Applied Mathematics & Information Sciences
An International Journal

http://dx.doi.org/10.12785/amis/090534

New Modification Of Laplace Decomposition Method for Seventh Order KdV Equation

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Received: 3 Feb. 2015, Revised: 5 May 2015, Accepted: 6 May 2015
Published online: 1 Sep. 2015

Abstract: In this paper, we develop a new modification of Laplace decomposition method for solving the seventh order KdV equations. The numerical results show that the method converges rapidly and compared with the Adomian decomposition method. The conservation properties of solution are examined by calculating the first three invariants.

Keywords: seventh order KdV Equation, Laplace decomposition method , Adomian decomposition method , conservation laws.

1 Introduction

The general seventh-order KdV equation [1, 2] reads

$$u_t + au^3u_x + bu_x^3 + cuu_xu_{2x} + du^2u_{3x} + eu_{2x}u_{3x} + fu_xu_{4x} + guu_{5x} + u_{7x} = 0 \quad (1)$$

Where $a,b,c,d,e,f,$ and g are nonzero parameters and $u_{kk} = \frac{\partial^k}{\partial x^k}$. Some particular cases of (1)are:

(i) Seventh-order Lax equation $(a = 140, b = 70, c = 280, d = 70, e = 70, f = 42, g = 14)$:

$$u_t + 140u^3u_x + 70u_x^3 + 280uu_xu_{2x} + 70u^2u_{3x} + 70u_{2x}u_{3x} + 42u_xu_{4x} + 14uu_{5x} + u_{7x} = 0 \quad (2)$$

(ii) Seventh-order Sawada-Kotera equation $(a = 252, b = 63, c = 378, d = 126, e = 63, f = 42, g = 21)$:

$$u_t + 252u^3u_x + 63u_x^3 + 378uu_xu_{2x} + 126u^2u_{3x} + 63u_{2x}u_{3x} + 42u_xu_{4x} + 21uu_{5x} + u_{7x} = 0 \quad (3)$$

The ADM [3] is applied to generalized KdV equation [4, 5] , fifth order KdV equation [6–9] fifth order KdV equation [4, 5] , and seventh order KdV equation [10]. The modified Laplace decomposition method [11–13] is applied to KdV equation [14]. In this work, We apply the new modification of Laplace - Adomian decomposition method to investigate propagating traveling solitary wave solutions of seventh-order KdV equations, which play a very important role in mathematical physics, engineering, and applied sciences. The proposed iterative scheme finds the solution without any discretization, perturbation, linearization, or restrictive assumptions. As is well known, conservation laws (CLaws) play an important role in mathematical physics. The first few CLaws have a physical meaning, such as conservation of momentum and energy [15]. From the objectives of this paper are studying the properties of CLaws of seventh-order KdV equation [16, 17].

2 The numerical method

In this section, we outline the new modification of Laplace ADM to obtain explicit solution of equation (1) with initial condition $u(x,0) = f(x)$.
Let us consider the standard form of a seventh order KdV equation (1) in an operator form

$$L_t(u) + a(Ku) + b(Mu) + c(Nu) + d(Pu) + e(Qu) + f(Ru) + g(Vu) + L_x(u) = 0 \quad (4)$$

where the notation $Ku = u^3u_x$, $Mu = u_x^3$, $Nu = uu_xu_{2x}$, $Pu = u^2u_{3x}$, $Qu = u_{2x}u_{3x}$, $Ru = u_xu_{4x}$ and $Vu = uu_{5x}$ symbolize the nonlinear term, respectively. The notation $L_t = \frac{\partial}{\partial t}$ and $L_x = \frac{\partial^7}{\partial x^7}$ symbolize the linear

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differential operators. We represent solution as an infinite series given below,

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) \tag{5}$$

The nonlinear terms Ku, Mu, Nu, Pu, Qu, Ru and Vu can be decomposed into infinite series of polynomial given by $Ku = u^3 u_x = \sum_{n=0}^{\infty} A_n$, $Mu = u_x^3 = \sum_{n=0}^{\infty} B_n$, $Nu = uu_x u_{2x} = \sum_{n=0}^{\infty} C_n$, $Pu = u^2 u_{3x} = \sum_{n=0}^{\infty} D_n$, $Qu = u_{2x} u_{3x} = \sum_{n=0}^{\infty} E_n$, $Ru = u_x u_{4x} = \sum_{n=0}^{\infty} F_n$ and $Vu = uu_{5x} = \sum_{n=0}^{\infty} G_n$, where $A_n, B_n, C_n, D_n, E_n, F_n$ and G_n are Adomian polynomials [3] of $u_0, u_1, \cdots, u_n$ and it can be calculated by the formula given below

$$\Psi_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n}\Psi\left(\sum_{i=0}^{\infty}\lambda^i u_i\right)\right]_{\lambda=0} \; n \geq 0 \tag{6}$$

