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Upper and Lower Control Limits for Mean under Non-Normal and Correlated Data

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Abstract: For conducting the design of control charts, one usually assumes the measurements within a subgroup are independently and normally distributed. However, this assumption may not be tenable. In this paper, an attempt has been made to develop the control limits for mean chart under non-normal and correlated data by making use of an Edgeworth series. Control limits are calculated for different values of correlation coefficient and standardized cumulants.

Keywords: Control Chart, Correlation, Edgeworth Series, Standardized Cumulants.

1 Introduction

Statistical methods provide a useful application in process control. One of the key methods is control chart technique, which may be considered as a graphical expression of statistical hypothesis testing. Since 1924 when Shewart presented the first control chart, various control chart technique have been developed and widely applicable in process control. The major function of control charting is to detect the accuracy of assignable cause so that the necessary corrective action may be taken before a large quantity of non-conforming products is manufactured.

Traditionally, when conducting the design of control charts, one usually assumes the measurements within the subgroup are independently and normally distributed. However, this assumption may not be tenable in some specific processes. In practice, the normality assumption is often violated. Alwan and Roberts [1] examine 235 quality control applications and find that in most cases the assumptions of normality and independence are not fulfilled, resulting in incorrect control limits. The impact of non-normality on the performance of the control chart can be substantial. Shewhart shows that the probability of false signaling of the $\overline{X}$ control chart with 3σ limits is smaller than 0.11 irrespective of the underlying distribution, and smaller than 0.05 for distributions likely to be encountered in practice, i.e. for strongly unimodal distributions. Padgett et al. [2] examine the impact of non-normality on the design scheme when μ and σ are estimated by their usual estimators, i.e. for μ the mean of the sample means and for σ the mean sample standard deviation or the mean sample range. They also conclude that the in-control probability of signaling of both charts greatly increases under non-normality. Several researchers correct the control limits based on the shape of the underlying distribution. Burr [3] studies the effect of non-normality on the $\overline{X}$ control chart considering various degrees of skewness and kurtosis. He determines constants for each degree of non-normality. Albers and Kallenberg [4] use the normal power family to model the underlying distribution. When the production process consists of multiple but similar units on a single part, such as several cavities on a single casting, multiple pins on an integral circuit chips or multiple contact pads on a single machine mount (grant and Leavenworth [5]) ,the collected measurements within a subgroup may be correlated. Neuhardt [6] studied the effect of correlation within a subgroup on control charts. Yang and Honcock [7] extended Neuhardt's work to conduct simulation studies to determine the effect of correlated data on $\overline{X}$, R and S^2 chart. For the assumption of normality, if the measurement are really normally distributed the statistic $\overline{X}$ is also normally distributed. If the measurements are asymmetrically distributed, the statistic $\overline{X}$ will be approximately normally distributed only when the sample size n is sufficiently large. Unfortunately, when a control chart is applied to monitor the process, the sample size n is always not sufficiently large due to sampling cost. Therefore if measurements are not normally distributed the traditional way for designing the control chart may reduce the ability that a control chart detects the assignable cause. Yourstone and Zimmer

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[8] Used the burr distribution to represent various non-normal distributions. Singh and Kulkarni [9] have studied the problem of setting up control limits for mean in case of non-normal variation and EWMA model. They represented non-normal distribution by the first four times of an edgeworth series.

The aim of the present chapter is to study the problem of setting up control limits for means in case of non- normal and correlated data. The non-normal distribution has been represented by first four terms of an Edgeworth series. The values of standardized cummulant $\lambda_3 = \sqrt{\beta_1}$ and $\lambda_4 = \beta_2 - 3$ considered are with Barton and Dennis [10] limits, which means that for such values the population is positive definite and unimodel. For various non-normal populations and different values of correlation coefficient (ρ), the values of upper and lower control limits are tabulated and compared with those of the normal population.

2 Effect of Non- Normality and Correlation on Control Limits for Mean

We consider the frequency function for the quality characteristics to be represented by the Edgeworth form of type A series then the probability integral for it has been given by Cornish and Fisher in terms of the normal probability levels. Let ξ denote the variable for the quality characteristic in standard form i.e., having zero mean and unit standard variate. The p percent probability levels of the standard normal variable $X = \frac{(W_t - \mu)}{\sigma}$ by the expression

$$\begin{aligned}\xi = X &+ \frac{1}{6}\sqrt{\beta_1}\left(X^2-1\right)+\frac{1}{24}(\beta_2-3)\left(X^3-3X\right)-\frac{1}{36}\beta_1\left(2X^3-5X\right)+\frac{1}{120}\lambda_5\left(X^4-6X^2+3\right)\\ &-\frac{1}{24}\lambda_3\lambda_4\left(X^4-5X^2+2\right)+\frac{1}{324}\lambda_3^2\left(12X^4-53X^2+17\right)+\frac{1}{720}\lambda_6\left(X^5-10X^3+15X\right)\\ &-\frac{1}{384}\lambda_4^2\left(3X^5-24X^3+29X\right)-\frac{1}{180}\lambda_3\lambda_5\left(2X^5-17X^3+21X\right)+\frac{1}{288}\lambda_3^2\lambda_4\left(14X^5-103X^3+107X\right)\\ &-\frac{1}{7776}\lambda_3^4\left(252X^5-1688X^3+151X\right)\end{aligned} \tag{2.1}$$

Where $\lambda_3 = \sqrt{\beta_1}$, $\lambda_4 = (\beta_2 - 3)$, λ_5 and λ_6 are standardized cumulants defined by $\lambda_i = \frac{K_i}{K_2^{1/2}}$ of the distribution of a statistic we should find out the first four few λ-coefficients of the statistic for the determination of the probability integral in terms of the normal deviate X by the equation (2.1). In the case of control chart for average ξ we should therefore, obtain standardized cumulant of ξ and substitute then in the above expressions to obtain the necessary control limits.

