD}(x)=&\ (1+\lambda)\left[1-\frac{\alpha\left(\theta+\alpha^2\right)+\theta\alpha^2x}{\alpha\left(\theta+\alpha^2\right)}e^{-\frac{\theta}{\alpha}x}\right]-\lambda\left[1-\frac{\alpha\left(\theta+\alpha^2\right)+\theta\alpha^2x}{\alpha\left(\theta+\alpha^2\right)}e^{-\frac{\theta}{\alpha}x}\right]^2\ ;|\lambda|\leq 1\\ =&\ \frac{e^{-\frac{2\theta x}{\alpha}}\left(\alpha^2+\theta+\alpha\theta x\right)\left[(3\lambda+1)\left(\alpha^2+\theta\right)e^{\frac{\theta x}{\alpha}}-\lambda\left(\alpha^2+\theta+\alpha\theta x\right)\right]}{\left(\alpha^2+\theta\right)^2}-2\lambda.\end{aligned} \tag{6}$$

The corresponding pdf of the transmuted Janardan distribution is

$$\psi_{TJD}(x)=\frac{\theta^2(\alpha x+1)e^{-\frac{2\theta x}{\alpha}}}{\alpha(\alpha^2+\theta)^2}\left[2\lambda\left(\alpha^2+\theta+\alpha\theta x\right)-(\lambda-1)\left(\alpha^2+\theta\right)e^{\frac{\theta x}{\alpha}}\right]. \tag{7}$$

Figures (1) and (2) showed the shapes of the cdf and pdf of the TJD for fixed $\alpha=6,\ \theta=2$ and $\lambda=-1,-0.3,0,0.3,1$. Based on Figure (2) we can note that the TJD is a non symmetric distribution.

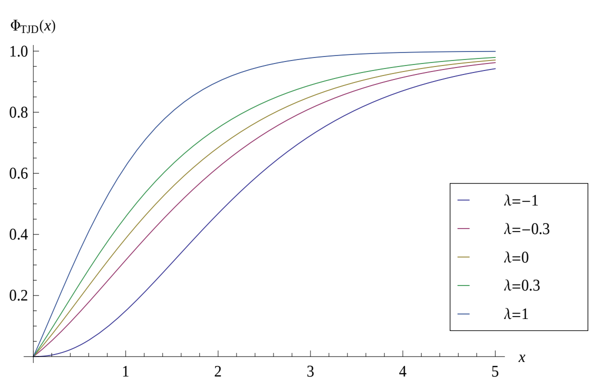


**Fig. 1:** The cdf $\Phi_{TJD}(x)$ with $\alpha=2,\ \theta=2$ and $\lambda=-1,-0.3,0,0.3,1$.

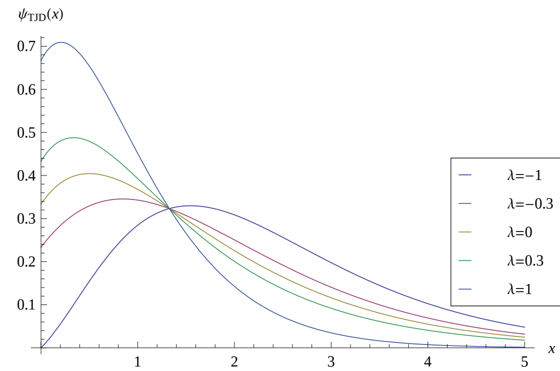


**Fig. 2:** The pdf $\psi_{TJD}(t)$ with $\alpha = 2,\ \theta = 2$ and $\lambda = -1, -0.3, 0, 0.3, 1$.

## 3 Reliability analysis

The transmuted Janardan distribution can be considered to a real data set. The reliability function $R(t) = 1 - \Phi(t)$, which is known as the probability of an item not failing prior to some time t, and the hazard rate function $H(t) = \frac{\psi(t)}{1-\Phi(t)}$, which is defined as instantaneous failure rate, are derived for the transmuted Janardan distribution.

**Theorem 1:** The reliability and hazard rate (survival) functions of the transmuted Janardan distribution, respectively are

$$R_{TJD}(t) = 1 - \Phi_{TJD}(t) = \frac{e^{-\frac{2\theta t}{\alpha}}\left(\alpha^2 + \theta + \alpha\theta t\right)\left[\lambda\left(\alpha^2 + \theta + \alpha\theta t\right) - (\lambda - 1)\left(\alpha^2 + \theta\right)e^{\frac{\theta t}{\alpha}}\right]}{\left(\alpha^2 + \theta\right)^2}, \tag{8}$$

and

$$H_{TJD}(t) = \frac{\psi_{TJD}(t)}{1 - \Phi_{TJD}(t)} = \frac{\theta^2(\alpha t + 1)\left[(\lambda - 1)\left(\alpha^2 + \theta\right)e^{\frac{\theta t}{\alpha}} - 2\lambda\left(\alpha^2 + \theta + \alpha\theta t\right)\right]}{\alpha\left(\alpha^2 + \theta + \alpha\theta t\right)\left[(\lambda - 1)(\alpha^2 + \theta)e^{\frac{\theta t}{\alpha}} - \lambda\left(\alpha^2 + \theta + \alpha\theta t\right)\right]}. \tag{9}$$

**Proof:** The proofs of (8) and (9) are straightforward by using $\psi_{TJD}(x)$ and $\Phi_{TJD}(x)$. Figures (3) and (4) showed the shapes of the reliability and hazard functions for the TJD for fixed values of $\alpha = \theta = 2$ and $\lambda = -1, -0.3, 0, 0.3, 1$.