Where Ψ_n is the Adomian polynomials for the nonlinear term $\Psi(u)$.
By applying the Laplace transform to both sides of equation (4), we obtain

$$\begin{aligned}\mathscr{L}\{u(x,t)\} &= \frac{1}{s}u(x,0) - \frac{1}{s}(a\mathscr{L}\{Ku\} \\ &+ b\mathscr{L}\{Mu\} + c\mathscr{L}\{Nu\} + d\mathscr{L}\{Pu\} + e\mathscr{L}\{Qu\} \\ &+ f\mathscr{L}\{Ru\} + g\mathscr{L}\{Vu\} + \mathscr{L}\{L_x(u)\}\end{aligned} \tag{7}$$

A sum of components defined by the decomposition series

$$\mathscr{L}\{u(x,t)\} = U(x,s) = \sum_{n=0}^{\infty} U_n(x,s) \tag{8}$$

In the new modification [18] , the initial condition can expressed as a series of infinite components i.e.,

$$u(x,0) = \sum_{n=0}^{\infty} u_0^n(x) \tag{9}$$

and the new recursive relationship can be expressed in the form

$$U_0(x,s) = \frac{1}{s}u_0^0(x)$$

$$\begin{aligned}U_{n+1}(x,s) &= \frac{1}{s}u_0^{n+1}(x) - \frac{1}{s}(a\mathscr{L}\{A_n\} \\ &+ b\mathscr{L}\{B_n\} + c\mathscr{L}\{C_n\} + d\mathscr{L}\{D_n\} + e\mathscr{L}\{E_n\} \\ &+ f\mathscr{L}\{F_n\} + g\mathscr{L}\{G_n\} + \mathscr{L}\{L_x(u_n(x,t))\}), \quad n \geq 0\end{aligned} \tag{10}$$

Finally, by applying inverse Laplace transformation we get:

$$u_0(x,0) = \mathscr{L}^{-1}\{U_0(x,s)\}$$

$$u_{n+1}(x,t) = \mathscr{L}^{-1}\{U_{n+1}(x,s)\}, \quad n \geq 0 \tag{11}$$

Using (11) the series solution follows immediately. In some cases the exact solution in the closed form may be obtained. In practice all terms of the series u_i can't be determined, so we have an approximation of the solution by the following truncated series,

$$\Phi_k(x,t) = \sum_{n=0}^{k-1} u_n(x,t) \tag{12}$$

$$\lim_{k\to\infty} \Psi_k(x,t) = u(x,t) \tag{13}$$

3 Numerical illustrations

To demonstrate the applicability of the proposed method, it has been applied to tow classical problems concerning the motion of solutions, their interactions and their generations from an arbitrary initial condition. Numerical results of the corresponding problems are presented. The accuracy of the method is measured using L_2 and L_∞ error norms defined by

$$L_2 = [h\sum_{j}^{N} |U_j^{exact} - U_j^n|^2]^{\frac{1}{2}} \tag{14}$$

$$L_\infty = max_j |U_j^{exact} - U_j^n| \tag{15}$$

where h is the distance, and U_j^{exact}, U_j^n the exact solution and the numerical solution at $x = hj$ respectively. The conservation properties of the solution are examined by calculating the Claws [17] :
In the case of Rank$(\rho_1) = 6$

$$\rho_1 = u_x^2 + \frac{3-2\tilde{f}}{21}u^3 \tag{16}$$

In the case of Rank$(\rho_2) = 8$

$$\begin{aligned}\rho_2 &= \frac{10\tilde{f}^2 - 45\tilde{f} + 49\tilde{c} + 20}{1764}u^4 \\ &+ \frac{1-2\tilde{f}}{7}uu_x^2 + u_{2x}^2\end{aligned} \tag{17}$$

