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On the Convergence of the Preconditioned Group Rotated Iterative Methods In The Solution of Elliptic PDEs

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The convergence rates of the explicit group methods derived from the standard and skewed (rotated) finite difference operators depend on the spectral properties of the coefficient matrices resulted from these group discretization formulas. By applying appropriate preconditioner, we may transform the resulting linear system into another equivalent system that has the same solution, but has a better spectral property than its unpreconditioned form. In Saeed and Ali [11], the application of a specific splitting-type block preconditioner to the Explicit Decoupled Group Successive Over-Relaxation (EDG SOR) method was presented where the preconditioned scheme was shown to have a better rate of convergence compared to its unpreconditioned counterpart. In this paper, some new Fundamental theorems and lemmas related to the convergence properties of this preconditioned scheme will be established and presented.

Keywords: Preconditioning method, Explicit decoupled group (EDG) method, π-consistently ordered matrix.

1 Introduction

In solving systems of linear equations arising from practical scientific and engineering modelling and simulations such as electromagnetic applications, it is critical to choose a fast and robust solver. Improved techniques using explicit group methods derived from the standard and skewed (rotated) finite difference operators have been developed over the last few years in solving the linear systems that arise from the discretization of the elliptic partial differential equation ([1], [2], [3], [4], [5], [9], [10], [16]). It is widely recognized that preconditioning is the most critical ingredient in the development of efficient solvers for challenging problems in scientific computation. A good preconditioner should be constructed inexpensively and should be a good approximation to the inverse of coefficient matrix of the iterative method. Many researchers have investigated preconditioners applied to linear systems. For example, Gunawardena et al. [6] introduced a preconditioner that improves the convergence rate of the Gauss-Seidel method. Such work has been further enhanced by Usui et al. [13]. Martins et al. [8] analyzed and verified the superiority of the preconditioner proposed by Usui et al. [13] theoretically. In Lee [7], preconditioners have been successfully applied to the rotated five point formula in solving the Poisson problem with promising results. Recently, Saeed and Ali [11], presented a $(I + \bar{S})$ -type preconditioning matrix applied to the original system obtained from the four point Explicit Decoupled Group (EDG) method for solving the elliptic partial differential equation, where $\bar{S}$ is obtained by taking the first upper diagonal groups of the iteration matrix of the original system. The focus of this study is to establish the convergence properties of the preconditioning techniques for improving the performance and reliability of the explicit group methods derived from the rotated finite difference formula. In this paper we will prove that, under certain conditions, the rate of convergence of the EDG SOR iterative method can be enlarged if the above preconditioner $(I + \bar{S})$ applied to this method. The proof depends on the spectral properties of the coefficient matrices resulted from these group discretization formulas. This work is structured in 5 Sections: In Section 2, we present some definitions and preliminary results related to the under study preconditioning techniques. A brief description of the application of the preconditioner in block formulation to the EDG SOR is given in Section 3. The theoretical convergence analysis of this method is discussed in Section 4. Finally, we report a brief conclusion in Section 5.

2 Preparatory Knowledge

For convenience, we shall now briefly explain some of the definitions and theorems used in this paper. We will denote the spectral radius of a matrix by $\rho(.)$, which is defined as the largest of the moduli of the eigenvalues of the iteration matrix.

Theorem 2.1. *([14]). Let A and B be two n× n matrices with* $0 \leq |B| \leq A$. *Then,* $\rho(B) \leq \rho(A)$.

For the following theorem we consider expressing the matrix *A* in the form $A = M - N$, where *M* and *N* are also *n* × *n* matrices. If *M* is non-singular, we say that this expression represent a splitting of the matrix *A*, and associated with this splitting is an iterative method

$$Mx^{(m+1)} = Nx^{(m)} + k, \quad m \geq 0 \tag{2.1}$$

which we write equivalently as $x^{(m+1)} = M^{-1}Nx^{(m)} + M^{-1}k, \quad m \geq 0$

Theorem 2.2. *([14]). If A=M-N is a regular splitting of the matrix A and* $A^{-1} \geq 0$, *then* $\rho(M^{-1}N) = \frac{\rho(A^{-1}N)}{1+\rho(A^{-1}N)} < 1$.

Thus, the matrix $M^{-1}N$ is convergent, and the iterative method of (2.1) converges for any initial vector $x^{(0)}$.