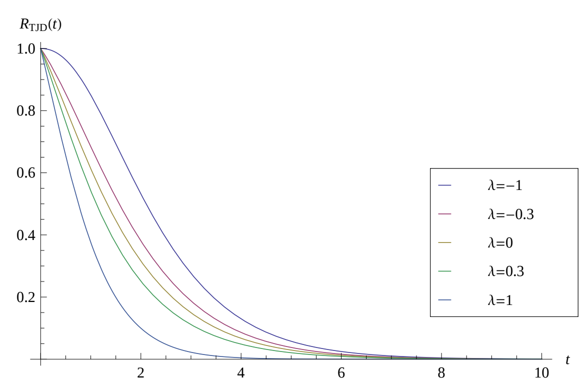


**Fig. 3:** $H_{TJD}(t)$ with $\alpha = 2,\ \theta = 2$ and $\lambda = -1, -0.3, 0, 0.3, 1$.

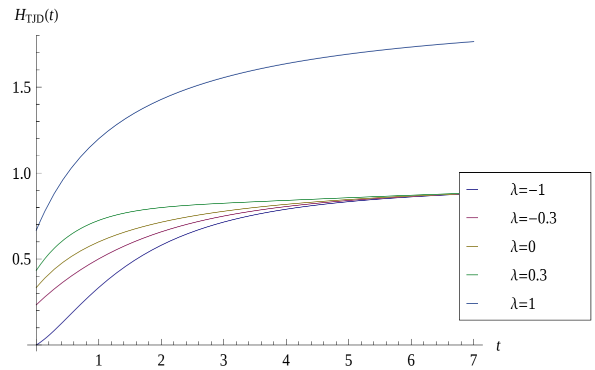


**Fig. 4:** $H_{TJD}(x)$ with $\alpha = 2,\ \theta = 2$ and $\lambda = -1, -0.3, 0, 0.3, 1$.

## 4 Order Statistics

Order statistics are crucial in numerous areas of training and statistical theory. Let $X_1, X_2, ..., X_n$ be a random sample of size $n$ from a pdf $\Psi(x)$ and cdf $\Phi(x)$ , and $X_{[1]}, X_{[2]}, ..., X_{[n]}$ be its order statistics. The pdf of the $i$th order statistics is defined as

$$\psi_{(j)}(x) = \frac{n!}{(j-1)!(n-j)!}\psi(x)[\Phi(x)]^{j-1}[1-\Phi(x)]^{n-j}, i = 1,2,...,n. \tag{10}$$

Let, $X_{[1]} = \min\{X_1, X_2, ..., X_n\}$ and $X_{[n]} = \max\{X_1, X_2, ..., X_n\}$ with pdfs $\psi_{(1)}(x)$ and $\psi_{(n)}(x)$ defined as follow

$$\psi_{(1)}(x) = \frac{\theta^2 n(\alpha x+1)e^{-\frac{2\theta x}{\alpha}}}{\alpha(\alpha^2+\theta)^{2n}}\left[2\lambda\left(\alpha^2+\theta+\alpha\theta x\right) - (\lambda-1)\left(\alpha^2+\theta\right)e^{\frac{\theta x}{\alpha}}\right]$$
$$\times\left\{e^{-\frac{2\theta x}{\alpha}}\left(\alpha^2+\theta+\alpha\theta x\right)\left[\lambda\left(\alpha^2+\theta+\alpha\theta x\right)-(\lambda-1)\left(\alpha^2+\theta\right)e^{\frac{\theta x}{\alpha}}\right]\right\}^{n-1}, \tag{11}$$

and

$$\psi_{(n)}(x) = \frac{\theta^2 n(\alpha x+1)e^{-\frac{2\theta x}{\alpha}}}{\alpha(\alpha^2+\theta)^2}\left[2\lambda\left(\alpha^2+\theta+\alpha\theta x\right) - (\lambda-1)\left(\alpha^2+\theta\right)e^{\frac{\theta x}{\alpha}}\right]$$
$$\times\left\{\frac{e^{-\frac{2\theta x}{\alpha}}\left(\alpha^2+\theta+\alpha\theta x\right)\left[(\lambda-1)\left(\alpha^2+\theta\right)e^{\frac{\theta x}{\alpha}}-\lambda\left(\alpha^2+\theta+\alpha\theta x\right)\right]}{\left(\alpha^2+\theta\right)^2}+1\right\}^{n-1}. \tag{12}$$

Furthermore, for any value of $j$ the common form of $f_{(j)}(x)$ can be obtained as

$$\psi_{(j)}(x) = \frac{\theta^2 n!(\alpha x+1)e^{-\frac{2\theta x}{\alpha}}\left[2\lambda\left(\alpha^2+\theta+\alpha\theta x\right) - (\lambda-1)\left(\alpha^2+\theta\right)e^{\frac{\theta x}{\alpha}}\right]}{\alpha(\alpha^2+\theta)^{2n-2j+2}\Gamma(j)\Gamma(n-j+1)}$$
$$\times\left[e^{-\frac{2\theta x}{\alpha}}\left(\alpha^2+\theta+\alpha\theta x\right)\left(\lambda\left(\alpha^2+\theta+\alpha\theta x\right)-(\lambda-1)\left(\alpha^2+\theta\right)e^{\frac{\theta x}{\alpha}}\right)\right]^{n-j}$$
$$\times\left\{\frac{e^{-\frac{2\theta x}{\alpha}}\left(\alpha^2+\theta+\alpha\theta x\right)\left[(\lambda-1)\left(\alpha^2+\theta\right)e^{\frac{\theta x}{\alpha}}-\lambda\left(\alpha^2+\theta+\alpha\theta x\right)\right]}{\left(\alpha^2+\theta\right)^2}+1\right\}^{j-1}. \tag{13}$$

## 5 Statistical Properties

In this section we discussed some of statistical properties of the transmuted Janardan distribution. Also, some numerical values of the subject model are obtained.