Each of the Lax and Sawada-Kotera equations admit ρ_1 and ρ_2, where $\tilde{f} = \frac{f}{g}, \tilde{c} = \frac{c}{g^2}$.
In the case of Rank$(\rho_3) = 12$
For the seventh order Lax equation we have

$$\rho_3 = \frac{3}{2744}u^6 + u_{4x}^2 - \frac{5}{28}u_x^4 + \frac{10}{7}u_{2x}^3 - \frac{9}{7}uu_{3x}^2 - \frac{15}{98}u^3u_x^2 + \frac{9}{14}u^2u_{2x}^2 \quad (18)$$

For the seventh order Sawada-Kotera equation we have

$$\rho_3 = \frac{4}{21609}u^6 + u_{4x}^2 - \frac{17}{147}u_x^4 + \frac{16}{21}u_{2x}^3 - uu_{3x}^2 - \frac{50}{1029}u^3u_x^2 + \frac{16}{49}u^2u_{2x}^2 \quad (19)$$

Since the conservation constants are expected to remain constant during the run of the algorithm to have accurate numerical scheme, conservation constants will be monitored. As various problems of science were modeled by non linear partial differential equations and since therefore the seventh order KdV equation is of high importance, the following examples have been considered.

Example 3.1. Let us consider seventh-order Lax equation (2) with the initial condition

$$u(x,0) = 2k^2 sech^2(kx) \quad (20)$$

By applying Laplace transform and using given initial condition we get:

$$\mathscr{L}\{u(x,t)\} = \frac{1}{s}u(x,0) - \frac{1}{s}(140\mathscr{L}\{Ku\} + 70\mathscr{L}\{Mu\} + 280\mathscr{L}\{Nu\} + 70\mathscr{L}\{Pu\} + 70\mathscr{L}\{Qu\} + 42\mathscr{L}\{Ru\} + 14\mathscr{L}\{Vu\} + \mathscr{L}\{L_x(u)\}) \quad (21)$$

The initial condition $u(x,0)$ can expressed as a series of infinite components i.e.

$$u(x,0) = 2k^2 - 2k^4x^2 + \frac{4k^6x^4}{3} - \frac{34k^8x^6}{45} + \frac{124k^{10}x^8}{315} - \frac{2764k^{12}x^{10}}{14175} + O[x]^{11} \quad (22)$$

Using recursive relation (11) with Adomian polynomials yield the components

$$u_0(x,t) = \mathscr{L}^{-1}\{U_0(x,s)\} = \mathscr{L}^{-1}\{\frac{2k^2}{s}\} = 2k^2 \quad (23)$$

$$u_1(x,t) = \mathscr{L}^{-1}\{U_1(x,s)\} = \mathscr{L}^{-1}\{\frac{-2k^4x^2}{s}\} = -2k^4x^2 \quad (24)$$

Table 1: L_2 and L_∞ errors.

t	L_2	L_∞
0.1	6.42564e-10	1.67514e-9
0.2	1.5258e-9	3.73702e-9
0.3	2.67549e-9	6.18565e-9
0.4	4.10451e-9	9.02102e-9
0.5	5.81966e-9	1.22431e-9

Table 3: Result of the proposed method compared with results in [10]. We will take $t = 0.3$ and $x \in [0.1, 0.5]$. ENML denotes the absolute error by the new modification of Laplace decomposition method and EADM denotes the absolute error by Adomian decomposition method

x	ENML	EADM
0.1	5.10963e-10	3.50406e-8
0.2	1.40584e-9	6.95085e-8
0.3	2.68464e-9	1.03998e-7
0.4	4.34734e-9	1.38303e-7
0.5	6.39397e-9	1.72205e-7

$$u_2(x,t) = \mathscr{L}^{-1}\{U_2(x,s)\} = \mathscr{L}^{-1}\{\frac{4480k^{10}x}{s^2} + \frac{4k^6x^4}{3s}\} = 4480k^{10}tx + \frac{4k^6x^4}{3} \quad (25)$$

$$u_3(x,t) = \mathscr{L}^{-1}\{U_3(x,s)\} = \mathscr{L}^{-1}\{-\frac{34k^8x^6}{45s} - \frac{\frac{17920k^{10}x}{s} + 70(\frac{71680k^{16}}{s^2} + \frac{832k^{12}x^3}{3s})}{s} = -\frac{2}{45}(564480k^{16}t^2 + 403200k^{10}tx + 436800k^{12}tx^3 + 17k^8x^6) \quad (26)$$

