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Bayesian Estimation of Exponentiated Gamma Parameter for Progressive Type II Censored Data With Binomial Removals

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Abstract: In the present study, we have considered the exponentiated gamma distribution as an important life time model for the situations where hazard rate function is either monotonic increasing or in the bathtub shape. We propose Bayes estimators of the parameter of the exponentiated gamma distribution under general entropy loss function, squared error loss function and we have also derived its maximum likelihood estimator. The estimators have been compared through their simulated risks.

Keywords: Bayesian Inferences, Life Time Distributions, Censoring and Reliability.

1 Introduction

[1] introduced exponentiated gamma distribution as a life time model applicable to the life data having either monotonic increasing or bathtub shaped hazard rate function. It seems to be flexible in this sense and it has single parameter, the shape. Therefore, it is parsimonious in parameters and hence simple to use.

The probability density function (pdf) of exponentiated gamma distribution EG(θ) is of the form,

$$f(x) = \theta x e^{-x}[1-e^{-x}(x+1)]^{\theta-1} \quad ; \quad x>0,\ \theta>0 \tag{1}$$

where θ is the shape parameter.

The cumulative distribution function (cdf), survival function and hazard rate function corresponding to the pdf (1) are given by,

$$F(x) = [1-e^{-x}(x+1)]^{\theta} \tag{2}$$

$$S(x) = 1-[1-e^{-x}(x+1)]^{\theta} \tag{3}$$

and

$$h(x) = \frac{\theta x e^{-x}[1-e^{-x}(x+1)]^{\theta-1}}{1-[1-e^{-x}(x+1)]^{\theta}} \tag{4}$$

respectively.

It has increasing hazard rate function when $\theta > \frac{1}{2}$ and its hazard rate function takes Bathtub shape when $\theta \leq \frac{1}{2}$. It is important to mention that when $\theta = 1$, pdf (1) is that of gamma distribution with shape parameter $\alpha = 2$ and scale parameter $\beta = 1$, i.e. $G(2,1)$. For more details about this distribution, see [2].

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The rest of the paper is organized as follows: In section 2, we have explained Progressive Type II (PT-II) Censored Sample with Binomial Removals and derived MLE of θ for such a sample from EG(θ) distribution. Further, in section 3, we have derived the Bayes estimators of θ under General Entropy Loss Function (GELF) and Squared Error Loss Function (SELF) using gamma distribution as prior for θ. Comparison between the considered estimators of θ, through simulated risks under GELF is shown in section 4. Finally, conclusions have been shown in the last section.

2 PT-II Censored Sample with Binomial Removals

In medical field and reliability analysis, most of the data are not coming either in the form of the complete sample or in the form of Type I or Type II censored samples. The observations may be removed from the experiment at any point of time other than the termination point. This is what is known as progressive censoring scheme (see, [9] and [10]). For example, patients are registered for visiting to a doctor on a certain day, after getting the time of appointment, some patient(s) may be removed any time, due to some unavoidable reasons. The sample, thus obtained is the patients, who successfully completed his task of visiting the doctor, is called the PT-II censored sample. It may also be noted that the no. of patients removed from the process is not fixed, therefore it may be supposed to be a non-negative integral valued discrete random variable. If the no. of removals follow Binomial distribution with certain parameters, then the corresponding sample is called PT-II censored sample with Binomial removals. For more applications of PT-II censored with Binomial removal, the readers are referred to [11,12,13,14,15,16,17]. More precisely, this scheme is planed as follows;

Suppose n identical items are put on a life testing experiment and we will terminate the experiment as soon as $m \leq n$ items are failed (i.e. m is the effective sample size). Also, suppose that the probability p of removal is constant for each individual. Let R_i be the no. of removals after the i^{th} failure occured at time X_i. Then $(X_1,R_1),(X_2,R_2),\ldots,(X_m,R_m)$ constitute a PT-II censored sample with the Binomial removals, if