### 5.1 *The* r*th moment*

**Theorem 2:** If the random variable $X$ is from a $\Phi_{TJD}(x,\alpha,\theta,\lambda)$, then the $r$th moment of $X$ is defined as

$$E\left(X^r\right)=\left(\frac{\alpha}{\theta}\right)^r\left\{\begin{array}{c}\left[1-\lambda+\frac{\lambda\alpha^2-\alpha^2}{\theta+\alpha^2}+\frac{\lambda}{2^r}\left(\frac{\theta}{\theta+\alpha^2}\right)\right]\Gamma(r+1)+\left[\frac{\alpha^2-\lambda\alpha^2}{\theta+\alpha^2}+\frac{\lambda\alpha^2}{2^r}\left(\frac{8\theta+7\alpha^2}{2\left(\theta+\alpha^2\right)^2}\right)\right]\Gamma(r+2)\\ +\lambda\left(\frac{\alpha}{2\theta}\right)^2\left(\frac{\theta\alpha}{\theta+\alpha^2}\right)^2\Gamma(r+3)\end{array}\right\}, \tag{14}$$

where $\Gamma(*)$ is the gamma function, $\Gamma(\vartheta)=\int_0^\infty t^{\vartheta-1}Exp(-t)dt$.

**Proof:**
The $r$th moment is defined as

$$\begin{aligned}E\left(X^r\right)&=\int_0^\infty x^r\Psi(x)dx\\ &=\int_0^\infty\left[\begin{array}{l}\frac{\theta}{\alpha}x^r e^{-\frac{\theta}{\alpha}x}-\lambda\frac{\theta}{\alpha}x^r e^{-\frac{\theta}{\alpha}x}+\frac{\theta^2x}{\theta+\alpha^2}x^r e^{-\frac{\theta}{\alpha}x}-\frac{\theta\alpha}{\theta+\alpha^2}x^r e^{-\frac{\theta}{\alpha}x}\\ -\lambda\frac{\theta^2x}{\theta+\alpha^2}x^r e^{-\frac{\theta}{\alpha}x}+\lambda\frac{\theta\alpha}{\theta+\alpha^2}x^r e^{-\frac{\theta}{\alpha}x}+\lambda\frac{2\theta}{\alpha}x^r e^{-\frac{2\theta}{\alpha}x}+4\lambda\frac{\theta^2x}{\theta+\alpha^2}x^r e^{-\frac{2\theta}{\alpha}x}\\ -2\lambda\frac{\theta\alpha}{\theta+\alpha^2}x^r e^{-\frac{2\theta}{\alpha}x}-2\lambda x^r e^{-\frac{2\theta}{\alpha}x}\left(\frac{(\theta\alpha)^2x}{\left(\theta+\alpha^2\right)^2}\right)+\lambda x^r\left(\frac{2\theta}{\alpha}e^{-\frac{2\theta}{\alpha}x}\right)\left(\frac{\theta\alpha x}{\theta+\alpha^2}\right)^2\end{array}\right]dx.\end{aligned}$$

Then, integrate each part as follow

$$\begin{aligned}\frac{\theta}{\alpha}\int_0^\infty x^r e^{-\frac{\theta}{\alpha}x}dx&=\frac{\theta}{\alpha}\frac{\alpha}{\theta}\left(\frac{\alpha}{\theta}\right)^r\int_0^\infty u^r e^{-u}du\\ &=\left(\frac{\alpha}{\theta}\right)^r\int_0^\infty u^r e^{-u}du.\end{aligned}$$

Let

$$u=\frac{\theta}{\alpha}x,\ du=\frac{\theta}{\alpha}dx,\ dx=\frac{\alpha}{\theta}du,\ and\ x=\frac{\alpha}{\theta}u.$$

Then,

$$\begin{aligned}\frac{\lambda\theta}{\alpha}\int_0^\infty x^r e^{-\frac{\theta}{\alpha}x}dx&=\frac{\lambda\theta}{\alpha}\int_0^\infty\left(\frac{\alpha}{\theta}u\right)^r e^{-u}\frac{\alpha}{\theta}du\\ &=\lambda\left(\frac{\alpha}{\theta}\right)^r\int_0^\infty u^r e^{-u}du\\ &=-\lambda\left(\frac{\alpha}{\theta}\right)^r\Gamma(r+1),\end{aligned}$$

$$\begin{aligned}\frac{4\lambda\theta^2}{\theta+\alpha^2}\int_0^\infty x^{r+1}e^{-\frac{2\theta}{\alpha}x}dx&=\frac{4\lambda\theta^2}{(\theta+\alpha^2)}\frac{\alpha}{2\theta}\left(\frac{\alpha}{2\theta}\right)^{r+1}\int_0^\infty u^{r+1}e^{-u}du\\ &=\frac{4\lambda\theta^2}{(\theta+\alpha^2)}\frac{\alpha}{2\theta}\left(\frac{\alpha}{2\theta}\right)^{r+1}\Gamma(r+2).\end{aligned}$$

Let

$$u=\frac{2\theta}{\alpha}x,\ du=\frac{2\theta}{\alpha}dx,\ dx=\frac{\alpha}{2\theta}du,\ x=\frac{\alpha}{2\theta}u.$$