And so on, in this manner the rest of components of the decomposition series were obtained. Substituting (23)-(26) into (5)gives the solution $u(x,t)$ in a series form and in a close form by

$$u(x,t) = 2k^2 sech^2(k(x - 64k^6t)) \quad (27)$$

This result can be verified through substitution. The L_2, L_∞ errors norm and the three invariants ρ_1, ρ_2, ρ_3 for the problem are documented in Table 1 and Table 2 respectively. The results produced by the new modification of Laplace decomposition method are in a very good agreement with the best of the results of the methods listed in Table 3. The profile of the solitary wave

Table 2: The invariants ρ_1, ρ_2, ρ_3

$t \setminus x$	ρ_1			ρ_2			ρ_3		
	0.1	0.3	0.5	0.1	0.3	0.5	0.1	0.3	0.5
0.1	-1.142e-7	-3.411e-7	-5.634e-7	1.513e-8	4.529e-8	7.509e-8	1.156e-10	3.450e-10	5.692e-10
0.2	-1.142e-7	-3.411e-7	-5.634e-7	1.513e-8	4.529e-8	7.509e-8	1.156e-10	3.449e-10	5.690e-10
0.3	-1.142e-7	-3.411e-7	-5.634e-7	1.513e-8	4.529e-8	7.510e-8	1.156e-10	3.448e-10	5.688e-10
0.4	-1.142e-7	-3.411e-7	-5.634e-7	1.513e-8	4.530e-8	7.511e-8	1.156e-10	3.446e-10	5.686e-10
0.5	-1.142e-7	-3.411e-7	-5.634e-7	1.513e-8	4.530e-8	7.512e-8	1.156e-10	3.447e-10	5.684e-10

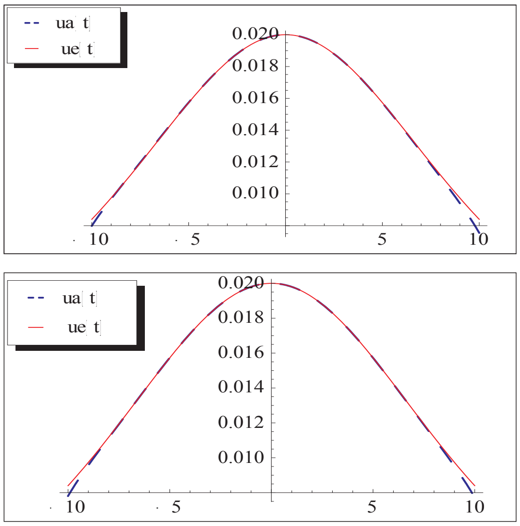

Fig. 1: The graph of the exact (ue) and approximate (ua) solution, (top) $t = 0.5$, $x \in [-10,\ 10]$, (bottom) $t = 1.0$, $x \in [-10,\ 10]$.

at times $t = 0.5$ and $t = 1.0$ compared in Figure 1.

Example 3.2.

Let us consider seventh-order Sawada-Kotera equation (3) with the initial condition

$$u(x,0) = \frac{4k^2}{3}(2 - 3tanh^2(kx)) \tag{28}$$

By applying Laplace transform and using given initial condition we get:

$$\mathscr{L}\{u(x,t)\} = \frac{1}{s}u(x,0) - \frac{1}{s}(252\mathscr{L}\{Ku\} + 63\mathscr{L}\{Mu\} + 378\mathscr{L}\{Nu\} + 126\mathscr{L}\{Pu\} + 63\mathscr{L}\{Qu\} + 42\mathscr{L}\{Ru\} + 21\mathscr{L}\{Vu\} + \mathscr{L}\{L_x(u)\}) \tag{29}$$

The initial condition $u(x,0)$ can expressed as a series of infinite components i.e.

$$u(x,0) = \frac{8k^2}{3} - 4k^4x^2 + \frac{8k^6x^4}{3} - \frac{68k^8x^6}{45} + \frac{248k^{10}x^8}{315} - \frac{5528k^{12}x^{10}}{14175} + O[x]^{11} \tag{30}$$