Definition 2.1. ([14]). A matrix *A* of order *n* has property *A* if there exists two disjoint subsets *S* and *T* of $W = \{1, 2, ..., n\}$ such that if *i* ≠ *j* and if either $a_{ij} \neq 0$ and $a_{ji} \neq 0$, then *i* ∋ *S* and *j* ∋ *T* or else *i* ∋ *T* and *j* ∋ *S*.

Definition 2.2. ([15]). An ordered grouping π of $W = \{1, 2, ..., n\}$ is a subdivision of *W* into disjoint subsets $R_1, R_2, ..., R_q$ such that $R_1 + R_2 + ... + R_q = W$.

Given a matrix *A* and and an ordered grouping π we define the submatrices $A_{m,n}$ for $m, n = 1, 2, ..., q$ as follows: $A_{m,n}$ is formed from *A* deleting all rows except those corresponding to R_m and all columns except those corresponding to R_n.

Definition 2.3. ([15]). Let π be an ordered grouping with *q* groups. A matrix *A* has Property $A^{(\pi)}$ if the $q \times q$ matrix $Z = (z_{r,s})$ defined by $z_{r,s} = \{0 \ if \ A_{r,s} = 0 \ or \ 1 \ if \ A_{r,s} \neq 0\}$ has Property *A*.

Definition 2.4. ([15]). A matrix *A* of order *n* is consistently ordered if for some *t* there exist disjoint subsets $S_1, S_2, ..., S_t$ of $W = \{1, 2, ..., n\}$ such that $\sum_{k=1}^{t} S_k = W$ and such that if *i* and *j* are associated, then $j \in S_{k+1}$ if $j > i$ and $j \in S_{k-1}$ if $j < i$,where S_k is the subset containing *i*. Note that a matrix A is a π-consistently ordered matrix if the matrix *Z* is consistently ordered.

An accurate analysis of convergence properties of the SOR method is possible if the matrix *A* is consistently ordered in the following sense (see [12]).

Definition 2.5. For given positive integers *q* and *r*, the matrix *A* of ordered *N* is a *(q,r)*-consistently ordered matrix (a CO*(q,r)*-matrix) if for some *t*, there exist disjoint subsets $S_1, S_2, ..., S_t$ of $W = \{1, 2, ..., N\}$ such that $\sum_{k=1}^{t} S_k = W$ and such that: if $a_{i,j} \neq 0$ and $i < j$, then $i \in S_1 + S_2 + ... + S_{t-r}$ and $j \in S_{k+r}$, where S_k is the subset containing *i*; if $a_{i,j} \neq 0$ and $i > j$, then $i \in S_{q+1} + S_{q+2} + ... + S_t$ and $j \in S_{k-q}$ where S_k is the subset containing *i*.

Definition 2.6. A matrix *A* is a generalized *(q,r)*-consistently ordered matrix (a GCO*(q,r)*-matrix) if: $\Delta = \det(\alpha^q E + \alpha^{-r} F - kD)$ is independent of α for all $\alpha \neq 0$ and for all *k*. Here *D=diag A* and *E* and *F* are strictly lower and strictly upper triangular matrices, respectively, such that: *A=D- E-F* .

In this paper, we will denote the block Jacobi iterative of *A* by $B_J(A)$ such that $B_J = L + U$.

3 Preconditioned Explicit Decoupled Group SOR (EDG SOR)

The linear system

$$A\tilde{u} = \tilde{b} \tag{3.1}$$

is obtained after the discretisation of the Poisson equation using the EDG finite difference scheme [11] where

$$A = \begin{bmatrix} R_0 & R_1 & & & \\ R_2 & R_0 & R_1 & & \\ & R_2 & R_0 & \ddots & \\ & & \ddots & \ddots & R_1 \\ & & & R_2 & R_0 \end{bmatrix}_{\frac{(N-1)^2}{2} \times \frac{(N-1)^2}{2}},$$

$$R_0 = \begin{bmatrix} R_{00} & R_{01} & & \\ R_{02} & R_{00} & \ddots & \\ & \ddots & \ddots & R_{01} \\ & & R_{02} & R_{00} \end{bmatrix}_{(N-1)\times(N-1)},$$

$$R_{00} = \begin{bmatrix} 1 & -\frac{1}{4} \\ -\frac{1}{4} & 1 \end{bmatrix}, R_{01} = \begin{bmatrix} 0 & 0 \\ -\frac{1}{4} & 0 \end{bmatrix}, R_{02} = R_{01}^T,$$

$$R_1 = \begin{bmatrix} R_{01} & R_{01} & & \\ & R_{01} & \ddots & \\ & & \ddots & R_{01} \\ & & & R_{01} \end{bmatrix}_{(N-1)\times(N-1)},$$

$$R_2 = \begin{bmatrix} R_{02} & & & \\ R_{02} & R_{02} & & \\ & \ddots & \ddots & \\ & & R_{02} & R_{02} \end{bmatrix}_{(N-1)\times(N-1)}$$