Similarly, integrating each part of the integration we get

$$
\begin{aligned}
E\left(X^{r}\right) &= \left(\frac{\alpha}{\theta}\right)^{r}\Gamma(r+1) - \lambda\left(\frac{\alpha}{\theta}\right)^{r}\Gamma(r+1) + \frac{\alpha^{2}}{(\theta+\alpha^{2})}\left(\frac{\alpha}{\theta}\right)^{r}\Gamma(r+2) \\
&- \frac{\theta\alpha}{(\theta+\alpha^{2})}\frac{\alpha}{\theta}\left(\frac{\alpha}{\theta}\right)^{r}\Gamma(r+1) - \frac{\lambda\alpha^{2}}{(\theta+\alpha^{2})}\left(\frac{\alpha}{\theta}\right)^{r}\Gamma(r+2) \\
&+ \frac{\lambda\alpha^{2}}{(\theta+\alpha^{2})}\left(\frac{\alpha}{\theta}\right)^{r}\Gamma(r+1) + \lambda\left(\frac{\alpha}{2\theta}\right)^{r}\Gamma(r+1) + \frac{4\lambda\alpha^{2}}{(\theta+\alpha^{2})}\left(\frac{\alpha}{2\theta}\right)^{r}\Gamma(r+2) \\
&- \frac{\lambda\alpha^{2}}{(\theta+\alpha^{2})}\left(\frac{\alpha}{2\theta}\right)^{r}\Gamma(r+1) - \frac{\lambda\alpha^{4}}{2(\theta+\alpha^{2})^{2}}\left(\frac{\alpha}{2\theta}\right)^{r}\Gamma(r+2) + \lambda\left(\frac{\alpha}{2\theta}\right)^{2+r}\left(\frac{\theta\alpha}{\theta+\alpha^{2}}\right)^{2}\Gamma(r+3) \\
&= \left[1-\lambda+\frac{\lambda\alpha^{2}-\alpha^{2}}{\theta+\alpha^{2}}+\frac{\lambda}{2^{r}}\left(\frac{\theta}{\theta+\alpha^{2}}\right)\right]\left(\frac{\alpha}{\theta}\right)^{r}\Gamma(r+1) \\
&+ \left[\frac{\alpha^{2}-\lambda\alpha^{2}}{\theta+\alpha^{2}}+\frac{\lambda\alpha^{2}}{2^{r}}\left(\frac{4}{\theta+\alpha^{2}}-\frac{\alpha^{2}}{2(\theta+\alpha^{2})^{2}}\right)\right]\left(\frac{\alpha}{\theta}\right)^{r}\Gamma(r+2) + \lambda\left(\frac{\alpha}{2\theta}\right)^{2+r}\left(\frac{\theta\alpha}{\theta+\alpha^{2}}\right)^{2}\Gamma(r+3).
\end{aligned}
$$

## *5.2 Variance, Skewness, Kurtosis, Coefficient of Variation*

By (14), the first and second moments can be computed by using $r = 1, 2$, respectively as

$$E\left(X\right) = -\frac{\alpha\left(\alpha^{4}(3\lambda-8)+6\alpha^{2}\theta(\lambda-2)+2\theta^{2}(\lambda-2)\right)}{4\theta\left(\alpha^{2}+\theta\right)^{2}}, \Re\left(\frac{\theta}{\alpha}\right) > 0,$$

and

$$E\left(X^{2}\right) = \frac{\alpha^{2}\left(3\alpha^{4}(8-5\lambda)+8\alpha^{2}\theta(4-3\lambda)+2\theta^{2}(4-3\lambda)\right)}{4\theta^{2}\left(\alpha^{2}+\theta\right)^{2}}, \Re\left(\frac{\theta}{\alpha}\right) > 0.$$

The third and fourth moments of the TJD random variable are

$$E\left(X^{3}\right) = \frac{3\alpha^{3}\left(\alpha^{4}(32-25\lambda)+5\alpha^{2}\theta(8-7\lambda)+\theta^{2}(8-7\lambda)\right)}{4\theta^{3}\left(\alpha^{2}+\theta\right)^{2}}, \Re\left(\frac{\theta}{\alpha}\right) > 0,$$

and

$$E\left(X^{4}\right) = \frac{3\alpha^{4}\left(\alpha^{4}(80-70\lambda)+6\alpha^{2}\theta(16-15\lambda)+\theta^{2}(16-15\lambda)\right)}{2\theta^{4}\left(\alpha^{2}+\theta\right)^{2}}, \Re\left(\frac{\theta}{\alpha}\right) > 0.$$

The variance of $X$ is

$$Var(X) = \frac{\alpha^{2}\begin{pmatrix} 4\left(\alpha^{2}+\theta\right)^{2}\left(3\alpha^{4}(8-5\lambda)+8\alpha^{2}\theta(4-3\lambda)+2\theta^{2}(4-3\lambda)\right) \\ -\left(\alpha^{4}(3\lambda-8)+6\alpha^{2}\theta(\lambda-2)+2\theta^{2}(\lambda-2)\right)^{2} \end{pmatrix}}{16\theta^{2}\left(\alpha^{2}+\theta\right)^{4}}, \Re\left(\frac{\theta}{\alpha}\right) > 0. \quad (15)$$

The Skewness (Sk) and Kurtosis (Ku) of the TJD random variable, respectively, are defined as

$$Sk = -\frac{2\begin{pmatrix} 6\lambda^{2}\left(3\alpha^{4}+6\alpha^{2}\theta+2\theta^{2}\right)\left(3\alpha^{6}+9\alpha^{4}\theta+10\alpha^{2}\theta^{2}+2\theta^{3}\right)\left(\alpha^{2}+\theta\right) \\ -64\left(2\alpha^{6}+6\alpha^{4}\theta+6\alpha^{2}\theta^{2}+\theta^{3}\right)\left(\alpha^{2}+\theta\right)^{3}+\lambda^{3}\left(3\alpha^{4}+6\alpha^{2}\theta+2\theta^{2}\right)^{3} \\ +24\lambda\left(\alpha^{8}+4\alpha^{6}\theta+7\alpha^{4}\theta^{2}+7\alpha^{2}\theta^{3}+\theta^{4}\right)\left(\alpha^{2}+\theta\right)^{2} \end{pmatrix}}{\begin{pmatrix} -\lambda^{2}\left(3\alpha^{4}+6\alpha^{2}\theta+2\theta^{2}\right)^{2}+16\left(2\alpha^{4}+4\alpha^{2}\theta+\theta^{2}\right)\left(\alpha^{2}+\theta\right)^{2} \\ -4\lambda\left(3\alpha^{6}+9\alpha^{4}\theta+10\alpha^{2}\theta^{2}+2\theta^{3}\right)\left(\alpha^{2}+\theta\right) \end{pmatrix}^{3/2}}, \Re\left(\frac{\theta}{\alpha}\right) > 0, \quad (16)$$