Using recursive relation (11) with Adomian polynomials yield the components

$$u_0(x,t) = \mathscr{L}^{-1}\{U_0(x,s)\} = \mathscr{L}^{-1}\{\frac{8k^2}{3s}\} = \frac{8k^2}{3} \tag{31}$$

$$u_1(x,t) = \mathscr{L}^{-1}\{U_1(x,s)\} = \mathscr{L}^{-1}\{\frac{-4k^4x^2}{s}\} = -4k^4x^2 \tag{32}$$

$$u_2(x,t) = \mathscr{L}^{-1}\{U_2(x,s)\} = \mathscr{L}^{-1}\{\frac{114688k^{10}x}{3s^2} + \frac{8k^6x^4}{3s}\} = \frac{8}{3}(1433k^{10}tx + k^6x^4) \tag{33}$$

$$u_3(x,t) = \mathscr{L}^{-1}\{U_3(x,s)\} = \mathscr{L}^{-1}\{\frac{-68k^8x^6}{45s} - \frac{\frac{121856k^{10}x}{s} + 126(\frac{117440512k^{16}}{81s^2} + \frac{143360k^{12}x^3}{81s})}{s}\} = -\frac{4}{45}(1027604480k^{16}t^2 + 1370880k^{10}tx + 2508800k^{12}tx^3 + 17k^8x^6) \tag{34}$$

And so on, in this manner the rest of components of the decomposition series were obtained. Substituting (31)-(34) into (5) gives the solution $u(x,t)$ in a series form and in a close form by

$$u(x,t) = \frac{4k^2}{3}(2 - 3tanh^2(k(x - \frac{256k^6}{3}t))) \tag{35}$$

This result can be verified through substitution.

The L_2, L_∞ errors norm and the three invariants ρ_1, ρ_2, ρ_3

Table 4: L_2 and L_∞ errors.

t	L_2	L_∞
0.1	7.35372e-9	1.44404e-8
0.2	2.14924e-8	3.81436e-8
0.3	4.26366e-8	7.11095e-8
0.4	7.08343e-8	1.13338e-7
0.5	1.06102e-7	1.6483e-7

Table 6: Result of the proposed method compared with results in [10]. We will take $t = 0.4$ and $x \in [0.1,\ 0.5]$. ENML denotes the absolute error by the new modification of Laplace decomposition method and EADM denotes the absolute error by Adomian decomposition method

x	ENML	EADM
0.1	1.15762e-8	3.82381e-8
0.2	3.23494e-8	6.68601e-8
0.3	6.23195e-8	9.49483e-7
0.4	1.01486e-7	1.22581e-7
0.5	1.49851e-7	1.49860e-7

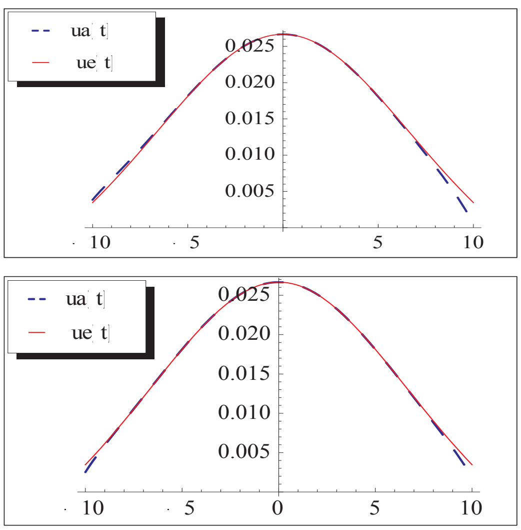

Fig. 2: The graph of the exact (ue) and approximate (ua) solution, (top) $t = 0.5$, $x \in [-10,\ 10]$, (bottom) $t = 1.0$, $x \in [-10,\ 10]$.

for the problem are documented in Table 4 and Table 5 respectively. The results produced by the results produced by the new modification of Laplace decomposition method are in a very good agreement with the best of the results of the methods listed in Table 6. The profile of the solitary wave at times $t = 0.5$ and $t = 1.0$ are compared in Figure 2.

4 Conclusion

In this paper, the new modification of Laplace decomposition method has been successfully applied to finding the solutions of the seventh order KdV equations, Lax equation and Sawada-Kotera equation. The obtained solutions are compared with those of ADM [10]. All the examples show that the results of the present method are in approximate agreement with those of ADM. The conservation of the invariants can be seen to be almost constant. The results show that the new modification of Laplace decomposition method is a powerful mathematical tool for solving linear and nonlinear partial differential equations, and therefore, can be widely applied in engineering problems.

Acknowledgement

This work was funded by the Deanship of Scientific Research (DSR), King Abdulaziz University, Jeddah, under grant No. (363-008-D1433). The authors, therefore, acknowledge with thanks DSR technical and financial support.