When the basic variable ξ follow the Edgeworth series the mean $\bar{\xi}$ also follow the same law but with different values of standardized cumlant whose expression are already known. If we consider moderately non-normal population, terms upto that in β_1 will provide good approximation. This is particularly so far the distribution of mean even in cases where basic population may need more terms of edgeworth series for a satisfactory representation. Owing to the fact that standardized cumulant of the distribution of the mean are of order n^{-1} where n is the size of the sample and $i = \frac{1}{2}, 1, \frac{3}{2}, \ldots$ when we stick to the first four moments and correlated data ,we obtain the simpler expression, by neglecting power of λ_4 , λ_3^2 and terms of order higher than those in λ_4 and λ_3^2 such as

$$\begin{aligned}\bar{\xi} &= X + \frac{1}{6}\lambda_3 T\left(X^2-1\right)+\frac{1}{24}\lambda_4 T^2\left(X^3-3X\right)-\frac{1}{36}\lambda_3^2 T^2\left(2X^3-5X\right)\\ \bar{\xi} &= \bar{X} + \lambda_3 T M_3\left(\bar{X}\right)+\lambda_4 T^2 M_4\left(\bar{X}\right)+\lambda_3^2 T^2 M_{33}\left(\bar{X}\right)\end{aligned} \tag{2.2}$$

Where, $M_3\left(\bar{X}\right) = \frac{1}{6}\left(\bar{X}^2-1\right)$, $M_4\left(\bar{X}\right) = \frac{1}{24}\left(\bar{X}^3-3\bar{X}\right)$ and $M_{33}\left(\bar{X}\right) = -\frac{1}{36}\left(2\bar{X}^3-5\bar{X}\right)$

And $T^2 = \left[1+(n-1)\rho\right]$.

In fact β_1 and β_2 for the basic variable may even be considerably large, for those of the statistic (here, mean) is less in magnitude. We know for the mean

$$\beta_1(\bar{\xi}) = \frac{\beta_1(\bar{\xi}_1)}{n} \quad \text{And} \quad \beta_2(\bar{\xi}) = 3 + \frac{\beta_2(\bar{\xi} - 3)}{n}$$

And note that the probability integral for non-normal variable ξ and correlated model.

$$\int_{-\infty}^{\zeta_1} P(\xi)d\xi = \int_{\xi_2}^{\infty} P(\xi)d\xi = 0.005$$

In which ξ_1 and ξ_2 are given by

$$\bar{\xi} = \bar{X} + \frac{\lambda_3}{T} M_3(\bar{X}) + \frac{\lambda_4}{T^2} M_4(\bar{X}) + \frac{\lambda_3^2}{T^2} M_{33}(\bar{X}) \tag{2.3}$$

Where $\bar{X} = \pm 2.576T$

2.1 Upper and Lower Control Limits for Non-Normal and Un-Correlated Population (ρ=0 and n=5)

λ3→ λ4↓	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
-0.5	-2.2811	-2.1694	-2.0459	-1.9105	-1.7634	-1.6043	-1.4335	-1.2508	-1.0563	-0.8499
	2.4689	2.5451	2.6094	2.6619	2.7026	2.7314	2.7484	2.7535	2.7468	2.7283
-0.4	-2.3201	-2.2084	-2.0849	-1.9496	-1.8024	-1.6434	-1.4725	-1.2898	-1.0953	-0.8889
	2.5079	2.5841	2.6484	2.7009	2.7416	2.7704	2.7874	2.7925	2.7858	2.7673
-0.3	-2.3591	-2.2474	-2.1239	-1.9886	-1.8414	-1.6824	-1.5115	-1.3288	-1.1343	-0.9279
	2.5469	2.6231	2.6874	2.7399	2.7806	2.8094	2.8264	2.8315	2.8249	2.8063
-0.2	-2.3981	-2.2864	-2.1629	-2.0276	-1.8804	-1.7214	-1.5505	-1.3678	-1.1733	-0.9670
	2.5860	2.6621	2.7265	2.7790	2.8196	2.8484	2.8654	2.8706	2.8639	2.8454
--0.1	-2.4371	-2.3255	-2.2020	-2.0666	-1.9194	-1.7604	-1.5896	-1.4069	-1.2123	-1.0060
	2.6250	2.7011	2.7655	2.8180	2.8586	2.8875	2.9044	2.9096	2.9029	2.8844
0	-2.4762	-2.3645	-2.2410	-2.1056	-1.9585	-1.7994	-1.6286	-1.4459	-1.2514	-1.0450
	2.6640	2.7402	2.8045	2.8570	2.8977	2.9265	2.9435	2.9486	2.9419	2.9234
0.1	-2.5152	-2.4035	-2.28	-2.1447	-1.9975	-1.8385	-1.6676	-1.4849	-1.2904	-1.084
	2.7030	2.7792	2.8435	2.8960	2.9367	2.9655	2.9825	2.9876	2.9809	2.9624
0.2	-2.5542	-2.4425	-2.3190	-2.1837	-2.0365	-1.8775	-1.7066	-1.5239	-1.3294	-1.1230
	2.7420	2.8182	2.8825	2.9350	2.9757	3.0045	3.0215	3.0266	3.0200	3.0014
0.3	-2.5932	-2.4815	-2.3580	-2.2227	-2.0755	-1.9165	-1.7456	-1.5629	-1.3684	-1.1621
	2.7811	2.8572	2.9216	2.9741	3.0147	3.0435	3.0605	3.0657	3.0590	3.0405
0.4	-2.6322	-2.5206	-2.3971	-2.2617	-2.1145	-1.9555	-1. 7848	-1.6020	-1.4074	-1.2011
	2.8201	2.8962	2.9606	3.0131	3.0537	3.0826	3.0995	3.1047	3.0980	3.0795
0.5	-2.6713	-2.5596	-2.4361	-2.3007	-2.1536	-1.9945	-1.8237	-1.6410	-1.4465	-1.2401
	2.8591	2.9353	2.9996	3.0521	3.0928	3.1216	3.1386	3.1437	3.1370	3.1185
0.6	-2.7103	-2.5986	-2.4751	-2.3398	-2.1926	-2.0336	-1.8627	-1.6800	-1.4855	-1.2791
	2.8981	2.9743	3.0386	3.0911	3.1318	3.1606	3.1776	3.1827	3.1760	3.1575
0.7	-2.7493	-2.6376	-2.5141	-2.3788	-2.2316	-2.0726	-1.9017	-1.7190	-1.5245	-1.3181
	2.9371	3.0133	3.0776	3.1301	3.1708	3.1996	3.2166	3.2217	3.2151	3.1965
0.8	0. 7890	-2.6766	-2.5531	-2.4178	-2.2706	-2.1116	-1.9407	-1.7580	-1.5635	-1.3572
	2.9762	3.0523	3.1167	3.1692	3.2098	3.2386	3.2556	3.2608	3.2541	3.2356