It is observed that the partitioning of A is in the following block form:

$$A = \begin{bmatrix} A_{11} & A_{12} & & & \\ A_{21} & A_{22} & A_{23} & & \\ & A_{32} & A_{33} & \ddots & \\ & & \ddots & \ddots & A_{(p-1)p} \\ & & & A_{p(p-1)} & A_{pp} \end{bmatrix} \tag{3.2}$$

with *p=(N-1)*, where $A_{ii} \in \mathbb{C}_{\pi,p}^{n_i,n_i}$, $i = 1, 2, ..., p$ and $\sum_{i=1}^{p} n_i = n$. Let $\mathbb{C}_{\pi,p}^{n_i,n_i}$ denote the set of all matrices in $\mathbb{C}^{n_i,n_i}$ which are of the form (3.2) relative to some given block partitioning π. Let $A \in \mathbb{C}_{\pi,p}^{n_i,n_i}$ be written as *A = D - E - F*, where $D = diag(A_{11}, A_{22}, ..., A_{pp})$ and

$$E = (E_{ij}) = \begin{cases} -A_{ij} \; for \; j < i \\ 0 \quad for \quad j \geqslant i \end{cases}, \quad F = (F_{ij}) = \begin{cases} -A_{ij} \; for \; j > i \\ 0 \quad for \quad j \leqslant i \end{cases} \tag{3.3}$$

are block matrices consisting of the block diagonal, strict block lower triangular, and strict block upper triangular parts of *A*. Here the diagonal entries A_{ii} are nonsingular. The block Jacobi iteration matrix is $B_J = D^{-1}(E + F) = L + U$, where $L = D^{-1}E, U = D^{-1}F$, the block Gauss-seidel iteration matrix is $B_{GS} = (I - L)^{-1}U$, and the Block Successive Over-Relaxation method (BSOR) iteration matrix is

$$B_{\ell_w} = (I - wL)^{-1}\{(1 - w)I + wU\} \tag{3.4}$$

Since the matrix *A* of Eq. (3.2) is a π- consistently ordered and possesses property $A^{(\pi)}$ (Abdullah [1]), therefore the theory of block SOR is valid for this iterative method.
For convenience, we refer to the generally consistently ordered *(π,q,r)* of a matrix *A* as the following:

Definition 3.1. ([12]). A matrix *A* of the form (3.2) is said to be generally consistently ordered *(π,q,r)* or simply GCO*(π,q,r)*,where *q* and *r* are positive integers, if for the partitioning π of *A*, the diagonal submatrices A_{ii}, $i = 1, 2, ..., p(\geqslant 2)$ are non-singular, and the eigenvalues of:

$$B_J(\alpha) = \alpha^r L + \alpha^{-q} U \tag{3.5}$$

are independent of α, for all $\alpha \neq 0$, where *L* and *U* are given in (3.4).

For any matrix $C = (c_{ij})$ in $\mathbb{C}_{\pi,p}^{n_i,n_i}$, let $|C|$ denote the block matrix in $\mathbb{C}_{\pi,p}^{n_i,n_i}$ with entries $|c_{i,j}|$. Given the matrix:

$$B_J = L + U \tag{3.6}$$

Let $\bar{\mu}$ denote the spectral radius of matrix:

$$|B_J| = |L + U| = |L| + |U| \tag{3.7}$$

Namely, $\bar{\mu} := \rho(|B_J|$. For the following part of this paper we assume that the matrix *A* of (3.2) belongs to the matrix set

$F = \{A \in \mathbb{C}_{\pi,p}^{n_i,n_i} / \, |B_J| = |L + U| = |L| + |U| \, is \; a \; GCO(\pi, q, r) - matrix\}$