References

[1] W. Hereman, P.P. Banerjee, A. Korpel, G. Assanto, A. van Immerzeele and A. Meerpoel, Exact solitary wave solutions of nonlinear evolution and wave equations using a direct algebraic method, J. Phys. A: Math. Gen 19, 607-628 (1986).

[2] E. J. Parkes and B.R. Duffy, An automated tanh-function method for ending solitary wave solutions to non-linear evolution equations, Comput.Phys. Commun., 98, 288-300 (1996).

[3] G. Adomian, Solving Frontier Problems of Physics: The Decomposition Method, Kluwer Academic Publisher, Boston, MA, 1994..

[4] M.I. Syam, Adomian decomposition method for approximating the solution of the Korteweg-deVries equation, Applied Mathematics and Computation, 162, 1465-1473(2005).

[5] T. Geyikli, D. Kaya, An application for a modified KdV equation by the decomposition method and finite element method, Applied Mathematics and Computation, 169, 971-981(2005).

[6] G.Adomian, The fifth-order Korteweg-De Vries equation. Internat. J. Math and Math. Sci. 19(2), 415-415 (1996).

[7] D. Kaya, An explicit and numerical solutions of some fifth-order KdV equation by decomposition method, Applied Mathematics and Computation, 144(2-3), 353-363 (2003).

[8] D. Kaya, An application for the higher order modified KdV equation by decomposition method, Communications in Nonlinear Science and Numerical Simulation 10(6), 693-702 (2005).

[9] D. Kaya and S. M. El-Sayed, On a generalized fifth order KdV equations, Physics Letters A. 310(1) 44-51, (2003).

Table 5: The invariants ρ_1, ρ_2, ρ_3

$t \setminus x$	ρ_1			ρ_2			ρ_3		
	0.1	0.3	0.5	0.1	0.3	0.5	0.1	0.3	0.5
0.1	-9.008e-8	-2.648e-7	-4.232e-7	6.192e-8	1.853e-7	3.073e-8	3.851e-10	1.148e-9	1.892e-9
0.2	-9.007e-8	-2.648e-7	-4.232e-7	6.192e-8	1.853e-7	3.074e-7	3.849e-10	1.147e-9	1.888e-9
0.3	-9.007e-8	-2.648e-7	-4.232e-7	6.192e-8	1.853e-7	3.074e-7	3.848e-10	1.146e-9	1.885e-9
0.4	-9.007e-8	-2.648e-7	-4.232e-7	6.192e-8	1.853e-7	3.074e-7	3.846e-10	1.144e-9	1.882e-9
0.5	-9.007e-8	-2.648e-7	-4.232e-7	6.193e-8	1.854e-7	3.075e-7	3.845e-10	1.143e-9	1.878e-9

[10] S. M. El-Sayed and Dogan Kaya, An application of the ADM to seventh order Sawada Kotara equations, Applied Mathematics and Computation 157, 93-101 (2004).

[11] E. Yusfoglu, Numerical solution of Dung equation by the Laplace decomposition algorithm, Appl. Math. Comput. 177, 572-580 (2006).

[12] O. Kiymaz, An algorithm for solving initial value problems using Laplace Adomian decomposition method, Applied Mathematical sciences. 3(30), 1453-1459 (2009).

[13] H. Jafari, C. M. Khalique, M. Khan and Ghasemi, A two-step Laplace decom-position method for solving nonlinear partial differential equations, International Journal of the Physical Sciences. 6(16), 4102-4109 (2011).

[14] M. M. Khader and R. F. Al-Bar, Approximate Method for Studying the Waves Propagating along the Interface between Air-Water, Mathematical Problems in Engineering, Volume 2011 , Article ID 147327, 21 pages (2011).

[15] R. M. Miura, C. S. Gardner, and M. D. Kruscal, Korteweg-de Vries Equation and Generalizations. II. Existence of Conservation Laws and Constants of Motion, Journal of mathematical physics. 9(8), 1204-1209 (1968).

[16] Y. RX, X. GQ, and L. ZB, Conservation Laws and Soliton Solutions for Generalized Seventh Order KdV Equation, Commun. Theor. Phys. (Beijing, China). 41(4), 487-492 (2004).

[17] Y. RX and L. ZB, Conservation laws and new exact solutions for the generalized seventh order KdV equation, Chaos, Solitons and Fractals 20, 259-266 (2004).

[18] A. M. Wazwaz and S. M. El-Sayed, A new modification of the Adomian decomposition method for linear and nonlinear operators, Applied Mathematics and Computation, 122, 393-405 (2001).

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