At the time of first failure X_1, let R_1 items are randomly removed with fixed probability p of removals from the remaining $n-1$ survived items. Here, R_1 follows $B(n-m,p)$. Similarly, at the time of second failure X_2, let R_2 items from the remaining $n-2-R_1$ survived items are randomly removed with the same removal probability p. Here, R_2 follows $B(n-m-R_1,p)$. Similarly, at the time of third failure X_3, let R_3 items are randomly removed from the remaining $n-3-\sum_{i=1}^{2} R_i$ alived items with the same removal probability p. Here, R_3 follow $B\left(n-m-\sum_{i=1}^{2} R_i, p\right)$. Continuing in this way, at the time of $(m-1)^{th}$ failure X_{m-1}, let R_{m-1} items are randomly removed with prefixed probability p of removals from the remaining $n-(m-1)-\sum_{i=1}^{m-2} R_i$ survived items and also R_{m-1} follow $B\left(n-m-\sum_{i=1}^{m-2} R_i, p\right)$. Finally, at the time of m^{th} failure X_m, all the remaining $R_m = n-m-\sum_{i=1}^{m-1} R_i$ survived items will be removed. Therefore, $(X_1,R_1),(X_2,R_2),\ldots,(X_m,R_m)$ denote PT-II censored sample with Binomial removals, where $X_1 < X_2 < X_3 < \ldots < X_m$ and $R_1,R_2,\ldots,R_m$ are the corresponding no. of removals.

The full likelihood function of PT-II censored sample with Binomial removals is given by (see, [9])

$$L(\theta) = L_1(\theta,\mathbf{X}|R=r)P(R=r) \tag{5}$$

where $L_1(\theta,\mathbf{X}|R=r)$ is the likelihood function with pre-determined number of removals, say $R_1=r_1,R_2=r_2,\ldots,R_m=r_m$ and is given by

$$L_1(\theta,\mathbf{X}|R=r) = c^*\prod_{i=1}^{m} f(x_i)[S(x_i)]^{r_i} \tag{6}$$

and

$$P(R=r) = P(R_{m-1}=r_{m-1}|R_{m-2}=r_{m-2},\ldots,R_1=r_1)\ldots P(R_2=r_2|R_1=r_1)P(R_1=r_1) \tag{7}$$

The constant c^* involved in (6) is given by

$$c^* = n\prod_{l=1}^{m-1}\left(n-l-\sum_{i=1}^{l} r_i\right)$$

It is known that

$$P(R_1 = r_1) = \binom{n-m}{r_1} p^{r_1}(1-p)^{n-m-r_1} \tag{8}$$

and also for every $i = 2, 3, \cdots, m-1$,

$$P(R_i = r_i | R_{i-1} = r_{i-1}, \ldots, R_1 = r_1) = \binom{n-m-\sum_{l=1}^{i-1} r_l}{r_i} p^{r_i}(1-p)^{n-m-\sum_{l=1}^{i} r_l} \tag{9}$$

Putting the values from (8) and (9) in (7), after some simplification, we get

$$P(R = r) = \frac{(n-m)!\, p^{\sum_{i=1}^{m-1} r_i}(1-p)^{(m-1)(n-m)-\sum_{i=1}^{m-1}(m-i)r_i}}{\left(n-m-\sum_{i=1}^{m-1} r_i\right)!\left(\prod_{i=1}^{m-1} r_i!\right)} \tag{10}$$

Again, putting the values from (1) and (3) in (6), we get

$$L_1(\theta, \mathbf{X}|R = r) = c^* \theta^m \prod_{i=1}^{m} x_i e^{-\sum_{i=1}^{m} x_i} \prod_{i=1}^{m} [1 - e^{-x_i}(x_i+1)]^{\theta-1} \prod_{i=1}^{m} [1 - (1 - e^{-x_i}(x_i+1))^{\theta}]^{r_i} \tag{11}$$