The preconditioner *P* of Saeed and Ali [11] applied to this linear system (3.1) and transformed it into another equivalent system

$$PA\tilde{u} = P\tilde{b} \tag{3.8}$$

with $P = (I + \overline{S})$, where *I* is the identity matrix which has the same dimension as *A* while $\bar{S}$ is obtained by taking the first upper diagonal groups of R_0 in the original system above as the following:

$$\overline{S} = \begin{bmatrix} Z_1 & & & \\ & Z_1 & & \\ & & \ddots & \\ & & & Z_1 \end{bmatrix}_{\frac{(N-1)^2}{2} \times \frac{(N-1)^2}{2}}, Z_1 = \begin{bmatrix} \underset{\sim}{0} & -R_{01} & & \\ & \underset{\sim}{0} & \ddots & \\ & & \ddots & -R_{01} \\ & & & \underset{\sim}{0} \end{bmatrix}_{(N-1)\times(N-1)}$$

Therefore, the preconditioner, $(I + \overline{S})$ matrix will become

$$I+\overline{S} = \begin{bmatrix} Z_2 & & & \\ & Z_2 & & \\ & & \ddots & \\ & & & Z_2 \end{bmatrix}_{\frac{(N-1)^2}{2} \times \frac{(N-1)^2}{2}}, Z_2 = \begin{bmatrix} I_0 & -R_{01} & & \\ & I_0 & \ddots & \\ & & \ddots & -R_{01} \\ & & & I_0 \end{bmatrix}_{(N-1)\times(N-1)}$$

Here I_0 is a (2×2) identity matrix and the system (3.1) become

$$(I + \bar{S})A\tilde{u} = (I + \bar{S})\tilde{b} \tag{3.9}$$

Hence, the SOR method can be applied to the linear system of equations

$$\bar{A}\tilde{u} = \bar{\tilde{b}} \tag{3.10}$$

where $\bar{A} = (I + \bar{S})A = I - L - \bar{S}L - (U - \bar{S} + \bar{S}U)$ and $\bar{\tilde{b}} = (I + \bar{S})\tilde{b}$.

The SOR iteration matrix can be obtained, we can call it a Modified Block Successive Over-Relaxation iteration matrix (MBSOR) and it is given by

$$\tilde{B}_{\ell_w} = \{I - w(L + \bar{S}L)\}^{-1}[(1 - w)I + w(U - \bar{S} + \bar{S}U)] \tag{3.11}$$

In the next section, we will present some theoretical results for the SOR method applied to the preconditioned linear system (3.10).

4 Convergence of the Preconditioned EDG SOR

Lemma 4.1 (12). *. Let $|B_J|$ of (3.7) be a GCO (q,r)-matrix and p := q + r. Then for any real nonnegative constant α, β and γ with $\gamma \neq 0$ satisfying: $\alpha^r \beta^q \bar{\mu}^p < \gamma^p$, the matrix $T := \gamma I - \alpha |L| - \beta |U|$ is such that: $T^{-1} \geqslant 0$.*

Lemma 4.2. *Suppose A= I -L-U is a GCO(π,q,r), where -L and -U are strictly lower and upper triangular matrices respectively. Let B_{ℓ_w} be the block iteration matrix of the SOR method given by (3.4). If $0 < w < 2$, then the block SOR method converges, i.e. $\rho(B_{\ell_w}) < 1$.*

Proof. Let a matrix *A* with partitioning π be given as in (3.2) and let the block SOR iteration matrix B_{ℓ_w} be given as in (3.4). Setting: $B'_{\ell_w} = (I - |wL|)^{-1}\{|1-w|\, I + |w|\, U\}$ Clearly, we can see that $|B_{\ell_w}| < B'_{\ell_w}$ and hence by theorem 2.1, $\rho(B_{\ell_w}) \leqslant \rho(B'_{\ell_w})$. Consider the matrix $\hat{A} \in \mathbb{C}^{n_i,n_i}_{\pi,p}$ defined by: $\hat{A} := \bar{M} - \bar{N}$, where $\bar{M} := I - |w|\,|L|$ and $\bar{N} := |1-w|\, I + |w|\, |U|$. It is easily seen that $\bar{M}$ is nonsingular and $B'_{\ell_w} = \bar{M}^{-1}\bar{N}$. Moreover, since $\bar{M}^{-1} \geqslant 0$ and $\bar{N} \geqslant 0$, $\bar{M} - \bar{N}$ is a regular splitting of $\hat{A}$ (cf.[14]). In addition to this, for *w* satisfying the condition $0 < w < 2$, Lemma (4.1) above implies that $\hat{A}^{-1} \geqslant 0$ Therefore, recalling Theorem 2.2 above, we have $\rho(B'_{\ell_w}) < 1$. Hence, $\rho(B_{\ell_w}) < 1$.