and

$$Ku = \frac{768\theta^4(\alpha^2+\theta)^8}{\alpha^4}\frac{\begin{pmatrix} -8\lambda^3\left(3\alpha^4+6\alpha^2\theta+2\theta^2\right)^2\left(3\alpha^6+9\alpha^4\theta+10\alpha^2\theta^2+2\theta^3\right)\left(\alpha^2+\theta\right) \\ -32\lambda^2\left(\alpha^2+2\theta\right)\left(3\alpha^4+6\alpha^2\theta+2\theta^2\right)\left(8\alpha^6+16\alpha^4\theta+13\alpha^2\theta^2+2\theta^3\right)\left(\alpha^2+\theta\right)^2 \\ +256\left(8\alpha^8+32\alpha^6\theta+44\alpha^4\theta^2+24\alpha^2\theta^3+3\theta^4\right)\left(\alpha^2+\theta\right)^4-\lambda^4\left(3\alpha^4+6\alpha^2\theta+2\theta^2\right)^4 \\ -128\lambda\left(6\alpha^{10}+30\alpha^8\theta+60\alpha^6\theta^2+58\alpha^4\theta^3+27\alpha^2\theta^4+3\theta^5\right)\left(\alpha^2+\theta\right)^3 \end{pmatrix}}{\begin{pmatrix} \alpha^8(3\lambda-4)(3\lambda+8)+4\alpha^6\theta(3\lambda-4)(3\lambda+8)+4\alpha^4\theta^2(\lambda(12\lambda+19)-44) \\ +24\alpha^2\theta^3(\lambda(\lambda+2)-4)+4\theta^4(\lambda(\lambda+2)-4) \end{pmatrix}^4}, \Re\left(\frac{\theta}{\alpha}\right)>0. \tag{17}$$

The coefficient of variation (CV) of the TJD is

$$CV = -\frac{\sqrt{\alpha^2\begin{pmatrix} 4\left(\alpha^2+\theta\right)^2\left(3\alpha^4(8-5\lambda)+8\alpha^2\theta(4-3\lambda)+2\theta^2(4-3\lambda)\right) \\ -\left(\alpha^4(3\lambda-8)+6\alpha^2\theta(\lambda-2)+2\theta^2(\lambda-2)\right)^2 \end{pmatrix}}}{\alpha^5(3\lambda-8)+6\alpha^3\theta(\lambda-2)+2\alpha\theta^2(\lambda-2)}, \Re\left(\frac{\theta}{\alpha}\right)>0. \tag{18}$$

In Table (1), we summarized some values of the mean, variance, coefficient of variation, skewness and kurtosis of the TJD.

**Table 1:** Some values of the mean, variance, coefficient of variation, skewness and kurtosis of the TJD with $\lambda = 1, 0.8, 0.7, 0.3$, 0,- 0.3, - 0.7, - 0.8, - 1.

| $\lambda$ | **Mean** | **Var** | CV | Sk | Ku |
|---|---|---|---|---|---|
| | | $\theta = 3$ and $\alpha = 5$ | | | |
| 1 | 1.90955 | 1.88351 | 0.718710 | 1.28595 | 1.521620 |
| 0.8 | 2.15859 | 2.85963 | 0.783403 | 1.80361 | 1.070450 |
| 0.7 | 2.28311 | 3.30117 | 0.795806 | 1.81677 | 0.786177 |
| 0.6 | 2.40763 | 3.71171 | 0.800197 | 1.78350 | 0.592897 |
| 0.5 | 2.53215 | 4.09123 | 0.798798 | 1.72989 | 0.461531 |
| 0 | 3.15476 | 5.52367 | 0.744984 | 1.42560 | 0.197783 |
| -0.5 | 3.77737 | 6.18083 | 0.658163 | 1.22320 | 0.139494 |
| -0.6 | 3.90189 | 6.21922 | 0.639135 | 1.20185 | 0.136514 |
| -0.7 | 4.02641 | 6.22661 | 0.619737 | 1.18859 | 0.135639 |
| 0.8 | 4.15094 | 6.20298 | 0.600004 | 1.18459 | 0.136818 |
| -1 | 4.39998 | 6.06270 | 0.559610 | 1.21103 | 0.145720 |
| | | $\theta = 6$ and $\alpha = 2$ | | | |
| 1 | 0.246667 | 0.052489 | 0.928802 | 1.71257 | 2605.79 |
| 0.8 | 0.290667 | 0.086180 | 1.009970 | 2.27783 | 1570.35 |
| 0.7 | 0.312667 | 0.101573 | 1.019310 | 2.25307 | 1078.81 |
| 0.6 | 0.334667 | 0.115998 | 1.017680 | 2.18382 | 772.889 |
| 0.5 | 0.356667 | 0.129456 | 1.008780 | 2.09921 | 577.448 |
| 0 | 0.466667 | 0.182222 | 0.914732 | 1.69887 | 216.008 |
| -0.5 | 0.576667 | 0.210789 | 0.796158 | 1.44331 | 137.127 |
| -0.6 | 0.598667 | 0.213597 | 0.771994 | 1.41180 | 131.293 |
| -0.7 | 0.620667 | 0.215440 | 0.747832 | 1.38758 | 127.522 |
| -0.8 | 0.642667 | 0.216313 | 0.723695 | 1.37128 | 125.611 |
| -1 | 0.686670 | 0.215160 | 0.675510 | 1.36616 | 127.059 |

From Table (1), we can see that the suggested TJD is asymmetric and the values of the variance when $\theta = 6$ and $\alpha = 2$ are less than that of $\theta = 3$ and $\alpha = 5$, while the mean values are smaller.

### 5.3 Moment Generating Function

The moment generating function $E\left(e^{tX}\right)$ of the transmuted Janardan distribution is given by the following theorem.