Finally, substituting the values from (10) and (11) in (5), we get

$$L(\theta) = c^{**} \theta^m \prod_{i=1}^{m} x_i e^{-\sum_{i=1}^{m} x_i} \prod_{i=1}^{m} [1 - e^{-x_i}(x_i+1)]^{\theta-1} \prod_{i=1}^{m} [1 - (1 - e^{-x_i}(x_i+1))^{\theta}]^{r_i} \tag{12}$$

where

$$c^{**} = c^* \frac{(n-m)!\, p^{\sum_{i=1}^{m-1} r_i}(1-p)^{(m-1)(n-m)-\sum_{i=1}^{m-1}(m-i)r_i}}{\left(n-m-\sum_{i=1}^{m-1} r_i\right)!\left(\prod_{i=1}^{m-1} r_i!\right)}$$

which is independent of θ.

2.1 Maximum Likelihood Estimator of θ

The log-likelihood function can be obtained by taking ln of both sides of (12). The same is obtained as follows

$$l(\theta) = C + m\ln\theta + \sum_{i=1}^{m} \ln(x_i) - \sum_{i=1}^{m} x_i + (\theta-1)\sum_{i=1}^{m} \ln[1 - e^{-x_i}(x_i+1)] + \sum_{i=1}^{m} r_i \ln[1 - (1 - e^{-x_i}(x_i+1))^{\theta}] \tag{13}$$

The log- likelihood equation for estimating θ is given by,

$$\frac{\partial l(\theta)}{\partial \theta} = 0 \tag{14}$$

and after simplification, it reduces to,

$$\frac{m}{\theta} + \sum_{i=1}^{m} \ln[1 - e^{-x_i}(x_i+1)] - \sum_{i=1}^{m} \frac{r_i[1 - e^{-x_i}(x_i+1)]^{\theta} \ln[1 - e^{-x_i}(x_i+1)]}{[1 - \{1 - e^{-x_i}(x_i+1)\}^{\theta}]} = 0 \tag{15}$$

If $\hat{\theta}_M$ be the Maximum Likelihood Estimator (MLE) of θ, then it will be the solution of (15) for θ. The same is, therefore given by,

$$\hat{\theta}_M = \frac{m}{\sum_{i=1}^{m} \frac{r_i[1-e^{-x_i}(x_i+1)]^{\hat{\theta}_M} \ln[1-e^{-x_i}(x_i+1)]}{[1-\{1-e^{-x_i}(x_i+1)\}^{\hat{\theta}_M}]} - \sum_{i=1}^{m} \ln[1 - e^{-x_i}(x_i+1)]} \tag{16}$$

Above is an implicit equation in $\hat{\theta}_M$, so it cannot be solved analytically. For solving it, we propose to use some numerical iteration method, particularly we have used Newton-Raphson method.

3 Bayes Estimators of θ under GELF and SELF

3.1 Loss Function

An important element of statistical inference, which can always be viewed as a statistical decision problem, is the loss function (see, [3]). The incorporation of a loss function into statistical analysis was first studied extensively by [4]. Though, it is often argued that it is too difficult to define functions that truly represent the losses. Besides this, a single loss function is not appropriate for all the situations. However, a statistical loss function has to be non-negative function taking smallest value zero when the inference matches exactly with the reality. A number of loss functions have been suggested in the statistical literature depending upon the various considerations related to the type of inference to be drawn and the requirement of the problem in hand. One among them is squared error loss function (SELF), as defined by

$$L_S(\hat{\theta},\theta) = (\hat{\theta}-\theta)^2 \tag{17}$$

The Bayes estimator of θ under SELF (17) is given by

$$\hat{\theta}_S = E[\theta|\mathbf{X}] \tag{18}$$

provided that the posterior expectation $E(\theta|\mathbf{X})$ exists and is finite.

