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Comparative Analysis of Stock Price Simulation and its European-Styled Call Options

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Abstract: With the popularization of computer application technology, simulation in finance has become an important method for financial risk management. We'll start with the application of the random walk model in simulation of stock price. Then, we'll provide an analysis of the rate of return of China Petroleum, the biggest listing corporation in China, and a simulation of it. Furthermore, we obtain expressions of European-styled call options based on Ito's lemma. And then a simulation is made. We analyze the effect of the variance of stock simulated price and the time for European-styled call options which have a critical and practical significance in building refined pricing model for the financial derivation.

Keywords: European Style call options, simulation, Ito's lemma

1. Application of the Model of Random walk in Stock Price Simulation

With the popularization of computer application technology, simulation in finance becomes an important method for financial risk management[1,4]. Here we choose a typical stock for research, for the reason that there is a close relationship between the price of European style call options and the variance of stock price[2,3]. And meanwhile we provide a simulation[7] for the stock's price. Thereafter, based on the European style call options of the simulated price of the stock, we simulate the price variance of the options. This simulation has a practical significance in development and application of the financial derivation[2], and also a practical significance for the financial derivation in exerting a creative effect in modern financial market[4].

In the case of corporation's good operation and management and good operation of security market, based on experience, we discover a close relationship between annual average rate of return of a stock on investment and investment time[5].

Firstly, we analyze the relationship between the mean value of rate of return and the time period, so that we find a concrete relationship between annual average rate of return of a stock and time interval step.

Let δt be the smaller time interval step. How does time interval affect return distribution when the step turns to be short interval sampling? Without any involve of randomness, we have known that there is a proportion relationship between bonds return and time step. Let μ be its annual average rate of return of a stock on investment, then unit rate of return with a step of δt can be: $\mu\delta t$. Or:

$$\frac{S_{i+1}}{S_i} = 1 + \mu\delta t \qquad (1)$$

Let S_0 be the initial stock price, then stock price after one time step turns to be:

$$S_1 = S_0(1 + \mu\delta t) \qquad (2)$$

Stock price after two time steps is:

$$S_2 = S_0(1 + \mu\delta t)^2 \qquad (3)$$

We can easily derive that stock price after M time steps turns to be:

$$S_M = S_0(1 + \mu\delta t)^M = S_0 e^{M\ln(1+\mu\delta t)} \approx S_0 e^{M\mu\delta t} = S_0 e^{\mu T} \qquad (4)$$

This is the approximate exponential relationship between stock price and time, where $T = M\delta t$.

With further consideration for volatility of rate of return, no matter how small we subdivide the time, the variance of volatility for rate of return in a fixed observation period will not increase infinitely. By average, when time step δt turns to be very small, the expression for standard deviation of rate of return is:

$$s = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left(R_i - \bar{R}\right)^2} \qquad (5)$$

Where R_i is the rate of return in a single step.

In order to ensure the finite of variance, every item in the bracket in (5) must have the same order with δt, in other word, that is $O(\delta t)$. By average, the standard deviation should have the same order with $\delta t^{\frac{1}{2}}$, in other word, that is $O\left(\delta t^{\frac{1}{2}}\right)$.

In order to show the volatility of the rate of return, we let the volatility satisfies the condition of normality and refine the model as follows:

$$\frac{S_{i+1} - S_i}{S_i} = \mu\delta t + \sigma\varphi\delta t^{\frac{1}{2}} \qquad (6)$$

where σ is the standard deviation of annual return on assets, we use σ to show the uncertainty of return; φ is the random number of standard normal distribution.

Therefore we can get the expression of the model of random walk:

$$S_{i+1} = S_i\left(1 + \mu\delta t + \sigma\varphi\delta t^{\frac{1}{2}}\right) \qquad (7)$$

or: $\delta S / S = \mu\delta t + \sigma\varphi\delta t^{\frac{1}{2}}$ (8)

Where $\delta S = S_{i+1} - S_i$, S represents renewals of S_i.

Let $\varphi\delta t^{\frac{1}{2}}$ be dX, then dX has a mean value of zero, and a variance of dt.

According to Ito's lemma, expression (8) can be transformed into:

$$d(\log S) = \left(\mu - \frac{1}{2}\sigma^2\right)dt + \sigma dX \qquad (9)$$

Let $S(0)$ be the initial value, then the stock price at time t is:

$$S(t) = S(0)\exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma\int_0^t dX\right) \qquad (10)$$

When the time step increase by δt, the increment δS of $S(t)$ will be:

$$S(t+\delta t) = S(t) + \delta S =$$

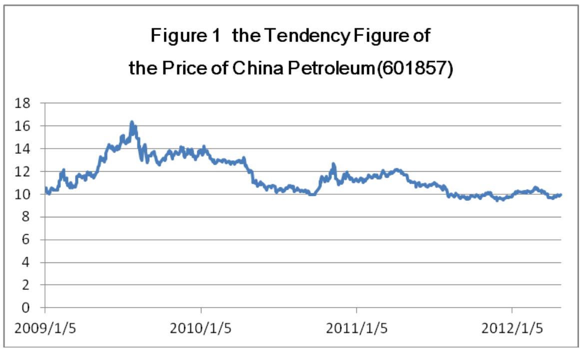
$$S(t)\exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)\delta t + \sigma\varphi\delta t^{\frac{1}{2}}\right) \qquad (11)$$

2 Selection of Stock and Simulation of Stock Price

We select China Petroleum (601857)[8], which exert a significant effect on economy and which has a biggest market value, as research object. The economic significance of this kind of study is very important.