Theorem 4.1. *Suppose A= I -L-U is a GCO(π,q,r), where -L and -U are strictly lower and upper triangular matrices respectively. Let B_{ℓ_w} and $\tilde{B}_{\ell_w}$ be the iteration matrices of the SOR method given by (3.4) and (3.11) respectively. If $0 < w < 2$, then*

(i) $\rho(\tilde{B}_{\ell_w}) < \rho(B_{\ell_w})$ *if* $\rho(B_{\ell_w}) < 1$

(ii) $\rho(\tilde{B}_{\ell_w}) = \rho(B_{\ell_w})$ *if* $\rho(B_{\ell_w}) = 1$

(iii) $\rho(\tilde{B}_{\ell_w}) > \rho(B_{\ell_w})$ *if* $\rho(B_{\ell_w}) > 1$

Proof. From Lemma 4.2 and since the matrix *A* of (3.2) is a GCO*(π,q,r)* and $B_{\ell_w} = (I - wL)^{-1}\{(1-w)I + wU\}$, there exists a positive vector x such that: $B_{\ell_w}x = \lambda x$, where: $\lambda = \rho(B_{\ell_w})$ or: equivalently,

$$[(1-w)I + wU]x = \lambda(I - wL)x \tag{4.1}$$

thus,

$$\begin{aligned}\tilde{B}_{\ell_w}x - \lambda x &= \{I - w(L + \bar{S}L)\}^{-1}[(1-w)I + w(U - \bar{S} + \bar{S}U) - \lambda\{I - w(L + \bar{S}L)\}]x \\ &= \{I - w(L + \bar{S}L)\}^{-1}[(1 - (1-\bar{S})w - \lambda)I + (w\lambda - \lambda\bar{S})L + (w + \bar{S})U]x\end{aligned} \tag{4.2}$$

But, from (4.1) we have:

$$[\lambda wL + wU]x = [(\lambda - 1 + w)I]x$$

Hence,

$$\tilde{B}_{\ell_w}x - \lambda x = \{I - w(L + \bar{S}L)\}^{-1}[(1-\lambda) - (1-\bar{S})wI + (\lambda - 1 + w)I - \bar{S}(U - \bar{S})]x$$

In addition to this, for $0 < w < 2$ and from [8] we can get:

(i) $\lambda < 1$, then $\tilde{B}_{\ell_w}x - \lambda x < 0$ and from [6] we have $\rho(\tilde{B}_{\ell_w}) < \rho(B_{\ell_w})$.

(ii) $\lambda = 1$, then $\tilde{B}_{\ell_w}x = \lambda x$ and from [6] we have $\rho(\tilde{B}_{\ell_w}) = \rho(B_{\ell_w}) = 1$.

(iii) $\lambda > 1$, then $\tilde{B}_{\ell_w}x - \lambda x > 0$ and from [6] we have $\rho(\tilde{B}_{\ell_w}) > \rho(B_{\ell_w})$.

Thus, the proof is complete.

Remark 4.1. In view of Theorem 4.1 the superiority of the preconditioned EDG SOR against unpreconditioned EDG SOR in [11] is theoretically confirmed.

5 Conclusions

In this work, we introduced a theoretical convergence analysis of the application of a specific splitting-type preconditioner in block formulation for the EDG SOR iterative method. A new theoretical framework for the method of preconditioned EDG SOR has been presented. We have proven that the spectral radius of the iteration matrix of preconditioned EDG SOR method is smaller than that of the unpreconditioned EDG SOR method, if the relaxation parameter $\omega \in (0, 2)$. We conclude that the rate of convergence of the preconditioned EDG SOR method is faster than the rate of convergence of the unpreconditioned EDG SOR iterative method. Finally, we have also point out that the block successive over-relaxation(BSOR) leads to a great improvement when the underlying iteration matrix is generally consistently ordered.

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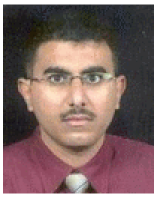

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