Continued…

λ3→ λ4↓	-1	-0.9	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0
-0.5	-2.7283	-2.7468	-2.7535	-2.7484	-2.7314	-2.7026	-2.6619	-2.6094	-2.5451	-2.4689	-2.3809
	0.8499	1.0563	1.2508	1.4335	1.6043	1.7634	1.9105	2.0459	2.1694	2.2811	2.3809
-0.4	-2.7673	-2.7858	-2.7925	-2.7874	-2.7704	-2.7416	-2.7009	-2.6484	-2.5841	-2.5079	-2.4199
	0.8889	1.0953	1.2898	1.4725	1.6434	1.8024	1.9496	2.0849	2.2084	2.3201	2.4199
-0.3	-2.8063	-2.8249	-2.8315	-2.8264	-2.8094	-2.7806	-2.7399	-2.6874	-2.6231	-2.5469	-2.4589
	0.9279	1.1343	1.3288	1.5115	1.6824	1.8414	1.9886	2.1239	2.2474	2.3591	2.4589
-0.2	-2.8454	-2.8639	-2.8706	-2.8654	-2.8484	-2.8196	-2.7790	-2.7265	-2.6621	-2.5860	-2.4980
	0.9670	1.1733	1.3678	1.5505	1.7214	1.8804	2.0276	2.1629	2.2864	2.3981	2.4980
-0.1	-2.8844	-2.9029	-2.9096	-2.9044	-2.8875	-2.8586	-2.8180	-2.7655	-2.7011	-2.6250	-2.5370
	1.0060	1.2123	1.4069	1.5896	1.7604	1.9194	2.0666	2.2020	2.3255	2.4371	2.5370
0	-2.9234	-2.9419	-2.9486	-2.9435	-2.9265	-2.8977	-2.8570	-2.8045	-2.7402	-2.6640	-2.5760
	1.0450	1.2514	1.4459	1.6286	1.7994	1.9585	2.1056	2.2410	2.3645	2.4762	2.5760
0.1	-2.9624	-2.9809	-2.9876	-2.9825	-2.9655	-2.9367	-2.8960	-2.8435	-2.7792	-2.7030	-2.6150
	1.0840	1.2904	1.4849	1.6676	1.8385	1.9975	2.1447	2.2800	2.4035	2.5152	2.6150
0.2	-3.0014	-3.0200	-3.0266	-3.0215	-3.0045	-2.9757	-2.9350	-2.8825	-2.8182	-2.7420	-2.6540
	1.1230	1.3294	1.5239	1.7066	1.8775	2.0365	2.1837	2.3190	2.4425	2.5542	2.6540
0.3	-3.0405	-3.0590	-3.0657	-3.0605	-3.0435	-3.0147	-2.9741	-2.9216	-2.8572	-2.7811	-2.6931
	1.1621	1.3684	1.5629	1.7456	1.9165	2.0755	2.2227	2.3580	2.4815	2.5932	2.6931
0.4	-3.0795	-3.0980	-3.1047	-3.0995	-3.0826	-3.0537	-3.0131	-2.9606	-2.8962	-2.8201	-2.7321
	1.2011	1.4074	1.6020	1.7847	1.9555	2.1145	2.2617	2.3971	2.5206	2.6322	2.7321
0.5	-3.3111	-3.2096	-3.1126	-3.0202	-2.9324	-2.8491	-2.7704	-2.6962	-2.6266	-2.5615	-2.5010
	1.8409	1.9424	2.0394	2.1318	2.2196	2.3029	2.3816	2.4558	2.5254	2.5905	2.6510
0.6	-3.1575	-3.1760	-3.1827	-3.1776	-3.1606	-3.1318	-3.0911	-3.0386	-2.9743	-2.8981	-2.8101
	1.2791	1.4855	1.6800	1.8627	2.0336	2.1926	2.3398	2.4751	2.5986	2.7103	2.8101
0.7	-3.1965	-3.2151	-3.2217	-3.2166	-3.1996	-3.1708	-3.1301	-3.0776	-3.0133	-2.9371	-2.8491
	1.3181	1.5245	1.7190	1.9017	2.0726	2.2316	2.3788	2.5141	2.6376	2.7493	2.8491
0.8	-3.2356	-3.2541	-3.2608	-3.2556	-3.2386	-3.2098	-3.1692	-3.1167	-3.0523	-2.9762	-2.8882
	1.3572	1.5635	1.758	1.9407	2.1116	2.2706	2.4178	2.5531	2.6766	2.7883	2.8882