No doubt, it is a suitable loss function, in a situation where over estimation and under estimation of the equal magnitudes are of the equal importance. Hence, it is suitable for estimation of the location parameter. For estimation of scale parameter, a modified form of it may be defined as follows,

$$L_{MS}(\hat{\theta},\theta) = \left(\frac{\hat{\theta}}{\theta}-1\right)^2 \tag{19}$$

But, one criticism behind its use is that it penalizes over estimation more heavily than under estimation. Likewise, several loss functiuons have been defined in literature and they have their own advantages/disadvantages. A useful asymmetric loss function available in the literature is general entropy loss function (GELF), proposed by [5]. It is defined as follows;

$$L_G(\hat{\theta},\theta) \propto \left(\frac{\hat{\theta}}{\theta}\right)^c - c\,\ln\left(\frac{\hat{\theta}}{\theta}\right) - 1 \tag{20}$$

where c is the loss parameter. When $c>0$, over estimation is more serious than under estimation and when $c<0$, under estimation is more serious than over estimation. For detailed discussion about its seriousness, readers may refer to [8].

The Bayes estimator of θ under GELF (20) is given as,

$$\hat{\theta}_G = [E(\theta^{-c}|\mathbf{X})]^{-\frac{1}{c}} \tag{21}$$

It is easy to see that when $c=-1$, the Bayes estimator (21) under GELF reduces to the Bayes estimator(18) under SELF.

3.2 Prior and Posterior Distributions

For Bayes estimators, another important element is to specify a prior distribution for the parameter θ. Hence, we consider Gamma prior (see [6]) for θ having pdf

$$g(\theta) = \frac{\delta^\gamma}{\Gamma(\gamma)} e^{-\delta\theta}\theta^{\gamma-1}; \quad \theta>0, \delta>0, \gamma>0 \tag{22}$$

where γ and δ are the hyper- parameters. These can be obtained, if any two independent informations on θ are available, say prior mean and prior variance are known (see [7]). The mean and variance of the prior distribution (22) are $\frac{\gamma}{\delta}$ and $\frac{\gamma}{\delta^2}$ respectively. Thus, we take $M=\frac{\gamma}{\delta}$ and $V=\frac{\gamma}{\delta^2}$, yielding $\gamma=\frac{M^2}{V}$ and $\delta=\frac{M}{V}$.

The formula for the evaluation of the posterior pdf of θ is given by,

$$h(\theta|\mathbf{X}) = \frac{L(\theta)g(\theta)}{\int_0^\infty L(\theta)g(\theta)\partial\theta} \tag{23}$$

Putting the value from (12) and (22) in (23), after simplification, we get

$$h(\theta|\mathbf{X}) = \frac{\theta^{m+\gamma-1}e^{-\delta\theta}\prod_{i=1}^{m}\{1-e^{-x_i}(x_i+1)\}^{\theta-1}\prod_{i=1}^{m}[1-\{1-e^{-x_i}(x_i+1)\}^{\theta}]^{r_i}}{\int_0^\infty \theta^{m+\gamma-1}e^{-\delta\theta}\prod_{i=1}^{m}\{1-e^{-x_i}(x_i+1)\}^{\theta-1}\prod_{i=1}^{m}[1-\{1-e^{-x_i}(x_i+1)\}^{\theta}]^{r_i}\partial\theta} \tag{24}$$

Now, to have an idea about the shape of the prior and posterior pdfs for different confidence levels in the guessed value of θ, we have generated a PT-II censored sample with Binomial removals from EG(θ) distribution for fixed $n = 20$, $m = 16$, $\theta = 2$, $p = 0.7$, $M = 2$ (guess/ expected value of θ as its true value) and $V = 0.1$ (showing a higher confidence in the guessed value) and $V = 100$ (showing a weak confidence in the guessed value). The sample generated is

$(\mathbf{X}, R) = (0.7610091, 3), (1.0066683, 1), (1.5228306, 0), (1.8726164, 0), (1.8920417, 0),$
$(1.9040363, 0), (2.1048797, 0), (2.4016176, 0), (2.6582957, 0), (2.6584566, 0), (3.0237240, 0),$
$(3.0540966, 0), (3.1027171, 0), (3.7851181, 0), (3.9574827, 0), (4.4831233, 0).$