We choose the closing price from Jan.1st 2009 to Apr. 27th of China Petroleum as underlying stock price, S ,(on the date we choose, there must be a trade).

The data derive from Netease database. There are 800 valid data point except the date without trades.



Firstly, we take on the test for normal distribution, we choose k-s One-Sample Kolmogorov-Smirnor test for these 800 data point, and get results as follows:

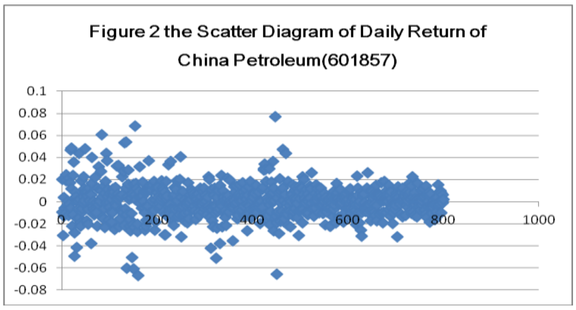


Figure 2 the Scatter Diagram of Daily Return of China Petroleum(601857)

With a significance level of 0.05, we consider that the daily return of this stock varies significantly.

Table 1:One-Sample Kolmogorov-Smirnov Test

		x
	N	800
Normal Parametersa	Mean	0.0000567
	Std. Deviation	0.01499082
Most Extreme Differences	Absolute	0.071
	Positive	0.071
	Negative	-0.058
Kolmogorov-Smirnov Z		1.998
Asymp. Sig. (2-tailed)		0.001

a. Test distribution is Normal.

Let μ be the annual average rate of return, our nation's one year fixed deposit rate is 3.5%, with consideration of inflation, we assume risk-free interest rate into three levels: high (7%), medium (5%), low (3%). For the reason that our nation is now in the period of inflation, we set risk-free interest rate over one year. The step is 0.01, fluctuation rate is added up to 25%. We simulate in three situations, and choose one path in every situation, as shown in follow figure 3.

Therefore, by means of a large number of simulations, we can calculate the average path of stock price variation, which exert an important effect in financial risk management.[9]

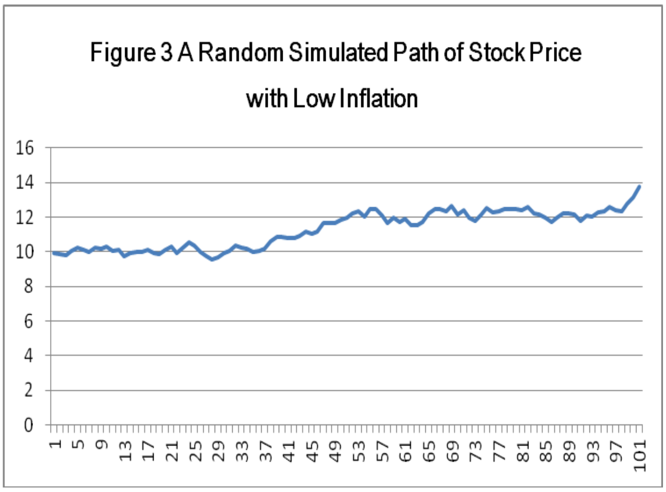


Figure 3 A Random Simulated Path of Stock Price with Low Inflation

The average stock price according to high inflation rate, medium inflation rate, and low inflation rate after one year (in other words, 252 trade day) could be 12.44536 Yuan, 11.57726 Yuan, and 10.53918 Yuan.

3 Fundamental Analysis of European Style call Options

Based on Ito's lemma, we derive the formula of Black- Scholes:

$$\frac{\partial V}{\partial t}+\frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2}+rS\frac{\partial V}{\partial S}-rV=0 \qquad (12)$$

Where $V(S,t)$ is the present value of the derivation which has a value of S and varies by time t. We found the European Style call Options expression as follows under the condition of the formula of Black-Scholes, the maturity date T of European Style call Options satisfies:

$c=\max\{S-X_0,0\}$, according to expression (12), we found:

$$c=SN(d_1)-X_0e^{-r(T-t)}N(d_2) \qquad (13)$$

Where c is the present value of the call options, t is the present time.

$$d_1=\frac{\log(S/X_0)+\left(r+\frac{1}{2}\sigma^2\right)(T-t)}{\sigma(T-t)}$$

$$d_2=\frac{\log(S/X_0)+\left(r-\frac{1}{2}\sigma^2\right)(T-t)}{\sigma(T-t)}$$

where X_0 is the strike price of maturity date T of the underlying stock $N(d)$ is standard normal distribution function.