2.2 Upper and Lower Control Limits for Non-Normal and Correlated Population (for n=5 and ρ =0. 4)

λ3→ λ4↓	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
-0.5	-2.5951	-2.5436	-2.4968	-2.4545	-2.4167	-2.3835	-2.3548	-2.3307	-2.3112	-2.2962
	2.5569	2.6084	2.6552	2.6975	2.7353	2.7685	2.7972	2.8213	2.8408	2.8558
-0.4	-2.5801	-2.5286	-2.4818	-2.4395	-2.4017	-2.3685	-2.3398	-2.3157	-2.2962	-2.2812
	2.5719	2.6234	2.6702	2.7125	2.7503	2.7835	2.8122	2.8363	2.8558	2.8708
-0.3	-2.5651	-2.5136	-2.4668	-2.4244	-2.3867	-2.3535	-2.3248	-2.3007	-2.2812	-2.2662
	2.5869	2.6384	2.6852	2.7276	2.7653	2.7985	2.8272	2.8513	2.8708	2.8858
-0.2	-2.5500	-2.4986	-2.4518	-2.4094	-2.3717	-2.3385	-2.3098	-2.2857	-2.2661	-2.2511
	2.6020	2.6534	2.7002	2.7426	2.7803	2.8135	2.8422	2.8663	2.8859	2.9009
-0.1	-2.5350	-2.4836	-2.4367	-2.3944	-2.3567	-2.3235	-2.2948	-2.2707	-2.2511	-2.2361
	2.6170	2.6684	2.7153	2.7576	2.7953	2.8285	2.8572	2.8813	2.9009	2.9159
0	-2.5200	-2.4686	-2.4217	-2.3794	-2.3417	-2.3085	-2.2798	-2.2557	-2.2361	-2.2211
	2.6320	2.6834	2.7303	2.7726	2.8103	2.8435	2.8722	2.8963	2.9159	2.9309
0.1	-2.5050	-2.4536	-2.4067	-2.3644	-2.3267	-2.2934	-2.2648	-2.2407	-2.2211	-2.2061
	2.6470	2.6984	2.7453	2.7876	2.8253	2.8586	2.8872	2.9113	2.9309	2.9459
0.2	-2.4900	-2.4386	-2.3917	-2.3494	-2.3116	-2.2784	-2.2498	-2.2257	-2.2061	-2.1911
	2.662	2.7134	2.7603	2.8026	2.8404	2.8736	2.9022	2.9263	2.9459	2.9609
0.3	-2.4750	-2.4236	-2.3767	-2.3344	-2.2966	-2.2634	-2.2348	-2.2107	-2.1911	-2.1761

	2.6770	2.7284	2.7753	2.8176	2.8554	2.8886	2.9172	2.9413	2.9609	2.9759
0.4	-2.4600	-2.4086	-2.3617	-2.3194	-2.2816	-2.2484	-2.2198	-2.1957	-2.1761	-2.1611
	2.6920	2.7434	2.7903	2.8326	2.8704	2.9036	2.9322	2.9563	2.9759	2.9909
0.5	-2.4450	-2.3936	-2.3467	-2.3044	-2.2666	-2.2334	-2.2048	-2.1806	-2.1611	-2.1461
	2.7070	2.7584	2.8053	2.8476	2.8854	2.9186	2.9472	2.9714	2.9909	3.0059
0.6	-2.4300	-2.3786	-2.3317	-2.2894	-2.2516	-2.2184	-2.1897	-2.1656	-2.1461	-2.1311
	2.7220	2.7734	2.8203	2.8626	2.9004	2.9336	2.9623	2.9864	3.0059	3.0209
0.7	-2.4150	-2.3636	-2.3167	-2.2744	-2.2366	-2.2034	-2.1747	-2.1506	-2.1311	-2.1161
	2.7370	2.7884	2.8353	2.8776	2.9154	2.9486	2.9773	3.0014	3.0209	3.0359
0.8	-2.4000	-2.3485	-2.3017	-2.2594	-2.2216	-2.1884	-2.1597	-2.1356	-2.1161	-2.1011
	2.7520	2.8035	2.8503	2.8926	2.9304	2.9636	2.9923	3.0164	3.0359	3.0509