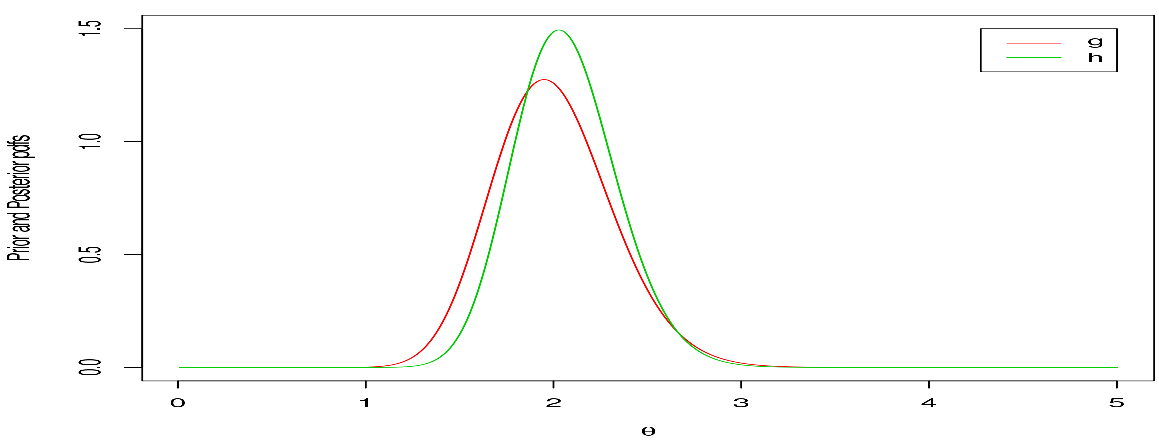

Fig. 1: Prior and Posterior pdfs of θ for a randomly generated sample from EG(θ) for fixed $n = 20$, $m = 16$, $p = 0.7$, $\theta = 2$, $M = 2$ and **V=0.1**

Now, the Bayes estimator of θ under GELF (20) is obtained as follows,

$$\begin{aligned}\hat{\theta}_G &= \left[E\{\theta^{-c}|\mathbf{X}\}\right]^{-\frac{1}{c}} \\ &= \left[\int_0^\infty \theta^{-c}h(\theta|\mathbf{x})\partial\theta\right]^{-\frac{1}{c}} \\ &= \left[\frac{\int_0^\infty \theta^{m-c+\gamma-1}e^{-\delta\theta}\prod_{i=1}^{m}\{1-e^{-x_i}(x_i+1)\}^{\theta-1}\prod_{i=1}^{m}[1-\{1-e^{-x_i}(x_i+1)\}^{\theta}]^{r_i}\partial\theta}{\int_0^\infty \theta^{m+\gamma-1}e^{-\delta\theta}\prod_{i=1}^{m}\{1-e^{-x_i}(x_i+1)\}^{\theta-1}\prod_{i=1}^{m}[1-\{1-e^{-x_i}(x_i+1)\}^{\theta}]^{r_i}\partial\theta}\right]^{-\frac{1}{c}}\end{aligned} \tag{25}$$