4. Calculations for Related Parameter of Equity Options

The condition for building equity options is rigor, therefore we can only build approximate the model of pricing by means of Black-Scholes formula. Here we only analyze the pricing model of European style call options. According to our statistical research, we consider that stock which has a relatively small market value cannot satisfy the condition of Black-Scholes formula because of its volatility is relatively big.

Therefore we still use China Petroleum whose market value is big as the underlying of the equity options to build analysis model.

We give a conversion for the rate of return in Part 2 as follows: $\hat{r} = 0.020682$, $\hat{\sigma} = 5.472$. Let $T-t$ be a typical analysis period of one year, X_0 be the strike price of call options. We assume that is 10% higher than recent average price as the fundamental analysis condition. Based on it, we provide simulations for call options.

5. Simulation Analysis of Call Options

After calculations of related parameter, we build the model of the stock of China Petroleum by means of Black-Scholes Pricing Model and *SAS* software, and achieve relevant figures as follows.

5.1 the influence of variance of stock price on the value of call options

We select the closing price, 9.93 Yuan, of China Petroleum on Apr. 28th 2012 as standard price, and strike price as $X_0 = 10.923$ with an increment of 10% for the closing price. Therefore we provide the value variance of European style call options influenced by underlying stock price. We finished expected simulation figure as follows fig 4.

5.2 affections of time and price of the stock changes in call Options

Let the time period be 12 months, in other words, 252 trade days, where $c = SN(d_1) - X_0 e^{-r(T-t)} N(d_2)$. We should note that d_1 and d_2 are still functions of time $T-t$. We provide the simulation figure of relationship between the value of European style call options and the price of underlying stock, as well as relationship between the value of European style call options and time (as follows fig 5).

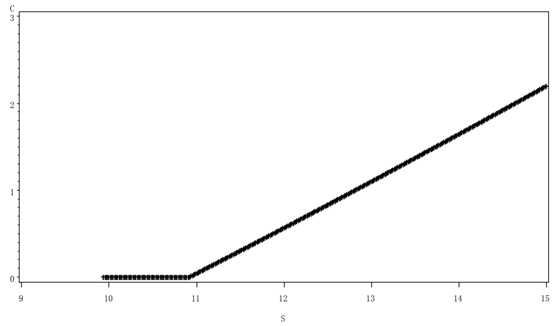


Figure 4 Relationship between European style call options and the value of stock

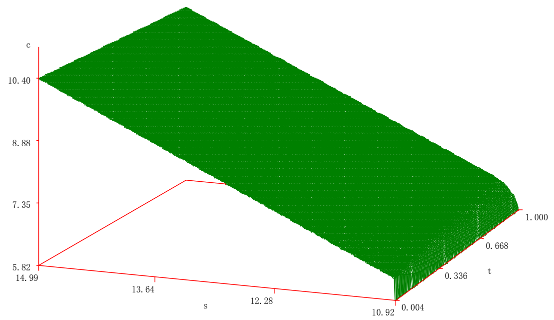


Figure5 Relationship between European call options and the price of underlying stock(s), time (t)

6 Comparative Analysis of Results of the two Methods and the Conclusion of our Research

(1) According to the definition of European style call options, the maturity date T satisfies $c = \max\{S - X_0, 0\}$, and cannot be operated in advance. Without the consideration of trading fees, based on the definition of European style call options we can derive that the value of European style call options on maturity day are

$$c_1 = 12.44536 - 10.923 = 1.52236$$

$$c_2 = 11.55726 - 10.923 = 0.65426 \quad c_3 = 0$$

(2) (high inflation, medium inflation, low inflation).

which means that the variation of inflation can influence the value of equity options.

(3) Value of options calculated by the expressions of European style options

We provide figures under high, medium, and low inflation, and simulation results are as follows:

$$c_1 = 1.533772 \quad c_2 = 0.6584631 \quad c_3 = 0$$

(4) Comparative analysis between absolute error and relative error

The simulation results based on European-styled options are slightly larger than those directly derived from the definition of equity options. The absolute error is below 2% while the relative error is below 1%. The comparison tells us that these two results are closely related, but whether the simulation results are related to actual values depends on the evaluation of inflation. Of course, the simple assumption of high, medium, and low inflation is far from enough and from this perspective, the calculation expression of European-styled options is better than that by definition. Of course, call options can be influenced by the rate of interest, the volatility of underlying stock price, the sensitivity of call options to underlying stock price and so on. These discussions are practically beneficial in building complete pricing model of financial derivation and financial market which will exert effect on improving economy.

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