Continued…

$\lambda 3 \rightarrow$ $\lambda 4 \downarrow$	-1	-0.9	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0
-0.5	-3.4611	-3.3596	-3.2627	-3.1703	-3.0825	-2.9992	-2.8090	-2.8463	-2.7766	-2.7116	-2.6510
	1.6909	1.7924	1.8893	1.9817	2.0695	2.1528	2.2315	2.3057	2.3754	2.4404	2.5010
-0.4	-3.4461	-3.3446	-3.2477	-3.1553	-3.0675	-2.9842	-2.9054	-2.8313	-2.7616	-2.6966	-2.6360
	1.7059	1.8074	1.9043	1.9967	2.0845	2.1678	2.2466	2.3207	2.3904	2.4554	2.5160
-0.3	-3.4311	-3.3296	-3.2327	-3.1403	-3.0525	-2.9692	-2.8904	-2.8163	-2.7466	-2.6815	-2.6210
	1.7209	1.8224	1.9193	2.0117	2.0995	2.1828	2.2616	2.3357	2.4054	2.4705	2.5310
-0.2	-3.4161	-3.3146	-3.2177	-3.1253	-3.0374	-2.9542	-2.8754	-2.8012	-2.7316	-2.6665	-2.6060
	1.7359	1.8374	1.9343	2.0267	2.1146	2.1978	2.2766	2.3508	2.4204	2.4855	2.5460
-0.1	-3.4011	-3.2996	-3.2027	-3.1103	-3.0224	-2.9392	-2.8604	-2.7862	-2.7166	-2.6515	-2.5910
	1.7509	1.8524	1.9493	2.0417	2.1296	2.2128	2.2916	2.3658	2.4354	2.5005	2.5610
0	-3.3861	-3.2846	-3.1877	-3.0953	-3.0074	-2.9241	-2.8454	-2.7712	-2.7016	-2.6365	-2.5760
	1.7659	1.8674	1.9643	2.0567	2.1446	2.2279	2.3066	2.3808	2.4504	2.5155	2.5760
0.1	-3.3711	-3.2696	-3.1727	-3.0803	-2.9924	-2.9091	-2.8304	-2.7562	-2.6866	-2.6215	-2.5610
	1.7809	1.8824	1.9793	2.0717	2.1596	2.2429	2.3216	2.3958	2.4654	2.5305	2.5910
0.2	-3.3561	-3.2546	-3.1576	-3.0653	-2.9774	-2.8941	-2.8154	-2.7412	-2.6716	-2.6065	-2.5460
	1.7959	1.8974	1.9944	2.0867	2.1746	2.2579	2.3366	2.4108	2.4804	2.5455	2.606
0.3	-3.3411	-3.2396	-3.1426	-3.0502	-2.9624	-2.8791	-2.8004	-2.7262	-2.6566	-2.5915	-2.5310
	1.8109	1.9124	2.0094	2.1018	2.1896	2.2729	2.3516	2.4258	2.4954	2.5605	2.6210
0.4	-3.3261	-3.2246	-3.1276	-3.0352	-2.9474	-2.8641	-2.7854	-2.7112	-2.6416	-2.5765	-2.5160
	1.8259	1.9274	2.0244	2.1168	2.2046	2.2879	2.3666	2.4408	2.5104	2.5755	2.6360
0.5	-3.3111	-3.2096	-3.1126	-3.0202	-2.9324	-2.8491	-2.7704	-2.6962	-2.6266	-2.5615	-2.5010
	1.8409	1.9424	2.0394	2.1318	2.2196	2.3029	2.3816	2.4558	2.5254	2.5905	2.6510
0.6	-3.2961	-3.1946	-3.0976	-3.0052	-2.9174	-2.8341	-2.7554	-2.6812	-2.6116	-2.5465	-2.4860
	1.8559	1.9574	2.0544	2.1468	2.2346	2.3179	2.3966	2.4708	2.5404	2.6055	2.6660
0.7	-3.2810	-3.1796	-3.0826	-2.9902	-2.9024	-2.8191	-2.7404	-2.6662	-2.5965	-2.5315	-2.4709
	1.8710	1.9724	2.0694	2.1618	2.2496	2.3329	2.4116	2.4858	2.5555	2.6205	2.6811
0.8	-3.2660	-3.1645	-3.0676	-2.9752	-2.8874	-2.8041	-2.7254	-2.6512	-2.5815	-2.5165	-2.4559
	1.8860	1.9875	2.0844	2.1768	2.2646	2.3479	2.4266	2.5008	2.5705	2.6355	2.6961
0.9	-3.2510	-3.1495	-3.0526	-2.9602	-2.8724	-2.7891	-2.7103	-2.6362	-2.5665	-2.5015	-2.4409
	1.9010	2.0025	2.0994	2.1918	2.2796	2.3629	2.4417	2.5158	2.5855	2.6505	2.7111

2.3 Upper and Lower Control Limits for Non-Normal and Un-Correlated Population (ρ=0.8 and n=5)

$\lambda 3 \rightarrow$ $\lambda 4 \downarrow$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
-0.5	-2.5778	-2.5360	-2.4970	-2.4608	-2.4275	-2.3969	-2.3692	-2.3443	-2.3222	-2.3030
	2.5742	2.6160	2.6550	2.6912	2.7245	2.7551	2.7828	2.8077	2.8298	2.8490
-0.4	-2.5685	-2.5267	-2.4877	-2.4515	-2.4182	-2.3877	-2.3599	-2.3350	-2.3129	-2.2937
	2.5835	2.6253	2.6643	2.7005	2.7338	2.7643	2.7921	2.8170	2.8391	2.8583
-0.3	-2.5592	-2.5174	-2.4784	-2.4423	-2.4089	-2.3784	-2.3506	-2.3257	-2.3037	-2.2844