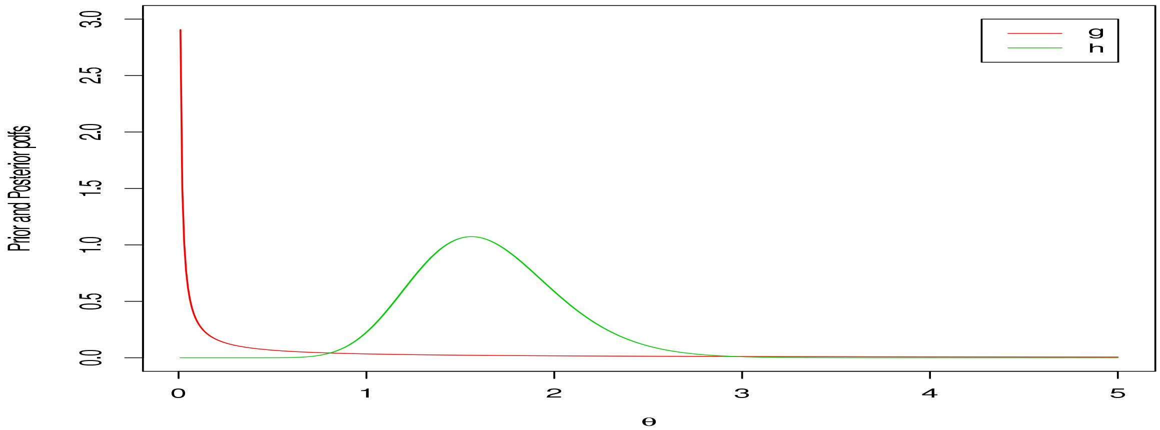

Fig. 2: Prior and Posterior pdfs of θ for a randomly generated sample from EG(θ) for fixed $n = 20$, $m = 16$, $p = 0.7$, $\theta = 2$, $M = 2$ and **V=100**

Further, putting $c = -1$ in (25), we will get the Bayes estimator $\hat{\theta}_S$ of θ under SELF (17) as follows,

$$\hat{\theta}_S = \frac{\int_0^\infty \theta^{m+\gamma} e^{-\delta\theta} \prod_{i=1}^{m} \{1-e^{-x_i}(x_i+1)\}^{\theta-1} \prod_{i=1}^{m} [1-\{1-e^{-x_i}(x_i+1)\}^{\theta}]^{r_i} \partial\theta}{\int_0^\infty \theta^{m+\gamma-1} e^{-\delta\theta} \prod_{i=1}^{m} \{1-e^{-x_i}(x_i+1)\}^{\theta-1} \prod_{i=1}^{m} [1-\{1-e^{-x_i}(x_i+1)\}^{\theta}]^{r_i} \partial\theta} \tag{26}$$

The above integrals can not be solved in analytic form, and we have used Gauss- Laguerre's quadrature formula for their evaluation.

4 Comparison of the Estimators

In this section, we compare the various estimators i.e. $\hat{\theta}_M$, $\hat{\theta}_S$, $\hat{\theta}_G$ in terms of simulated risks (average loss over sample space) under GELF. It is clear that the expressions for the risks cannot be obtained in nice closed form. So, the risks of the estimators are estimated on the basis of Monte Carlo simulation study of 15000 samples. It may be noted that the risks of the estimators will be a function of number of items put on test n, number of failure items m, parameter θ of the model, the hyper parameters γ and δ of the prior distribution, the probability of removals p and the loss parameter c. In order to consider the variation of these values, we obtained the simulated risks for $n = 20$, $m = 16$, $c = \pm 2$, $p = 0.1[0.1]0.9$, $(\theta, M) = (0.45, 0.45), (1,1), (2,2), (3,3)$ and $V = 0.1, 0.5, 1, 2, 5, 10, 100$.

Tables 1-2 shows the simulated risks of the estimators of θ for different values of p to know the effect of variation of probability of removals on the behaviour of the estimators and rest Table shows the behaviour of the estimators for different confidence (in terms of the prior variance V) in the guessed value of θ as the prior mean M. For Table 1, the prior variance V is taken as 0.1 (showing highest confidence in the guessed/ expected value of θ as $M = 2$) and for Table 2, it is taken as 100 (showing weak confidence in the guessed value of θ as $M = 2$). Tables 3-6 shows the simulated risks of the estimators of θ for different confidence (in terms of the prior variance V) in the guessed/ expected value of θ as the prior mean M. The values of (θ, M) are taken as (0.45, 0.45), (1, 1), (2, 2) and (3, 3) respectively.