**Theorem 3:** If the random variable $X$ is from a $\Phi_{TJD}(x,\alpha,\theta,\lambda)$, then the moment generating function is defined as

$$E\left(e^{tX}\right)=\frac{(1-\lambda)\theta}{\frac{\theta}{\alpha}-t}\left(\frac{1}{\alpha}-\frac{\alpha}{\theta+\alpha^2}\right)+\frac{(1-\lambda)\theta^2}{\left(\frac{\theta}{\alpha}-t\right)^2(\theta+\alpha^2)}+\frac{2\theta^2\lambda}{\left(\frac{2\theta}{\alpha}-t\right)^2}\left(\frac{2\theta+\alpha^2}{(\theta+\alpha^2)^2}\right)+\frac{2\theta\lambda}{\frac{2\theta}{\alpha}-t}\left(\frac{1}{\alpha}-\frac{\alpha}{\theta+\alpha^2}\right)+\frac{4\alpha\lambda\theta^3}{\alpha\left(\frac{2\theta}{\alpha}-t\right)^3(\theta+\alpha^2)^2}. \tag{19}$$

**Proof:**
Assume that $X$ is a random variable follow the TJD, then the moment generating function is

$$E(e^{tx})=\int_0^\infty e^{tx}\left\{\begin{array}{c}\frac{\theta}{\alpha}e^{-\frac{\theta}{\alpha}x}-\lambda\frac{\theta}{\alpha}e^{-\frac{\theta}{\alpha}x}+\frac{\theta^2x}{\theta+\alpha^2}e^{-\frac{\theta}{\alpha}x}-\frac{\theta\alpha}{\theta+\alpha^2}e^{-\frac{\theta}{\alpha}x}-\lambda\frac{\theta^2x}{\theta+\alpha^2}e^{-\frac{\theta}{\alpha}x}+\lambda\frac{\theta\alpha}{\theta+\alpha^2}e^{-\frac{\theta}{\alpha}x}\\ +\lambda\frac{2\theta}{\alpha}e^{-\frac{2\theta}{\alpha}x}+4\lambda\frac{\theta^2x}{\theta+\alpha^2}e^{-\frac{2\theta}{\alpha}x}-2\lambda\frac{\theta\alpha}{\theta+\alpha^2}e^{-\frac{2\theta}{\alpha}x}-2\lambda e^{-\frac{2\theta}{\alpha}x}\left[x\left(\frac{\theta\alpha}{\theta+\alpha^2}\right)^2\right]+\lambda\frac{2\theta}{\alpha}e^{-\frac{2\theta}{\alpha}x}\left(\frac{\theta\alpha x}{\theta+\alpha^2}\right)^2\end{array}\right\}dx$$

$$=\left(\frac{\theta}{\alpha}-\lambda\frac{\theta}{\alpha}-\frac{\theta\alpha}{\theta+\alpha^2}+\lambda\frac{\theta\alpha}{\theta+\alpha^2}\right)\int_0^\infty e^{tx}e^{-\frac{\theta}{\alpha}x}dx+\left(\frac{\theta^2}{\theta+\alpha^2}-\lambda\frac{\theta^2}{\theta+\alpha^2}\right)\int_0^\infty xe^{tx}e^{-\frac{\theta}{\alpha}x}dx$$

$$+\left(4\lambda\frac{\theta^2}{\theta+\alpha^2}-2\lambda\frac{(\theta\alpha)^2}{(\theta+\alpha^2)^2}\right)\int_0^\infty xe^{tx}e^{-\frac{2\theta}{\alpha}x}dx+\left(\lambda\frac{2\theta}{\alpha}-2\lambda\frac{\theta\alpha}{\theta+\alpha^2}\right)\int_0^\infty e^{tx}e^{-\frac{2\theta}{\alpha}x}dx$$

$$+\lambda\frac{2\theta}{\alpha}\left(\frac{\theta\alpha}{\theta+\alpha^2}\right)^2\int_0^\infty x^2e^{tx}e^{-\frac{2\theta}{\alpha}x}dx.$$

Then, each part of the integral can be calculated by

$$\left(\frac{\theta}{\alpha}-\lambda\frac{\theta}{\alpha}-\frac{\theta\alpha}{\theta+\alpha^2}+\lambda\frac{\theta\alpha}{\theta+\alpha^2}\right)\int_0^\infty e^{-x\left(\frac{\theta}{\alpha}-t\right)}dx=\frac{1}{\frac{\theta}{\alpha}-t}\left(\frac{\theta}{\alpha}-\lambda\frac{\theta}{\alpha}-\frac{\theta\alpha}{\theta+\alpha^2}+\lambda\frac{\theta\alpha}{\theta+\alpha^2}\right), \tag{20}$$

$$\left(\frac{\theta^2}{\theta+\alpha^2}-\lambda\frac{\theta^2}{\theta+\alpha^2}\right)\int_0^\infty xe^{-\left(x\frac{\theta}{\alpha}-t\right)}dx=\frac{1}{\left(\frac{\theta}{\alpha}-t\right)^2}\left(\frac{\theta^2}{\theta+\alpha^2}-\lambda\frac{\theta^2}{\theta+\alpha^2}\right), \tag{21}$$

$$\lambda\frac{2\theta}{\alpha}\left(\frac{\theta\alpha}{\theta+\alpha^2}\right)^2\int_0^\infty x^2e^{-x\left(\frac{2\theta}{\alpha}-t\right)}dx=\lambda\frac{2\theta}{\alpha}\frac{2}{\left(\frac{2\theta}{\alpha}-t\right)^3}\left(\frac{\theta\alpha}{\theta+\alpha^2}\right)^2. \tag{22}$$

Similarly, integrate all the integrals and combine Equations (20), (21) and (22) to get the proof.

## 6 Parameters Estimation and Entropy

The maximum likelihood estimators (MLEs) of the distribution parameters and the entropy are derived in this section.

### 6.1 Maximum likelihood estimation

The maximum likelihood estimators (MLEs) of the TJD parameters $\theta,\alpha,\lambda$ are derived from complete samples only. Let $X_1,X_2,\ldots,X_n$ be a random sample of size $n$ from a $\Phi_{TJD}(x,\alpha,\theta,\lambda)$ distribution, then the likelihood function can be defined as

$$\begin{aligned}L(x;\theta,\alpha,\lambda)&=\prod_{i=1}^n\psi(x_i)\\&=\prod_{i=1}^n\frac{\theta^2(\alpha x_i+1)e^{-\frac{2\theta x_i}{\alpha}}}{\alpha(\alpha^2+\theta)^2}\left[2\lambda\left(\alpha^2+\theta+\alpha\theta x_i\right)-(\lambda-1)\left(\alpha^2+\theta\right)e^{\frac{\theta x_i}{\alpha}}\right]\\&=\frac{\theta^{2n}}{\alpha^n(\alpha^2+\theta)^{2n}}\prod_{i=1}^n(1+\alpha x_i)e^{-\frac{2\theta x_i}{\alpha}}\left[2\lambda\left(\alpha^2+\theta+\alpha\theta x_i\right)-(\lambda-1)\left(\alpha^2+\theta\right)e^{\frac{\theta x_i}{\alpha}}\right].\end{aligned} \tag{23}$$