	2.5928	2.6346	2.6736	2.7097	2.7431	2.7736	2.8014	2.8263	2.8483	2.8676
-0.2	-2.5500	-2.5081	-2.4691	-2.4330	-2.3996	-2.3691	-2.3413	-2.3164	-2.2944	-2.2751
	2.6020	2.6439	2.6829	2.7190	2.7524	2.7829	2.8107	2.8356	2.8576	2.8769
-0.1	-2.5407	-2.4988	-2.4599	-2.4237	-2.3903	-2.3598	-2.3321	-2.3072	-2.2851	-2.2658
	2.6113	2.6532	2.6921	2.7283	2.7617	2.7922	2.8199	2.8448	2.8669	2.8862
0	-2.5314	-2.4896	-2.4506	-2.4144	-2.3810	-2.3505	-2.3228	-2.2979	-2.2758	-2.2565
	2.6206	2.6624	2.7014	2.7376	2.7710	2.8015	2.8292	2.8541	2.8762	2.8955
0.1	-2.5221	-2.4803	-2.4413	-2.4051	-2.3717	-2.3412	-2.3135	-2.2886	-2.2665	-2.2472
	2.6299	2.6717	2.7107	2.7469	2.7803	2.8108	2.8385	2.8634	2.8855	2.9048
0.2	-2.5128	-2.4710	-2.4320	-2.3958	-2.3624	-2.3319	-2.3042	-2.2793	-2.2572	-2.2379
	2.6392	2.6810	2.7200	2.7562	2.7896	2.8201	2.8478	2.8727	2.8948	2.9141
0.3	-2.5035	-2.4617	-2.4227	-2.3865	-2.3532	-2.3226	-2.2949	-2.2700	-2.2479	-2.2286
	2.6485	2.6903	2.7293	2.7655	2.7988	2.8294	2.8571	2.8820	2.9041	2.9234
0.4	-2.4942	-2.4524	-2.4134	-2.3772	-2.3439	-2.3133	-2.2856	-2.2607	-2.2386	-2.2194
	2.6578	2.6996	2.7386	2.7748	2.8081	2.8387	2.8664	2.8913	2.9134	2.9326
0.5	-2.4849	-2.4431	-2.4041	-2.3679	-2.3346	-2.3040	-2.2763	-2.2514	-2.2293	-2.2101
	2.6671	2.7089	2.7479	2.7841	2.8174	2.8480	2.8757	2.9006	2.9227	2.9419
0.6	-2.4756	-2.4338	-2.3948	-2.3586	-2.3253	-2.2947	-2.2670	-2.2421	-2.2200	-2.2008
	2.6764	2.7182	2.7572	2.7934	2.8267	2.8573	2.8850	2.9099	2.9320	2.9512
0.7	-2.4663	-2.4245	-2.3855	-2.3494	-2.3160	-2.2855	-2.2577	-2.2328	-2.2107	-2.1915
	2.6857	2.7275	2.7665	2.8026	2.8360	2.8665	2.8943	2.9192	2.9413	2.9605
0.8	-2.4570	-2.4152	-2.3762	-2.3401	-2.3067	-2.2762	-2.2484	-2.2235	-2.2015	-2.1822
	2.6950	2.7368	2.7758	2.8119	2.8453	2.8758	2.9036	2.9285	2.9505	2.9698

Continued…

$\lambda3\rightarrow$ $\lambda4\downarrow$	-1	-0.9	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0
-0.5	-3.2237	-3.1509	-3.0809	-3.0138	-2.9494	-2.8879	-2.8292	-2.7733	-2.7202	-2.6699	-2.6225
	1.9283	2.0011	2.0711	2.1382	2.2026	2.2641	2.3228	2.3787	2.4318	2.4821	2.5295
-0.4	-3.2145	-3.1416	-3.0717	-3.0045	-2.9401	-2.8786	-2.8199	-2.7640	-2.7109	-2.6606	-2.6132
	1.9375	2.0104	2.0803	2.1475	2.2119	2.2734	2.3321	2.3880	2.4411	2.4914	2.5388
-0.3	-3.2052	-3.1324	-3.0624	-2.9952	-2.9308	-2.8693	-2.8106	-2.7547	-2.7016	-2.6513	-2.6039
	1.9468	2.0196	2.0896	2.1568	2.2212	2.282	2.3414	2.3973	2.4504	2.5007	2.5481
-0.2	-3.1959	-3.1231	-3.0531	-2.9859	-2.9215	-2.8600	-2.8013	-2.7454	-2.6923	-2.6420	-2.5946
	1.9561	2.0289	2.0989	2.1661	2.2305	2.2920	2.3507	2.4066	2.4597	2.5100	2.5574
-0.1	-3.1866	-3.1138	-3.0438	-2.9766	-2.9123	-2.8507	-2.7920	-2.7361	-2.6830	-2.6327	-2.5853
	1.9654	2.0382	2.1082	2.1754	2.2397	2.3013	2.3600	2.4159	2.4690	2.5193	2.5667
0	-3.1773	-3.1045	-3.0345	-2.9673	-2.9030	-2.8414	-2.7827	-2.7268	-2.6737	-2.6234	-2.5760
	1.9747	2.0475	2.1175	2.1847	2.2490	2.3106	2.3693	2.4252	2.4783	2.5286	2.5760
0.1	-3.1680	-3.0952	-3.0252	-2.9580	-2.8937	-2.8321	-2.7734	-2.7175	-2.6644	-2.6142	-2.5667
	1.9840	2.0568	2.1268	2.1940	2.2583	2.3199	2.3786	2.4345	2.4876	2.5378	2.5853
0.2	-3.1587	-3.0859	-3.0159	-2.9487	-2.8844	-2.8228	-2.7641	-2.7082	-2.6551	-2.6049	-2.5574
	1.9933	2.0661	2.1361	2.2033	2.2676	2.3292	2.3879	2.4438	2.4969	2.5471	2.5946
0.3	-3.1494	-3.0766	-3.0066	-2.9394	-2.8751	-2.8136	-2.7548	-2.6989	-2.6458	-2.5956	-2.5481
	2.0026	2.0754	2.1454	2.2126	2.2769	2.3384	2.3972	2.4531	2.5062	2.5564	2.6039
0.4	-3.1401	-3.0673	-2.9973	-2.9302	-2.8658	-2.8043	-2.7455	-2.6896	-2.6366	-2.5863	-2.5388
	2.0119	2.0847	2.1547	2.2218	2.2862	2.3477	2.4065	2.4624	2.5154	2.5657	2.6132
0.5	-3.1308	-3.0580	-2.9880	-2.9209	-2.8565	-2.7950	-2.7362	-2.6803	-2.6273	-2.5770	-2.5295
	2.0212	2.0940	2.1640	2.2311	2.2955	2.3570	2.4158	2.4717	2.5247	2.5750	2.6225
0.6	-3.1216	-3.0487	-2.9787	-2.9116	-2.8472	-2.7857	-2.7270	-2.6711	-2.6180	-2.5677	-2.5203
	2.0304	2.1033	2.1733	2.2404	2.3048	2.3663	2.4250	2.4809	2.5340	2.5843	2.6317
0.7	-3.1123	-3.0395	-2.9695	-2.9023	-2.8379	-2.7764	-2.7177	-2.6618	-2.6087	-2.5584	-2.5110
	2.0397	2.1125	2.1825	2.2497	2.3141	2.3756	2.4343	2.4902	2.5433	2.5936	2.6410
0.8	-3.1030	-3.0302	-2.9602	-2.8930	-2.8286	-2.7671	-2.7084	-2.6525	-2.5994	-2.5491	-2.5017