Table 1: Risks of the estimators of θ under GELF for fixed $n = 20$, $m = 16$, $\theta = 2$, $M = 2$, $V = 0.1$

p	c=+2			c=-2		
	$R_G(\hat{\theta}_G)$	$R_G(\hat{\theta}_S)$	$R_G(\hat{\theta}_M)$	$R_G(\hat{\theta}_G)$	$R_G(\hat{\theta}_S)$	$R_G(\hat{\theta}_M)$
0.1	0.01806085	0.02280319	0.2616136	0.02224671	0.02081016	0.1432867
0.2	0.01753692	0.02051633	0.2217551	0.02130798	0.02034693	0.1349945
0.3	0.01747559	0.01966373	0.2040761	0.02061367	0.01991531	0.1302015
0.4	0.0175328	0.01936451	0.1973288	0.01978936	0.01931158	0.1275748
0.5	0.01746726	0.01892542	0.1895154	0.01967025	0.01926669	0.1269812
0.6	0.01725819	0.01840503	0.180719	0.01940204	0.01908038	0.1271345
0.7	0.01735347	0.01847842	0.1786356	0.01922389	0.01900441	0.12498
0.8	0.0173863	0.01842922	0.1782316	0.01929778	0.0190292	0.1244912
0.9	0.0174571	0.01819471	0.1711512	0.01935479	0.01919445	0.1242753

Table 2: Risks of the estimators of θ under GELF for fixed $n = 20$, $m = 16$, $\theta = 2$, $M = 2$, $V = 100$

p	c=+2			c=-2		
	$R_G(\hat{\theta}_G)$	$R_G(\hat{\theta}_S)$	$R_G(\hat{\theta}_M)$	$R_G(\hat{\theta}_G)$	$R_G(\hat{\theta}_S)$	$R_G(\hat{\theta}_M)$
0.1	0.1842305	0.2613134	0.2627249	0.1540899	0.1432171	0.1457917
0.2	0.1643036	0.2249233	0.2261695	0.1385676	0.1313545	0.1334275
0.3	0.1494717	0.2002807	0.2013577	0.1321424	0.126318	0.1289029
0.4	0.1447854	0.191031	0.1921082	0.1292502	0.1249286	0.1269216
0.5	0.1407876	0.1823119	0.1834088	0.1260229	0.1221056	0.1249726
0.6	0.1443127	0.1859287	0.1869588	0.1265856	0.1233041	0.1255811
0.7	0.1382853	0.1759059	0.1767949	0.1253101	0.1228271	0.1254824
0.8	0.1404795	0.177176	0.1783076	0.1249847	0.1232442	0.1253196
0.9	0.1377727	0.1717521	0.1727641	0.1231351	0.1211229	0.1236936

Table 3: Risks of the estimators of θ under GELF for fixed $n = 20$, $m = 16$, $\theta = 0.45$, $M = 0.45$, $p = 0.7$

V	c=+2			c=-2		
	$R_G(\hat{\theta}_G)$	$R_G(\hat{\theta}_S)$	$R_G(\hat{\theta}_M)$	$R_G(\hat{\theta}_G)$	$R_G(\hat{\theta}_S)$	$R_G(\hat{\theta}_M)$
0.1	0.1522145	0.2597943	20.5683	0.1430109	0.1283871	2.250325
0.5	0.1982171	0.3231893	20.5683	0.1683532	0.1511554	2.250325
1	0.2002473	0.3301908	20.5683	0.1691168	0.1517257	2.250325
2	0.2058107	0.3365795	20.5683	0.1671809	0.1502441	2.250325
5	0.2078822	0.3378962	20.5683	0.170149	0.1528192	2.250325
10	0.2062502	0.3368736	20.5683	0.1729808	0.1553371	2.250325
100	0.2047746	0.3356332	20.5683	0.1720665	0.1548754	2.250325

Table 4: Risks of the estimators of θ under GELF for fixed $n = 20$, $m = 16$, $\theta = 1$, $M = 1$, $p = 0.7$