Hence, the log of the likelihood function $\ell = \ln L$ is

$$\ell = 2n\ln\theta - n\ln\alpha - 2n\ln\left(\alpha^2+\theta\right) + \sum_{i=1}^{n}\ln\left(1+\alpha x_i\right) - \frac{2\theta}{\alpha}\sum_{i=1}^{n}x_i + \sum_{i=1}^{n}\left[2\lambda\left(\alpha^2+\theta+\alpha\theta x_i\right) - (\lambda-1)\left(\alpha^2+\theta\right)e^{\frac{\theta x_i}{\alpha}}\right]. \quad (24)$$

The normal equations become

$$\frac{\partial\ell}{\partial\theta} = \frac{2n}{\theta} - \frac{2n}{\alpha^2+\theta} - \frac{2}{\alpha}\sum_{i=1}^{n}x_i + \sum_{i=1}^{n}\left\{\frac{2\lambda\left(1+\alpha x_i\right) - (\lambda-1)\left[\left(\alpha^2+\theta\right)\frac{x_i}{\alpha}e^{\frac{\theta x_i}{\alpha}}\right]}{2\lambda\left(\alpha^2+\theta+\alpha\theta x_i\right) - (\lambda-1)\left(\alpha^2+\theta\right)e^{\frac{\theta x_i}{\alpha}}}\right\}, \quad (25)$$

$$\frac{\partial\ell}{\partial\lambda} = \sum_{i=1}^{n}\left\{\frac{2\left[\left(\alpha^2+\theta+\alpha\theta x_i\right)\right] - \left(\alpha^2+\theta\right)e^{\frac{\theta x_i}{\alpha}}}{2\lambda\left(\alpha^2+\theta+\alpha\theta x_i\right) - (\lambda-1)\left(\alpha^2+\theta\right)e^{\frac{\theta x_i}{\alpha}}}\right\}, \quad (26)$$

$$\frac{\partial\ell}{\partial\alpha} = \frac{-n}{\alpha} + \sum_{i=1}^{n}\left(\frac{x_i}{1+\alpha x_i}\right) + \frac{2\theta}{\alpha^2}\sum_{i=1}^{n}x_i + \sum_{i=1}^{n}\left\{\frac{2\lambda\left(2\alpha+\theta x_i\right) - (\lambda-1)\left[\left(\alpha^2+\theta\right)\frac{-\theta x_i}{\alpha^2} + 2\alpha\right]e^{\frac{\theta x_i}{\alpha}}}{2\lambda\left(\alpha^2+\theta+\alpha\theta x_i\right) - (\lambda-1)\left(\alpha^2+\theta\right)e^{\frac{\theta x_i}{\alpha}}}\right\}. \quad (27)$$

The MLEs of $\alpha, \theta$ and $\lambda$ can be obtained by setting the Equations (25), (26) and (27) to zero and solving them simultaneously.

## *6.2 Entropy*

**Theorem 4:** The Renyi entropy of the transmuted Janardan distribution can be defined as

$$E_R(\rho) = \frac{1}{1-\rho}\log\,(1-\lambda)^{\rho}\sum_{i=0}^{\rho}\sum_{k=0}^{\rho}\sum_{m=0}^{\rho-i}\sum_{j}^{-\rho+i}\sum_{n}^{m}2^{\rho-i}\binom{\rho}{i}\binom{\rho}{k}\binom{\rho-i}{m}\binom{i-\rho}{j}\binom{m}{n} \times\left(\frac{\lambda}{1-\lambda}\right)^{\rho-i}\left(\theta+\alpha^2\right)^{-m-\rho}\left(\theta+\alpha^2-\alpha\right)^{-k+i-j}(2\rho-i)^{-D}\alpha^{R}\theta^{S}\Gamma(D). \quad (28)$$

where $D = k+m+j+n+1$, $R = 3m-\rho+k+j+n+1$, $S = 2\rho-i-n-1$.
**Proof:** The Renyi entropy is defined as

$$E_R(\rho) = \frac{1}{1-\rho}\log\int_0^{\infty}f(x)^{\rho}dx.$$

Now,

$$(g(x))^{\rho} = \left\{(1-\lambda)\left(\frac{\theta(\theta+\alpha^2)+\theta^2x-\theta\alpha}{\alpha(\theta+\alpha^2)}\right)e^{-\frac{2\theta}{\alpha}x}\left[\frac{\lambda\left[2\theta\left(\theta+\alpha^2\right)+2\alpha\theta x\left[2\theta+\alpha\left(\alpha+\theta x\right)\right]\right]}{(1-\lambda)(\theta+\alpha^2)\left(\theta\left[1+x\right]+\alpha\left[\alpha-1\right]\right)}+e^{\frac{\theta}{\alpha}x}\right]\right\}^{\rho}$$
$$= (1-\lambda)^{\rho}\left(\frac{\theta(\theta+\alpha^2)+\theta^2x-\theta\alpha}{\alpha(\theta+\alpha^2)}\right)^{\rho}\left(e^{-\frac{2\theta}{\alpha}x}\right)^{\rho}\left[\frac{\lambda\left[2\theta\left(\theta+\alpha^2\right)+2\alpha\theta x\left[2\theta+\alpha\left(\alpha+\theta x\right)\right]\right]}{(1-\lambda)(\theta+\alpha^2)\left(\theta\left[1+x\right]+\alpha\left[\alpha-1\right]\right)}+e^{\frac{\theta}{\alpha}x}\right]^{\rho}.$$