	2.0490	2.1218	2.1918	2.2590	2.3234	2.3849	2.4436	2.4995	2.5526	2.6029	2.6503

2.4 Upper and Lower Control Limits for Non-Normal and Un-Correlated Population (ρ= 1 and n=5)

λ3→ λ4↓	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
-0.5	-2.5742	-2.5357	-2.4997	-2.4659	-2.4346	-2.4056	-2.3790	-2.3547	-2.3329	-2.3133
	2.5778	2.6163	2.6523	2.6861	2.7174	2.7464	2.7730	2.7973	2.8191	2.8387
-0.4	-2.5664	-2.5279	-2.4919	-2.4581	-2.4268	-2.3978	-2.3712	-2.3469	-2.3251	-2.3055
	2.5856	2.6241	2.6601	2.6939	2.7252	2.7542	2.7808	2.8051	2.8269	2.8465
-0.3	-2.5586	-2.5201	-2.4841	-2.4503	-2.4190	-2.3900	-2.3634	-2.3391	-2.3173	-2.2977
	2.5934	2.6319	2.6679	2.7017	2.7330	2.7620	2.7886	2.8129	2.8347	2.8543
-0.2	-2.5508	-2.5123	-2.4762	-2.4425	-2.4112	-2.3822	-2.3556	-2.3313	-2.3094	-2.2899
	2.6012	2.6397	2.6758	2.7095	2.7408	2.7698	2.7964	2.8207	2.8426	2.8621
-0.1	-2.5430	-2.5045	-2.4684	-2.4347	-2.4034	-2.3744	-2.3478	-2.3235	-2.3016	-2.2821
	2.6090	2.6475	2.6836	2.7173	2.7486	2.7776	2.8042	2.8285	2.8504	2.8699
0	-2.5352	-2.4967	-2.4606	-2.4269	-2.3956	-2.3666	-2.3400	-2.3157	-2.2938	-2.2743
	2.6168	2.6553	2.6914	2.7251	2.7564	2.7854	2.8120	2.8363	2.8582	2.8777
0.1	-2.5274	-2.4889	-2.4528	-2.4191	-2.3878	-2.3588	-2.3322	-2.3079	-2.2860	-2.2665
	2.6246	2.6631	2.6992	2.7329	2.7642	2.7932	2.8198	2.8441	2.8660	2.8855
0.2	-2.5196	-2.4811	-2.4450	-2.4113	-2.3800	-2.3510	-2.3244	-2.3001	-2.2782	-2.2587
	2.6324	2.6709	2.7070	2.7407	2.7720	2.8010	2.8276	2.8519	2.8738	2.8933
0.3	-2.5118	-2.4733	-2.4372	-2.4035	-2.3722	-2.3432	-2.3166	-2.2923	-2.2704	-2.2509
	2.6402	2.6787	2.7148	2.7485	2.7798	2.8088	2.8354	2.8597	2.8816	2.9011
0.4	-2.5040	-2.4655	-2.4294	-2.3957	-2.3644	-2.3354	-2.3088	-2.2845	-2.2626	-2.2431
	2.6480	2.6865	2.7226	2.7563	2.7876	2.8166	2.8432	2.8675	2.8894	2.9089
0.5	-2.4962	-2.4577	-2.4216	-2.3879	-2.3566	-2.3276	-2.3010	-2.2767	-2.2548	-2.2353
	2.6558	2.6943	2.7304	2.7641	2.7954	2.8244	2.8510	2.8753	2.8972	2.9167
0.6	-2.4884	-2.4499	-2.4138	-2.3801	-2.3487	-2.3198	-2.2931	-2.2689	-2.2470	-2.2275
	2.6636	2.7021	2.7382	2.7719	2.8033	2.8322	2.8589	2.8831	2.9050	2.9245
0.7	-2.4806	-2.4421	-2.4060	-2.3723	-2.3409	-2.3120	-2.2853	-2.2611	-2.2392	-2.2197
	2.6714	2.7099	2.7460	2.7797	2.8111	2.8400	2.8667	2.8909	2.9128	2.9323
0.8	-2.4727	-2.4343	-2.3982	-2.3645	-2.3331	-2.3042	-2.2775	-2.2533	-2.2314	-2.2119
	2.6793	2.7177	2.7538	2.7875	2.8189	2.8478	2.8745	2.8987	2.9206	2.9401