V	c=2			c=-2		
	$R_G(\hat{\theta}_G)$	$R_G(\hat{\theta}_S)$	$R_G(\hat{\theta}_M)$	$R_G(\hat{\theta}_G)$	$R_G(\hat{\theta}_S)$	$R_G(\hat{\theta}_M)$
0.1	0.01998175	0.03513954	0.1772896	0.02743486	0.02357507	0.1254698
0.5	0.08176263	0.1149959	0.1757939	0.0816942	0.07709967	0.1249649
1	0.1027509	0.1397839	0.1810192	0.09569857	0.09148601	0.1261698
2	0.1145359	0.1559517	0.1845201	0.1009505	0.09688423	0.1240642
5	0.1125766	0.1552062	0.1739459	0.1071259	0.1026709	0.1254877
10	0.1206365	0.1649486	0.1806348	0.1062286	0.102354	0.1234283
100	0.1198332	0.1629902	0.1759634	0.1109909	0.1069599	0.1248256

Table 5: Risks of the estimators of θ under GELF for fixed $n = 20$, $m = 16$, $\theta = 2$, $M = 2$, $p = 0.7$

V	c=+2			c=-2		
	$R_G(\hat{\theta}_G)$	$R_G(\hat{\theta}_S)$	$R_G(\hat{\theta}_M)$	$R_G(\hat{\theta}_G)$	$R_G(\hat{\theta}_S)$	$R_G(\hat{\theta}_M)$
0.1	0.01738049	0.01849966	0.1809	0.01928572	0.01902396	0.1226408
0.5	0.05263576	0.06856368	0.1800376	0.05624724	0.05315186	0.1244203
1	0.08196411	0.1032413	0.1824513	0.07914178	0.07612692	0.1229493
2	0.1040259	0.1298358	0.1763877	0.09843685	0.09536179	0.1234137
5	0.1250234	0.1580818	0.1796584	0.1153303	0.1125645	0.1258732
10	0.1303433	0.1672054	0.1784685	0.120486	0.1178791	0.1255648
100	0.1391073	0.177199	0.1781687	0.1254443	0.1228761	0.1251145

Table 6: Risks of the estimators of θ under GELF for fixed $n = 20$, $m = 16$, $\theta = 3$, $M = 3$, $p = 0.7$

V	c=2			c=-2		
	$R_G(\hat{\theta}_G)$	$R_G(\hat{\theta}_S)$	$R_G(\hat{\theta}_M)$	$R_G(\hat{\theta}_G)$	$R_G(\hat{\theta}_S)$	$R_G(\hat{\theta}_M)$
0.1	0.002441971	0.002372834	0.1779936	0.002452271	0.002487627	0.1236747
0.5	0.02418586	0.02894724	0.1793605	0.02651089	0.02496056	0.1235788
1	0.05407217	0.0626433	0.1808652	0.05448362	0.05292322	0.1249691
2	0.08221695	0.09848584	0.1790107	0.08108148	0.07933753	0.1251812
5	0.107614	0.1324118	0.1703554	0.1045946	0.1024301	0.1236653
10	0.1235384	0.1542973	0.1752024	0.1146213	0.1120572	0.1229152
100	0.142346	0.1810237	0.1811093	0.129197	0.1266361	0.1270126

5 Conclusion

In this paper, we proposed the classical and the Bayesian approaches to estimate the parameter of EG(θ)- distribution. In classical approach, we have derived MLE. Bayes estimators are obtained using both GELF and SELF. To compare the considered estimators, extensive simulation studies have been performed. The hyper parameters were chosen as per method suggested in sub-section 3.2. The results shows that, when over estimation is more serious than under estimation, in almost all the cases, the estimator $\hat{\theta}_G$ performs better than the estimators $\hat{\theta}_S$ and $\hat{\theta}_M$; while in the reverse situation, in almost all the cases, $\hat{\theta}_S$ performs better than $\hat{\theta}_G$ and $\hat{\theta}_M$.

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