Use the binomial series to expand the sum expressions in the integral and rearrange them, the entropy becomes as follows

$$E_R(\rho) = \frac{1}{1-\rho}\log(1-\lambda)^{\rho}\sum_{i=0}^{\rho}\binom{\rho}{i}\left(\frac{1}{\alpha(\theta+\alpha^2)}\right)^{\rho}\theta^{2\rho}\left(\frac{\lambda}{(1-\lambda)(\theta+\alpha^2)}\right)^{\rho-i}$$
$$\times\sum_{k=0}^{\rho}\sum_{m=0}^{\rho-i}\sum_{j}^{-\rho+i}\sum_{n}^{m}\binom{\rho}{k}\binom{\rho-i}{m}\binom{i-\rho}{j}\binom{m}{n}\left(\frac{\theta+\alpha^2-\alpha}{\theta}\right)^{\rho-k}\left[2\theta\left(\theta+\alpha^2\right)\right]^{\rho-i-m}(2\alpha\theta)^{m}\theta^{i-\rho}(\alpha\theta)^{m}$$
$$\times\left(\frac{\theta+\alpha^2-\alpha}{\theta}\right)^{-\rho+i-j}\left(\frac{2\theta+\alpha^2}{\alpha\theta}\right)^{m-n}\int_0^{\infty}\left[e^{-x\left(\frac{2\theta\rho-\theta i}{\alpha}\right)}x^{k+m+j+n}\right]dx$$

Since the integral is very sophisticated, again we use the binomial series and we use some manipulation algebra, we get

$$E_R(\rho) = \frac{1}{1-\rho}\log(1-\lambda)^{\rho}\sum_{i=0}^{\rho}\binom{\rho}{i}\left(\frac{1}{\alpha(\theta+\alpha^2)}\right)^{\rho}\theta^{2\rho}\left(\frac{\lambda}{(1-\lambda)(\theta+\alpha^2)}\right)^{\rho-i}$$
$$\times\sum_{k=0}^{\rho}\sum_{m=0}^{\rho-i}\sum_{j}^{i-\rho}\sum_{n}^{m}\binom{\rho}{k}\binom{\rho-i}{m}\binom{i-\rho}{j}\binom{m}{n}\left(\frac{\theta+\alpha^2-\alpha}{\theta}\right)^{\rho-k}\left[2\theta\left(\theta+\alpha^2\right)\right]^{\rho-i-m}(2\alpha\theta)^m\theta^{i-\rho}(\alpha\theta)^m$$
$$\times\left(\frac{\theta+\alpha^2-\alpha}{\theta}\right)^{i-j-\rho}\left(\frac{2\theta+\alpha^2}{\alpha\theta}\right)^{m-n}\int_0^{\infty}\left[e^{-x\left(\frac{2\theta\rho-\theta i}{\alpha}\right)}x^{k+m+j+n}\right]dx$$

Integrate the integrals separately as follows. For

$$\int_0^{\infty}e^{-x\left(\frac{2\theta\rho-\theta i}{\alpha}\right)}x^{k+m+j+n}dx.$$

Let

$$u = x\left(\frac{2\theta\rho-\theta i}{\alpha}\right),\quad then\quad x=\frac{\alpha}{2\theta\rho-\theta i}u\ and\ dx=\frac{\alpha}{2\theta\rho-\theta i}du$$

Therefore,

$$\int_0^{\infty}e^{-u}\left(\frac{\alpha}{2\theta\rho-\theta i}u\right)^{k+m+j+n}\frac{\alpha}{2\theta\rho-\theta i}du=\left(\frac{\alpha}{2\theta\rho-\theta i}\right)^{k+m+j+n+1}\Gamma(k+m+j+n+1),$$

and hence

$$E_R(\rho) = \frac{1}{1-\rho}\log(1-\lambda)^{\rho}\sum_{i=0}^{\rho}\sum_{k=0}^{\rho}\sum_{m=0}^{\rho-i}\sum_{j}^{i-\rho}\sum_{n}^{m}\binom{\rho}{i}\binom{\rho}{k}\binom{\rho-i}{m}\binom{i-\rho}{j}\binom{m}{n}\left(\frac{1}{\alpha(\theta+\alpha^2)}\right)^{\rho}$$
$$\times\theta^{2\rho}\left(\frac{\lambda}{(1-\lambda)(\theta+\alpha^2)}\right)^{\rho-i}\left(\frac{\theta+\alpha^2-\alpha}{\theta}\right)^{\rho-k}\left[2\theta\left(\theta+\alpha^2\right)\right]^{\rho-i-m}(2\alpha\theta)^m\theta^{-\rho+i}(\alpha\theta)^m$$
$$\times\left(\frac{\theta+\alpha^2-\alpha}{\theta}\right)^{-\rho+i-j}\left(\frac{\alpha}{2\theta\rho-\theta i}\right)^{k+m+j+n+1}\Gamma(k+m+j+n+1).$$

Simplify and rearrange we get

$$E_R(\rho) = \frac{1}{1-\rho}\log(1-\lambda)^{\rho}\sum_{i=0}^{\rho}\sum_{k=0}^{\rho}\sum_{m=0}^{\rho-i}\sum_{j=0}^{-\rho+i}\sum_{n=0}^{m}2^{\rho-i}\binom{\rho}{i}\binom{\rho}{k}\binom{\rho-i}{m}\binom{i-\rho}{j}\binom{m}{n}$$
$$\times\left(\frac{\lambda}{1-\lambda}\right)^{\rho-i}\left(\theta+\alpha^2\right)^{-m-\rho}\left(\theta+\alpha^2-\alpha\right)^{-k+i-j}(2\rho-i)^{-(k+m+j+n+1)}$$
$$\times\alpha^{3m-\rho+k+j+n+1}\theta^{2\rho-i-n-1}\Gamma(k+m+j+n+1).$$

## 7 Conclusion

In the present study, we have introduced a new distribution called a transmuted Janardan distribution has been discussed. Some statistical properties of this distribution are addressed as: the moments, reliability and hazard functions, moment generating function, order statistics, estimation of parameters by the method of maximum likelihood, and entropy. We hope that suggested distribution may attract several applications in modeling of real data.

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