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λ3→ λ4↓	-1	-0.9	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0
-0.5	-3.1534	-3.0889	-3.0268	-2.9670	-2.9097	-2.8546	-2.8020	-2.7517	-2.7038	-2.6582	-2.6150
	1.9986	2.0631	2.1252	2.1850	2.2423	2.2974	2.3500	2.4003	2.4482	2.4938	2.5370
-0.4	-3.1456	-3.0811	-3.0190	-2.9592	-2.9018	-2.8468	-2.7942	-2.7439	-2.6960	-2.6504	-2.6072
	2.0064	2.0709	2.1330	2.1928	2.2502	2.3052	2.3578	2.4081	2.4560	2.5016	2.5448
-0.3	-3.1378	-3.0733	-3.0112	-2.9514	-2.8940	-2.8390	-2.7864	-2.7361	-2.6882	-2.6426	-2.5994
	2.0142	2.0787	2.1408	2.2006	2.2580	2.3130	2.3656	2.4159	2.4638	2.5094	2.5526
-0.2	-3.1300	-3.0655	-3.0034	-2.9436	-2.8862	-2.8312	-2.7786	-2.7283	-2.6803	-2.6348	-2.5916
	2.0220	2.0865	2.1486	2.2084	2.2658	2.3208	2.3734	2.4237	2.4717	2.5172	2.5604
-0.1	-3.1222	-3.0577	-2.9956	-2.9358	-2.8784	-2.8234	-2.7708	-2.7205	-2.6725	-2.6270	-2.5838
	2.0298	2.0943	2.1564	2.2162	2.2736	2.3286	2.3812	2.4315	2.4795	2.5250	2.5682
0	-3.1144	-3.0499	-2.9878	-2.9280	-2.8706	-2.8156	-2.7630	-2.7127	-2.6647	-2.6192	-2.5760
	2.0376	2.1021	2.1642	2.2240	2.2814	2.3364	2.3890	2.4393	2.4873	2.5328	2.5760
0.1	-3.1066	-3.0421	-2.9800	-2.9202	-2.8628	-2.8078	-2.7551	-2.7049	-2.6569	-2.6114	-2.5682
	2.0454	2.1099	2.1720	2.2318	2.2892	2.3442	2.3969	2.4471	2.4951	2.5406	2.5838
0.2	-3.0988	-3.0343	-2.9722	-2.9124	-2.8550	-2.8000	-2.7473	-2.6971	-2.6491	-2.6036	-2.5604
	2.0532	2.1177	2.1798	2.2396	2.2970	2.3520	2.4047	2.4549	2.5029	2.5484	2.5916
0.3	-3.0910	-3.0265	-2.9644	-2.9046	-2.8472	-2.7922	-2.7395	-2.6893	-2.6413	-2.5958	-2.5526

	2.0610	2.1255	2.1876	2.2474	2.3048	2.3598	2.4125	2.4627	2.5107	2.5562	2.5994
0.4	-3.0832	-3.0187	-2.9566	-2.8968	-2.8394	-2.7844	-2.7317	-2.6814	-2.6335	-2.5880	-2.5448
	2.0688	2.1333	2.1954	2.2552	2.3126	2.3676	2.4203	2.4706	2.5185	2.5640	2.6072
0.5	-3.0754	-3.0109	-2.9488	-2.8890	-2.8316	-2.7766	-2.7239	-2.6736	-2.6257	-2.5802	-2.5370
	2.0766	2.1411	2.2032	2.2630	2.3204	2.3754	2.4281	2.4784	2.5263	2.5718	2.6150
0.6	-3.0676	-3.0031	-2.9410	-2.8812	-2.8238	-2.7688	-2.7161	-2.6658	-2.6179	-2.5724	-2.5292
	2.0844	2.1489	2.2110	2.2708	2.3282	2.3832	2.4359	2.4862	2.5341	2.5796	2.6228
0.7	-3.0598	-2.9953	-2.9332	-2.8734	-2.8160	-2.7610	-2.7083	-2.6580	-2.6101	-2.5646	-2.5214
	2.0922	2.1567	2.2188	2.2786	2.3360	2.3910	2.4437	2.4940	2.5419	2.5874	2.6306
0.8	-3.0520	-2.9875	-2.9253	-2.8656	-2.8082	-2.7532	-2.7005	-2.6502	-2.6023	-2.5568	-2.5136
	2.1000	2.1645	2.2267	2.2864	2.3438	2.3988	2.4515	2.5018	2.5497	2.5952	2.6384

3 Conclusion

In this article, we have shown the effect of non-normality and correlation on the contol limits. For various non-normal population with various non-normality parameters (λ_3, λ_4) and correlation coefficient ρ, the values of upper and lower control limits are given in table 3.1 to 3.4 above. If ρ =0, we get tabulated values of Gayen [11] which is shown in table 3.1. On examining the effect of non-normality and correlation coefficient, we conclude that non –normality is not a problem for sub-group size of four or more but there is substantial effect of correlation on upper and lower control limits. From the tables it is evident that upper control limits for constant ρ = 0.2, 0.4, 0.6, 0.8, 1.0 and (λ_3, λ_4) = (-0.60. -0.50) are 1.9199, 4.1134, 4.7104, 5.2507, 5.7520 respectively, whereas lower control limits for above constants are -3.5513, -4.1918, -4.7894, -5.3073 and -5.7682. By comparing entries of table 3.1 for ρ =0 with table 3.2 to 3.4 i.e, when ρ =0.2, 0.4, 0.8 and 1, one can easily see that the effect of correlation coefficient is quite serious on the upper and lower control limits. When ρ =0, (λ_3, λ_4) = (0, 0), the upper and lower control limits are respectively given by 2.576 and -2.576. When ρ =1 and (λ_3, λ_4) = (0, 0), the upper and lower control limits are 5.7601 and -5.7601. When ρ =1 and (λ_3, λ_4) = (1, 0.2) the upper and lower control limits are respectively given by (9.832, -1.6878). On comparing these values we see that there is a substantial effect of correlation on control limits as the upper control limits get broadened and lower control limit get shrinked as the value of (λ_3, λ_4) and ρ are increased and the upper and lower control limits are far away from the limits given by Gayen [11] i.e, for the population with ρ =0 and conclude that the non-normal and correlated data severely affects the performance of control charts as the control limits are very different from the limits given by Gayen[11] and there are chances that false items may fall inside the specified control limits and good items may remain out of specified control limits.In general, the stronger the correlation is, the wider the control limits will be. If the main concern is to obtain narrow and broad control limits the population choosen must be normal and uncorrelated otherwise non- normal and correlated.

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